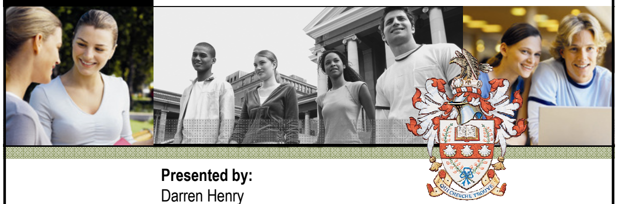
DEBT SECURITIES

Topic 4: Bond valuation

LA TROBE UNIVERSITY Faculty of Law and Management



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LA TROBE UNIVERSITY Bond valuation

Student learning objectives

4.1 Explain the steps in the bond valuation process (i.e., estimate expected cash flows, determine an appropriate discount rate or rates, and compute the present value of the cash flows);

4.2 Identify the types of bonds for which estimating the expected cash flows is difficult, and explain the problems encountered when estimating the cash flows for these bonds;

4.3 Compute the value of a bond, given the expected annual or semi-annual cash flows and the appropriate single (constant) discount rate, explain how the value of a bond changes if the discount rate increases or decreases, and compute the change in value that is attributable to the rate change;

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Student learning objectives

4.4 Explain how the price of a bond changes as the bond approaches its maturity date, and compute the change in value that is attributable to the passage of time;

4.5 Compute the value of a zero-coupon bond;

4.6 Explain the arbitrage-free valuation approach and the market process that forces the price of a bond toward its arbitrage-free value, and explain how a dealer could generate an arbitrage profit if a bond is mispriced.

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References

> **Fabozzi F. J. (2007).** *Fixed Income Analysis*. John Wiley & Sons Inc. New Jersey. Chapter 5.

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Objective 4.1

Valuation

> Valuation is the process of determining the fair value of a financial asset

> The fundamental principle of financial asset valuation is that the value of a financial security is equal to the present value of its expected cash flows

> There are three steps in the valuation process
  - Estimate the expected cash flows
  - Determine the appropriate interest rate or interest rates that should be used to discount the cash flows
  - Calculate and sum the present value of the expected cash flows using the appropriate discount rates

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Objective 4.1 & 4.2

Estimating expected cash flows

> **Cash flow** is simply the cash, including principal and interest, that is expected to be received in the future from an investment

> **Non-callable U.S. Treasury securities** have known cash flows

> Cash flows of other securities may vary depending on future changes in interest rates and their impact on any embedded option attached to the securities
  - **Callable bonds, putable bonds, mortgage-backed securities and asset-backed securities**: offer the issuer or investor the option to change the contractual due date for the repayment of principal
  - **Floating rate securities**: reset the coupon payments periodically
  - **Securities with coupon-related embedded options**: such as caps and floors, step-up or step-down coupons
  - **Convertible bonds and exchangeable bonds**: offer the investor the option to convert or exchange the security into common stock

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Objective 4.1

Determining the appropriate discount rate or rates

- The minimum interest rate that an investor should require is the yield available in the market on a default-free cash flow; i.e. an on-the-run U.S. Treasury security
- For a non-U.S. Treasury security, investors will require a yield premium over the yield available on an on-the-run Treasury security
  - As discussed in Lecture 3, this premium is compensation for the additional risk associated with the particular debt security (for credit risk, as well as other potential elements impacting on cash flow certainty – such as liquidity, call and volatility risks)
- Although the traditional valuation method uses a common discount rate for all cash flows regardless of their maturity, a more appropriate approach would be to use multiple interest rates, with each one being appropriate for a specific maturity

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Objective 4.1 & 4.3

Discounting the expected cash flows

- The value of a single cash flow to be received in the future is the amount of money that must be invested today to generate that future value
- The present value of a cash flow depends on:
  - The timing of the cash flow
  - The discount rate used
- The value of a bond is given by:

$$B_0 = \sum_{t=1}^{n} \frac{C_t}{(1+i)^t}$$

where:
- C_t = cash flow at time t
- i = discount rate

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Example 4.3.1

Bond valuation

- A simple bond matures in four years, has a coupon rate of 10% pa and has a maturity value of $100
- Assume that the bond pays interest annually and that the appropriate discount rate equals 8% pa

$$B_0 = \sum_{t=1}^{n} \frac{C_t}{(1+i)^t}$$

$$= \frac{\$10}{(1+0.08)} + \frac{\$10}{(1+0.08)^2} + \frac{\$10}{(1+0.08)^3} + \frac{\$110}{(1+0.08)^4}$$

$$= \$9.2593 + \$8.5734 + \$7.7983 + \$80.8533$$

$$= \$106.62$$

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Example 4.3.1

Bond valuation

- A simple bond matures in four years, has a coupon rate of 10% pa and has a maturity value of $100
- Assume that the bond pays interest annually and that the appropriate discount rate equals 8% pa

