CE2004: Circuits & Signal Analysis Lab Report 1&2

Do Anh Tu

U1721947F

Group SE2

School of Computer Engineering

Nanyang Technological University

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Abstract

Lab 1: Application of Ohm Law, KCL, KVL & Thévenin's Theorems.

Lab 2: Characteristics of RC Circuit and 1st order Transient Circuit

1. Experiment 1 – Circuit Analysis Techniques

1.1. Measurement of Series-Parallel Circuit (based on Ohm's Law, Kirchhoff's Laws)

Objective 1: Measure the current and voltage readings for the resistors shown in Figure 1.

To measure the voltage readings of the resistors, we simply probe the two terminals of the multimeter across the metallic parts of the resistors connected to the circuit.

On the other hand, when measuring the current passing through the resistors, we need to open the circuit by removing a side of the connected resistors from the circuit, and then using the two probes of multimeter to close the open circuit by connecting them to the disconnected points.

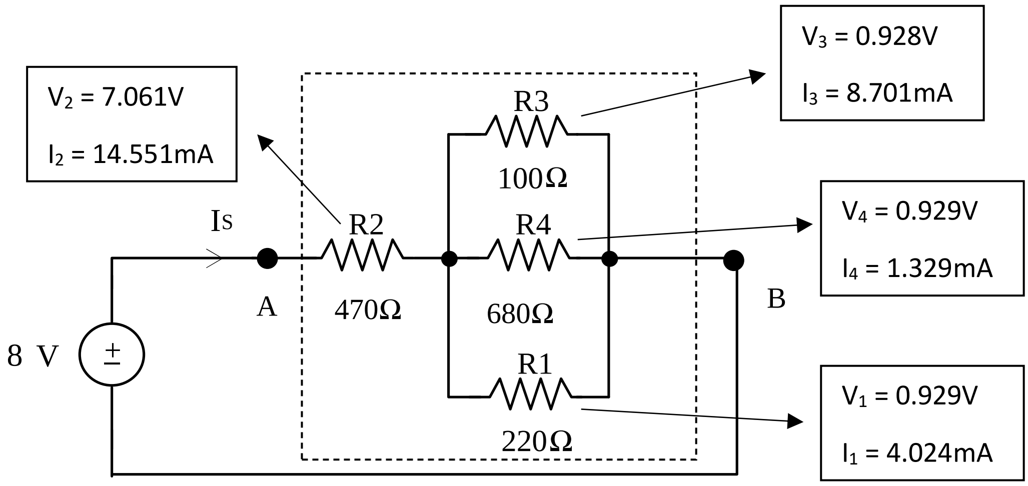


Figure 1. Series-Parallel Circuit with Voltage and Current Readings for The Resistors.

Using this methodology in measuring current and voltage readings of the resistors, the results obtained are as displayed on the above Figure 1 (to 3 d.p).

Objective 2: State Ohm's Law. Apply Ohm's law in part i) and ii)

Ohm's Law states that the current (I) flowing through an ideal conductor is directly proportional to the potential difference/voltage (V) applied across it ends.

$$v = iR$$

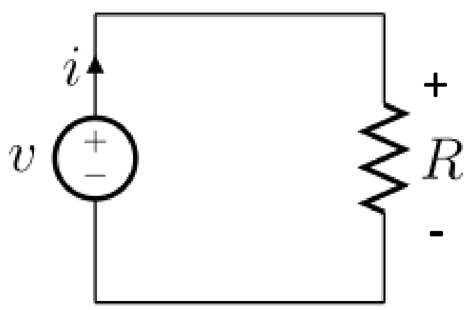


Figure 2. Ohm's law

Where:

v is voltage (V), *i* is current (A), *R* is resistance of conductor (Ω) and *R*≥0.

i. *From the measurements made in Objective 1, determine the resistance between terminals A-B of the given circuit. Verify your answer through calculation using the known circuit configuration and resistor values*

By Calculation of measured values:

Referring to figure 1 for the readings of voltages and currents for each resistor, by Ohm's law:

$$R_2 = \frac{V_2}{I_2} = \frac{7.061}{14.551m} = 0.48526\ k\Omega = 485.26\ \Omega$$
$$R_1 = \frac{V_1}{I_1} = \frac{0.929}{4.024m} = 0.23086\ k\Omega = 230.86\ \Omega$$
$$R_3 = \frac{V_3}{I_3} = \frac{0.928}{8.701m} = 0.10665\ k\Omega = 106.65\ \Omega$$
$$R_4 = \frac{V_4}{I_4} = \frac{0.929}{1.329m} = 0.69902\ k\Omega = 699.02\ \Omega$$

Given R1, R4 and R3 are connected in parallel, and this portion is connected in series with R2, by Ohm's law:

$$R_3//R_4 = \frac{106.65*699.02}{106.65+699.02} = 92.53\ \Omega$$
$$R_1//(R_3//R_{4)} = \frac{230.86*92.53}{230.86+92.53} = 66.06\ \Omega$$
$$R_{AB} = R_2 + R_1//(R_3//R_{4)} = 485.26 + 66.06 = 551.32\ \Omega$$

By Calculating the effective resistor from the given resistors value in figure 1:

Given R1, R4 and R3 are connected in parallel, and this portion is connected in series with R2, by Ohm's law:

$$R_3//R_4 = \frac{100*680}{100+680} = 87.18\Omega$$
$$R_1//(R_3//R_{4)} = \frac{220*87.18}{220+87.18} = 62.44\Omega$$
$$R_{AB} = R_2 + R_1//(R_3//R_{4)} = 470 + 62.44 = 532.44\Omega$$

***Observation**: The **value of R_{AB} obtained from measurements and from calculation of given resistor value in figure 1 is close to each other's but not exactly the same**. This is because the **resistors used in experiment** for constructing the circuit **is not ideal and has a tolerance value** (indicated by the 4^{th} band on its body). Thus, their **resistances are not exact** like displayed in figure 1, which will **carry some error into the measurement of effective resistance R_{AB}** as indicated in the effective resistance value obtained from the measurements*

ii. *If all resistors between terminals A-B are allowed to be connected in any configuration except for all 4 resistors in parallel, re-arrange the resistors so that the current I_s is maximum. What is this maximum value of I_s*

By Ohm's law:

$I_S = \frac{V}{R_{AB}}$, given that V of the independent voltage source is constant value of 8V, for maximum value of I_s, R_{AB} must be at minimum value possible.

