DEBT SECURITIES

Topic 9: Valuing bonds with embedded options

LA TROBE UNIVERSITY

Faculty of Law and Management

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Student learning objectives

9.1 Evaluate whether a security is undervalued or overvalued, given an appropriate benchmark, using relative value analysis;

9.2 Evaluate the importance of the benchmark interest rates in interpreting spread measures;

9.3 Illustrate the backward induction valuation methodology within the binomial interest rate tree framework;

9.4 Compute the value of a callable bond from an interest rate tree, given the call schedule and the rule for calling a bond;

9.5 Illustrate the relationship among the values of a callable (putable) bond, the corresponding option-free bond, and the embedded option;

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Student learning objectives

9.6 Explain the effect of volatility on the arbitrage-free value of an option;

9.7 Interpret an option-adjusted spread with respect to a nominal spread and to benchmark interest rates;

9.8 Calculate the value of a putable bond, using an interest rate tree.

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References

> **Fabozzi, F.J. (2007)** *Fixed Income Analysis*. John Wiley and Sons. Chapter 9.

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Objective 9.1

Relative value of securities

- Yield spread is used in assessing the relative value of a security
- Relative value analysis involves identifying securities that can potentially enhance return relative to a benchmark
- Different spread measures begin with different benchmark interest rates, which can be drawn from one of the following sources:
  - The Treasury market
  - A specific bond sector with a given credit rating
  - A specific issuer
- Benchmark interest rates can be based either on:
  - An estimated yield curve
  - An estimated spot rate curve

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Objective 9.1

Treasury market benchmark

- In the US, yields in the Treasury market are used as the benchmark
- The benchmark can either be the Treasury yield curve or the Treasury spot rate curve
- Spreads measure the compensation for the additional credit risk, option risk and liquidity risk associated with a non-Treasury security
  - **Nominal spread** is measured relative to the Treasury yield curve, being the YTM of a non-Treasury bond less the yield on a Treasury bond of the same maturity
  - **Zero-volatility spread (Z-spread)** is measured relative to the Treasury spot rate curve, being the spread that an investor would realise over the entire Treasury spot rate curve if the bond is held to maturity
  - **Option Adjusted Spread (OAS)** is measured relative to the Treasury spot rate curve and is calculated as being equal to the Z-spread less the option cost

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Objective 9.1

Treasury market benchmark

- If the OAS is greater than what the market requires for credit risk and liquidity risk, the security is undervalued
  - The security in this situation is referred to as "cheap"
- If the OAS is less than what the market requires for credit risk and liquidity risk, the security is overvalued
  - The security in this situation is referred to as "rich"
- If the OAS is equal to what the market requires for credit risk and liquidity risk, the security is considered to be fairly valued

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Example 9.1.1

Treasury market benchmark

- Assume the following for a non-Treasury security, Bond W, a triple B rated corporate bond with an embedded call option:
  - Benchmark: Treasury market
  - Nominal spread: 170 basis points
  - Z-spread: 160 basis points
  - OAS: 125 basis points
  - Nominal spread on equivalent, option-free bonds = 145 basis points
- The nominal spread (and the Z-spread) on Bond W are higher than the nominal spread on equivalent option-free bonds
- Hence, it may appear that Bond W is underpriced
- However, after allowing for the value of the embedded option, the OAS is less than the option-free spread; hence, Bond W is "rich"

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Example 9.1.2

Specific bond sector

- Assume the following for a non-Treasury security, Bond W, a triple B rated corporate bond with an embedded call option:
  - Benchmark: Double A rated corporate bond sector
  - Nominal spread: 110 basis points
  - Z-spread: 100 basis points
  - OAS: 80 basis points
  - Nominal spread on equivalent, option-free bonds = 90 basis points
- Once again, the nominal spread (and the Z-spread) on Bond W are higher than the nominal spread on equivalent option-free bonds, indicating that Bond W is underpriced
- Once again, after allowing for the value of the embedded option, the OAS is less than the option-free spread; hence, Bond W is rich

