Phụ lục:

1. Code vẽ đồ thị bài toán động học thuận

```
close all; clear all; clc;

l1 = 0.25; l2 = 0.194; l3 = 0.265; a1 = 0.03;

%Thoi gian t
t = 0:120;

%Toa do diem thao tac
xe = l3.*cos(cos(0.05.*t)).*cos(0.8.*cos(0.1.*t)) - cos(0.8.*cos(0.1.*t)).*(a1 -
l2.*cos(0.5.*cos(0.05.*t)));

ye = l3.*cos(cos(0.05.*t)).*sin(0.8.*cos(0.1.*t)) - sin(0.8.*cos(0.1.*t)).*(a1 -
l2.*cos(0.5.*cos(0.05.*t)));

ze = l1 + l3.*sin(cos(0.05.*t)) + l2.*sin(0.5.*cos(0.05.*t));

%Van toc diem thao tac E
vxe = (l2.*sin(cos(t./20)./2).*sin(t./20).*cos((4.*cos(t./10))./5))./40 -
(2.*sin((4.*cos(t./10))./5).*sin(t./10).*(a1 - l2.*cos(cos(t./20)./2)))./25 +
(2.*l3.*sin((4.*cos(t./10))/5).*sin(t./10).*cos(cos(t./20)))./25 +
(l3.*sin(t./20).*sin(cos(t./20)).*cos((4.*cos(t./10))./5))./20;

vye = (2.*sin(t./10).*cos((4.*cos(t./10))./5).*(a1 - l2.*cos(cos(t./20)./2)))./25 +
(l2.*sin((4.*cos(t./10))./5).*sin(cos(t./20)./2).*sin(t./20))./40 -
(2.*l3.*sin(t./10).*cos(cos(t./20)).*cos((4.*cos(t./10))./5))./25 +
(l3.*sin((4.*cos(t./10))./5).*sin(t./20).*sin(cos(t./20)))./20;

vze = - (l2.*sin(t./20).*cos(cos(t./20)./2))./40 -
(l3.*sin(t./20).*(2.*cos(cos(t./20)./2).^2 - 1))./20;

%Gia toc diem thao tac E
axe = (4.*sin(t./10).^2.*cos((4.*cos(t./10))./5).*(a1 -
l2.*cos(cos(t./20)./2)))./625 - (sin((4.*cos(t./10))./5).*cos(t./10).*(a1 -
l2.*cos(cos(t./20)./2)))./125 +
(l2.*sin(cos(t./20)./2).*cos(t./20).*cos((4.*cos(t./10))./5))./800 -
(4.*l3.*sin(t./10).^2.*cos(cos(t./20)).*cos((4.*cos(t./10))./5))./625 -
(l3.*sin(t./20).^2.*cos(cos(t./20)).*cos((4.*cos(t./10))./5))./400 -
(l2.*sin(t./20).^2.*cos((4.*cos(t./10))./5).*cos(cos(t./20)./2))./1600 +
(l3.*sin((4.*cos(t./10))./5).*cos(t./10).*cos(cos(t./20)))./125 +
(l3.*cos(t./20).*sin(cos(t./20)).*cos((4.*cos(t./10))./5))./400 +
(l3.*sin((4.*cos(t./10))./5).*sin(t./10).*sin(t./20).*sin(cos(t./20)))./125 +
(l2.*sin((4.*cos(t./10))./5).*sin(cos(t./20)./2).*sin(t./10).*sin(t./20))./250;
aye = (4.*sin((4.*cos(t./10))./5).*sin(t./10).^2.*(a1 + l3 -
2.*l3.*cos(cos(t./20)./2).^2 - l2.*cos(cos(t./20)./2)))./625 +
(cos(t./10).*cos((4.*cos(t./10))./5).*(a1 + l3 - 2.*l3.*cos(cos(t./20)./2).^2 -
```

```
l2.*cos(cos(t./20)./2))./125 +
(sin((4.*cos(t./10))./5).*sin(cos(t./20)./2).*cos(t./20).*(l2 +
4.*l3.*cos(cos(t./20)./2)))./800 -
(sin((4.*cos(t./10))./5).*sin(t./20).^2.*cos(cos(t./20)./2).*(l2 +
4.*l3.*cos(cos(t./20)./2)))./1600 -