Since 4 parallel resistors is not allowed, we must use the circuit configuration same as given in figure 1, where the 470Ω and 680Ω are in parallel and this component is parallel to the 100Ω and 220Ω.

In this scenario, the effective resistor of the 4 resistors must be minimum to yield the minimum value of R_{AB}. Thus, connect the resistors as shown in figure 3.

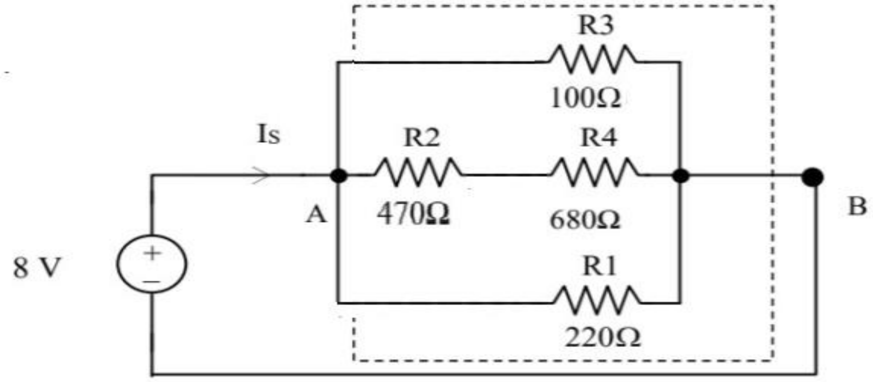


Figure 3. Circuit configuration for minimum R_{AB}, hence maximum I_s

R_{AB} = ((470+680)//100)//220 = 64.87Ω

Max. value of $I_S = \frac{V}{R_{AB}} = \frac{8}{64.87} = 0.123A = 123mA$

1.2. Thevenin's Theorem

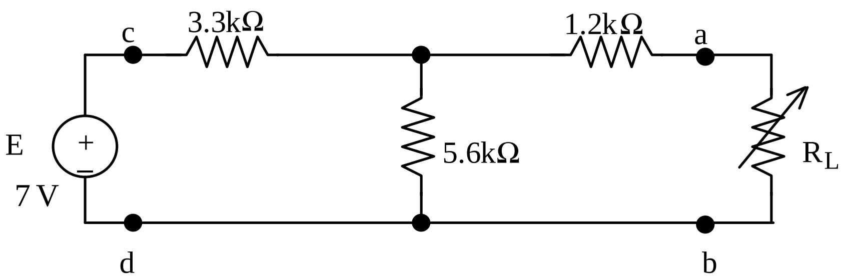
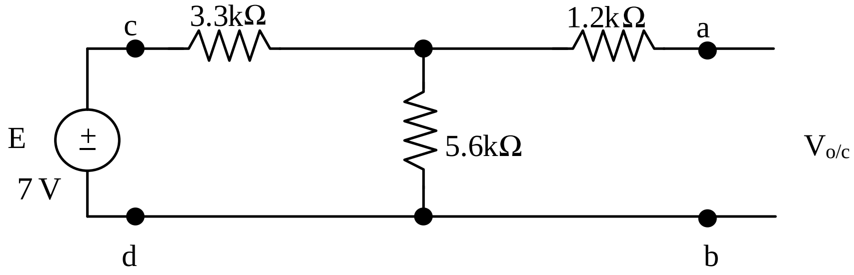


Figure 4

Given connected circuit with configuration such as in figure 4. V_E = 7.005V (from multi-meter across terminal c-d).

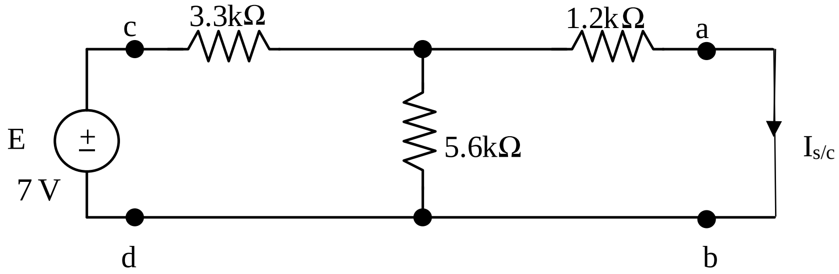
When load resistor R_L is removed, the circuit becomes:



i. *Measure the $V_{o/c}$ or Thevenin's equivalent voltage of the circuit as seen by load resistor at terminals **a-b***

$$V_{o/c} = 4.434V = V_{TH}$$

ii. *Short circuit the terminals **a-b** and measure current flowing from terminal **a** to terminal **b**. This $I_{s/c}$ is Norton's equivalent current.*