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Example 9.1.3

Issuer specific benchmark

- Assume the following for a non-Treasury security, Bond W, a triple B rated corporate bond, issued by RJK Corporation, with an embedded call option:
  - Benchmark: RJK's bond issues
  - Nominal spread: 30 basis points
  - Z-spread: 20 basis points
  - OAS: -25 basis points
- If the nominal spread (and the Z-spread) on Bond W are positive when compared to equivalent, option-free bonds from the same issuer, this would suggest that Bond W is underpriced
- However, a negative OAS indicates that the option-adjusted yield on Bond W is less than the yield on its other option-free bonds; hence, Bond W is overvalued, or rich

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Objective 9.2

OAS, the benchmark and relative valuation

- We can summarise how to interpret OAS as a relative value measure based on the benchmark
- Using the **Treasury spot rate curve** as a benchmark, if the OAS is…
  - Zero: The security offers no spread over equivalent, option-free Treasuries, and should be avoided
  - Negative: The offers a negative spread – a lower yield – than equivalent, option-free securities and should be avoided
  - Positive: The value of the security depends on the spread that the market is demanding on comparable securities
    - If the security OAS is greater than the required OAS, the security is cheap
    - If the security OAS is less than the required OAS, the security is rich
    - If the security OAS is equal to the required OAS, the security is fairly priced

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Objective 9.2

OAS, the benchmark and relative valuation

- Using a **sector specific benchmark**, if the OAS is…
  - Zero or negative: The security offers no spread, or a negative spread, compared to the bond sector benchmark, and is overvalued
  - Positive: The value of the security depends on the spread that the market is demanding on comparable securities
    - If the security OAS is greater than the required OAS, the security is cheap
    - If the security OAS is less than the required OAS, the security is rich
    - If the security OAS equals the required OAS, the security is fairly priced
- Using an **issuer specific benchmark**, if the OAS is…
  - Positive: The security is cheap (relative to other securities from the same issuer)
  - Negative: The security is rich
  - Zero: The security is fairly priced

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Example 9.3.1

Review of how to value an option-free bond

?

> Consider an option-free bond with four years remaining to maturity and a coupon rate of 6.5% paid annually

> We will use as our benchmark the securities of the issuer whose bond is being valued

> To obtain a particular issuer's on-the-run yield curve an appropriate credit spread is added to each on-the-run Treasury issue

> The on-the-run yield curve for this issuer is as shown at right

Maturity	YTM	Price
1 year	3.5%	100
2 years	4.2%	100
3 years	4.7%	100
4 years	5.2%	100

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Example 9.3.1

Review of how to value an option-free bond

?

Year	YTM
1	3.5000%
2	4.2148%
3	4.7352%
4	5.2706%

> Based on the issuer's on-the-run yield curve the issuer's spot rates are calculated using the bootstrapping method

> The spot rates are as shown at left

> Based on the issuer's spot rate curve the following forward rates can be calculated

Year	YTM
1-year forward	3.500%
1-year forward one year hence	4.935%
1-year forward two years hence	5.784%
1-year forward three years hence	6.893%

> **Based on the above data calculate the value of a 6.5% 4-year option-free bond using both the spot rates and the forward rates**

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Example 9.3.1

Review of how to value an option-free bond

✓

> Consider an option-free bond with four years remaining to maturity and a coupon rate of 6.5% paid annually

> Bond value based on spot rates:

$$D = \frac{6.50}{1.035} + \frac{6.50}{(1.042148)^2} + \frac{6.50}{(1.047352)^3} + \frac{106.50}{(1.052706)^4} = \$104.643$$

Spot rates

Year	YTM
1	3.5000%
2	4.2148%
3	4.7352%
4	5.2706%

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Example 9.3.1

Review of how to value an option-free bond

✓

> Consider an option-free bond with four years remaining to maturity and a coupon rate of 6.5% paid annually

