# Equity Securities

TOPIC 2: RISKLESS RATES AND RISK PREMIUMS

(ALL LECTURE SLIDES WITH SOLUTIONS)

## Student learning outcomes

2.1 Distinguish among the following return concepts: holding period return, realised return and expected return, required return, discount rate, the return from convergence of price to intrinsic value (given that price does not equal value), and internal rate of return.

2.2 Calculate and interpret an equity risk premium using historical and forward looking estimation approaches.



## References


- **Pinto J.E., E. Henry, T.R. Robinson and D.D. Stowe. (2010).** Equity Asset Valuation. (2nd edition) John Wiley & Sons: New Jersey. Chapter 2. Pages 37 – 57.




## Introduction

- All valuation models can be related to an estimated or imputed present value of a future stream of earnings
- A discount rate is required to calculate a present value
- The discount rate should represent the required return of the asset
- The required return of an asset should equal the risk-free rate plus a premium required to compensate for the non-diversifiable risk of an investment



## Introduction

- There exist a range of models of required return, which have some common assumptions
  - All define risk in terms of the variance of return around the mean or expected return
  - All measure risk from the viewpoint of the marginal investor whose investments are well diversified, which leads the models to define expected return as a function of non-diversifiable risk
- The models differ as to how each defines non-diversifiable risk



## Introduction

| Name | Assumptions | Model - $E(R_i)$ = … | Inputs |
|---|---|---|---|
| CAPM | No transaction costs, no private information | $R_f + \beta_i\left[E(R_m) - R_f\right]$ | Risk-free rate<br>Market risk premium<br>Beta relative to market portfolio |
| APM | Investments with the same risk must trade at the same price | $R_f + \sum_{j=1}^{k} \beta_{i,j}\left[E(R_j) - R_f\right]$ | Risk-free rate<br>Factor risk premiums<br>Beta relative to risk factors |
| MFM | Investments with the same risk must trade at the same price | $R_f + \sum_{j=1}^{k} \beta_{i,j}\left[E(R_j) - R_f\right]$ | Risk-free rate<br>Factor risk premiums<br>Beta relative to macro factors |
| RM | Higher returns must compensate for higher risk | $\alpha_i + \sum_{j=1}^{k} \beta_{i,j}(Y_j)$ | Proxies<br>Regression coefficients |

## Introduction

- CAPM

$$E(R_i) = R_f + \beta_i \left[E(R_m) - R_f\right]$$

Market risk premium

Beta of investment i relative to market risk premium

Risk free rate



Outcome 2.1

## Holding period return

- The return on an investment over the period of the investment (the *holding period*) is called the **holding period return**
- This can be expressed as:
  - A dollar figure
  - A percentage (where the dollar return is expressed as percentage of the initial investment
  - This is more useful, because it makes returns comparable irrespective of the size of the investment
- There is also an important difference between:
  - Realised return
  - Expected return



Outcome 2.1

## Realised holding period return

- The realised holding period return is equal to the dollar return *actually received* divided by the initial outlay

$$r = \frac{D_1 + (P_1 - P_0)}{P_0}$$

where
- $r$ = the realised holding period return on the stock
- $D_1$ = the dividend received per share
- $P_0$ = the initial price of the share
- $P_1$ = the price of the share at the end of the holding period



Outcome 2.1

## Expected holding period return

- If an investor wishes to buy a share today and hold it for one year, the value of that share today is the present value of the expected dividend to be received on the stock plus the present value of the expected selling price in one year's time

$$V_0 = \frac{D_1}{(1+r)^1} + \frac{P_1}{(1+r)^1} = \frac{D_1 + P_1}{(1+r)^1}$$

where
- $V_0$ = the value of a share of stock today, at $t = 0$
- $P_1$ = the expected price of the share in one year, at $t = 1$
- $D_1$ = the expected dividend per share for Year 1, paid at year end
- $r$ = the required rate of return on the stock



