DEBT SECURITIES

Topic 5: Yields, spot and forward rates

LA TROBE UNIVERSITY — Faculty of Law and Management

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Student learning objectives

5.1 Explain the sources of return from investing in a bond (i.e., coupon interest payments, capital gain/loss, reinvestment income);

5.2 Compute and interpret the traditional yield measures for fixed-rate bonds (e.g., current yield, yield to maturity, yield to first call, yield to first par call date, yield to refunding, yield to put, yield to worst, cash flow yield) and explain the assumptions underlying traditional yield measures and the limitations of the traditional yield measures;

5.3 Explain the importance of reinvestment income in generating the yield computed at the time of purchase, calculate the amount of income required to generate that yield, and discuss the factors that affect reinvestment risk;

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Student learning objectives

5.4 Describe the methodology for computing the theoretical Treasury spot rate curve and compute the value of a bond using spot rates;

5.5 Define a forward rate, and compute spot rates from forward rates and forward rates from spot rates.

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References

> **Fabozzi F. J. (2007).** *Fixed Income Analysis*. John Wiley & Sons Inc. New Jersey. Chapter 6.

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Objective 5.1

Sources of return

> An investor in a fixed income security can expect to receive a return from one or more of the following sources
  - The coupon interest payments (except for zero coupon bonds)
  - Any capital gain when the security matures, is called or is sold in the secondary market (being the difference between the price paid and the amount received when the security matures, is called or is sold)
  - Income from reinvestment of interim cash flows (except for zero coupon bonds)

> The **yield** is a measure of the rate of return gained from the above sources of return

> There are many different ways in which the yield can be measured

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Objective 5.2

Yield measures

> Current yield
> Yield to maturity
> Yield to call
> Yield to refunding
> Yield to put
> Yield to worst
> Cash flow yield

Objective 5.2

Current yield

- **Current yield** relates the annual dollar coupon interest to a bond's market price

$$\text{Current Yield} = \frac{\text{Annual Dollar Coupon Interest}}{\text{Market Price}}$$

- The current yield will exceed the coupon rate when the bond sells at a discount
- This measure has a number of limitations, including:
  - It considers only the coupon interest
  - It excludes any capital gain or loss if a security is purchased at a discount or premium
  - There is no consideration of reinvestment income

Example 5.2.1

Current yield

- On February 8, 2002, a 7% 8-year bond had a market price of $94.17

$$\text{Current Yield} = \frac{\text{Annual Dollar Coupon Interest}}{\text{Market Price}} = \frac{0.07 \times \$100}{\$94.17} = 0.0743 = 7.43\%$$

Objective 5.2

Yield to maturity

- **Yield to maturity** is the discount rate that will equate the present value of a bond's cash flows to its market price (plus accrued income)
  - First we need to determine the expected cash flows and then search, by trial and error, for the discount rate that will equate the present value of a bond's cash flows with its market price plus accrued income
  - The resulting yield to maturity will typically be a semi-annual yield, which according to market convention is doubled to give the **bond-equivalent yield**
- The following relationships hold:
  - If the bond sells at par: coupon rate = current yield = yield to maturity
  - If the bond sells at a discount: coupon rate < current yield < yield to maturity
  - If the bond sells at a premium: coupon rate > current yield > yield to maturity
- It assumes that coupon payments are reinvested at the YTM

Example 5.2.2

Yield to maturity

- On February 8, 2002, a 7% 8-year bond had a market price of $94.17
- The cash flows comprise: (1) 16 payments, one every 6 months, of $3.50
  (2) 1 payment of $100 in 16 six-month periods' time
- If we try a discount rate of 3.5%, the present value is given by:

$$PV = 3.50\left[\frac{1-\frac{1}{(1.035)^{16}}}{0.035}\right] + \frac{100}{(1.035)^{16}} = \$100$$

- Trial and error gives the following PVs for various discount rates:

Semi-annual YTM	3.5%	5.0%	4.5%	4.0%
Present value	100.00	83.74	88.77	94.17

- The semi-annual YTM must be multiplied by 2

Example 5.2.2

Yield to maturity

- On February 8, 2002, a 7% 8-year bond had a market price of $94.17
- Calculate the yield to maturity on the security.

