

Data Visualization

Lecture 5 Mathematics Visualization

Phuc Loi Luu, PhD
p.luu@garvan.org.au
luu.p.loi@gmail.com

Content

Plot a mathematics graph

- Calculus and Gradient Descent
- Linear Algebra
- Statistics

Defining a Function in R (1)

- Example 1: define a function `fahrenheit_to_celsius` that converts temperatures from Fahrenheit to Celsius:

```
fahrenheit_to_celsius <- function(temp_F) {  
  temp_C <- (temp_F - 32) * 5 / 9  
  return(temp_C)  
}  
fahrenheit_to_celsius(32)  
fahrenheit_to_celsius(212)
```

```
> fahrenheit_to_celsius <- function(temp_F) {  
  temp_C <- (temp_F - 32) * 5 / 9  
  return(temp_C)  
}  
> fahrenheit_to_celsius(32)  
[1] 0  
> fahrenheit_to_celsius(212)  
[1] 100
```

Defining a Function in R (2)

- Example 2: define a function SumNum that add all the digits of a number, i.e $\text{SumNum}(3)=1+2+3=6$:

```
SumNum(num) {
```

```
}
```

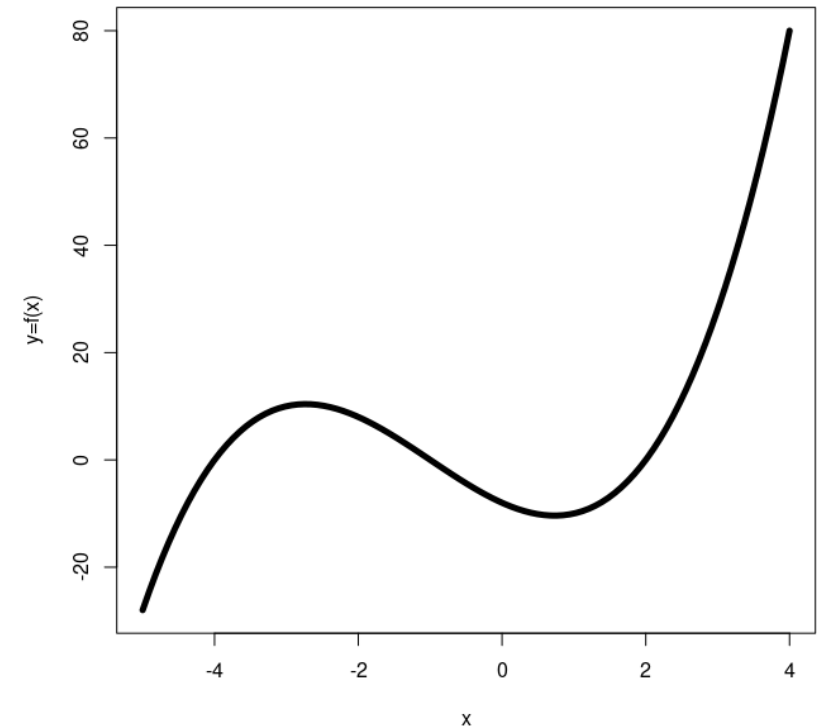
```
SumNum(10)
```

Calculus (1)

- Plot the polynomial function

$$y = f(x) = x^3 + 3x^2 - 6x - 8$$

```
# Define a function  
f <- function(x) (x^3 + 3 * x^2 - 6 * x - 8)  
# plot the curve  
curve(f, from=-5, to=4, ylab = "y=f(x)", lwd=5)
```



Calculus (2)

- Plot the polynomial function

$$y = f(x) = x^3 + 3x^2 - 6x - 8$$

Define a function

```
f <- function(x) (x^3 + 3 * x^2 - 6 * x - 8)
```

plot the curve

```
curve(f, from=-50, to=40, ylab = "y=f(x)", lwd=5)
```

Calculus (3)

Differentiate a function

$$y = f(x) = x^3 + 3x^2 - 6x - 8$$

The first derivative of y

$$y' = 3x^2 + 6x - 6$$

```
# Differentiate a function
# define function g which is the derivative of f(x)
g <- function(x) {}
body(g) <- D(body(f), 'x')
# The derivative of f(x) is: body(g)
```

```
> g
function (x)
3 * x^2 + 3 * (2 * x) - 6
> body(g)
3 * x^2 + 3 * (2 * x) - 6
```

Calculus (4)