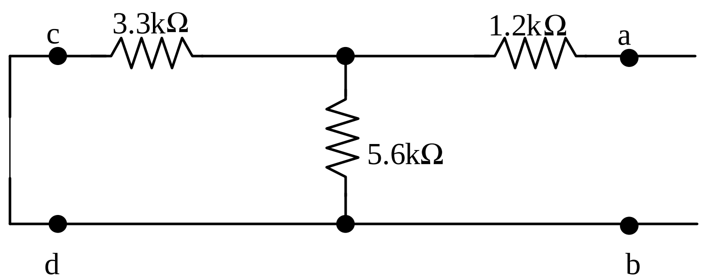


$$I_{s/c} = 1.356mA = I_N$$

iii. *Calculate Thevenin's equivalent resistance*

$$R_{TH} = \frac{V_{o/c}}{I_{s/c}} = \frac{4.434}{1.356m} \approx 3.27k\Omega$$

iv. *Removing the short circuit across **a-b** terminals and remove voltage source E, replace by a short circuit across **c-d** terminal, measure the resistance across **a-b**. Compare measured Thevenin's resistance with the calculated value.*



$$R_{AB} = 3.272k \approx 3.27k\Omega$$

=>R_{AB} is the same value as R_{TH}. This means the measured value and calculated value of Thevenin's resistance is the same to each other (very low error).

v. *Connecting different load resistor R_L at terminals a-b. Measure voltage across R_L and calculate current through them (refer to figure 4). Result in table 1*

R_L /Ω	V_L (across R_L)/V	I_L (through R_L)/mA
1k	1.031	1.042
4.7k	2.634	0.553
10k	3.335	0.338

Table 1

vi. *With the circuit in figure 5. Repeat the measurements as before with same R_L value. Compare and comment the results of v and vi. Are they expected?*

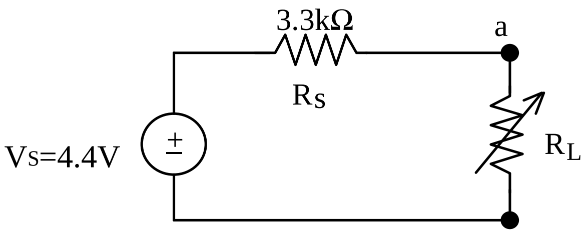


Figure 5. Thevenin's equivalent circuit

R_L /Ω	V_L (across R_L)/V	I_L (through R_L)/mA
1k	1.024	1.035
4.7k	2.616	0.549
10k	3.311	0.336

Table 2

The result of table 1 and table 2 are very close to each other in value such that we can assume them to be equivalent to each other. This mean the value of V_L and I_L obtained in both tables are equal.

This is expected in accordance with Thevenin's theorem. Based on this theorem, we can replace a portion of a complex circuit with a simple circuit of independent voltage source connected in series with a resistor. In this case we had replace everything in figure 4. (except the load resistor) by a V_{TH} source in series with R_{TH} shown in figure 5. This observation proved that Thevenin's theorem holds in experiment, consistent with the theory.

2. Experiment 2 (from Lab 2) – First Order Transient Circuits

2.1. Behaviour of a Capacitor (in a RC Circuit)

The Resistor – Capacitor circuit in figure 6 is given for investigation whether the voltage across a discharging capacitor shows exponential behaviour.

Here in the set-up, a square wave generator is used to simulate the switch for the circuit.

When signal from the generator is HIGH, the capacitor is charged up as there is current flowing through it, and when the signal is LOW, capacitor will be discharged.

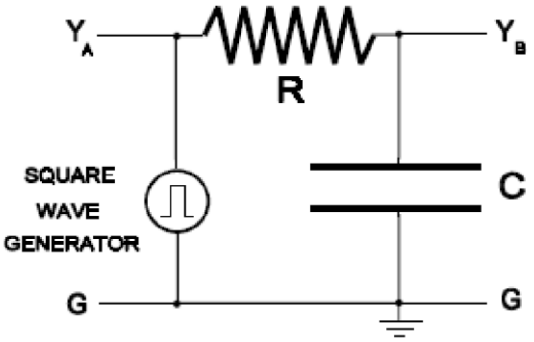


Figure 6 RC Circuit

To conduct the experiment, we need to leave the switch close/open for a long enough period for the capacitor to be fully charged or discharged. Else, an incomplete exponential curve will be observed, which hamper the objective of the experiment.

2.1.1. Measurement and calculation for time constant τ

To calculate τ, use step by step method where τ = R_{TH}*C, where R_{TH} is the Thevenin's resistors seen by the capacitor. In the circuit of figure 6, R_{TH} = R so just multiplication of Capacitance C and R will yield the value of τ:

For R=100 kΩ, C=100nF: $\tau = 100k * 100n = 10ms$

For R=100 kΩ, C=1µF: $\tau = 100k * 100n = 100ms$

For R= R=10 kΩ, C=1 µF: $\tau = 100k * 100n = 10ms$

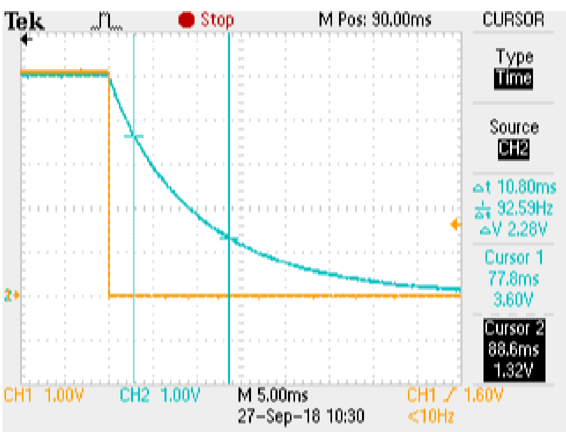


Figure 7

To obtain τ, measure from the oscilloscope screen (part of the discharging graph of capacitor) a random value V_1 at time t_1 and V_2 at time t_2, such that V_2 is 1/e of V_1.

For R=100 kΩ, C=100nF:

From oscilloscope (figure 7), taking time value t_1 at V_1 = 3.60V ,and t_2 at V_2 = V_1*1/e =1.324 ≈ 1.32V (where V_1 value is decreased by a factor of 1/e based on exponential response graph).

