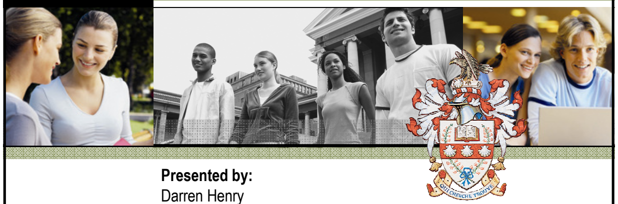
DEBT SECURITIES

Topic 8: Term structure of interest rates

Faculty of Law and Management



Presented by:
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School of Economics and Finance

Student learning objectives

8.1 Illustrate and explain parallel and non-parallel shifts in the yield curve, a yield curve twist and a change in the curvature of the yield curve (i.e. a butterfly shift);

8.2 Describe the factors that have been observed to drive U.S. Treasury security returns, and evaluate the importance of each factor;

8.3 Explain the various universes of Treasury securities that are used to construct the theoretical spot rate curve, and evaluate their advantages and disadvantages;

8.4 Explain the swap rate curve (LIBOR curve) and discuss the reasons that market participants have increasingly used the swap rate curve as a benchmark rather than a government bond yield curve;

Student learning objectives

8.5 Illustrate the various theories of the term structure of interest rates (i.e., pure expectations, liquidity and preferred habitat) and the implications of each theory for the shape of the yield curve;

8.6 Compute and interpret yield volatility, given historical yields;

8.7 Distinguish between historical yield volatility and implied yield volatility;

8.8 Explain how yield volatility is forecasted.

References

> **Fabozzi F. J. (2007).** *Fixed Income Analysis*. John Wiley & Sons Inc. New Jersey. Chapter 8.

Objective 8.1

Shape of the Treasury yield curve

- We introduced the **Treasury yield curve** in Lecture 3 and discussed its derivation in Lecture 5
- The Treasury yield curve depicts the relationship between the yield on Treasury securities and the maturity of those securities
- Historically four shapes have been observed for the yield curve:
  - Normal or positively sloped yield curve
  - Flat yield curve
  - Inverted or negatively sloped yield curve
  - Humped yield curve

Objective 8.1

Shape of the Treasury yield curve

- The slope of the yield curve is defined by the spread between the yield of a long-term Treasury yield and a short-term Treasury yield
  - The most common practice is to use the spread between the 30-year yield and the 2-year yield
- The slope of the yield curve varies over time
  - In the U.S., over the period from 1989 to 2000, the slope was steepest and positive at 348 basis points in September and October 1992 and steepest and negative at -65 basis points in May 2000
- Sometimes the slope is divided into short, medium and long sectors
  - In the U.S. the short end is measured as the spread between the 1-year and 5-year yield, the medium by the spread on the 5-year and 10-year yields and the long end as the spread on the 10-year and the 30-year yields

Term structure of interest rates

Objective 8.1

Yield curve shifts

- A shift in the yield curve refers to the relative change for each Treasury maturity
- A **parallel shift** refers to a shift in which the change in the yield of all maturities is the same
- A **non-parallel shift** means that the yield for different maturities does not change by the same number of basis points
  - A **twist in the slope of the yield curve** refers to a flattening (i.e. a reduction in the spread) or steepening (i.e. an increase in the spread) of the yield curve
  - A **butterfly shift** refers to a change in the curvature of the yield curve
    - Positive butterfly means that the yield curve becomes less curved
    - Negative butterfly mean that the yield curve becomes more curved

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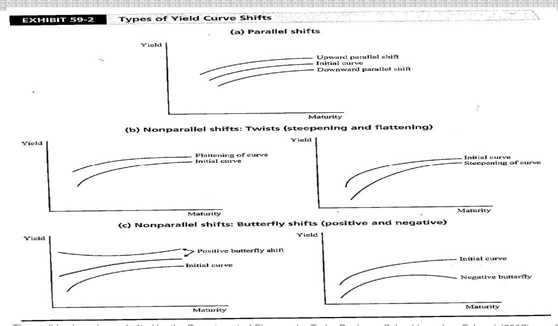
Term structure of interest rates

