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Research on the Control Method of Inverted Pendulum Based on Kalman Filter

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***Abstract*—Inverted pendulum is a nonlinear, multivariable and instability system. In the past, without considering the effects of measurement noise and output noise of the sensor in the processing of inverted pendulum control, the strong random jitter occur, and the control accuracy and stable of the inverted pendulum is reduced. Because the Kalman filter can estimate the signal is corrupted by the noise, the filtered system will have fine robustness and dynamic performance. Inverted pendulum in the straight line car is given as example, and the application of Kalman filter in the inverted pendulum control system is described in the paper.**

Keywords—kalman filter; inverted pendulum; state equation

I. Introduction

The initial study of inverted pendulum began in the 1950s, and cybernetics experts at the Massachusetts institute of technology according to the principle of the rocket booster design the experiment equipment of a single inverted pendulum. Hereafter, its control method and idea has a wide range of applications in the field of military industry, aerospace, robot and general industrial process such as the balance control in the process of the robot, verticality control of rocket and satellite attitude control in flight, etc. The control of the inverted pendulum is a common method of checking a new type of control theory and algorithms [2]. In the past, people control the inverted pendulum mainly adopting PID control, the LQR control, fuzzy control, neural network control, etc., but in these control methods the effects of measurement noise and output noise of the sensor are not given, which reduces the control precision. Kalman filter consists of a series of recursive mathematical formula, and the problem of linear discrete data filtering is solved by Kalman filter using recursive method. As a commonly used parametric estimation method, Kalman filter estimation, in fact, is a kind of unbiased estimation with the least variance [3]. Therefore, we can real-time online estimate the state in the process of movement of inverted pendulum using the Kalman filter and then get state feedback which can significantly reduce the effects of noise.

II. The Mathematical Model of Straight Line Car Inverted Pendulum System

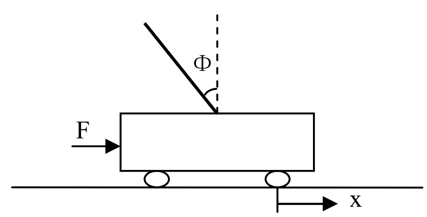

Fig. 1. Straight line car inverted pendulum system

A straight line car inverted pendulum system is shown in figure 1. The mathematical model of straight line car inverted pendulum system is provided as follows [4]. Analyzing the horizontal force of car, then we can get:

$$M\ddot{x} = F - b\dot{x} - N \tag{1}$$

where M is the quality of car, x is the car position, F is the force on the car and b is the friction coefficient of car. Analyzing the horizontal force of pendulum, then we can get:

$$N = m\frac{d^2}{dt^2}(x + l\sin\theta) \tag{2}$$

That is

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$$N = m\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta \qquad (3)$$

where m quality of pendulum, l is the length of rotation axis to the center of mass of the pendulum and θ is the angle of pendulum and vertical downward direction. According to the above equations, we can get the first equation of motion:

$$(M+m)\ddot{x} + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F \qquad (4)$$

Analyzing the vertical upward direction force of pendulum, then we can get:

$$P - mg = m\frac{d^2}{dt^2}(l\cos\theta) \qquad (5)$$

$$P - mg = -ml\ddot{\theta}\sin\theta - ml\dot{\theta}^2\cos\theta \qquad (6)$$

The torque balance equation is

$$-Pl\sin\theta - Nl\cos\theta = I\ddot{\theta} \qquad (7)$$

In equation, I is the pendulum inertia and ϕ is the angle of pendulum and vertical upward direction. According to the above equations, we can get the second equation of motion, under such condition:

$\theta = \pi + \phi$, $\cos\phi = -\cos\theta$, $\sin\phi = -\sin\theta$.

$$(I + ml^2)\ddot{\theta} + mgl\sin\theta = -ml\ddot{x}\cos\theta \qquad (8)$$

ϕ is assumed to be far smaller than 1, we so can get the approximate model of the system:

$$\begin{cases}(I + ml^2)\ddot{\phi} - mgl\,\theta = ml\,\ddot{x} \\ (M+m)\ddot{x} + b\dot{x} - ml\ddot{\phi} = u\end{cases} \qquad (9)$$

$\dot{x}$ and $\dot{\phi}$ are selected as state variables, state equation can be obtained from equation (9).With external forces as the input of the system state equations are as follows:

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\\ \dot{\phi}\\ \ddot{\phi}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & \frac{-(I+ml^2)b}{I(M+m)+Mml^2} & \frac{m^2gl^2}{I(M+m)+Mml^2} & 0\\ 0 & 0 & 0 & 1\\ 0 & \frac{-mlb}{I(M+m)+Mml^2} & \frac{mgl(M+m)}{I(M+m)+Mml^2} & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ \phi\\ \dot{\phi}\end{bmatrix} + \begin{bmatrix}0\\ \frac{(I+ml^2)}{I(M+m)+Mml^2}\\ 0\\ \frac{ml}{I(M+m)+Mml^2}\end{bmatrix}u$$

$$y = \begin{bmatrix}x\\ \phi\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ \phi\\ \dot{\phi}\end{bmatrix} + \begin{bmatrix}0\\ 0\end{bmatrix}u \qquad (10)$$

where

$$A = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & \frac{-(I+ml^2)b}{I(M+m)+Mml^2} & \frac{m^2gl^2}{I(M+m)+Mml^2} & 0\\ 0 & 0 & 0 & 1\\ 0 & \frac{-mlb}{I(M+m)+Mml^2} & \frac{mgl(M+m)}{I(M+m)+Mml^2} & 0\end{bmatrix},$$

$$B = \begin{bmatrix}0\\ \frac{(I+ml^2)}{I(M+m)+Mml^2}\\ 0\\ \frac{ml}{I(M+m)+Mml^2}\end{bmatrix}, C = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}.$$

III. THE DESIGN OF INVERTED PENDULUM SYSTEM CONTROLLER

In the control of the inverted pendulum, we can use many methods, such as PID control, the LQR optimal control and fuzzy control, etc. . But, the dynamic response of the PID control is less, and how to select suitable weighting matrix Q and R that makes the dynamic performance of the system expected as we do in LQR optimal control, there are still lack of general and effective guidelines. Usually we selected it only by experience. Determine the membership function of fuzzy control largely affect system performance. So we design the controller by pole assignment method of state space in the modern control theory [5].

A state equation of straight line car inverted pendulum system is described in formula (10). Due to rank [B | AB | A^2B | A^3B]=4 and rank[C^T | A^TC^T | $(A^T)^2C^T$ | $(A^T)^3C^T$]=4, the system is completely controllable and observable which satisfy the condition of state feedback [6]. Now all parameters in the small car inverted pendulum system is as follows: pendulum quality m＝1kg, length 2l＝2m, I＝1/3mL^2＝1/3kgm^2, the quality of the car M＝1kg, acceleration of gravity g≈10m/s^2。 Each parameter values are substituted in formula (10), then we get:

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A=[0 1 0 0;0 -0.1 6 0;0 0 0 1;0 -0.06 12 0]; B=[0;1.2;0;0.6]; C=[1 0 0 0;0 0 1 0].
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The sampling time is 0.1 s. Making the system discrete, then we get:

$$Ad=\begin{bmatrix}1.0606 & 0.1020 & 0 & 0\\ 1.2241 & 1.0606 & 0 & 0\\ -0.1683 & -0.0056 & 1 & 0.1\\ -3.4004 & -0.1683 & 0 & 1\end{bmatrix},\quad Bd=\begin{bmatrix}-0.0030\\ -0.0612\\ 0.0041\\ 0.0834\end{bmatrix},$$

$$Cd=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}$$

Considering pole assignment of straight line car inverted pendulum system at [0.5 0.5 0.5 0.5] we can get feedback matrix K=[-115.1371 27.6809 59.4988 38.6742].

IV. KALMAN FILTER IN THE APPLICATION OF STATE OBSERVATION

In the process of the above controller design, we did not consider the effect of system noise and measurement noise. But in fact these noises exist, and have an impact on the system. So we can use the Kalman filter to estimate the state of the system, get the status value to eliminate noise effects.

A discrete time system is described by the following difference equation.

$$x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1} \tag{11}$$

$$z_k = Hx_k + v_k \tag{12}$$

where random signal w_k denotes process noise excitation and v_k denotes observation noise. They are assumed to be independent each other, the normal distribution of the white noise is:

$$p(w) \sim N(0,Q) \tag{13}$$

$$p(v) \sim N(0,R) \tag{14}$$

The Kalman filter equations as follows:

$$\begin{aligned}
\hat{x}_{k|k-1} &= A\hat{x}_{k-1} + Bu_{k-1}\\
P_{k|k-1} &= AP_{k-1}A^T + Q\\
K_k &= P_{k|k-1}H^T\left(HP_{k|k-1}H^T + R\right)^{-1}\\
\hat{x}_k &= \hat{x}_{k|k-1} + K_k\left(z_K - H\hat{x}_{k|k-1}\right)\\
P_k &= \left(I - K_kH\right)P_{k|k-1}
\end{aligned} \tag{15}$$