To	Press	Display
Set all variables to default	2nd RESET ENTER	RST 0.00
Enter number of periods	4 N	N = 4.00◃
Enter discount rate	8 I/Y	I/Y = 8.00◃
Enter coupon payment	10 +/– PMT	PMT = -10.00◃
Enter face value	100 +/– FV	FV = -100.00◃
Compute present value	CPT PV	PV = 106.62*

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Objective 4.3 & 4.4

Properties of present values

- The further into the future the timing of the cash flow, the lower is its present value
- The higher the discount rate the lower the present value, and therefore the lower is the value of a security
- For example, if the discount rate in the previous example rises above 8%, the price would be expected to drop below $106.62. If it increased to 12%:

$$B_0 = \frac{\$10}{(1+0.12)} + \frac{\$10}{(1+0.12)^2} + \frac{\$10}{(1+0.12)^3} + \frac{\$110}{(1+0.12)^4}$$

$$= \$8.9286 + \$7.9719 + \$7.1178 + \$69.9070$$

$$= \$93.93$$

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Objective 4.3 & 4.4

Properties of present values

- The relationship between the price of a security and the discount rate is non-linear
- This relationship is described as **convex**, because the shape of the curve is convex with respect to the origin
- This non-linear, convex relationship is discussed in more detail in Lecture 7

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Objective 4.3

Relationship between coupon, discount rate and price

> In Lecture 1 we described the inverse relationship between the price of a bond and its yield
  - If the coupon rate = the yield required by the market → price = par value
    In this case we say that the bond is trading "at par"
  - If the coupon rate < the yield required by the market → price < par value
    In this case we say that the bond is trading "at a discount"
  - If the coupon rate > the yield required by the market → price > par value
    In this case we say that the bond is trading "at a premium"

> These are useful rules to remember, and can be used to check the reasonableness of bond valuation calculations

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Example 4.3.2

Relationship between coupon, discount rate, price

> A simple bond matures in four years, has a coupon rate of 10% pa and has a maturity value of $100

> Assume that the bond pays interest annually. Calculate the values of the bond if the discount rate is 6%, 8%, 10%,12% and 14% pa. respectively

Discount rate	Bond value
6%	$113.86
8%	$106.62
10%	$100.00
12%	$93.93
14%	$88.34

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Objective 4.4

Relationship between maturity and price

> At maturity, the bond's value is equal to its par value (unless an alternative repayment value is specified); hence its value will move towards par as it moves towards maturity

> Assuming the discount rate does not change, the bond's value:
  - Decreases over time if the bond is selling at a premium
  - Increases over time if the bond is selling at a discount
  - Remains unchanged if the bond is selling at par

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Example 4.4.1

Relationship between maturity and price

> A simple bond matures in **three** years, has a coupon rate of 10% pa and has a maturity value of $100

> Assume that the bond pays interest annually and that the appropriate discount rate is 8% pa

$$B_0 = \sum_{t=1}^{n} \frac{C_t}{(1+i)^t}$$

$$= \frac{\$10}{(1+0.08)} + \frac{\$10}{(1+0.08)^2} + \frac{\$110}{(1+0.08)^3}$$

$$= \$105.15$$

> Compared to Example 4.3.1, the price has **fallen** from $106.62 to $105.15 as the bond approaches maturity

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Example 4.4.1

Relationship between maturity and price

> A simple bond matures in **three** years, has a coupon rate of 10% pa and has a maturity value of $100

> Assume that the bond pays interest annually and that the appropriate discount rate is 8% pa

To	Press	Display		
Set all variables to default	[2nd] [RESET] [ENTER]	RST		0.00
Enter number of periods	3 [N]	N	=	3.00◃
Enter discount rate	8 [I/Y]	I/Y	=	8.00◃
Enter coupon payment	10 [+/–] [PMT]	PMT	=	-10.00◃
Enter face value	100 [+/–] [FV]	FV	=	-100.00◃
Compute present value	[CPT] [PV]	PV	=	105.15*

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Example 4.4.2

Relationship between maturity and price

> A simple bond matures in **three** years, has a coupon rate of 10% pa and has a maturity value of $100