τ = Δt = $t_2 - t_1$ = 88.6 ms – 77.8 ms = 10.8 ms

For R=100 kΩ, C=1μF:

From oscilloscope (figure 8), taking time value t_1 at V_1 = 3.76V ,and t_2 at V_2 = V_1*1/e = 1.383 ≈ 1.36V (where V_1 value is decreased by a factor of 1/e based on exponential response graph).

$\tau = \Delta t = t_2 - t_1$ = 1.14s – 1.03s = 112.0 ms

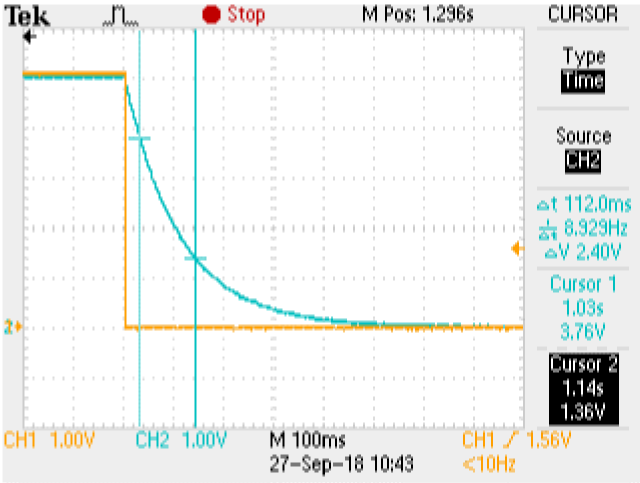


Figure 8

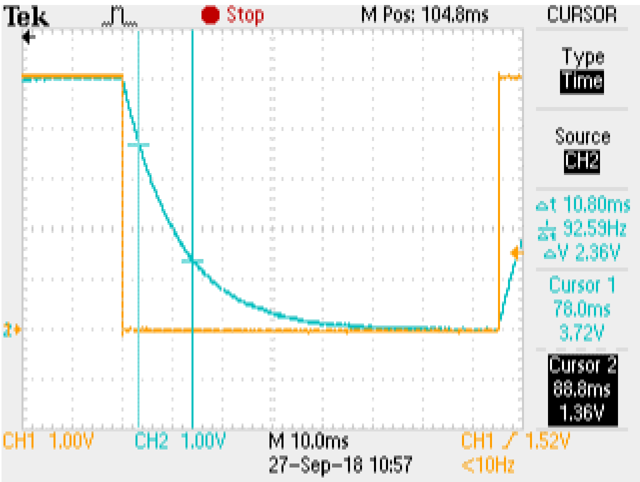


Figure 9

For R= R=10 kΩ, C=1 μF:

From oscilloscope (figure 9), taking time value t_1 at V_1 = 3.72V ,and t_2 at V_2 = V_1*1/e = 1.369 ≈ 1.36V (where V_1 value is decreased by a factor of 1/e based on exponential response graph).

$\tau = \Delta t = t_2 - t_1$ = 88.8 ms – 78.0 ms = 10.8 ms

The time constant obtained from both methods is shown in the table 3 below:

	R=100 kΩ, C=100nF	R=100 kΩ, C=1μF	R=10 kΩ, C=1 μF
Calculated τ	10.00 ms	100.00 ms	10.00 ms
Measured τ	10.80 ms	112.00 ms	10.80 ms
Sec/Div Used (for Oscilloscope)	5.00 ms	100 ms	10.0 ms

Table 3

Note: V_{max} , the value of the capacitors when it is fully charged is not used as V_1 as it is hard to trace the exact point when the capacitor starts discharging from the oscilloscope. Hence, a value at some moment within the discharging cycle of the capacitors is preferred as it is easier to pin point and reduce error in measurement from the experiment conductor.

Observation:

- τ value obtained from measurement is close and consistent to the calculated value from Step-by-Step method. However, measured value cannot be exact due to tolerance of the circuit. Also, as the oscilloscope has limitations on displaying scale used to measure voltage and time value, there will be certain error in measurement.

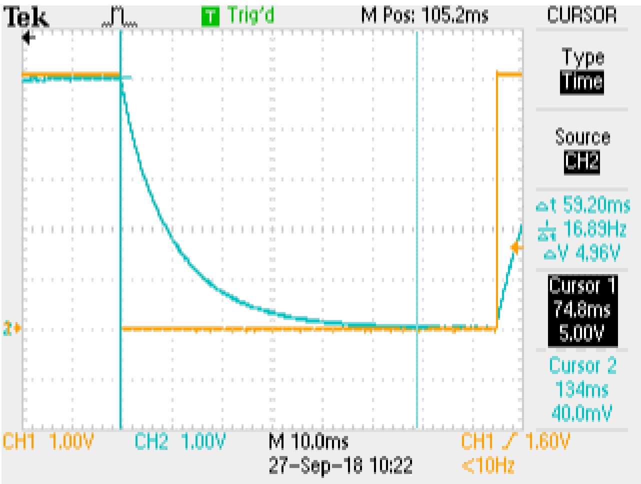


Figure 10 R=100 kΩ, C=100nF

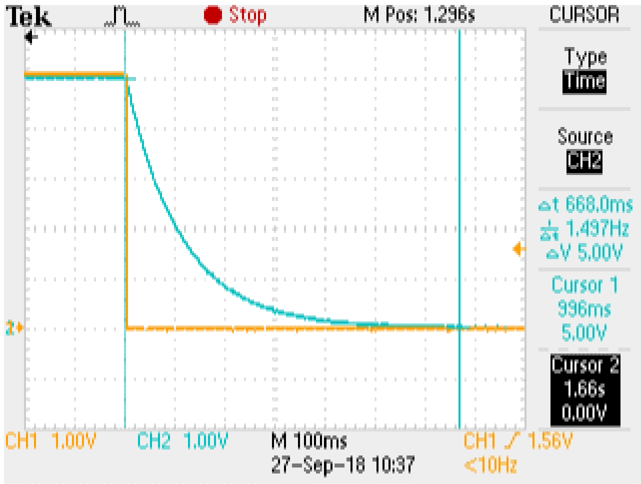


Figure 11 R=100 kΩ, C=1μF

- From measurement, we can conclude that time constant τ for discharging capacitors is directly proportional to the effective resistor in series with the capacitor and with its capacitance. This is shown on the above table where increasing the capacitance by 1000 times translates to 10 times increase in τ (case 1 and 2). Also, increasing resistor by 10 times yields 10 times increase in τ (case 3 and 2).