Objective 8.1

Yield curve shifts



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Term structure of interest rates

Objective 8.2

Factors explaining historical returns of Treasuries

- Litterman and Scheinkman (1991, Journal of Fixed Income) identified three factors that explained historical returns for zero-coupon Treasury securities for all maturities
  - Changes in the level of interest rates: This had the largest explanatory power (R^2 = 0.90 for all maturities)
  - Changes in the slope of the yield curve (R^2 = 0.085 for all maturities)
  - Changes in the curvature of the yield curve, which contributes relatively little to explaining historical returns of Treasury zero-coupon securities
- These findings have generally been confirmed by more recent similar studies

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Term structure of interest rates

Objective 8.3

Constructing the theoretical spot rate curve for Treasuries

- Although many market participants focus on the yield curve for on-the-run Treasury securities, it is the Treasury spot-rate yield curve (introduced in Lecture 5) that is used to value fixed-income securities
- The Treasury spot rate yield curve is what we refer to as the **term structure of interest rates**
- To construct this curve, we firstly select the securities to be used:
  - Treasury coupon strips
  - On-the-run Treasury issues
  - On-the-run Treasury issues and selected off-the-run Treasuries
  - All Treasury coupon securities and bills
- Selection should ensure that the yields are not biased by default risk, embedded options, liquidity differences and pricing errors

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Term structure of interest rates

Objective 8.3

Issues in the choice of security to calculate spot rate curve

- Treasury coupon strips
  - There are certain problems associated with choosing solely Treasury strips:
    - The observed rate on strips reflects a premium for the lower liquidity of the strips
    - The yield reflects a tax disadvantage of strips due to the fact that the accrued interest on strips is taxed even though the cash is not received by the investor
    - Principal strips may have a tax advantage in some countries due to the capital gain being taxed at a more favourable rate than the coupon; hence coupon strips are preferred
- On-the-run Treasury issues
  - On-the-run Treasury issues might be preferred, but account must be taken for on-the-run issues which are trading at a discount or premium, to alleviate the tax effect, by estimating the yield necessary to make the issue trade at par
  - There may exist large gaps between maturities of on-the-run issues

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Term structure of interest rates

Objective 8.3

Issues in the choice of security to calculate spot rate curve

- On-the-run Treasury issues and selected off-the-run Treasury issues
  - Including a selection of off-the-run Treasury issues to supplement the on-the-run Treasury issues helps overcome the gaps in the on-the-run issues
  - Linear interpolation fills in any remaining maturity gaps
  - Bootstrapping is then used to construct the theoretical spot rate curve
- All Treasury coupon securities and bills
  - Limiting the analysis to a selected sample of Treasury securities ignores the information contained in the pricing of remaining Treasury issues
  - Some analysts use all outstanding Treasury issues to construct the theoretical spot rate curve
  - The construction then eliminates securities trading at lower yields in the repo market, adjusts for the tax effect and uses more complex methodologies than bootstrapping

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Objective 8.4

The swap curve (or LIBOR curve)

- In the U.S. it is common to use the Treasury spot rate curve for purposes of valuation
- Outside the U.S. the **swap curve** is often used for valuation
- This is based on the quoted yields for a **"plain vanilla" swap**
- An interest rate swap is arranged as follows
  - Two parties agree to exchange periodic interest payments
  - The dollar amount of the interest payments is based on a **notional principal** amount multiplied by an agreed periodic interest rate
  - The only cash flows exchanged are the interest payments – not the principal
  - Under a generic, or plain vanilla, swap, one party, the **fixed-rate payer**, agrees to pay a fixed rate, known as the **swap rate**, and the other party, the **fixed-rate receiver**, agrees to pay a variable rate aligned with a reference rate

Objective 8.4

The swap curve (or LIBOR curve)