> Bond value based on forward rates:

$$D = \frac{6.50}{1.035} + \frac{6.50}{(1.035)(1.04935)} + \frac{6.50}{(1.035)(1.04935)(1.05784)} + \frac{106.50}{(1.035)(1.04935)(1.05784)(1.06893)} = \$104.643$$

Forward rates

Year	YTM
1-year forward	3.500%
1-year fwd 1 year hence	4.935%
1-year fwd 2 years hence	5.784%
1-year fwd 3 years hence	6.893%

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Objective 9.3

Valuing a bond with an embedded option

> The **binomial model** is one of several models which can be used to value a bond with an embedded option

> The interest rates that are used in the valuation process are obtained from a **binomial interest rate tree**, constructed from an on-the-run yield curve

> Once we allow for embedded options, consideration must be given to interest rate volatility by introducing volatility into the valuation model, which is achieved by the derivation of the **binomial interest rate tree**

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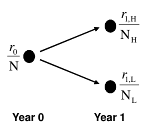
Objective 9.3

Binomial interest rate tree

> The dots refer to nodes, each of which, in a binomial interest rate tree, refers to a random event (i.e. a change in interest rates)

> It is assumed that each random event will take on only two possible values, each of which has a probability of being realised of 50%

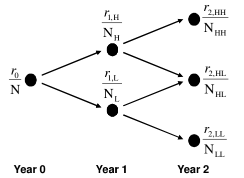


> There is a 50% chance that in Year 1 the interest rate will increase to $r_{1,H}$ (at Node N_H) and a 50% chance that the interest rate will fall to $r_{1,L}$ (at Node N_L)

Objective 9.3

Binomial interest rate tree

> The tree can be extended for any number of additional periods
- From Node N_H, there is a 50% chance that in Year 2 the interest rate will increase to $r_{2,HH}$ and a 50% chance that the interest rate will fall to $r_{2,HL}$
- From Node N_L, there is a 50% chance that in Year 2 the interest rate will increase to $r_{2,HL}$ and a 50% chance that the interest rate will fall to $r_{2,LL}$
- Hence there is an overall 25% chance that in Year 2 the interest rate will be $r_{2,HH}$, a 25% chance that the interest rate will be $r_{2,LL}$ and a 50% chance that the interest rate will be $r_{2,HL}$



Example 9.3.2

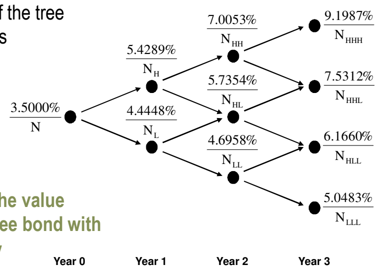
Valuing an option-free bond with a tree

?

> The diagram shows the 1-year forward rate at each node of the tree

> This model assumes 10% volatility for the 1-year rate

> **Based on the data shown, calculate the value of a 6.5% option-free bond with 4 years to maturity**



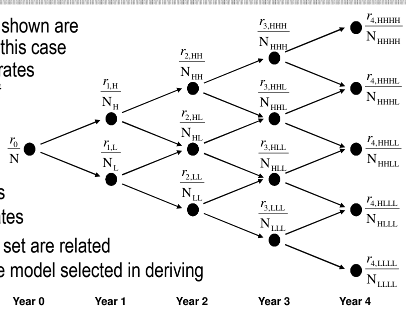
Objective 9.3

Binomial interest rate tree

> The interest rates shown are forward rates – in this case one-year forward rates starting at period *t*

> For any given period in a binomial interest rate model there is a set of forward rates

> The rates within a set are related by the interest rate model selected in deriving them



Example 9.3.2

Valuing an option-free bond with a tree

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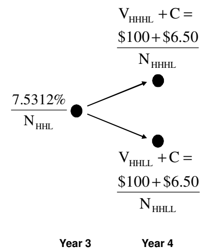
> Calculate the value of a 6.5% option-free bond with 4 years to maturity