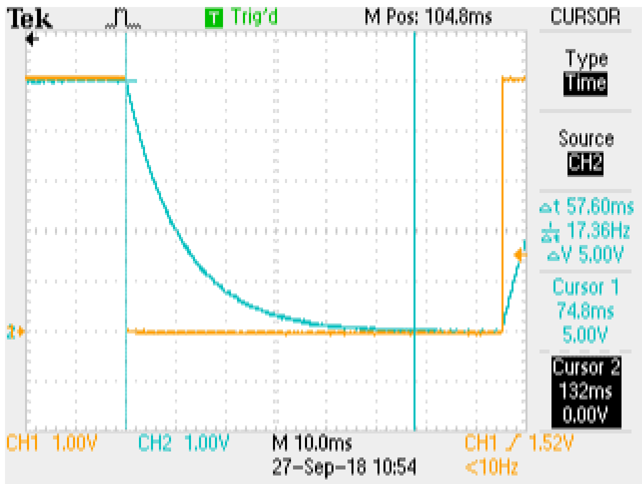


Figure 10 R=10 kΩ, C=1 μF

- The images from oscilloscope suggests the capacitors charges up at exponential rate when square wave is switched from low to high (from rising edge onwards), until a steady maximum value is reached. When the square wave function is switch to low (from falling edge onwards), the capacitor discharges from this max voltage and this voltage decreases at exponential rate, shown by the τ calculated, until minimum value is reached. Figure 10, 11 and 12 shown the max, min value (at steady state of the capacitors).

- Time take for the capacitors to be fully discharged (duration for voltage to drop from max, steady value to min, steady value) is more than 5τ. This corresponds to the exponential response time graph where the voltage value will be reduce by 99.9% after 5τ duration from its initial max value.

Given above that voltage of capacitor discharge like in an exponential response way

$$V_C(t) = V_C(0)e^{\frac{-t}{\tau}}$$

For the 3 cases, the $V_C(0)$ = 5V when the square function is changing from high value to low value, thus the capacitor's voltage can be expressed as:

$$V_C(t) = 5e^{\frac{-t}{\tau}}$$

Based on time constant value obtained above, $V_C(t)$ for:

- R=100 kΩ, C=100nF : $V_C(t) = 5e^{\frac{-t}{0.0108}}$
- R=100 kΩ, C=1μF: $V_C(t) = 5e^{\frac{-t}{0.112}}$
- R=10 kΩ, C=1 μF: $V_C(t) = 5e^{\frac{-t}{0.0108}}$

2.1.2. Square Wave function vs Sinusoidal Wave function

Figure 13, 14, and 15 show what happen when we switch the signal generator from generating square wave to sinusoidal wave as source voltage for the circuit of 3 set of data.

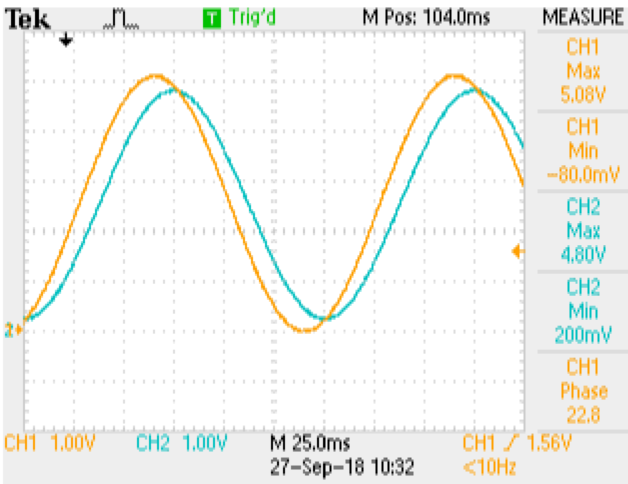


Figure 13. Sinusoidal Behaviour for V_C of R=100 kΩ, C=100nF

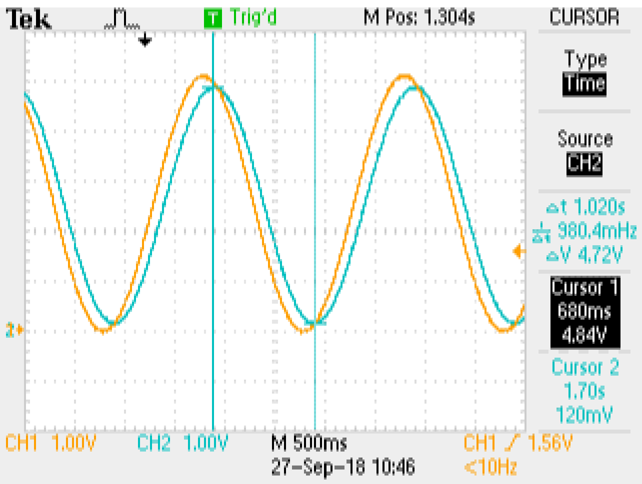


Figure 14. Sinusoidal Behaviour for V_C of R=100 kΩ, C=1μF

From the graphs, we deduce that the voltage across the resistors behave sinusoidally. This means the charging and discharging process is alternated continuously and the capacitors are never at their steady state (always either charged or discharged).