Differentiate a function

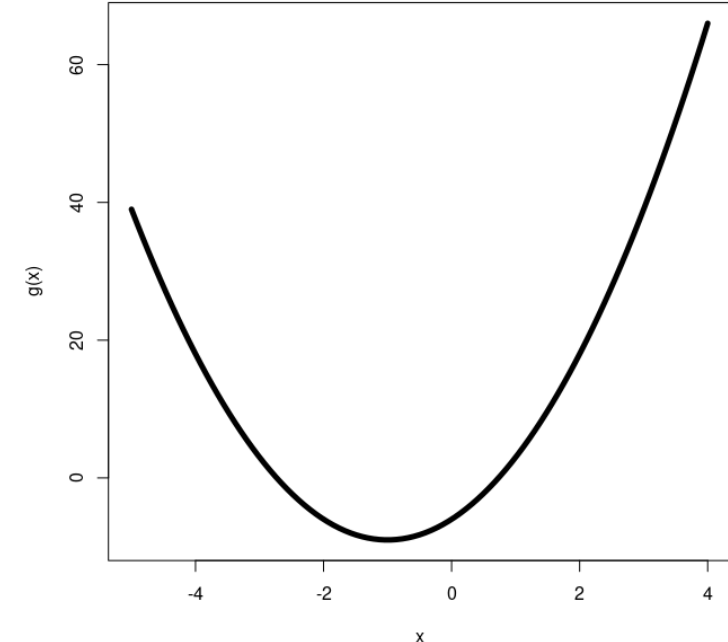
$$y = f(x) = x^3 + 3x^2 - 6x - 8$$

The first derivative of y

$$y' = 3x^2 + 6x - 6$$

Plot y'

```
# Differentiate a function  
# define function g which is the derivative of f(x)  
g <- function(x) {}  
body(g) <- D(body(f), 'x')  
# Plot the derivative.  
curve(g, -5, 4, ylab = "g(x)", lwd=5)
```



Calculus (5)

Differentiate a function

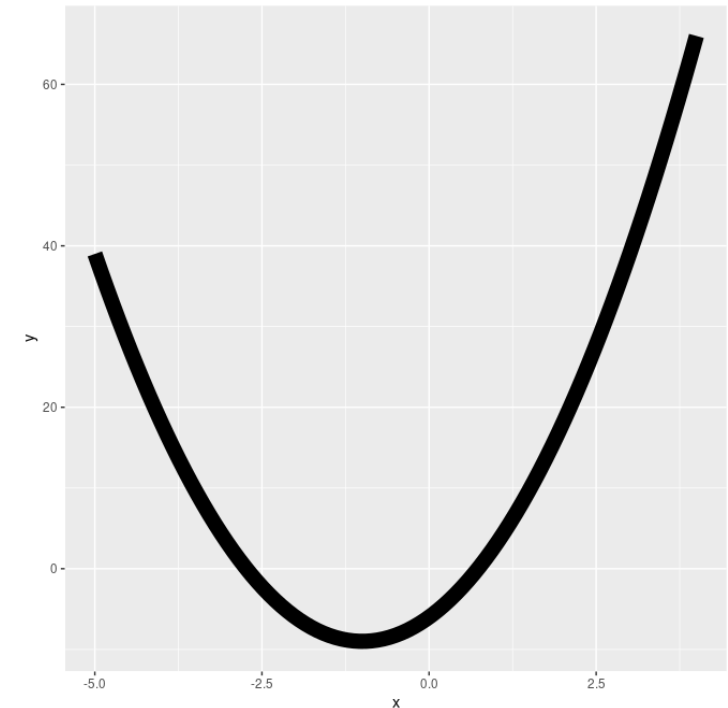
$$y = f(x) = x^3 + 3x^2 - 6x - 8$$

The first derivative of y

$$y' = 3x^2 + 6x - 6$$

Plot y' in ggplot

```
library(ggplot2)
# Plot the derivative.
ggplot(data.frame(x = c(-5, 4)), aes(x = x)) +
  stat_function(fun = g, lwd=5)
```