To	Press	Display
Set all variables to default	2nd RESET ENTER	RST 0.00
Enter number of periods	16 N	N = 16.00◃
Enter coupon payment	3.50 +/− PMT	PMT = -3.50◃
Enter face value	100 +/− FV	FV = -100.00◃
Enter present value	94.17 PV	PV = 94.17◃
Compute yield	CPT I/Y	I/Y = 4.00

Objective 5.2

Yield to call

- When a bond is callable, the practice has been to calculate a **yield to call** as well as a yield to maturity
- The yield to call assumes that the issuer will call the bond on some assumed call date, and that the call price is as per the call schedule
  - The **yield to first call** is computed for an issue that is not currently callable
  - The **yield to next call** is computed for an issue that is currently callable
  - The **yield to first par call** is computed based on the first date on which the issuer can call the bond at par
  - Some bonds are *non-refundable* for a period of time, which means that the issue cannot be called using the proceeds of debt issued at a lower interest rate (although the issue can be called using other sources of finance, such as cash)
  - The **yield to refunding** is computed based on the first date on which a bond can be *refunded* – i.e. called using lower-cost debt

Objective 5.2

Yield to call

- A yield to call is the discount rate that will equate the present value of a bond's cash flows up until a defined call date, including the defined call price, to its market price (plus accrued income)
  - First determine the expected cash flows and then search, by trial and error, for the discount rate that will equate the present value of a bond's cash flows, up until the defined call date, with its market price plus accrued income
- Limitations include
  - YTC assumes that coupon payments are reinvested at the YTC rate
  - YTC assumes the investor will hold the bond to the defined call date
  - YTC assumes the issuer will call the bond on the defined call date
  - Comparisons with YTM are meaningless as it is not known at what rate the redemption price is reinvested between the call date and the maturity date

Example 5.2.3

Yield to call

- On February 8, 2002, a 7% 8-year bond had a market price of \$106.36
- The first call date is three years hence and the strike price is \$103.00
- The cash flows comprise: (1) 6 payments, one every 6 months, of \$3.50
  (2) 1 payment of \$103 in 6 six-month periods' time
- For example, if YTC = 2.5%:

$$PV = 3.50\left[\frac{1-\frac{1}{(1.025)^6}}{0.025}\right] + \frac{103}{(1.025)^6} = \$108.10$$

- Trial and error gives the following PVs for various discount rates:

Semi-annual YTC	2.5%	3.0%	2.7%	2.8%
Present value	108.10	105.22	106.93	106.36

- Again, the YTC must be multiplied by 2

Objective 5.2

Yield to put

- **Yield to put** is the discount rate that will equate the present value of a bond's cash flows up until a defined put date, including the defined strike price, to its market price (plus accrued income)
  - First determine the expected cash flows and then search, by trial and error, for the discount rate that will equate the present value of a bond's cash flows, up until the defined put date, with its market price plus accrued income
- Limitations include
  - YTP assumes that coupon payments are reinvested at the YTP rate
  - YTP assumes the investor will hold the bond to the defined put date
  - YTP assumes the investor will put the bond on the defined put date
  - Comparisons with YTM are meaningless as it is not known at what rate the repaid bond is reinvested between the put date and the maturity date

Objective 5.2

Yield to worst

- **Yield to worst** is the lowest of all possible yields if a security has multiple call or put dates
  - Calculate the yield to call or yield to put for every possible date, as well as the yield to maturity. The yield to worst is the lowest of all the possible yields.
- Limitations include:
  - It holds little meaning as a measure of potential return
  - The limitations include all those associated with YTM, YTC and YTP
  - It also does not recognise that each yield calculation used in determining the yield to worst has different exposures to reinvestment risk

Objective 5.2

Cash flow yield

- **Cash flow yield** is the discount rate which equates the present value of projected cash flows, based on an assumed rate of prepayment, with the market price plus accrued interest
  - The cash flows for mortgage-backed and asset-backed securities are monthly; hence the discount rate will be a monthly rate, which is annualised according to convention, as follows:

$$\text{Annual cash flow yield} = 2\times\left[(1+\text{monthly yield})^6 - 1\right]$$

- Limitations include
  - All those associated with YTM
  - Additionally it is dependent on realising the projected cash flows according to the assumed projected prepayment rate

Example 5.2.5

Cash flow yield

- The monthly cash flow yield on a mortgage-backed security is 1.2%

$$\begin{aligned}\text{Annual cash flow yield} &= 2\times\left[(1+\text{monthly yield})^6 - 1\right]\\ &= 2\times\left[(1.012)^6 - 1\right]\\ &= 0.1484 = 14.84\%\end{aligned}$$