(l3.*sin((4.*cos(t./10))./5).*sin(t./20).^2.*(cos(cos(t./20))./2 - 1/2))./400 -
(sin(cos(t./20)./2).*sin(t./10).*sin(t./20).*cos((4.*cos(t./10))./5).*(l2 +
4.*l3.*cos(cos(t./20)./2)))./250;

aze =  - (l3.*cos(t./20).*(2.*cos(cos(t./20)./2).^2 - 1))./400 -
(l2.*sin(cos(t./20)./2).*sin(t./20).^2)./1600 -
(l2.*cos(t./20).*cos(cos(t./20)./2))./800 -
(l3.*sin(cos(t./20)/2).*sin(t./20).^2.*cos(cos(t./20)/2))./200;

%Do thi
figure(1);
plot(t,vxe,'k-.',t,vye,'b--',t,vze,'r-','linewidth',1);hold on;
xlabel('t(s)');ylabel('v(m/s)');
title('Do thi van toc diem E');
legend('vxE','vyE','vzE');

figure(2);
plot(t,axe,'k-.',t,aye,'b--',t,aze,'r-','linewidth',1);hold on;
xlabel('t(s)');ylabel('a(m/s^2)');
title('Do thi gia toc diem E');
legend('axE','ayE','azE');

figure(3);
plot3(xe,ye,ze,'r-','linewidth',1);
hold on;
title('Do thi quy dao diem E');

% ve van toc goc
% khau 1:
Vxw1 = 0;
Vyw1 = 0;
Vzw1 = -(2.*sin(t./10))/25;

figure(4);
plot(t,Vzw1,'r-','linewidth',1);hold on;
xlabel('t(s)');ylabel('w(rad/s)');
title('Do thi van toc goc w1');
legend('w1');

% khau 2:
Vxw2 = -(sin((4.*cos(t./10))/5).*sin(t./20))/40;
Vyw2 = (sin(t./20).*cos((4.*cos(t./10))/5))/40;
Vzw2 = -(2.*sin(t./10))/25;
```

```
figure(5);
plot(t,Vxw2,'k-.',t,Vyw2,'b--',t,Vzw2,'r-','linewidth',1);hold on;
xlabel('t(s)');ylabel('w(rad/s)');
title('Do thi van toc goc w2');
legend('w2x','w2y','w2z');


% khau 3:
Vxw3 = -(sin((4.*cos(t./10))/5).*sin(t./20))/20;
Vyw3 = (sin(t./20).*cos((4.*cos(t./10))/5))/20;
Vzw3 = -(2.*sin(t./10))/25;

figure(6);
plot(t,Vxw3,'k-.',t,Vyw3,'b--',t,Vzw3,'r-','linewidth',1);hold on;
xlabel('t(s)');ylabel('w(rad/s)');
title('Do thi van toc goc w3');
legend('w3');
```

2. Code vẽ đồ thị bài toán động học ngược

2.1. Function con

```
function K= rO1(q)
global l1 l2 l3
q1=q(1); q2=q(2); q3=q(3);
O1x=0;
O1y=0;
O1z=l1;
K=[O1x;O1y;O1z];
end

function K= rO2(q)
global l1 l2 l3 a1
q1=q(1); q2=q(2); q3=q(3);
O2x=-a1*cos(q1);
O2y=-a1*sin(q1);
O2z=l1;
K=[O2x;O2y;O2z];
end

function K= rO3(q)
global l1 l2 l3 a1
q1=q(1); q2=q(2); q3=q(3);
O3x= -cos(q1)*(a1 - l2*cos(q2));
O3y= -sin(q1)*(a1 - l2*cos(q2));
O3z= l1 + l2*sin(q2);
K=[O3x;O3y;O3z];
end
```

```
function K=rE(q)
global l1 l2 l3 a1
q1=q(1); q2=q(2); q3=q(3);
Ex = l3*cos(q2 + q3)*cos(q1) - cos(q1)*(a1 - l2*cos(q2));
Ey = l3*cos(q2 + q3)*sin(q1) - sin(q1)*(a1 - l2*cos(q2));
Ez = l1 + l2*sin(q2) + l3*sin(q2 + q3);