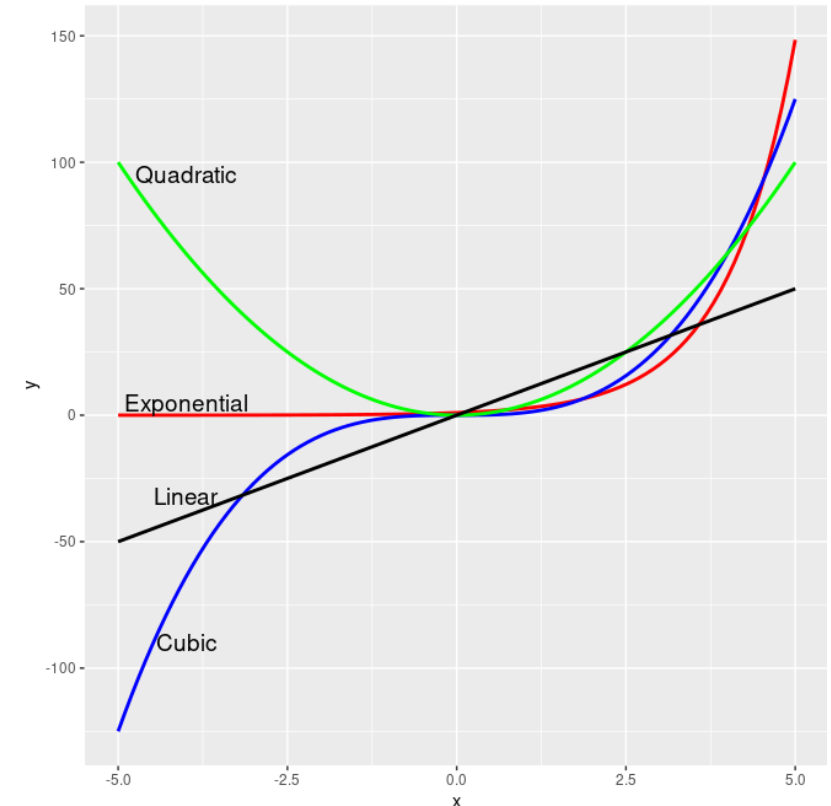
Calculus (6)

Differentiate a function

$$y = e^x$$
$$y = x^3$$
$$y = 4x^2$$
$$y = 10x$$

```
# Draw multiple functions in one plot
four_curves <- ggplot(data.frame(x = c(-5, 5)), aes(x = x)) +
  stat_function(fun = exp, color = "red", lwd = 1) +
  stat_function(fun = function(x){x^3}, color = "blue", lwd = 1) +
  stat_function(fun = function(x){4*x^2}, color = "green", lwd = 1) +
  stat_function(fun = function(x){10*x}, color = "black", lwd = 1) +
  annotate(geom = "text", label = "Exponential", x = -4, y = 5, size = 5) +
  annotate(geom = "text", label = "Quadratic", x = -4, y = 95, size = 5) +
  annotate(geom = "text", label = "Cubic", x = -4, y = -90, size = 5) +
  annotate(geom = "text", label = "Linear", x = -4, y = -32, size = 5)
```

`four_curves`



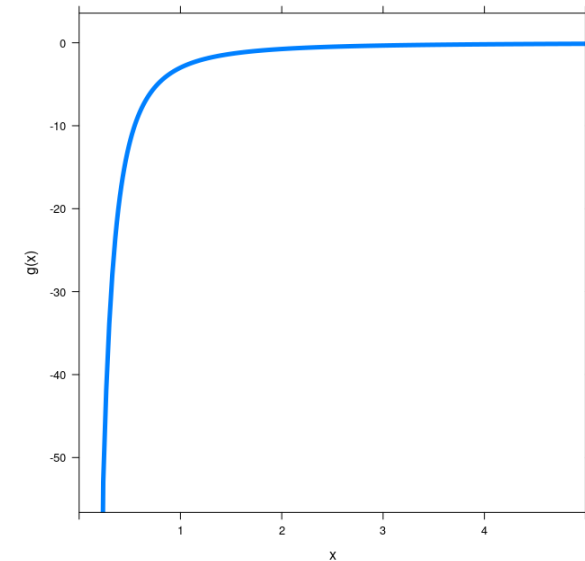
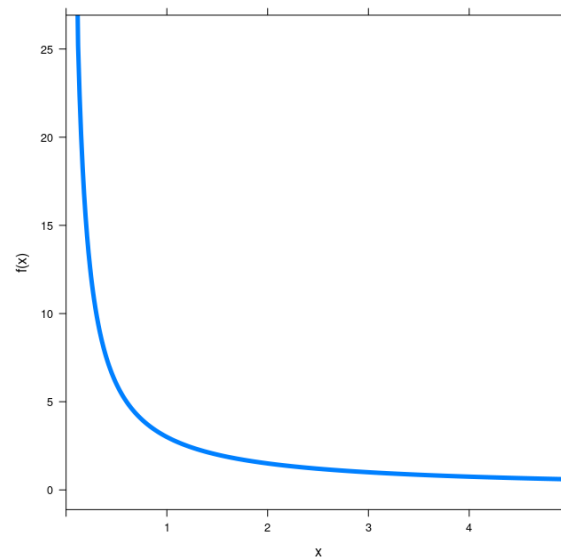
Calculus plot with mosaicCalc

Plot the function y and its first derivative

$$y = \frac{3}{x}$$

```
library(mosaic)
library(mosaicCalc)

# plot derivative function of  $y = 3/x$ 
# define function
f <- makeFun(a / x ~ x, a = 3)
# plot f
plotFun(f, xlim=c(0,5), lwd=5)
# cal derivative of f
g <- D(a / x ~ x, a = 3)
# plot g
plotFun(g, xlim=c(0,5), lwd=5)
```