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Objective 5.3

Reinvestment income

- As discussed previously, the sources of return from a debt security are:
  - Coupon payments
  - Capital gain or loss
  - Reinvestment income
- The computed YTM, YTC or YTP will only be realised if the coupon payments can be reinvested at the computed yield
- For example, suppose an investor placed $94.17 in an 8-year Certificate of Deposit paying 8% p.a., compounding semi-annually
- The future value of the deposit would be $94.17 × (1.04)^{16} = $176.38 and the total dollar return would be $176.38 – $94.17 = $82.21

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Objective 5.3

Reinvestment income

- If, instead, the investor purchased a 7%, 8-year, $100 coupon bond for $94.17, the YTM would be 8% p.a.
- Hence, the total dollar return from this investment should be $82.21
- However, the coupon payments total $3.50 × 16 = $56 and the capital gain is $100 – $94.17 = $5.83, totalling $61.83; $20.38 less than the $82.21 necessary for the 8% YTM to be realised
- The solution lies in the reinvestment income – 16 semi-annual payments of $3.50 at an assumed reinvestment rate of 8% p.a.
- The future value of this 16-period annuity is given by:

$$FV = 3.50\left[\frac{(1.04)^{16}-1}{0.04}\right] = \$76.39$$

and hence the reinvestment income is $76.39 – $56 = $20.39

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Example 5.3.1

Reinvestment income

- A 5% 2-year semi-annual coupon bond has a par value of $100 and is trading at a price of $98.14
- The first step is to determine the YTM, as shown in Example 5.2.2, using trial and error or a financial calculator
- In this case the YTM is 3% (semi-annual) or 6% (bond-equivalent)
- The total dollar return is composed of:

Coupon payments:	$2.50 × 4	= $10.00
Capital gain:	$100 – $98.14	= $1.86
Reinvestment income:	$FV = 2.50\left[\frac{(1.03)^{4}-1}{0.03}\right] = \$10.46 - \$10$	= $0.46
Total dollar return:	$98.14 × (1.03)^4 – $98.14	= $12.32

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Objective 5.3

Reinvestment risk

- In order to realise the YTM, it is assumed that coupon payments can be reinvested at the YTM rate
- **Reinvestment risk** is the risk that, because of changes in interest rates after a bond is purchased, the expected yield cannot be achieved
- There are two characteristics of a bond that affect the degree of reinvestment risk:
  - For a given YTM, for a non-zero coupon bond, the longer the maturity of a bond, the more the total dollar return depends on reinvestment income to realise the YTM; hence, the greater the reinvestment risk
  - For a given maturity and a given YTM, for a non-zero coupon bond, the greater the coupon rate the more the total dollar return depends on reinvestment income to realise the YTM; hence, the greater the reinvestment risk

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Objective 5.4

Theoretical spot rate curve

- The theoretical spot rates for Treasury securities represent the appropriate set of interest rates that should be used to value default-free cash flows
- A default-free theoretical spot rate curve can be constructed from the observed Treasury yield curve
- One approach to create a theoretical spot rate curve is called **bootstrapping**

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Objective 5.4

On-the-run Treasury yield curve

- Bootstrapping begins with the yield for on-the-run Treasury issues because there is no credit risk and no liquidity risk
- However, there is the problem of a lack of a sufficient number of data points to define the yield curve, because the U.S. Treasury currently issues 1-month, 3-month, 6-month and 1-year T-bills and 2-year, 3-year, 5-year, 7-year, 10-year and 30-year T-bonds
- The 23 missing whole year maturities are estimated using **linear interpolation** based on the **par yields** of the surrounding maturities
- The yield at time *t*, between two observed maturity points, is given by:

$$Yield_t = Yield_{t-1} + \frac{\text{Yield at higher maturity} - \text{yield at lower maturity}}{\text{Number of years between observed maturity points}}$$

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Example 5.4.1

On-the-run Treasury yield curve

- On September 5, 2003, Lehman Brothers reported the following values for the 5-year and 10-year Treasuries.
  - 5-year 3.25%
  - 10-year 4.35%
- The 6-year interpolated yield is:

$$Yield_6 = Yield_5 + \frac{Yield_{10} - Yield_5}{5} = 3.25 + \frac{4.35 - 3.25}{5} = 3.47\%$$

- The 8-year interpolated yield is:

$$Yield_8 = Yield_6 + 2\times\left(\frac{Yield_{10} - Yield_5}{5}\right) = 3.47 + 2\left(\frac{4.35 - 3.25}{5}\right) = 3.91\%$$