K=[Ex;Ey;Ez];
end


function K=X(t)
xE= 0.25 + 0.09*cos(t/0.75);
yE= 0.15 + 0.09*sin(t/0.75);
zE= 0;
K=[xE;yE;zE];
end


function K=dX(t)
vEx= -0.12*sin(1.3*t);
vEy= 0.12*cos(1.3*t);
vEz= 0;
K=[vEx;vEy;vEz];
end


function K=d2X(t)
d2xE= -0.16*cos(1.3*t);
d2yE= -0.16*sin(1.3*t);
d2zE= 0;
K=[d2xE;d2yE;d2zE];
end


function K=ptlk(q,t)
global l1 l2 l3 a1
q1=q(1); q2=q(2); q3=q(3);
x=X(t);
xE=x(1);yE=x(2);zE=x(3);
f1= l3*cos(q2 + q3)*cos(q1) - cos(q1)*(a1 - l2*cos(q2)) - xE;
f2= l3*cos(q2 + q3)*sin(q1) - sin(q1)*(a1 - l2*cos(q2)) - yE;
f3= l1 + l2*sin(q2) + l3*sin(q2 + q3) - zE;
K=[f1;f2;f3];
end
```

```
function K=Jq(q)
global l1 l2 l3 a1
q1=q(1);q2=q(2);q3=q(3);
K=zeros(3,3);

K(1,1)= sin(q1)*(a1 - l2*cos(q2)) - l3*cos(q2 + q3)*sin(q1);
K(1,2)= -l2*cos(q1)*sin(q2) - l3*sin(q2 + q3)*cos(q1);
K(1,3)= -l3*sin(q2 + q3)*cos(q1);

K(2,1)= l3*cos(q2 + q3)*cos(q1) - cos(q1)*(a1 - l2*cos(q2));
K(2,2)= -l2*sin(q1)*sin(q2) - l3*sin(q2 + q3)*sin(q1);
K(2,3)= -l3*sin(q2 + q3)*sin(q1);

K(3,1)= 0;
K(3,2)= l2*cos(q2) + l3*cos(q2 + q3);
K(3,3)= l3*cos(q2 + q3);
end


function K=dJq(q,dq)
global l1 l2 l3 a1
q1=q(1); q2=q(2); q3=q(3);
dq1=dq(1); dq2=dq(2); dq3=dq(3);
K=zeros(3,3);

K(1,1)= cos(q1)*dq1*(a1-
l2*cos(q2))+sin(q1)*l2*sin(q2)*dq2+l3*sin(q2+q3)*(dq2+dq3)*sin(q1)-
l3*cos(q2+q3)*cos(q1)*dq1;
K(1,2)= -
l3*cos(q2+q3)*(dq2+dq3)*cos(q1)+l3*sin(q2+q3)*sin(q1)*dq1+l2*sin(q1)*dq
1*sin(q2)-l2*cos(q1)*cos(q2)*dq2;
K(1,3)= -l3*cos(q2+q3)*(dq2+dq3)*cos(q1)+l3*sin(q2+q3)*sin(q1)*dq1;

K(2,1)= -l3*sin(q2+q3)*(dq2+dq3)*cos(q1)-
l3*cos(q2+q3)*sin(q1)*dq1+sin(q1)*dq1*(a1-l2*cos(q2))-
cos(q1)*l2*sin(q2)*dq2;
K(2,2)= -l3*cos(q2+q3)*(dq2+dq3)*sin(q1)-l3*sin(q2+q3)*cos(q1)*dq1-
l2*cos(q1)*dq1*sin(q2)-l2*sin(q1)*cos(q2)*dq2;
K(2,3)= -l3*cos(q2+q3)*(dq2+dq3)*sin(q1)-l3*sin(q2+q3)*cos(q1)*dq1;

K(3,1)=0;
K(3,2)= -l3*sin(q2+q3)*(dq2+dq3)-l2*sin(q2)*dq2;
K(3,3)= -l3*sin(q2+q3)*(dq2+dq3);
end
```

```
function K = Mq(q)
q1 = q(1); q2 = q(2); q3 = q(3);
K = zeros(3,3);
K(1,1) = 0.147e-2+0.52e-1*cos(q2)^2-0.135e-1*cos(q2)+0.1e-1*cos(q2+q3)^2-
0.3e-2*cos(q2+q3)+0.2e-1*cos(q2+q3)*cos(q2);
K(1,2) = 0;
K(1,3) = 0;

K(2,1) = 0;
K(2,2) = 0.62e-1+0.2e-1*sin(q2+q3)*sin(q2)+0.2e-1*cos(q2+q3)*cos(q2);
K(2,3) = 0.1e-1-0.4e-2*cos(q1)^2+0.9e-2*sin(q2+q3)*sin(q2)+0.9e-
2*cos(q2+q3)*cos(q2);

K(3,1) = 0;
K(3,2) = 0.1e-1-0.4e-2*cos(q1)^2+0.8e-2*sin(q2+q3)*sin(q2)+0.8e-