- Suppose a swap specifies that:
  - One party is to pay a fixed rate of 6% (times the notional principal) for 7 years
  - The other party is to pay 3-month LIBOR (the most common reference rate)
- This swap effectively locks in 3-month LIBOR at 6% for 7 years
- In terms of 3-month LIBOR, this implies that the 7-year maturity swap rate quote for 3-month LIBOR is 6%
- The **swap curve** is the schedule of swap rate quotes for different maturities
- Rather than quoting a swap rate, the convention in the swap market is to quote the **swap spread**; i.e. the difference between the swap rate and a **reference rate** (a Treasury yield), with the same term to maturity
- The swap spread reflects the credit risk of the banking sector

Objective 8.4

Reasons for increased use of the swap curve

- There are several advantages of using the swap curve as a benchmark for evaluating the performance of fixed income securities and for the pricing of fixed income securities
  - As investors borrow at LIBOR rather than the risk-free rate, the swap rate is more useful to funded investors than a government yield
  - There is almost no government regulation of the swap market
  - The supply of swaps depends only on the number of counterparties that are seeking or are willing to enter into a swap transaction at any given time
  - Comparisons across countries of government yield curves is difficult because of the differences in sovereign credit risk
  - There are more maturity points available to construct a swap curve

Objective 8.5

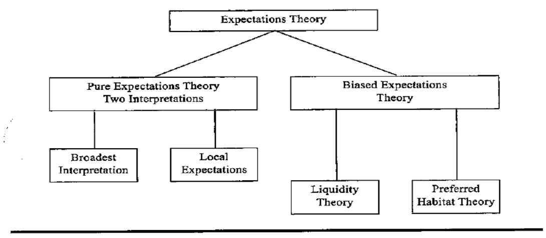
Term structure of interest rates

- As we noted earlier, the Treasury spot rate curve is also called the term structure of interest rates
- Analysts are also interested in the term structure of interest rates to ascertain whether there is any information implied in the term structure which may assist in making investment decisions
- The four main theories of the term structure of interest rates are the:
  - Pure expectations theory
  - Liquidity preference theory
  - Preferred habitat theory
  - Market segmentation theory

Objective 8.5

Pure expectations theory

EXHIBIT 59-3 Expectations Theories of the Term Structure of Interest Rates



Objective 8.5

Pure expectations theory

- According to this theory, the entire term structure at a given time reflects the market's current expectations of future short-term rates
  - A rising term structure indicates that the market expects short-term rates to rise throughout the relevant future
  - A flat term structure indicates the market expects short-term rates to remain constant
  - A falling term structure indicates the market expects short-term rates to fall
- Recall that the yield curve doesn't explicitly tells us what short-term rates will be in the future – it just tells us about short- and long-term rates available *now*
- The pure expectations theory says that long-term rates available now reflect market expectations about future short-term rates

Objective 8.5

Pure expectations theory

- The major drawback of the theory is that it neglects the risks inherent in investing in bonds
- If forward rates were perfect predictors of future interest rates, then the future price of bonds would be known with certainty, the return on any investment period would be certain and independent of maturity
- There are two risks which cause uncertainty about the return of an investment:
  - Interest rate risk: Uncertainty about the price of a bond at the end of the investment horizon
  - Reinvestment risk: Uncertainty about the rate at which cash flows can be reinvested until maturity

Objective 8.5

Interpretations of the pure expectations theory

- There are several interpretations of the theory which differ in the way they address the two risks identified:
  - Broadest interpretation
  - Local expectations interpretation
  - Forward rates and market consensus interpretation

Objective 8.5

Broadest interpretation

- Investors expect the return for any investment horizon to be the same, regardless of the maturity strategy selected
- For example, an investor investing for 5 years will be indifferent between purchasing a 5-year bond, a 12-year bond or a 30-year bond (selling the latter two after 5 years)
- They expect the return over each of the bonds to be identical over 5 years
- Empirically this is not the case due to the presence of interest rate risk – because future interest rates, and therefore the sale price of bonds in the future is unknown – expected returns from investment in securities with different maturities may differ significantly
  - In the above example, because of the different maturity terms, these three bond alternatives have different interest rate risk

Objective 8.5

Local expectations interpretation

- The return will be the same over a short-term investment horizon starting today; e.g. the return on an investment for 6 months will be the same whether you buy a 1-year, 5-year or 10-year bond and hold it for just 6 months

Example 8.5.1

Local expectations interpretation

?