> Consider the calculations for Node N_{HHL}
- The discount rate is given as 7.5312%
- The N_{HHHL} cash flow is $106.50, with a 50% probability of being realised
- The N_{HHLL} cash flow is $106.50, with a 50% probability of being realised

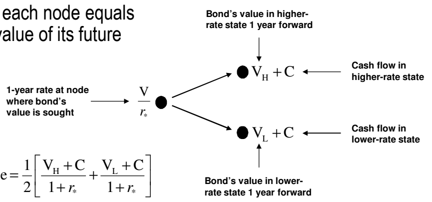
$$\text{Value at } N_{HHL} = \frac{1}{2}\left[\frac{106.50}{1.075312} + \frac{106.50}{1.075312}\right]$$
$$= \$99.041$$



Objective 9.3

Determining a value at a node

> The value at each node equals the present value of its future cash flows



$$\text{Value at a node} = \frac{1}{2}\left[\frac{V_H + C}{1 + r_*} + \frac{V_L + C}{1 + r_*}\right]$$

- The cash flows in each state are discounted to present value, with the discount rate being the 1-year rate at the node at which the value is being computed
- A weighted average is taken of each of the two possible future cash flows, where the weights are given by the probabilities of each cash flow being realised

Example 9.3.2

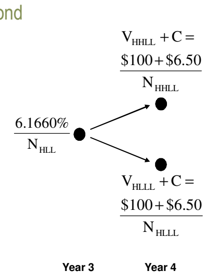
Valuing an option-free bond with a tree

✓

> Calculate the value of a 6.5% option-free bond with 4 years to maturity

> Consider the calculations for Node N_{HLL}
- The discount rate is given as 6.1660%
- The N_{HHLL} cash flow is $106.50, with a 50% probability of being realised
- The N_{HLLL} cash flow is $106.50, with a 50% probability of being realised

$$\text{Value at } N_{HLL} = \frac{1}{2}\left[\frac{106.50}{1.061660} + \frac{106.50}{1.061660}\right]$$
$$= \$100.315$$



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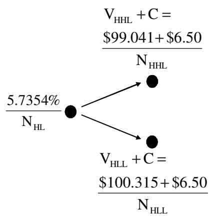
Example 9.3.2

Valuing an option-free bond with a tree

> Calculate the value of a 6.5% option-free bond with 4 years to maturity

> Now consider the calculations for Node N_{HL}
– The discount rate is given as 5.7354%
– The N_{HHL} cash flow is $105.541, with a 50% probability of being realised
– The N_{HLL} cash flow is $106.815, with a 50% probability of being realised



$$\text{Value at } N_{HL} = \frac{1}{2}\left[\frac{105.541}{1.057354} + \frac{106.815}{1.057354}\right] = \$100.418$$

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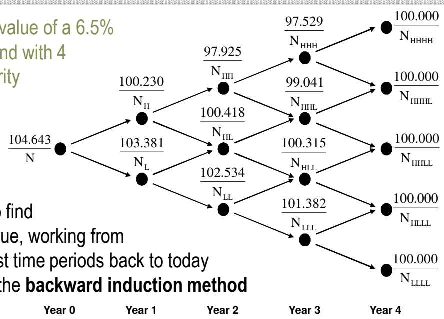
Example 9.3.2

Valuing an option-free bond with a tree

> Calculate the value of a 6.5% option-free bond with 4 years to maturity



> We continue the process to find each node value, working from the furthermost time periods back to today

> This is called the **backward induction method**

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Objective 9.4

Valuing a callable bond

> At each node in a binomial tree, either a random event or a decision occurs

> In the case of a binomial interest rate tree, there is a random event at each node

> When valuing a callable bond using binomial valuation, at each node a decision is made, by the issuer, as to whether or not to call the bond