Outcome 2.1

## Expected holding period return

- If we assume that the current price, $P_0$, of the share is fairly valued (i.e. equal to $V_0$) the expected holding period return equals the sum of the expected dividend yield ($D_1/P_0$) and the expected capital gain or *price appreciation yield* [$(P_1 - P_0)/P_0$]

$$P_0 = \frac{D_1 + P_1}{(1+r)^1}$$

$$\therefore r = \frac{D_1 + P_1}{P_0} - 1 = \frac{D_1}{P_0} + \frac{P_1 - P_0}{P_0} = \frac{D_1}{P_0} + g_1$$

where
- $V_0$ = the value of a share of stock today, at $t = 0$



Outcome 2.1

## Expected holding period return

- The current price of DaimlerChrysler AG ADR (DCX) is \$44.70. You expect a dividend of \$2.08 in one year. You forecast the stock to be \$49.00 in one year.

- **Calculate your expected one-year return on DCX.**

$$r = \frac{D_1 + P_1}{P_0} - 1 = \frac{2.08 - 49.00}{44.70} - 1 = 0.1427 = 14.27\%$$

$$r = \frac{D_1}{P_0} + \frac{P_1 - P_0}{P_0} = \frac{2.08}{44.70} + \frac{49.00 - 44.70}{44.70} = 14.27\%$$

Outcome 2.1

## Expected holding period return

- If an investor plans to hold a stock for two years, the value of the stock is the present value of the expected dividend in Year 1, plus the present value of the expected dividend in Year 2, plus the present value of the expected selling price at the end of Year 2

$$V_0 = \frac{D_1}{(1+r)^1} + \frac{D_2}{(1+r)^2} + \frac{P_2}{(1+r)^2}$$

- Generalised over *n* number of periods the Dividend Discount Model (DDM) formula becomes

$$V_0 = \sum_{t=1}^{n} \frac{D_t}{(1+r)^t} + \frac{P_n}{(1+r)^n}$$



Outcome 2.1

## Expected holding period return

- For the next five years the annual dividends of the stock are estimated to be \$2.00, \$2.10, \$2.20, \$3.50 and \$3.75. The stock price is expected to be \$40.00 in five years. The cost of equity is estimated to be 10%.

- **Calculate the value of the stock.**

$$V_0 = \sum_{t=1}^{n} \frac{D_t}{(1+r)^t} + \frac{P_n}{(1+r)^n}$$

$$= \frac{2.00}{(1.10)^1} + \frac{2.10}{(1.10)^2} + \frac{2.20}{(1.10)^3} + \frac{3.50}{(1.10)^4} + \frac{3.75}{(1.10)^5} + \frac{40.00}{(1.10)^5}$$

$$= \$34.76$$



Outcome 2.1

## Expected holding period return

| To | Press | Display | |
|---|---|---|---|
| Open cash flow worksheet | [CF] | | |
| Enter first cash flow | 0 [ENTER] | CFo= | 0.00 |
| Enter second cash flow | [↓] 2 [ENTER] | C01= | 2.00 |
| | [↓] | F01= | 1.00 |
| Enter remaining cash flows | [↓] 2.1 [ENTER] [↓] [↓] 2.3 [ENTER] [↓] | | |
| | [↓] 3.5 [ENTER] [↓] [↓] 43.75 [ENTER] [↓] | | |
| Enter discount rate | [NPV] 10 [ENTER] | I= | 10.00 |
| Compute PV | [↓] [CPT] | NPV= | 34.84 |
| Clear cash flow worksheet | [CF] [2nd] [CLR WORK] | CF0= | 0.00 |
| Clear discount rate | [NPV] [2nd] [CLR WORK] | | |



Outcome 2.1

## Expected holding period return

- As we increase the holding period by one year we have an extra dividend term in the DDM expression; hence, if we let the holding period extend into an indefinite future, the stock's value is the present value of all expected dividends

$$V_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t}$$



Outcome 2.1

## Expected return and required return

- **Expected return**: is the return we estimate a stock will generate based on our forecasts and the application of a valuation model such as DDM
- **Required rate of return**: is the return we estimate a stock should generate based on our estimation of the risk factors and the sensitivity of the stock to those risk factors and the application of an estimation model such as CAPM, APT or build-up models
- **Alpha**: is the difference between our expected return and the required rate of return for the stock
- **Efficient market**: only in an efficient market will alpha equal zero