- Based on the spot and forward rates presented in Exhibits 59-4 and 59-5, you invest in a 1-year coupon bond and hold it for six months
- Assume all forward rates are realised

- **Calculate the return on the six month investment.**

Example 8.5.1

Local expectations interpretation

?

EXHIBIT 59-4 Hypothetical Treasury Par Yield Curve

Period	Years	Annual Yield to Maturity (BEY)(%)*	Price	Spot Rate (BEY)(
1	0.5	3.00	—	3.0000
2	1.0	3.30	—	3.3000
3	1.5	3.50	100.00	3.5053
4	2.0	3.90	100.00	3.9164
5	2.5	4.40	100.00	4.4376
6	3.0	4.70	100.00	4.7520
7	3.5	4.90	100.00	4.9622
8	4.0	5.00	100.00	5.0650
9	4.5	5.10	100.00	5.1701
10	5.0	5.20	100.00	5.2772
11	5.5	5.30	100.00	5.3864
12	6.0	5.40	100.00	5.4976
13	6.5	5.50	100.00	5.6108
14	7.0	5.55	100.00	5.6643
15	7.5	5.60	100.00	5.7193
16	8.0	5.65	100.00	5.7755
17	8.5	5.70	100.00	5.8331
18	9.0	5.80	100.00	5.9584
19	9.5	5.90	100.00	6.0863
20	10.0	6.00	100.00	6.2169

* The yield to maturity and the spot rate are annual rates. They are reported as bond-equivalent yields. To obtain the semiannual yield rate, one half the annual yield or annual rate is used.

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Example 8.5.1

Local expectations interpretation

?

EXHIBIT 59-5 Six-Month Forward Rates: The Short-Term Forward Rate Curve (Annualized Rates on a Bond-Equivalent Basis)

Notation	Forward Rate	Notation	Forward Rate
$_1f_0$	3.00	$_1f_{10}$	6.48
$_1f_1$	3.60	$_1f_{11}$	6.72
$_1f_2$	3.92	$_1f_{12}$	6.97
$_1f_3$	5.15	$_1f_{13}$	6.36
$_1f_4$	6.54	$_1f_{14}$	6.49
$_1f_5$	6.33	$_1f_{15}$	6.62
$_1f_6$	6.23	$_1f_{16}$	6.76
$_1f_7$	5.79	$_1f_{17}$	8.10
$_1f_8$	6.01	$_1f_{18}$	8.40
$_1f_9$	6.24	$_1f_{19}$	8.72

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Example 8.5.1

Local expectations interpretation

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Example 8.5.1

Local expectations interpretation

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EXHIBIT 59-4 Hypothetical Treasury Par Yield Curve

Period	Years	Annual Yield to Maturity (BEY)(%)*	Price	Spot Rate (BEY)(
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6	3.0	4.70	100.00	4.7520
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Example 8.5.1

Local expectations interpretation

✓

- At the end of six months, the one-year bond is now a six-month bond
- The six-month forward rate when the bond was purchased was 3.6%
- Based on the pure expectations theory, we assume that this rate is realised when the bond is sold in six months time
- The price of the bond in six months time is therefore:

$$D = \frac{101.65}{1.018} = \$99.85$$

- Hence, the total return on the bond will be:

$$Return = \frac{P_1 - P_0 + Coupon}{P_0} = \frac{99.85 - 100 + 1.65}{100} = 1.5\% = 3\%\ p.a.$$

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Example 8.5.1

Local expectations interpretation

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- Recall that we assume that all bonds are trading at par
- The 1-year bond has a yield of 3.3% p.a., and if it is trading at par then the coupon rate must be 3.3% p.a.
- These are the bond-equivalent yield and the annual coupon rate, so the semi-annual yield and the semi-annual coupon rate are 1.65%
- The cash flows are, therefore, $1.65 in six months and $101.65 in one year
- The price of the bond when it is purchased is:

$$D = \frac{1.65}{1.0165} + \frac{101.65}{1.0165^2} = \$100$$

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Example 8.5.2

Local expectations interpretation

?