> Hence, the bond value at each node must be changed to reflect the lesser of:
– The value if it is not called (the value derived by the backward induction method)
– The call price

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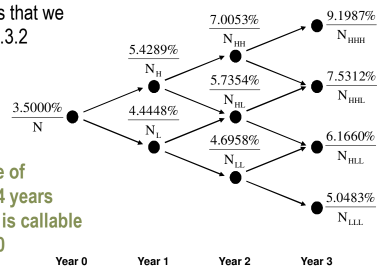
Example 9.4.1

Valuing a callable bond

> This diagram shows the same set of 1-year forward rates that we used for Example 9.3.2

> Based on the data shown, calculate the value of a 6.5% bond with 4 years to maturity, which is callable in one year at $100



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Example 9.4.1

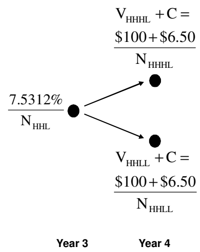
Valuing a callable bond

> Calculate the value of a 6.5% bond with 4 years to maturity, that is callable in 1 year at $100

> If we repeat the calculations for Node N_{HHL}, we again find that the computed value is $99.041

> Because the value of the bond is less than the call price of $100, it will not be called

> The bond value according to the valuation model is therefore $99.041



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Example 9.4.1

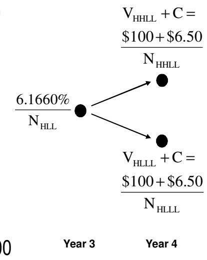
Valuing a callable bond

> Calculate the value of a 6.5% bond with 4 years to maturity, that is callable in 1 year at $100

> If we repeat the calculations for Node N_{HLL}, we again find that the computed value is $100.315

> Because the value of the bond is *greater than* the call price of $100, it *will* be called

> The bond value according to the valuation model is the lesser of the computed value and the call price – i.e. $100

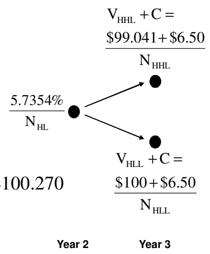


Example 9.4.1

Valuing a callable bond

> Calculate the value of a 6.5% bond with 4 years to maturity, that is callable in 1 year at $100

> Now consider the calculations for Node N_{HL}
- The discount rate is given as 5.7354%
- The N_{HHL} cash flow is $105.541 and the N_{HLL} cash flow is $106.50, each with a 50% probability of being realised



$$\text{Value at } N_{HL}=\frac{1}{2}\left[\frac{105.541}{1.057354}+\frac{106.50}{1.057354}\right]=\$100.270$$

- The computed value is $100.270, but since it would be called at that point, the value is $100

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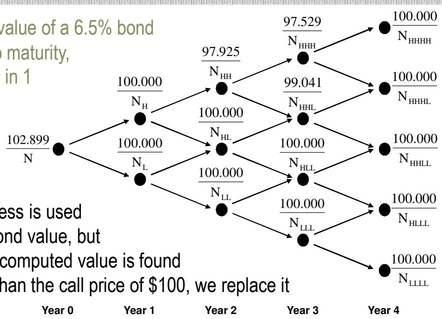
Example 9.4.1

Valuing a callable bond

> Calculate the value of a 6.5% bond with 4 years to maturity, that is callable in 1 year at $100



> The backward induction process is used to find each bond value, but each time the computed value is found to be greater than the call price of $100, we replace it with $100

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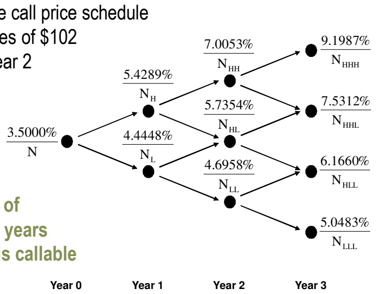
Example 9.4.2