Outcome 2.1

## Required return

- Recall the Capital Asset Pricing Model (CAPM)

$$E(R_i) = R_f + \beta_i \left[ E(R_m) - R_f \right]$$

- A valuation model such as the CAPM is actually specifying the *required return* (a function of the risk-free rate and a risk premium) – not the *expected return*
- If the required return derived from the model is equal to the investor's expected return, the market is in equilibrium as predicted by the model
- If they are not equal, then in the eyes of that investor the asset is mispriced – underpriced or overpriced – and alpha is non-zero

Outcome 2.1

## Risk-free rate

- CAPM

$$E(R_i) = R_f + \beta_i [E(R_m) - R_f]$$

Risk free rate



Outcome 2.1

## Risk-free rate

- An investment is 'risk free' if its actual return equals its expected return
- An investment is risk free if it has:
  - No default risk
  - No reinvestment risk



Outcome 2.1

## Proxy for a risk-free rate

- Government securities potentially have no default risk
- Zero coupon bonds have no reinvestment risk
- Theoretically:
  - Each cash flow discounted requires a time specific discount rate as the risk-free rate differs depending on the time horizon
  - The risk-free rate for a five year time horizon would be a government issued five-year zero coupon bond
- Practically:
  - The duration of the cash flows being discounted is used to select a risk-free rate based on the same duration



Outcome 2.1

## If a default-free entity exists…

- Use a long term government bond for longer term valuations and a Treasury bill for short term projects
- Calculate the risk premium based on the risk-free rate
- Ensure the currency of the risk-free rate is consistent with the currency of the cash flows being discounted
- If inflation is high or unstable, use a real risk-free rate to discount real cash flows:
  - Rate of an inflation-indexed Treasury, or
  - Rate of a nominal Treasury less expected inflation rate, or
  - Expected real growth rate of the economy



Outcome 2.2

## Risk premiums

- An incremental return over and above the risk-free rate of return, that investors require to be compensated for the non-diversifiable risk inherent in the investment
  - What are the factors that define the non-diversifiable risk associated with a particular investment?
  - What premium over the risk-free rate do investors require, on average, as compensation for taking on an exposure to each risk factor?



Outcome 2.2

## Market risk premium

- CAPM

$$E(R_i) = R_f + \beta_i \left[\boxed{E(R_m) - R_f}\right]$$

Market risk premium

Outcome 2.2

## Historical risk premiums

- In practice the market risk premium is usually estimated based on the historical risk premium
  - Most calculations of U.S. equity market risk premiums are based on data calculated by Ibbotson Associates
  - The usual method is to deduct the mean actual risk-free rate, calculated over a defined period of time, from the mean actual return on the market index, calculated over the same period of time



Outcome 2.2

## Historical risk premiums

Range of U.S. equity market premiums calculated by analysts at a given point in time according to Damodaran (2002)

**4% to 12%**



Outcome 2.2

## Historical risk premiums

- Reasons for differences in the calculations of historical equity market premium:
  - Choice of time interval
    - 50, 20, 10 years
  - Choice of risk-free security
    - Treasury bill, Treasury bond
  - Choice of averaging method
    - Arithmetic, geometric
  - Date of last revision



Outcome 2.2

## If calculating an historical premium…

- Use a geometric mean method to calculate the return of the market index and of the risk-free rate

$$Geometric\ average = \left(\frac{I_N}{I_0}\right)^{1/N} - 1$$

Historical risk premium for U.S. (1928 – 2000) using a geometric mean **5.51%**

Historical risk premium for U.S. (1928 – 2000) using an arithmetic mean **6.53%**



Outcome 2.2

## If calculating an historical premium…

- Calculate the mean over as long a period as possible
- Revise over time

Historical risk premium for U.S. (1928 – 2000) using a geometric mean **5.51%**

Historical risk premium for U.S. (1990 – 2000) using a geometric mean **7.09%**

Standard error of estimate over a ten year sample assuming 20% volatility **6.32%**



Outcome 2.2

## If calculating an historical premium…

- Use a long-term Treasury bond

Historical risk premium for U.S. (1928 – 2000) using an geometric mean and T bond **5.51%**