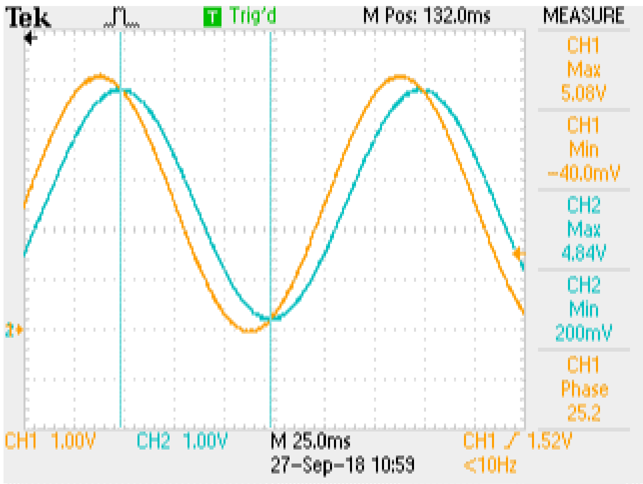


Figure 15. Sinusoidal Behaviour for VC of R=10 kΩ, C=1 μF

This behaviour can be explained by the equation characterised the voltage of the capacitor in a discharging phase:

$$V_C(t) = V_C(0) * e^{-\frac{t}{\tau}}$$

We take note that $V_C(0)$ refer to the initial voltage value of the capacitor and the fact that it is dependent on the voltage supplied by the signal generator.

In part 2.1.1, where the signal generator supplies a square wave function with discrete rising and falling edge and constant value of voltage in between, the $V_C(0)$ (voltage at the falling edge where capacitors will start discharging) is already at steady state (after charging cycle) and hence the exponential response of $V_C(t)$ can be clear

If the voltage supply by signal generator is varying sinusoidally, $V_C(0)$ will vary sinusoidally too. Thus $V_C(0)$ will never be at steady value. Hence, the $V_C(t)$ will also behave sinusoidally as described. In order word, capacitors are charging and discharging based on the sinusoidal cycle of the sine wave supplier.

2.2. A First Order Transient Ciruit

Given a 1st order transient circuit as follow in figure 16:

2.2.1. Measurement and calculation for time constant τ

Using the method described in part 2.1.1 to calculate and measure the time constant τ. In this experiment, we have two case to analyse: capacitor charging phase and capacitor discharging phase (figure 17 and 18)

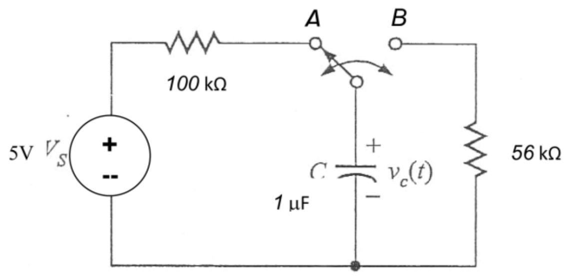


Figure 16. 1st Order Transient Circuit

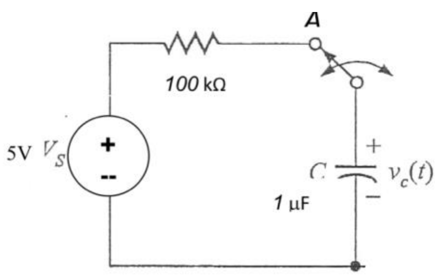


Figure 17. Capacitor in charging phase

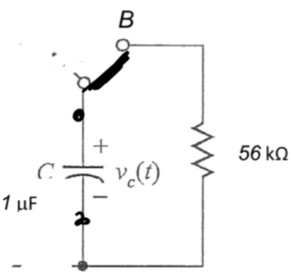


Figure 18. Capacitor in discharging phase

By Step-by-Step method:

$\tau_{charging\ phase}$= R*C = 100k * 1μ = 100ms

$\tau_{discharging\ phase}$= R*C = 56k * 1μ = 56ms

By measurement from oscilloscope:

For charging phase:

τ is defined as time taken for V_1 to increased by 1-1/e times.

In other word: $V_2 = V_1 + (V_{max} – V_1)*(1-1/e)$

Take V_1 = 0V

Then V_2 = 0 + 5*(1-1/e) = 3.161V ≈ 3.20V (for easier measurement)

$\tau = \Delta t = t_2 – t_1$= -930ms – (-1.05s) = 120.0ms

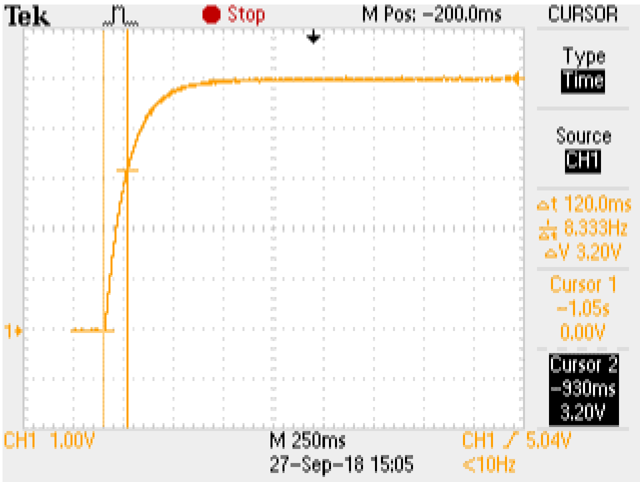


Figure 19. Oscilloscope reading for charging phase

For discharging phase:

τ is defined as time taken for V_1 to decreased by 1/e times of it value.