- Based on the spot and forward rates presented in Exhibits 59-4 and 59-5, you invest in a 5-year coupon bond and hold it for six months
- Assume all forward rates are realised

- **Calculate the return on the six month investment.**

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Example 8.5.2

Local expectations interpretation

EXHIBIT 59-4 Hypothetical Treasury Par Yield Curve				
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20	10.0	6.00	100.00	6.2169

* The yield to maturity and the spot rate are annual rates. They are reported as bond-equivalent yields. To obtain the semiannual yield rate, one half the annual yield or annual rate is used.

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Example 8.5.2

Local expectations interpretation

- The 5-year bond has an annual yield and coupon rate of 5.2% p.a., and therefore the semi-annual yield and coupon rate are 2.6%
- Once again we can show that the original price of the bond is $100
- At the end of six months, the 5-year bond is now a 4½-year bond
- To determine the value of this 4½-year bond in six months, we can use the six-month forward rates that will apply over the life of the bond to determine forward discount factors to discount cash flows to present value at that point in time

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Example 8.5.2

Local expectations interpretation

EXHIBIT 59-5 Six-Month Forward Rates: The Short-Term Forward Rate Curve (Annualized Rates on a Bond-Equivalent Basis)			
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Example 8.5.2

Local expectations interpretation

Period	CF	Fwd rate	1 + (f/2)	PV
1	2.60	3.60%	1.0180	2.5540
2	2.60	3.92%	1.0196	2.5049
3	2.60	5.15%	1.0258	2.4420
4	2.60	6.54%	1.0327	2.3647
5	2.60	6.33%	1.0317	2.2922
6	2.60	6.23%	1.0312	2.2229
7	2.60	5.79%	1.0290	2.1604
8	2.60	6.01%	1.0301	2.0974
9	102.60	6.24%	1.0312	80.2610
Total				98.8995

$$\text{Return} = \frac{P_1 - P_0 + \text{Coupon}}{P_0} = \frac{98.90 - 100 + 2.60}{100} = 1.5\% = 3\% \text{ p.a.}$$

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Objective 8.5

Forward rates and market consensus interpretation

- There are three ways we can interpret forward rates:
  - **Break-even rate**: the forward rate is the rate that will make an investor indifferent between investing for the full investment horizon or part of the investment horizon and rolling over the proceeds for the balance of the investment horizon
  - **Locked in rate**: the forward rate is the rate that is locked in by investing in a security with a longer maturity, rather than a security with a shorter maturity and then rolling over into the unknown future interest rate
  - **Market consensus rate**: the forward rate represents the market's consensus of future interest rates against which an investor can compare his/her own estimate of future interest rates to determine how to invest
    - If the investor expects future rates to be less than the forward rate, he should lock in the forward rate by investing long-term
    - If he expects future rates to be higher, he should invest short-term

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Objective 8.5

Liquidity preference theory

- One drawback of the pure expectations theory is that it does not consider the risks associated with investing in bonds
- The interest rate risk associated with holding a bond is greater the longer the maturity of the bond (i.e. the greater the duration)
- **The liquidity preference theory** states that investors will only hold longer-term maturities if they are offered a long-term rate that is higher than the average of expected future rates
- Forward rates will therefore reflect both interest rate expectations and a "liquidity" premium that reflects the risk of holding the security
- The liquidity premium is positively related to the term to maturity

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Objective 8.5

Liquidity preference theory

> According to this theory, forward rates will not be an unbiased estimate of the market's expectations of future interest rates, because they contain a liquidity premium

> Hence an upward sloping yield curve may reflect expectations that future interest rates either:

(1) will rise, or

(2) will be unchanged, or even fall, but with a liquidity premium increasing fast enough with maturity so as to produce an upward sloping yield curve

> The liquidity premium theory predicts that there will be an "upward bias" in yield curves, over and above what would be the case based on pure expectations

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Objective 8.5

Preferred habitat theory

> The **preferred habitat theory** also adopts the view that the term structure of interest rates reflects expectations of future interest rates as well as a risk premium to induce investors to switch from their preferred maturity

> However, unlike the liquidity premium theory, this theory rejects the assertion that the risk premium rises uniformly with maturity

> Instead, borrowers and lenders will have *preferred habitats* – maturities at which they would prefer to borrow or lend

> Yields at different maturities will therefore be determined by the supply and demand for securities of different maturities, which will in turn generate positive or negative risk premiums to induce borrowers or lenders to move from their preferred habitat

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Objective 8.6

Yield volatility

> So far we have used effective duration to measure interest rate risk, because it tells us how the value of a security will change for a given change in interest rates

> However, to adequately measure interest rate risk we need to combine effective duration with **yield volatility**

> Yield volatility measures how likely interest rates are to change, and how much they are likely to change

> Another reason for measuring yield volatility is because volatility is a critical component in estimating the value of embedded options (and therefore the price of a bond with an embedded option)

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Objective 8.6

Historical yield volatility

> **Historical yield volatility** is measured by the standard deviation

> The standard deviation of a random variable using historical data is calculated as follows:

$$\text{Standard Deviation} = \sqrt{\frac{\sum_{t=1}^{T}(X_t - \bar{X})^2}{T-1}}$$

> The variable in question is the relative yield change (the change in yield from one day to the next) which is given by:

$$\text{Relative Yield Change } (X_t) = \ln\left(\frac{y_t}{y_{t-1}}\right)$$

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Example 8.6.1

Historical yield volatility

Period (t)	Yield (y_t)	Period (t)	Yield (y_t)	Period (t)	Yield (y_t)
0	6.6945	9	6.593	18	6.515
1	6.699	10	6.620	19	6.533
2	6.710	11	6.568	20	6.543
3	6.675	12	6.575	21	6.559
4	6.555	13	6.646	22	6.500
5	6.583	14	6.607	23	6.546
6	6.569	15	6.612	24	6.589
7	6.583	16	6.575	25	6.539
8	6.555	17	6.552		

> **Based on the yields shown, calculate the historical yield volatility.**

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Example 8.6.1

Historical yield volatility

> The relative yield change from Day 0 to Day 1 is given by:

$$X_1 = \ln\left(\frac{y_t}{y_{t-1}}\right)\times 100 = \ln\left(\frac{6.699}{6.6945}\right)\times 100 = 0.0672$$

> We can calculate the relative yield change for each day, and then find the mean yield change, as shown on the next slide

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Example 8.6.1

Historical yield volatility

t	y_t	X_t	t	y_t	X_t
0	6.6945		13	6.646	1.07406
1	6.699	0.06720	14	6.607	-0.58855
2	6.710	0.16407	15	6.612	0.07565
3	6.675	-0.52297	16	6.575	-0.56116
4	6.555	-1.81411	17	6.552	-0.35042
5	6.583	0.42625	18	6.515	-0.56631
6	6.569	-0.21290	19	6.533	0.27590
7	6.583	0.21290	20	6.543	0.15295
8	6.555	-0.42625	21	6.559	0.24424
9	6.593	0.57804	22	6.500	-0.90360
10	6.620	0.40869	23	6.546	0.70520
11	6.568	-0.78860	24	6.589	0.65474
12	6.575	0.10652	25	6.539	-0.76173