Valuing a callable bond

> Assume now that the call price schedule provides for call prices of $102 in Year 1, $101 in Year 2 and $100 in Year 3



> **Based on the data shown, calculate the value of a 6.5% bond with 4 years to maturity, which is callable in one year**

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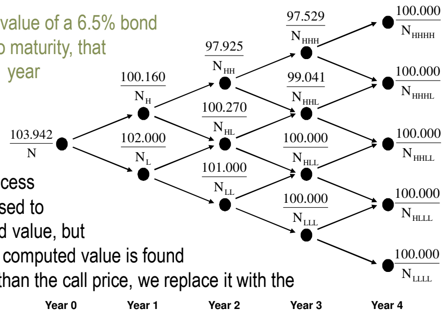
Example 9.4.2

Valuing a callable bond

> Calculate the value of a 6.5% bond with 4 years to maturity, that is callable in 1 year



> The same process as before is used to find each bond value, but each time the computed value is found to be greater than the call price, we replace it with the call price

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Example 9.5.1

Determining the call option value

> The value of a call option equals the value of an equivalent option-free bond less the value of a callable bond

> What is the value of the embedded call option in Examples 9.4.1 and 9.4.2?

	Example 9.4.1	Example 9.4.2
Value of option-free bond	$104.643	$104.643
Less Value of callable bond	–$102.899	–$103.942
Equals Value of call option	$1.744	$0.701

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Objective 9.6

Volatility and the call option value

> The model used to generate interest rates in the previous examples assumed a volatility of 10%

> The volatility assumption has an important impact on the value of an embedded option, and therefore the value of a bond with an embedded option

> The higher the volatility the higher the expected value of the option, and, therefore, since the value of an equivalent option-free bond doesn't change, the lower is the value of the bond with an embedded option

Valuing bonds with embedded options

Objective 9.7

Option adjusted spread

> The option-adjusted spread is the constant spread that, when added to all the 1-year rates on the binomial interest rate tree, will make the arbitrage-free value (the value produced by the binomial model) equal to the market price

> For example, suppose that:
  - The market price of the 4-year 6.5% callable bond used in Example 9.4.1 is $102.218
  - The theoretical value assuming 10% volatility is $102.899

> We can conclude from this that the bond is undervalued, or cheap

> In dollar terms it is undervalued by:

$102.899 – $102.218 = $0.681

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Objective 9.7

Option adjusted spread

> The option-adjusted spread is the spread that needs to be added to the interest rates on the binomial interest rate tree to generate a computed value equal to the market price

> In this case, if we add 35 basis points to all of the interest rates in the tree, the model yields a computed value of $102.218

> The OAS for this callable bond is therefore 35 basis points

> Bond market participants prefer to use the terms "cheap" or "expensive" in dollar terms rather than in terms of yield spread

> Hence, a cheap bond trades at a higher yield spread and an expensive bond trades at a lower yield spread

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Example 9.7.1

Option adjusted spread

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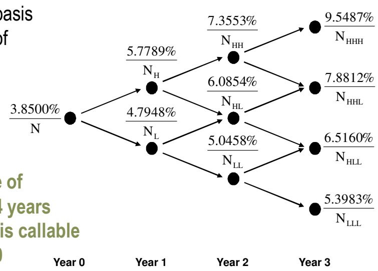
> This binomial interest rate tree shown has had 35 basis points added to all of the forward rates



> Based on the data shown, calculate the value of a 6.5% bond with 4 years to maturity, which is callable in one year at $100

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Example 9.7.1

Option adjusted spread

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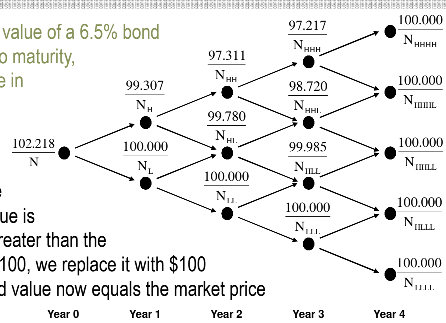
> Calculate the value of a 6.5% bond with 4 years to maturity, that is callable in 1 year