> Assume that the bond pays interest annually and that the appropriate discount rate is 12% pa

$$B_0 = \sum_{t=1}^{n} \frac{C_t}{(1+i)^t}$$

$$= \frac{\$10}{(1+0.12)} + \frac{\$10}{(1+0.12)^2} + \frac{\$110}{(1+0.12)^3}$$

$$= \$95.20$$

> The price has **increased** from $93.93 to $95.20 as the bond approaches maturity

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Example 4.4.2

Relationship between maturity and price

> A simple bond matures in **three** years, has a coupon rate of 10% pa and has a maturity value of $100

> Assume that the bond pays interest annually and that the appropriate discount rate is 12% pa

To	Press	Display
Set all variables to default	[2nd] [RESET] [ENTER]	RST 0.00
Enter number of periods	3 [N]	N = 3.00◁
Enter discount rate	12 [I/Y]	I/Y = 12.00◁
Enter coupon payment	10 [+/–] [PMT]	PMT = -10.00◁
Enter face value	100 [+/–] [FV]	FV = -100.00◁
Compute present value	[CPT] [PV]	PV = 95.20∗

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Example 4.4.3

Relationship between maturity and price

> A simple bond matures in **three** years, has a coupon rate of 10% pa and has a maturity value of $100

> Assume that the bond pays interest annually and that the appropriate discount rate has risen from 8% to 9% over the previous year

> *(Our objective is compare the price of this bond (3 years to maturity, 9% yield) with our earlier example (4 years to maturity, 8% yield)*

$$B_0 = \sum_{t=1}^{n} \frac{C_t}{(1+i)^t}$$

$$= \frac{\$10}{(1+0.09)} + \frac{\$10}{(1+0.09)^2} + \frac{\$110}{(1+0.09)^3}$$

$$= \$102.53$$

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Example 4.4.3

Relationship between maturity and price

> A simple bond matures in **three** years, has a coupon rate of 10% pa and has a maturity value of $100

> Assume that the bond pays interest annually and that the appropriate discount rate has risen from 8% to 9% over the previous year

To	Press	Display
Set all variables to default	[2nd] [RESET] [ENTER]	RST 0.00
Enter number of periods	3 [N]	N = 3.00◁
Enter discount rate	9 [I/Y]	I/Y = 9.00◁
Enter coupon payment	10 [+/–] [PMT]	PMT = -10.00◁
Enter face value	100 [+/–] [FV]	FV = -100.00◁
Compute present value	[CPT] [PV]	PV = 102.53∗

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Example 4.4.3

Relationship b/w maturity, discount rate, and price

4 years, 8% yield	3 years, 8% yield	3 years, 9% yield
$106.62	$105.15	$102.53

> The bond's value has fallen $4.09 from $106.62 to $102.53 as a result both of a one year reduction in maturity and a 1% increase in the discount rate

> This difference can be attributed as follows:
- – Change in maturity date $106.62 – 105.15 = $1.47
- – Change in discount rate $105.15 – 102.53 = <u>$2.62</u>

$4.09

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Objective 4.3

Valuing semi-annual cash flows

> Bonds, and particularly Government bonds, usually have semi-annual cash flows

> The bond's value then is calculated by halving the coupon and discount rates and doubling the number of periods

> The value of a bond with semi-annual coupon payments is given by:

$$B_0 = \sum_{t=1}^{n} \frac{C_t}{(1+i)^t}$$

where

- – n = the number of semi-annual periods (i.e. twice the number of years)
- – C = half the annual coupon amount
- – i = half the annual yield

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Example 4.3.3

Valuing semi-annual cash flows

> A simple bond matures in four years, has a coupon rate of 10% pa and has a maturity value of $100

> Assume that the bond pays interest semi-annually and that the appropriate discount rate is 8% pa

$$B_0 = \frac{\$5}{(1+0.04)} + \frac{\$5}{(1+0.04)^2} + \frac{\$5}{(1+0.04)^3} + \frac{\$5}{(1+0.04)^4} +$$

$$\frac{\$5}{(1+0.04)^5} + \frac{\$5}{(1+0.04)^6} + \frac{\$5}{(1+0.04)^7} + \frac{\$105}{(1+0.04)^8}$$

$$= \$106.73$$

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Example 4.3.3

Valuing semi-annual cash flows

> There are two ways to solve this problem using a financial calculator

> Using the first approach, we determine the number of periods and the discount rate per period, and enter these in the normal way