where $P_{k|k-1}$ denotes covariance of the prior estimation error, P_k denotes covariance of the posteriori estimation error , K_k denotes residual gain, and $\hat{x}_{k-1}$ and P_{k-1} denotes initial values. When we estimate the state of the inverted pendulum, considering the actual situation, we take $\hat{x}_{k-1}$ = [0 0 0 0] ^{T} and P_{k-1} order 4 unit matrix. Such not only can fast convergence, but can reduce the influence of white noise [7]. We will estimate the state of value as state feedback based on Kalman filter, and control the inverted pendulum system which improves the stability of the system and the control precision.

V. SIMULATION RESULTS

According to the above analysis, we under mallab7.1 of inverted pendulum system are simulated under the condition of white noise. The result is as follows:

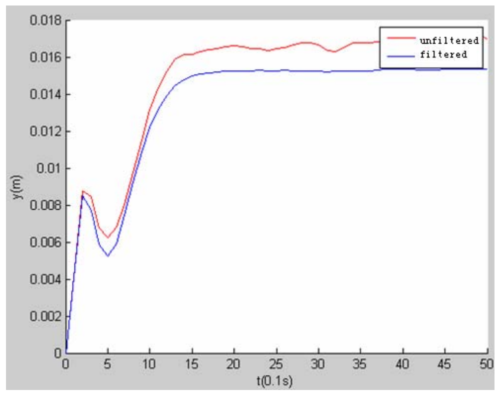

Fig. 2. the car displacement 1

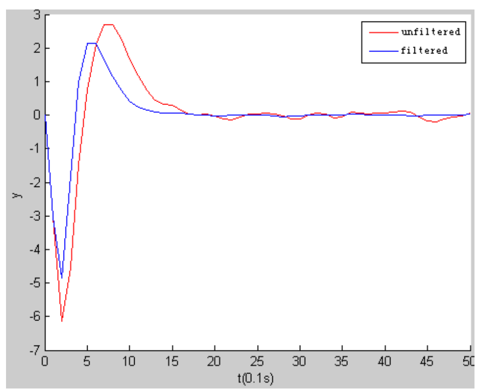

Fig. 3. pendulum angle one 1

Above two figures are the simulation diagrams of inverted pendulum system under step input. In figures, the red denotes the simulation output for not using Kalman filter and the blue denotes the simulation output for using Kalman filter. As you can see, the Kalman filter can be a very good estimate the states of the system, thus it greatly eliminate the influence of interference.

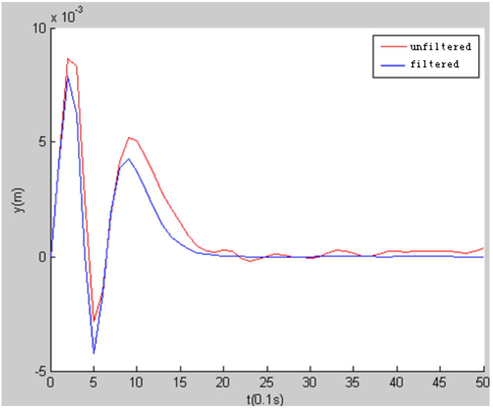

Fig. 4. the car displacement 2

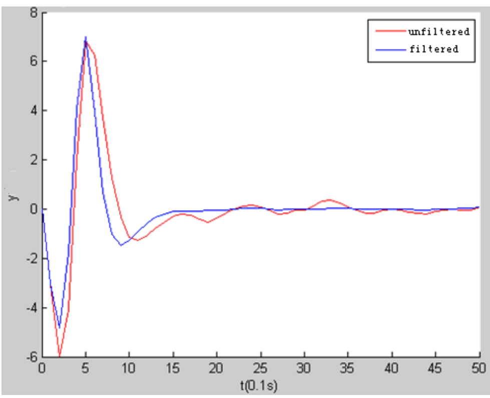

Fig. 5. pendulum angle 2

Above two figures are the simulation diagrams of inverted pendulum system under the wide pulse input. In figures, the red denotes the simulation output for not using Kalman filter and the blue denotes the simulation output for using Kalman filter. Also it can be seen that in the case of the application, filter greatly eliminate the influence of interference.

VI. CONCLUSION

Kalman filter is effective to estimate the states of the inverted pendulum. Compared with the control methods which do not adopt the kalman filter, the above method can reduce the noise influence on system, enhance the stability of the system, improve the control precision, and make the system have certain robustness [8].

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