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Example 5.4.2

Spot rate yield curve

- The bond equivalent (i.e. annual) yield for the 6-month and 12-month Treasury bills are 3.00% and 3.30%, respectively
- The annual coupon rate for the 1.5 year Treasury is 3.50% p.a.
- Estimate the spot rate for the 1.5 year maturity Treasury
- The present value of these cash flows is:

$$\frac{1.75}{(1+(0.03/2))} + \frac{1.75}{(1+(0.033/2))^2} + \frac{101.75}{(1+(z_3/2))^3} = 100$$

- Solving for z_3 gives:

$$z_3 = 0.0175265\times 2 = 0.035053 = 3.5053\%$$

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Objective 5.4

Spot rate yield curve

- The spot rate yield curve is found by bootstrapping, as follows:
  - The bond equivalent (i.e. annual) yields for the 6-month and 12-month Treasury bills, being zero-coupon securities, are equal to the 6 and 12-month spot rates
  - Given the spot rates for the 6 and 12-month maturities the spot rate for an 18-month zero-coupon Treasury can be estimated from the 18-month coupon bond
  - The value of a coupon Treasury security should equal the present value of the package of zero-coupon Treasury securities that duplicates the coupon bond's cash flows (as per the arbitrage-free valuation approach discussed last week)
  - The discount rate for each 6-month cash flow is the spot-rate for that maturity
  - In the case of a 1.5 year bond, we know the three cash flows, the 6 and 12-month spot rates and the value of the bond, which we assume is trading at par
  - The only unknown is the discount rate for the final cash flow – i.e. the 1.5 year spot rate – as demonstrated in the following example

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Example 5.4.3

Spot rate yield curve

- The bond equivalent (i.e. annual) yield for the 6-month and 12-month Treasury bills are 3.00% and 3.30%, respectively
- The 1.5 year spot rate is as calculated in Example 5.4.2
- The annual coupon rate for the 2 year Treasury is 3.90% p.a.
- Estimate the spot rate for the 2 year maturity Treasury
- The cash flows for the 2 year coupon Treasury are:

0.5 year	$0.039\times\$100\times0.5$	$=\$1.95$
1.0 year	$0.039\times\$100\times0.5$	$=\$1.95$
1.5 year	$0.039\times\$100\times0.5$	$=\$1.95$
2.0 years	$(0.039\times\$100\times0.5)+100$	$=\$101.95$

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Example 5.4.2

Spot rate yield curve

- The bond equivalent (i.e. annual) yield for the 6-month and 12-month Treasury bills are 3.00% and 3.30%, respectively
- The annual coupon rate for the 1.5 year Treasury is 3.50% p.a.
- Estimate the spot rate for the 1.5 year maturity Treasury
- The bonds are valued at par, so the coupon rate equals the yield
- The cash flows for the 1.5 year coupon Treasury are:

0.5 year	$0.035\times\$100\times0.5$	$=\$1.75$
1.0 year	$0.035\times\$100\times0.5$	$=\$1.75$
1.5 years	$(0.035\times\$100\times0.5)+100$	$=\$101.75$

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Example 5.4.3

Spot rate yield curve

- The bond equivalent (i.e. annual) yield for the 6-month and 12-month Treasury bills are 3.00% and 3.30%, respectively
- The 1.5 year spot rate is as calculated in Example 5.4.2
- The annual coupon rate for the 2 year Treasury is 3.90% p.a.
- Estimate the spot rate for the 2 year maturity Treasury
- The PV of these cash flows is: $\frac{1.95}{(1+(0.03/2))} + \frac{1.95}{(1+(0.033/2))^2} +$

$$\frac{1.95}{(1+(0.035053/2))^3} + \frac{101.95}{(1+(z_4/2))^4} = 100$$

- Solving for z_4 gives: $z_4 = 0.019582\times 2 = 0.039164 = 3.9164\%$

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Example 5.4.3

Spot rate yield curve

EXHIBIT 71-4 Hypothetical Treasury Yields (Interpolated)