> This is the approach that will be used from now on in these lecture notes

To	Press	Display
Set all variables to default	[2nd] [RESET] [ENTER]	RST 0.00
Enter number of periods	8 [N]	N = 8.00◃
Enter discount rate	4 [I/Y]	I/Y = 4.00◃
Enter coupon payment	5 [+/–] [PMT]	PMT = -5.00◃
Enter face value	100 [+/–] [FV]	FV = -100.00◃
Compute present value	[CPT] [PV]	PV = 106.73∗

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Example 4.3.3

Valuing semi-annual cash flows

> Using the second approach, we can enter the annual discount rate if we tell the calculator that payments occur twice a year, but we still need to enter the **total** number of semi-annual periods

To	Press	Display
Set all variables to default	[2nd] [RESET] [ENTER]	RST 0.00
Enter number of periods	8 [N]	N = 8.00◃
Enter discount rate	8 [I/Y]	I/Y = 8.00◃
Enter payments per year	[2nd] [P/Y] 2 [ENTER]	P/Y = 2.00◃
Enter coupon payment	[CE/C] 5 [+/–] [PMT]	PMT = -5.00◃
Enter face value	100 [+/–] [FV]	FV = -100.00◃
Compute present value	[CPT] [PV]	PV = 106.73∗

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Objective 4.3

Shortcut valuation

> For a fixed-rate coupon bond, the coupon payments represent an annuity

> We can, therefore, compute the value of a bond by computing the present value of the annuity and then adding the present value of the maturity value

$$B_0 = C \times \left[\frac{1 - \frac{1}{(1+i)^{\text{no. of periods}}}}{i} \right] + \frac{\text{maturity value}}{(1+i)^{\text{no. of periods}}}$$

where

– C = the value of each coupon payment

– i = the yield applicable to each payment period

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Example 4.3.4

Valuing semi-annual cash flows

> A simple bond matures in four years, has a coupon rate of 10% pa and has a maturity value of $100

> Assume that the bond pays interest semi-annually and that the appropriate discount rate is 8% pa

$$B_0 = C \times \left[\frac{1 - \frac{1}{(1+i)^{\text{no. of periods}}}}{i} \right] + \frac{\text{maturity value}}{(1+i)^{\text{no. of periods}}}$$

$$= \$5 \times \left[\frac{1 - \frac{1}{(1+0.04)^8}}{0.04} \right] + \frac{\$100}{(1+0.04)^8} = \$106.73$$

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Objective 4.5

Zero-coupon bond valuation

> The value of a zero-coupon bond is equal to the present value of the face value, discounted using the following equation:

$$B_0 = \frac{\text{maturity value}}{(1+i)^{\text{no. of years} \times 2}}$$

> Note that even though there are no coupon payments, i equals the semi-annual discount rate and the number of periods is equal to the number of years × 2

> This is because the pricing of a zero-coupon bond should be consistent with the pricing of a semi-annual coupon bond (for instance, if you are comparing bonds as investment alternatives)

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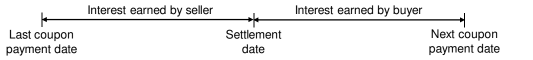
Objective 4.3

Valuing a bond between coupon payments

> If a bond is being valued between coupon payments the valuation must be adjusted to deduct the value of the interest which has accrued since the last coupon payment

> The reason for the adjustment is because an investor buying a bond between coupon payments will receive the next coupon payment in full

> However, the buyer has only earned the interest between the day the bond is sold and the next coupon date



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Objective 4.3

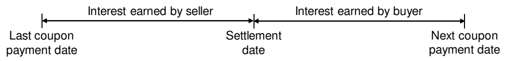
Valuing a bond between coupon payments

> The seller has earned an amount of interest (called **accrued interest**) between the last coupon payment and the date of sale

> The present value of the future cash flows – referred to as the **full price** or the **dirty price** – includes the accrued interest that is being paid by the buyer to the seller

> We, therefore, need to subtract the value of this accrued interest from the full price to find the true value of the bond at that point in time

> This value is referred to as the **clean price**



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Objective 4.3

Valuing a bond between coupon payments

> The first step is to find the present value of future cash flows when the valuation date occurs between cash flows

> This means that the number of periods is a fractional amount

> The formula requires the use of the variable w, which is the fraction of a period between the settlement date and the next coupon payment

$$w = \frac{\text{days between settlement date and next coupon payment date}}{\text{days in coupon period}}$$

> The present value of expected cash flows – the full price – is given by:

$$B_0 = \sum_{t=1}^{n} \frac{C_t}{(1+i)^{t-1+w}}$$

> Note that this includes the payment from the buyer to the seller for accrued interest