Calculus: parabola, tangent and secant

Plot the parabola $p: y = x^2$

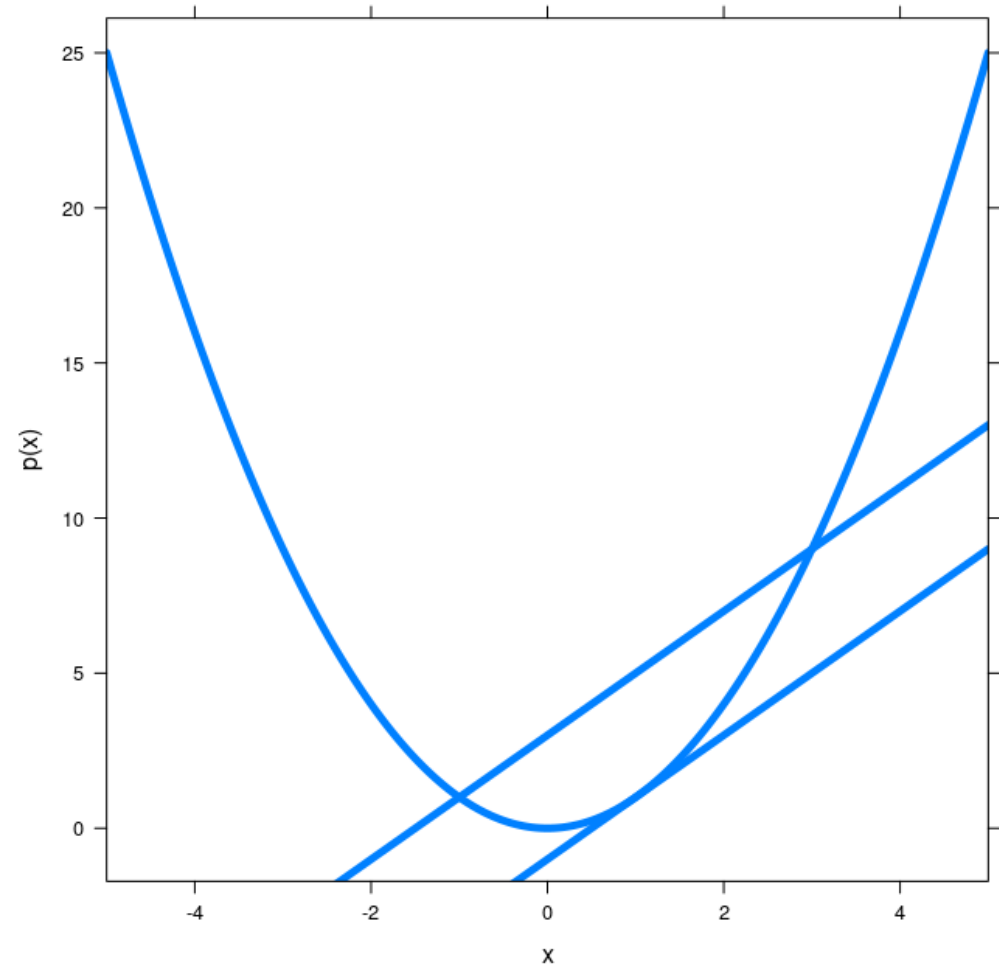
Plot the line $d1: y = 2x - 1$

Plot the line $d2: y = 2x + 3$

```
library(mosaic)
library(mosaicCalc)

# define function
p <- makeFun(x^2 + a ~ x, a = 0)
d1 <- makeFun(2*x + a ~ x, a = -1)
d2 <- makeFun(2*x + a ~ x, a = 3)

# plot p and d
plotFun(p, xlim=c(-5,5), lwd=5)
plotFun(d1, xlim=c(-5,5), lwd=5, add=T)
plotFun(d2, xlim=c(-5,5), lwd=5, add=T)
```



Calculus: min and max

Plot the parabola $p: y = x^2 - 2x + 2$

Calculate the first derivative of y

Plot y' in the same plot with p

```
library(mosaic)
library(mosaicCalc)

# define function
p <- makeFun(x^2 + a ~ x, a = 0)
d1 <- makeFun(2*x + a ~ x, a = -1)

# plot p and d
plotFun(p, xlim=c(-5,5), lwd=5)
plotFun(d1, xlim=c(-5,5), lwd=5, add=T)
```

Calculus: Gradient Descent

$$\hat{f}(x) = \min_x x^2 - 2x + 2$$

- 1) Take the derivative of the function
- 2) Study the derivative at the point we guessed (x=3)
$$x_{i+1} = x_i - \alpha f'(x_i)$$
- 3) Update n times (n=10) step 2)