Period	Years	Annual Par Yield to Maturity (BEY) (%)*	Price	Spot Rate (BEY) (%)*
1	0.5	3.00	—	3.0000
2	1.0	3.30	—	3.3000
3	1.5	3.50	100.00	3.5053
4	2.0	3.90	100.00	3.9164
5	2.5	4.40	100.00	4.4376
6	3.0	4.70	100.00	4.7520
7	3.5	4.90	100.00	4.9622
8	4.0	5.00	100.00	5.0650
9	4.5	5.10	100.00	5.1701
10	5.0	5.20	100.00	5.2772
11	5.5	5.30	100.00	5.3864
12	6.0	5.40	100.00	5.4976
13	6.5	5.50	100.00	5.6108
14	7.0	5.55	100.00	5.6643
15	7.5	5.60	100.00	5.7193
16	8.0	5.65	100.00	5.7755
17	8.5	5.70	100.00	5.8331
18	9.0	5.80	100.00	5.9584
19	9.5	5.90	100.00	6.0863
20	10.0	6.00	100.00	6.2169

* The yield to maturity and the spot rate are annual rates. They are reported as bond-equivalent yields. To obtain the semiannual yield or rate, one half the annual yield or annual rate is used

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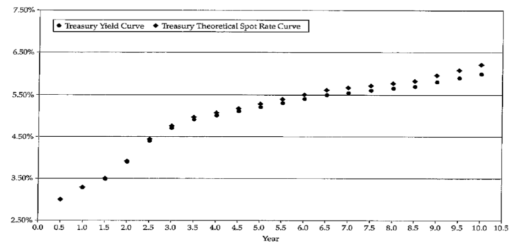
Example 5.4.3

Spot rate yield curve

EXHIBIT 71-6 Theoretical Spot Rate Curve and Treasury Yield Curve



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Objective 5.5

Forward rates

- A **forward rate** is the arbitrage-free rate available for a security for a defined period, beginning a defined number of periods in the future
- An investor seeking a one-year investment has two options
  - One option is to invest in a one-year zero coupon security and earn $X(1 + z_2)^2$, where X is the amount invested and z_2 is the one-year yield (divided by 2)
  - The other is to invest in a six-month security for the first six months, and reinvest the gross amount for a further six-month period on maturity of the initial investment, thereby earning $X(1 + z_1)(1 + f)$, where *f* is the six-month forward rate beginning six months from now
  - The forward rate, *f*, equalises the value of the two alternatives:

$$X(1+z_1)(1+f) = X(1+z_2)^2 \Rightarrow f = \frac{(1+z_2)^2}{1+z_1} - 1$$

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Example 5.5.1

Forward rates

- Based on the theoretical spot rates presented in Example 5.4.2, a 6-month bill spot rate of 3.0% and a 1-year bill spot rate of 3.3%:

$$f = \frac{(1+z_2)^2}{1+z_1} - 1 = \frac{(1+0.033/2)^2}{(1+0.03/2)} - 1 = 0.0180 = 1.8\%$$

- The semi-annual forward rate is 1.8% and the annual forward bond-equivalent rate is 2 × 1.8% = 3.6% p.a.
- This can be confirmed by calculating the return from investing X for six months at 3% p.a. and then reinvesting at the forward rate of 3.6% p.a.:

$$X(1.015)(1.018) = 1.03327X$$

- This can then be compared to the return from the one-year investment:

$$X(1.0165)^2 = 1.03327X$$

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Objective 5.5

Forward rates

- The notation that we use to indicate 6-month forward rates is:

$${}_1f_m$$

where the subscript 1 indicates a 1-period (6-month) forward rate and the subscript *m* indicates the period beginning *m* periods from now

- When *m* is equal to 0, this means the current rate
- In general, a six-month forward rate can be calculated as follows:

$${}_1f_m = \frac{(1+z_{m+1})^{m+1}}{(1+z_m)^m} - 1$$

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Example 5.5.2

Forward rates

- Based on the theoretical spot rates presented in Example 5.4.3:
- Calculate the six-month forward rate beginning four years from now.

- The 6-month forward rate beginning four years from now is given by:

$${}_1f_8 = \frac{(1+z_9)^9}{(1+z_8)^8} - 1 = \frac{(1+0.051701/2)^9}{(1+0.05065/2)^8} - 1 = 0.030064 = 3.0064\%$$

- Hence the semi-annual forward rate is 3.0064% and the annual forward bond-equivalent rate is 2 × 3.0064%, or 6.0128% p.a.