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Example 4.3.5

Valuing a bond between coupon payments

> A simple bond has five semi-annual coupon payments remaining, at a coupon rate of 10% pa, and has a maturity value of \$100

> The next coupon will be paid in 78 days time, there are 182 days in the current coupon period and the appropriate discount rate is 8% pa

$$\begin{aligned}\text{Full price} &= \sum_{t=1}^{n} \frac{C_t}{(1+i)^{t-1+w}} \\ &= \frac{\$5}{(1+0.04)^{1-1+78/182}} + \frac{\$5}{(1+0.04)^{1.4286}} + \frac{\$5}{(1+0.04)^{2.4286}} + \\ &\quad \frac{\$5}{(1+0.04)^{3.4286}} + \frac{\$105}{(1+0.04)^{4.4286}} \\ &= \$106.82\end{aligned}$$

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Objective 4.3

Valuing a bond between coupon payments

> To find the clean price, we must then deduct the accrued interest (AI), which is given by:

$$AI = C \times (1-w)$$

(Since w is the fraction of a period between the settlement date and the next coupon payment, $(1 - w)$ is the fraction of a period between the last coupon payment and the settlement date.)

> Hence, the clean price is given by:

$$\text{Clean price} = \sum_{t=1}^{n} \frac{C_t}{(1+i)^{t-1+w}} - C \times (1-w)$$

LA TROBE UNIVERSITY Bond valuation

Example 4.3.6

Valuing a bond between coupon payments

> A simple bond has five semi-annual coupon payments remaining, at a coupon rate of 10% pa, and has a maturity value of \$100

> The next coupon will be paid in 78 days time, there are 182 days in the current coupon period and the appropriate discount rate is 8% pa

$$\begin{aligned}\text{Clean price} &= \sum_{t=1}^{n} \frac{C_t}{(1+i)^{t-1+w}} - C \times (1-w) \\ &= \$106.82 - \$5 \times (1-0.4286) \\ &= \$106.82 - \$2.86 \\ &= \$103.96\end{aligned}$$

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Objective 4.6

The arbitrage-free valuation approach

> The traditional valuation approach discounts each cash flow from a security at the same discount rate

> The arbitrage-free valuation approach discounts each cash flow at the Treasury spot rate (plus a credit spread for non-Treasury securities) that is appropriate for the maturity of that cash flow

> Recall that the Treasury spot rate for a given maturity is the discount rate applicable to a zero-coupon bond of that maturity

> The arbitrage-free valuation approach considers a security as a portfolio of cash flows, similar to a portfolio of zero-coupon bonds, where the maturity of each zero-coupon bond matches the maturity of each cash flow from the security

Bond valuation

Objective 4.6

The arbitrage-free valuation approach

- The rationale behind this approach is that an investor could obtain a risk-free profit (i.e. implement arbitrage) by buying the bond and selling a replicating portfolio of zero-coupon bonds, if the aggregate value of the replicating portfolio is greater than the original bond
- The implementation of the arbitrage involves selling the replication portfolio and buying the bond
- This puts selling pressure on the replicating portfolio and buying pressure on the bond, ensuring that the price of the bond remains close to the value of its replicating portfolio
- Hence the arbitrage-free value is the theoretical value of the bond after any arbitrage opportunities have been eliminated

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Bond valuation

Objective 4.6

Bond valuation using multiple discount rates

- The correct way to value the cash flows of a bond under the arbitrage-free approach is to use a different discount rate that applies to the time period in which the cash flow will be received
- For Treasury securities, this will be the Treasury spot rate for a zero-coupon bond of that maturity
- The value of the bond is found by applying the formula for the present value of a zero-coupon bond to each of the bond's cash flows
- In the case of semi-annual coupon payments, the formula would be:

$$B_0 = \frac{\text{cash flow}}{(1+\text{spot rate}/2)^{\text{no. of semi-annual periods}}}$$

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Bond valuation

Example 4.6.1

Bond valuation using multiple discount rates

- A simple bond matures in four years, has a coupon rate of 10% pa and has a maturity value of $100
- Assume that the bond pays interest annually and that the appropriate discount rates are as follows:
  - Year one 6.8%
  - Year two 7.2%
  - Year three 7.6%
  - Year four 8.0%

$$B_0 = \frac{\$10}{(1+0.068)} + \frac{\$10}{(1+0.072)^2} + \frac{\$10}{(1+0.076)^3} + \frac{\$110}{(1+0.080)^4}$$
$$= \$106.95$$

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