Example 8.6.1

Historical yield volatility

- The total yield change is -2.35020
- The sample mean is given by:

$$\text{Sample Mean} = \frac{-2.35020}{25} = -0.09401\%$$

- We then find the square of the difference between each yield change and the sample mean
- For Day 1, this is given by:

$$(X_1 - \bar{X})^2 = (0.06720 + 0.09401)^2 = 0.02599$$

- We find the value of this for each day, as shown on the next slide

Example 8.6.1

Historical yield volatility

t	y_t	X_t	$(X_t - X)^2$	t	y_t	X_t	$(X_t - X)^2$
0	6.6945			13	6.646	1.07406	1.36438
1	6.699	0.06720	0.02599	14	6.607	-0.58855	0.24457
2	6.710	0.16407	0.06660	15	6.612	0.07565	0.02878
3	6.675	-0.52297	0.18401	16	6.575	-0.56116	0.21823
4	6.555	-1.81411	2.95875	17	6.552	-0.35042	0.06575
5	6.583	0.42625	0.27066	18	6.515	-0.56631	0.22307
6	6.569	-0.21290	0.01413	19	6.533	0.27590	0.13684
7	6.583	0.21290	0.09419	20	6.543	0.15295	0.06099
8	6.555	-0.42625	0.11038	21	6559	0.24424	0.11441
9	6.593	0.57804	0.45164	22	6500	-0.90360	0.65543
10	6.620	0.40869	0.25270	23	6546	0.70520	0.63873
11	6.568	-0.78860	0.48246	24	6589	0.65474	0.56063
12	6.575	0.10652	0.04021	25	6539	-0.76173	0.44586

Example 8.6.1

Historical yield volatility

- The total of the squared variances is 9.79094094
- The sample variance is given by:

$$\text{Variance} = \frac{9.79094094}{25-1} = 0.4045587$$

- The sample standard deviation is given by:

$$\begin{aligned}\text{Standard Deviation} &= \sqrt{\text{Variance}} \\ &= \sqrt{0.4045587} \\ &= 0.6360493\%\end{aligned}$$

Objective 8.6

Historical yield volatility

- Determining the number of observations
  - The appropriate number of observations will depend on the situation at hand
  - A trader concerned with an overnight position might use the last 10 trading days
  - A bond portfolio manager who is concerned with longer-term volatility might use 25 trading days
- Annualising volatility
  - The daily standard deviation can be annualised by multiplying it by the square root of the number of days in a year
  - The number of days in a year might be 365, 250 or 260 depending on the analyst

Example 8.6.2

Historical yield volatility

- The standard deviation for Example 8.6.1 was given by:

$$\begin{aligned}\text{Standard Deviation} &= \sqrt{\text{Variance}} \\ &= \sqrt{0.4045587} \\ &= 0.6360493\%\end{aligned}$$

- **Annualise the volatility of the Treasury zero security in Example 8.6.1, based on 250, 260 and 365 days per year.**

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Example 8.6.2
Historical yield volatility

- Based on 250 days, the annual standard deviation is given by:

$$0.6360493\% \times \sqrt{250} = 10.06\%$$

- Based on 260 days, the annual standard deviation is given by:

$$0.6360493\% \times \sqrt{260} = 10.26\%$$

- Based on 365 days, the annual standard deviation is given by:

$$0.6360493\% \times \sqrt{365} = 12.15\%$$

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Objective 8.7
Historical versus implied yield volatility

- Historical volatility is calculated based on the standard deviation of historical yields
- Implied volatility is based on the observed prices of interest rate options and caps
  - Volatility is one of the inputs to option pricing models
  - If the observed price of an interest rate option is assumed to be the fair price, and the option pricing model is assumed to produce that fair price, the implied volatility is the one that, when used as an input to the model, produces that price
- Problems in using implied volatility include the following:
  - It is assumed that the option pricing model is correct
  - Option pricing models typically assume that implied volatility is constant over the life of the option

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Objective 8.8
Forecasting yield volatility

- In forecasting yield volatility, greater weight is often given to more recent observations of return
- This can be done by revising the variance formula, as follows:

$$\text{Volatility} = \sqrt{\frac{\sum_{t=1}^{T}\left(W_t X_t^2\right)}{T-1}}$$

where W_t is a weight assigned to each observation X_t, with older observations carrying less weight, and all weights summing to 1