2*cos(q2+q3)*cos(q2);
K(3,3) = 0.1e-1-0.4e-2*cos(q1)^2;
end


function K = Cq(q,dq)
q1 = q(1); q2 = q(2); q3 = q(3); dq1 = dq(1); dq2 = dq(2); dq3 = dq(3);
K = zeros(3,3);
K(1,1) = (-0.1*cos(q2)*sin(q2)+0.1e-1*sin(q2)-0.2e-
1*cos(q2+q3)*sin(q2+q3)+0.3e-2*sin(q2+q3)-0.2e-1*sin(q2+q3)*cos(q2)-0.2e-
1*cos(q2+q3)*sin(q2))*dq2-0.2e-1*sin(q2+q3)*(cos(q2+q3)-
0.2+cos(q2))*dq3;
K(1,2) = -0.4e-2*cos(q1)*sin(q1)*dq3;
K(1,3) = -0.4e-2*cos(q1)*sin(q1)*dq2-0.4e-2*cos(q1)*sin(q1)*dq3;

K(2,1) = -(0.5*(-0.1*cos(q2)*sin(q2)+0.1e-1*sin(q2)-0.2e-
1*cos(q2+q3)*sin(q2+q3)+0.3e-2*sin(q2+q3)-0.2e-1*sin(q2+q3)*cos(q2)-0.2e-
1*cos(q2+q3)*sin(q2)))*dq1;
K(2,2) = (0.2e-1*cos(q2+q3)*sin(q2)-0.2e-1*sin(q2+q3)*cos(q2))*dq3;
K(2,3) = 0.8e-2*cos(q1)*sin(q1)*dq1+(0.9e-2*cos(q2+q3)*sin(q2)-0.9e-
2*sin(q2+q3)*cos(q2))*dq3;

K(3,1) = 0.1e-1*sin(q2+q3)*(cos(q2+q3)-0.2+cos(q2))*dq1;
K(3,2) = 0.8e-2*cos(q1)*sin(q1)*dq1+(0.8e-2*cos(q2+q3)*sin(q2)-0.8e-
2*sin(q2+q3)*cos(q2))*dq3-(0.5*(0.2e-1*cos(q2+q3)*sin(q2)-0.2e-
1*sin(q2+q3)*cos(q2)))*dq2-(0.5*(0.9e-2*cos(q2+q3)*sin(q2)-0.9e-
2*sin(q2+q3)*cos(q2)))*dq3;
K(3,3) = 0.8e-2*cos(q1)*sin(q1)*dq1-(0.5*(0.8e-2*cos(q2+q3)*sin(q2)-0.8e-
2*sin(q2+q3)*cos(q2)))*dq2;
end
```

```
function K = Gq(q)
q1 = q(1); q2 = q(2); q3 = q(3);
K = zeros(3,1);
K(1,1) = 0;
K(2,1) = 1.9*cos(q2)+0.41*cos(q2+q3);
K(3,1) = 0.41*cos(q2+q3);
1nd
```

2.2. main

```
%main
close all; clear all; clc;
global l1 l2 l3 a1
parameters;
% du doan nghiem dau
q=[0.7649928327;-0.3812343942;-0.3543431040];

% thoi gian chuyen dong
t = 1.1780972450:0.01:1.178097245*3.01;

%xac dinh vector toa do suy rong
for i=1:length(t)
   lk=ptlk(q,t(i));
   while(norm(lk,2)>1e-12) %norm(lk,2) = norm(lk)
      jc=Jq(q);
      delta=-inv(jc)*lk;
      q=q+delta;
      lk=ptlk(q,t(i));
   end
   % xac dinh Vector toa do suy rong tai thoi diem t(i)
   q1(i)=q(1); q2(i)=q(2);q3(i)=q(3);
   % xac dinh Vector van toc suy rong tai thoi diem t(i)
   dq=inv(jc)*dX(t(i));
   dq1(i)=dq(1); dq2(i)=dq(2);dq3(i)=dq(3);
   % xac dinh Vector gia toc suy rong tai thoi diem t(i)
   d2q=inv(jc)*(d2X(t(i))-dJq(q,dq)*dq);
   d2q1(i)=d2q(1); d2q2(i)=d2q(2);d2q3(i)=d2q(3);
   % Tinh cac momen dan dong
   tau = Mq(q)*d2q + Cq(q,dq)*dq + Gq(q);
   tau1(i) = tau(1); tau2(i) = tau(2); tau3(i) = tau(3);

end


% Ve do thi toa do suy rong
figure(1)
plot(t,q1,'k-',t,q2,'r-.',t,q3,'b--','linewidth',1);grid on;