In other word: $V_2 = V_1 * 1/e$

Take V1 = 5.00V

The V2 = 5.00*1/e =1.839 V ≈1.84V

$\tau = \Delta t = t_2 – t_1$= -384ms – (-446ms) = 62.00ms

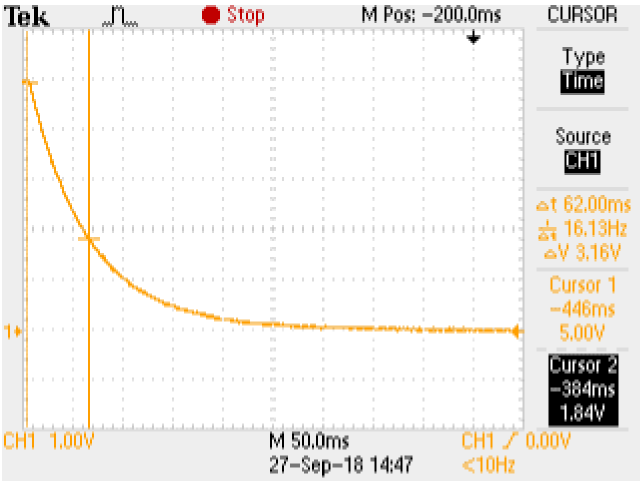


Figure 20. Oscilloscope reading for discharging phase

Observation:

- In both cases of charging and discharging for the capacitors, the waveform of voltage across them follows the form of exponential response.
- Calculated values of τ and measured values of τ are very closed to each other. However, as the circuit has some tolerance (elements of circuit are not of ideal condition assumed), the measured value is deviated from the calculated value with small range.

2.2.2. Plot V_C as a function of time t for both phases

Using the same method to measure τ for both charging and discharging cases, obtain the $V_C(t)$ value at different t = kτ (k = 1,2,3,4,5,…)

For charging phase, $V_2 = V_1 + (V_{max} - V_1)*(1-1/e)$ where V1 and V2 are $V_C(t)$ at two adjacent step of τ.

No. of τ	0	1	2	3	4	5
$V_C(t)$ / V	0	3.20	4.32	4.76	4.92	4.96
t / ms	-1050	-930	-820	-710	-600	-500

Table 4. $V_C(t)$ measurement in charging phase

Value of $V_C(t)$ for 0 and 1 τ (charging phase) is shown on figure 19. Value of $V_C(t)$ for 2, 3, 4, 5 τ are shown in figure 21 and 22. Value of $V_C(t)$ is rounded to the nearest sub-division of the oscilloscope for simplification of measurement.

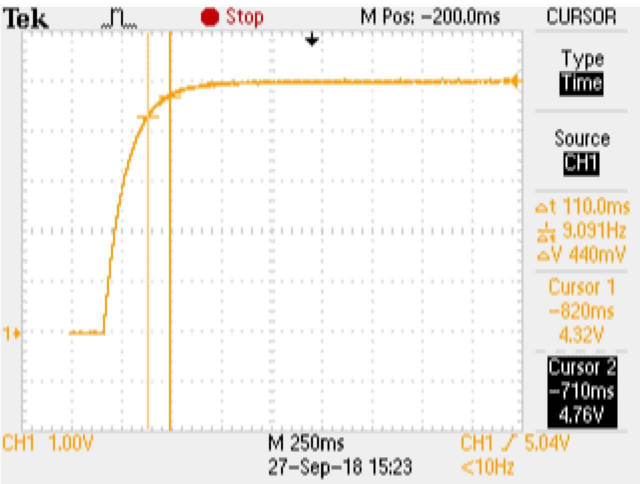


Figure 21. $V_C(t)$ at 2 and 3τ

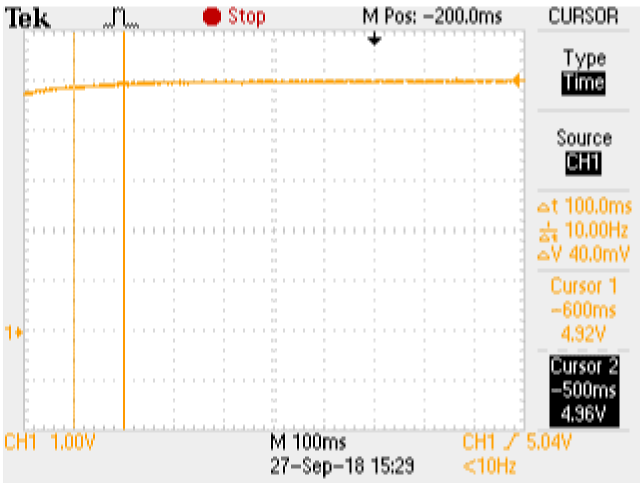


Figure 22. $V_C(t)$ at 4 and 5τ

For discharging phase, $V_2 = 1/e*V_1$ where V1 and V2 are $V_C(t)$ at two adjacent steps of τ.

No. of τ	0	1	2	3	4	5
$V_C(t)$ / V	5.00	1.84	0.680	0.240	0.080	0.040
t / ms	-446	-384	-320	-254	-172	-106

Table 5 $V_C(t)$ measurement in discharging phase

Value of $V_C(t)$ for 0 and 1 τ (charging phase) is shown on figure 20. Value of $V_C(t)$ for 2, 3, 4, 5 τ are shown in figure 23, 24 and 25. Value of $V_C(t)$ is rounded to the nearest sub-division of the oscilloscope for simplification of measurement.