Historical risk premium for U.S. (1928 – 2000) using an geometric mean and T bill **7.17%**

Outcome 2.2

## Historical risk premiums – caveats

- Risk premiums vary over time
- Risk premiums vary across countries
- The equity markets in most countries outside the U.S. do not lend themselves to providing reliable historical estimates of market risk premiums

| Country | Risk Premium |
|---|---|
| Australia | 1.48% |
| Germany | -0.80% |
| Hong Kong | 7.73% |
| Japan | 3.04% |
| Mexico | 1.17% |
| United Kingdom | 4.61% |

Ibbotson Associates
1970 - 1996



Outcome 2.2

## Implied risk premium

- The risk premium derived from the required rate of return implied by:
  - the current market price of the stock/index,
  - an appropriate valuation model, and
  - estimated inputs
- Does not rely on:
  - historical data
  - an adjustment for country risk
- Assumes:
  - the market is fairly priced
  - the valuation model from which it is derived is appropriate
  - the input forecasts are accurately estimated



Outcome 2.2

## Implied risk premium

- The Gordon Growth Model can be used to estimate an implied market return from which the risk-free rate can be deducted to give an implied equity risk premium

$$V_0 = \frac{D_1}{(r-g)} \Rightarrow r = \frac{\left[D_1 + (g \times V_0)\right]}{V_0} = \frac{D_1}{V_0} + \frac{g \times V_0}{V_0} = \frac{D_1}{P_0} + g$$

- A multi-stage discount model could be used if appropriate and solved for *r* using trial and error, or Excel's Solver function



Outcome 2.2

## Implied risk premium

- Assume the current level of the S&P 500 is 900, the expected dividend yield is 2% and the sustainable growth rate is 7%
- Assume also the U.S. Treasury bond rate is 6%
- Calculate the implied risk premium using a Gordon Growth Model.

$$r_e = \frac{D_1}{P_0} + g = 2\% + 7\% = 9\%$$

$$\Rightarrow \text{Premium} = r_e - r_f = 9\% - 6\% = 3\%$$



Outcome 2.2

## Implied risk premium

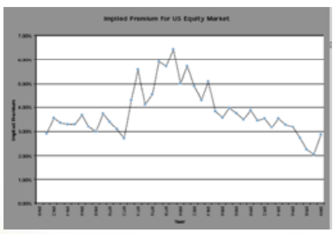
- Advantages
  - Reflects the market's perception of risk
  - Reflects current levels of risk aversion
  - No historical data required
- Disadvantages
  - Assumes the market is fairly priced
  - Assumes the appropriateness of the valuation model
  - Assumes accuracy of the forecast inputs



Outcome 2.2

## Implied risk premium

- Risk premiums change over time
  - Implied risk premiums are seldom as high as historical premiums
  - Note the possibility of a relationship between macroeconomic variables and implied risk premium



Outcome 2.2

## Implied risk premium

- Financial markets exhibit mean reversion tendencies
- We can allow for this by combining historical and implied risk premiums
  - Average historical implied risk premiums over 10 to 15 years
  - Regress historical implied risk premiums against macroeconomic factors

$$\text{Equity risk premium} = \alpha + \sum_{j=1}^{k} \beta_j \left(Y_j\right)$$



Outcome 2.2

## Historical and implied risk premiums

- If your valuation requires you to be market neutral use average implied risk premium

| | |
|---|---|
| Historical risk premium for U.S. (1928 – 2000) using a geometric mean | 5.51% |
| Implied risk premium for U.S. at the end of 2000 Damodaran (2002) | 2.87% |
| Implied risk premium for U.S. averaged (1960 – 2000) Damodaran (2002) | 4.00% |



## Key points

- Required rate of return builds on the risk-free rate by adding premiums for the non-diversifiable risk factors associated with the investment
- Long-term government bond rates offer a proxy for the risk-free rate
- Historical risk premiums are used in practice to estimate the risk premium for a mature market
- Adjustments need to be made to the risk premium of a mature market to account for the country risk of less mature markets
- Implied risk premiums provide a more market driven up-to-date estimate for risk premiums