hold on
xlabel('t[s]');ylabel('q[rad]');
```

```
title('Do thi cac toa do suy rong');
legend('q_1','q_2','q_3');

% Ve do thi cac van toc suy rong
figure(2)
plot(t,dq1,'k-',t,dq2,'r-.',t,dq3,'b--','linewidth',1);grid on;
xlabel('t[s]');ylabel('dq/dt[rad/s]');
title('Do thi cac van toc suy rong');
legend('dq_1/dt','dq_2/dt','dq_3/dt');

% Ve do thi cac gia toc suy rong
figure(3)
plot(t,d2q1,'k-',t,d2q2,'r-.',t,d2q3,'b--','linewidth',1);grid on;
xlabel('t[s]');ylabel('dq^2/dt[rad/s^2]');
title('Do thi cac gia toc suy rong');
legend('dq_1^2/dt','dq_2^2/dt','dq_3^2/dt');

% Ve do thi toa do suy rong q1
figure(4)
plot(t,q1,'k-',t,dq1,'r-.',t,d2q1,'b--','linewidth',1);grid on;
xlabel('t[s]');ylabel('q[rad],dq/dt[rad/s],dq^2/dt[rad/s^2]');
title('Do thi toa do suy rong q_1');
legend('q_1','dq_1/dt','dq_1^2/dt');

% Ve do thi toa do suy rong q2
figure(5)
plot(t,q2,'k-',t,dq2,'r-.',t,d2q2,'b--','linewidth',1);grid on;
xlabel('t[s]');ylabel('q[rad],dq/dt[rad/s],dq^2/dt[rad/s^2]');
title('Do thi toa do suy rong q_2');
legend('q_2','dq_2/dt','dq_2^2/dt');

% Ve do thi toa do suy rong q3
figure(6)
plot(t,q3,'k-',t,dq3,'r-.',t,d2q3,'b--','linewidth',1);grid on;
xlabel('t[s]');ylabel('q[rad],dq/dt[rad/s],dq^2/dt[rad/s^2]');
title('Do thi toa do suy rong q_3');
legend('q_3','dq_3/dt','dq_3^2/dt');

%ve cau hinh cua robot
x0=0; y0=0; z0=0; z1=l1;
OAx= [x0,-a1]; OAy=[y0,y0]; OAz=[z0,z1];


j = 1.1780972450:0.01:1.178097245*3.01;
ABx = 0.250 + 0.090.*cos(j./0.750);
ABy = 0.150 + 0.090.*sin(j./0.750);
ABz = zeros(1, length(ABy));
```

```
figure(7)
hold on;
plot3(x0,y0,z0,'ob','linewidth',2,'Markersize',6);grid on
plot3(ABx,ABy,ABz,'r-','linewidth',1);
for i=1:11:length(t)
    qq1=[q1(i);q2(i);q3(i)];
    RO3=rO3(qq1);
    RO2=rO2(qq1);
    RE=rE(qq1);
    AEx=[0,RO2(1),RO3(1),RE(1)];
    AEy=[0,RO2(2),RO3(2),RE(2)];
    AEz=[0,RO2(3),RO3(3),RE(3)];
    plot3(AEx,AEy,AEz,'k-','linewidth',1);
    plot3(RO2(1),RO2(2),RO2(3),'ob-','linewidth',1);
    plot3(RO3(1),RO3(2),RO3(3),'ob-','linewidth',1);
    plot3(RE(1),RE(2),RE(3),'or-','linewidth',1);
    axis equal
end


figure(8)
plot(t,tau1,'k-',t,tau2,'b--',t,tau3,'r-.','linewidth',1); grid on
xlabel('t[s]'); ylabel('tau[Nm]');
title('Do thi cac momen dan dong');
legend('tau_1','tau_2','tau_3');


% Gia tri lon nhat cua cac momen dan dong
Tau1Max = max(tau1)
Tau2Max = max(tau2)
Tau3Max = max(tau3)
```