Problem Set 2

Exercise 1. Determine if each function below is convex or concave:

1. $f(x, y) = 2x - y - x^2 + 2xy - y^2$

2. $f(x, y, z) = x^2 + 2y^2 + 3z^2 + 2xy + 2xz$

3. $f(x, y) = x + y - e^x - e^{x+y}$

4. $f(x, y) = 12x^{1/3}y^{1/2}, x, y \geq 0$

Exercise 2.

1. Let $f(x, y) = x^3 + 2x^2 + 2xy + \frac{1}{2}y^2 - 8x - 2y - 8$. Find the range of values (x, y) for which the function is convex.

2. Let $f(x, y, z) = 2x^2 + 2xz + 2ayz + 2z^2$. Determine the values of a for which the function is concave and for which it is convex.

Exercise 3. Let

$$f(x) = \begin{cases} -x - 1 & \text{if } x < -1 \\ x - 1 & \text{if } x > 1 \\ 0 & \text{if } -1 \leq x \leq 1. \end{cases}$$

Check if the point $x = 0$ is a global maximizer/global minimizer/local maximizer/local minimizer of f?

Exercise 4. Find global maximum and global minimum of the function:

1. $f(x, y) = x^2 + y^2 + y - 1$ subject to $x^2 + y^2 \leq 1$

2. $f(x, y) = x^2 + 2y^2 - x$ subject to $x^2 + y^2 \leq 1$

3. $f(x, y) = 3 + x^3 - x^2 - y^2$ subject to $x^2 + y^2 \leq 1$ and $x \geq 0$

4. $f(x, y) = xy$ subject to $x + y = 6$

5. $f(x, y) = xy^2$ subject to $2x^2 + y^2 = 3$

6. $f(x, y) = x^2 + y^2$ subject to $x^2 + xy + y^2 = 3$

7. $f(x, y) = (x - 1)^2 + y^2$ subject to $y^2 - 8x = 0$

8. $f(x, y) = (x - 4)^2 + (y - 4)^2$ subject to $x + y \leq 4$ and $x + 3y \leq 9$

9. $f(x, y) = x^2y^2$ subject to $2x + y \leq 2$ and $x, y \geq 0$

10. $f(x, y) = x^2 + y^2 - 4x - 4y$ subject to $x^2 \leq y$, and $x + y \geq 2$ and $x, y \geq 0$