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Example 5.5.2

Forward rates

EXHIBIT 71-12 Six-Month Forward Rates (Annualized Rates on a Bond-Equivalent Basis)

Notation	Forward Rate
${}_1f_0$	3.00
${}_1f_1$	3.60
${}_1f_2$	3.92
${}_1f_3$	5.15
${}_1f_4$	6.54
${}_1f_5$	6.33
${}_1f_6$	6.23
${}_1f_7$	5.79
${}_1f_8$	6.01
${}_1f_9$	6.24
${}_1f_{10}$	6.48
${}_1f_{11}$	6.72
${}_1f_{12}$	6.97
${}_1f_{13}$	6.36
${}_1f_{14}$	6.49
${}_1f_{15}$	6.62
${}_1f_{16}$	6.76
${}_1f_{17}$	8.10
${}_1f_{18}$	8.40
${}_1f_{19}$	8.71

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Objective 5.5

Calculating spot rates from forward rates

- We can derive a relationship between current long-term spot rates, current short-term rates and implied forward rates
- Investing for t periods at the current t-period spot rate will provide:

$$X(1+z_t)^t$$

- Alternatively, investing at the current 6-month spot rate and then reinvesting at each successive forward rate will provide:

$$X(1+z_1)(1+{}_1f_1)(1+{}_1f_2)\ldots(1+{}_1f_{t-1})$$

- Based on an arbitrage-free approach, the proceeds should be equal, and hence the t-period spot rate should be a geometric average of the current 6-month spot rate and succeeding 6-month forward rates:

$$z_t = [(1+z_1)(1+{}_1f_1)(1+{}_1f_2)\ldots(1+{}_1f_{t-1})]^{1/t} - 1$$

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Example 5.5.3

Calculating spot rates from forward rates

- Based on the theoretical spot and forward rates presented in Examples 5.4.2 and 5.5.2 respectively:
- Calculate the three-year spot rate.
- The three-year spot rate is calculated as follows:

$$z_t = [(1+z_1)(1+{}_1f_1)(1+{}_1f_2)\ldots(1+{}_1f_{t-1})]^{1/t} - 1$$

$$z_6 = [(1+z_1)(1+{}_1f_1)(1+{}_1f_2)\ldots(1+{}_1f_5)]^{1/6} - 1$$

$$= [(1.015)(1.018)(1.0196)(1.02575)(1.0327)(1.03165)]^{1/6} - 1$$

$$= 0.023761 = 2.3761\%$$

- Hence the semi-annual spot rate is 2.3761% and the annual (bond-equivalent) spot rate is 2 × 2.3761%, or 4.7522% p.a.

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Objective 5.5

Calculating forward rates from spot rates

- A similar relationship can be found between different long-term spot rates and implied forward rates
- Investing for $m+t$ periods at the current $m+t$-period spot rate provides:

$$X(1+z_{m+t})^{m+t}$$

- Alternatively, investing at the current m-period spot rate and then reinvesting at the t-period forward rate applicable at period m provides:

$$X(1+z_m)^m(1+{}_tf_m)^t$$

- Based on an arbitrage-free approach, the proceeds should be equal, and hence the t-period forward rate applicable in m periods should be:

$${}_tf_m = \left[\frac{(1+z_{m+T})^{m+t}}{(1+z_m)^m}\right]^{1/t} - 1$$

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Example 5.5.4

Calculating forward rates from spot rates

- Based on the theoretical spot rates presented in Examples 5.4.3:
- The two-year forward rate beginning three years from now is calculated as follows:

$${}_tf_m = \left[\frac{(1+z_{m+T})^{m+t}}{(1+z_m)^m}\right]^{1/t} - 1$$

$${}_4f_6 = \left[\frac{(1+z_{10})^{10}}{(1+z_6)^6}\right]^{1/4} - 1$$

$$= \left[\frac{(1+0.052772/2)^{10}}{(1+0.04752/2)^6}\right]^{1/4} - 1 = 0.030338$$

- Hence the semi-annual forward rate is 3.0338% and the annual forward bond-equivalent rate is 2 × 3.0338%, or 6.0675% p.a.

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Tutorial assessment task 2 in Tutorial 5 (Next week)

- Tutorial assessment task 2 will be a practical (calculation-based) exercise relating to bond valuation. You will be required to:
  - Calculate the value of a non-zero coupon bond based on fixed coupon rates with multiple compounding
  - Determine the impact of cash flow and/or yield changes on bond valuation
  - Calculate the valuation of a zero-coupon bond
  - Formulas (equations) will not be provided – you will need to know how to calculate the present value of single sum and annuity cash flows. For calculations made using a financial calculator, the answer provided should outline the relevant calculator inputs or specify the valuation equation.
  - Content is based on material in Lecture 4 and Tutorial 4

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