> Each time the computed value is found to be greater than the call price of $100, we replace it with $100

> The computed value now equals the market price

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Objective 9.7

Option adjusted spread

> The OAS seeks to remove from the Z-spread the amount that is due to the option risk

> The interpretation of the OAS depends on the benchmark used

> In Example 9.4.1 the benchmark was the issuer's own securities, and the OAS was positive; hence, the bond is cheap
  - It is trading for less than what is predicted by the valuation model
  - A positive spread is needed in order for the computed value to fall until it is equal to the market value

> Note that this conclusion is subject to the assumption that interest rate volatility is 10%
  - If it was 20%, the OAS would be negative and the bond would be relatively expensive

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Valuing bonds with embedded options

Objective 9.8

Valuing a putable bond

> A putable bond is a bond in which the investor has the right to put, or sell, the bond back to the issuer

> The value of a putable bond equals the value of an option-free bond plus the value of the put option

> Hence, the value of the put option equals the value of a putable bond less the value of an option-free bond

> The value of a put option (just like that of a call option) increases with interest rate volatility

These slides have been drafted by the Department of Finance, La Trobe Business School based on Fabozzi (2007). 8.42

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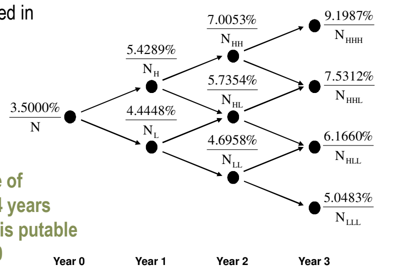
Example 9.8.1

Valuing a putable bond

?

> This diagram shows the binomial interest rate tree used in previous examples

> **Based on the data shown, calculate the value of a 6.5% bond with 4 years to maturity, which is putable in one year at $100**



LA TROBE UNIVERSITY — Valuing bonds with embedded options

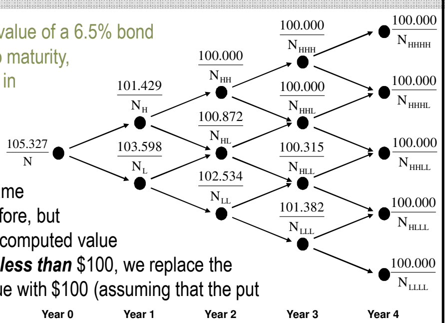
Example 9.8.1

Valuing a putable bond

✓

> Calculate the value of a 6.5% bond with 4 years to maturity, that is putable in 1 year

> We use the same process as before, but each time the computed value is found to be ***less than*** $100, we replace the computed value with $100 (assuming that the put is exercised)



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Objective 9.8

Valuing a putable bond

> Because the put option is valuable from the point of view of the investor, it increases the value of a putable bond compared to an equivalent option-free bond

Value of putable bond	$105.327
Less Value of option-free bond	–$104.643
Equals Value of put option	$0.684

> The procedure for valuing a bond that is both callable and putable is to adjust the value at each node to reflect whether the issue would be put or called

LA TROBE UNIVERSITY — Valuing bonds with embedded options

Tutorial assessment task 4 in Tutorial 9

> Tutorial assessment task 4 in Tutorial 9 will be a practical (calculation-based) exercise requiring you to:
- Calculate the effective duration for a bond
- Calculate the convexity adjustment for the bond based on a specified interest rate shock
- Determine the percentage price change for the bond based on a specified interest rate shock using the duration and convexity adjustment results
- Small theory component requiring interpretation of interest rate risk based on the duration/convexity outcome
- The equations for duration and the convexity adjustment will not be provided. You will not, however, need to determine the effects of the interest rate shocks on bond price

> Content relates to the Lecture 7 and Tutorial 7 material on interest rate risk measurement