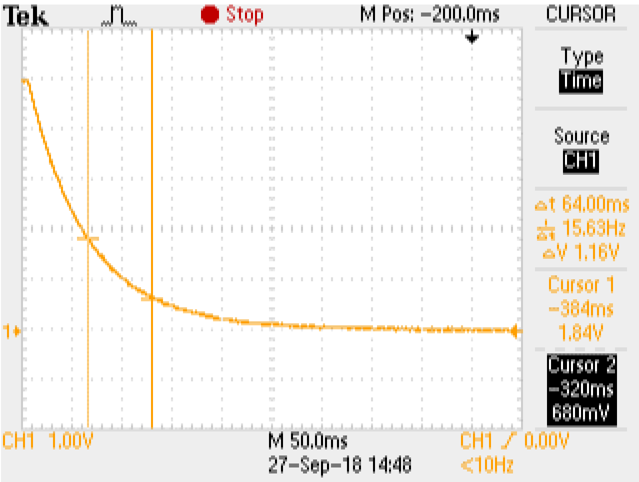


Figure 23. $V_C(t)$ at 1 and 2τ

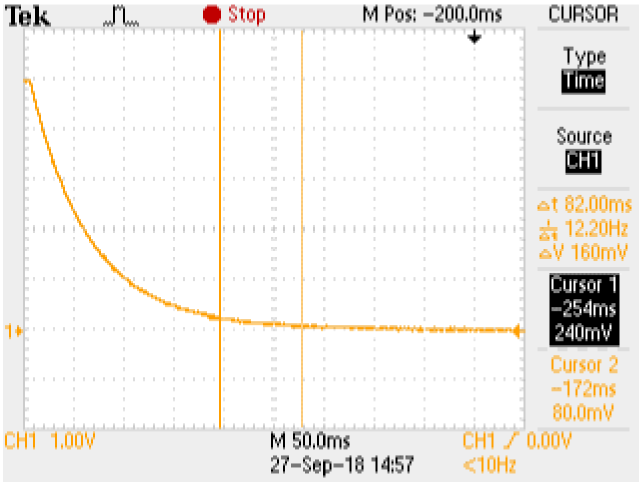


Figure 24. VC(t) at 3 and 4τ

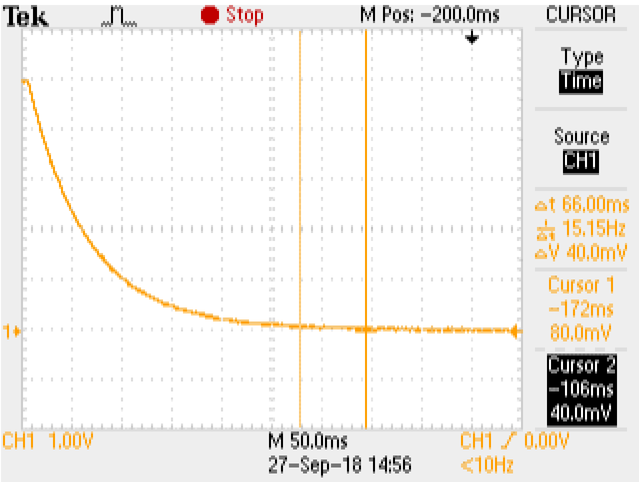


Figure 25. $V_C(t)$ at 4 and 5τ

From table 4 and table 5, and assuming the capacitors reach stable state after 5 steps of time constant τ (by observation from oscilloscope, thus extrapolate the data from there), we can plot a graph of $V_C(t)$ below in figure 27. τ for charging phase is taken from the quick value we obtained and is 120 ms and 62 ms respectively for charging and discharging phase. Also, in this plot, the switch is switched at t = 8τ of charging phase (t=960ms)

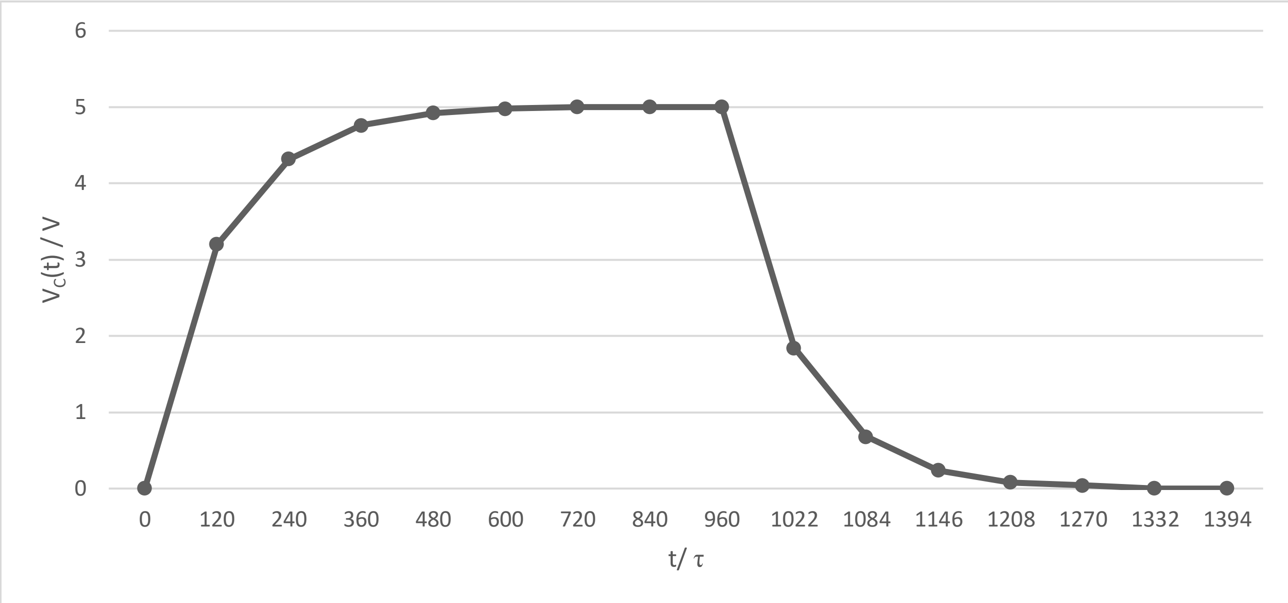


Figure 27. Plot of $V_C(t)$ as a function of t (both phases)