Lec14 竞争性分析、自组织表

self-organizing lists

list L of n elements

- operation Access(x) costs rank(x) = distance of x from the head of the list
- L can be reordered by transposing adjacent elements, the cost is 1

Example:

$L \to 12 \to 3 \to 50 \to 14 \to 17 \to 4$

Access(14). cost = 4

Transpose(3, 50). cost = 1

Def

a sequence S of operations is provided one at a time,
for each operation, an online algorithm must execute the operation immediately.
offline algorithm may see all of S in advance

Goal:

to minimize the total cost $C_A(S)$

worst-case analysis

adversary always accesses tail element of L. $C_A(S) = \Omega(|S| \cdot n)$ if online

average-case analysis

suppose element x is accessed with probability $p(x)$,

$$E[C_A(S)] = \sum_{x \in L} p(x) \cdot \text{rank}(x),$$

which is minimized when L is sorted in decreasing order with respect to p

Heuristic

keep count of the number of times each element is accessed,
and maintain the list in order of decreasing count

Practice

"move-to-front" heuristic
after accessing x, move x to head of list using transposes, cost $= 2 \times$ rank(x
respond well to locality in S

Competitive Analysis

Def: an on-line algorithm A is α-competitive if there exists a constant k, such that for any sequence S of operations, the cost of S using algorithm A

$$C_A(S) \le \alpha \cdot \underbrace{C_{opt}}_{\text{"optimal off-line algorithm"}}(S) + k$$

Theorem: Move-To-Front is 4-competitive for self-organizing lists

proof:

let L_i be MTF's list after the i-th access

let L_i^* be OPT's list after the i-th access

let c_i be MTF's cost for the i-th operation, equal $2 \times rank_{L_{i-1}}(x)$

let c_i^* be OPT's cost for the i-th operation, equal $rank_{L_{i-1}^*}(x) + \underbrace{t_i}_{\text{t次置换}}$

define the potential function $\phi : \{L_i\} \to R$ by:

$$\phi(L_i) = 2 \cdot \left|\{(x, y) : x <_{L_i} y \text{ and } y <_{L_i^*} x\}\right|$$

"MTF表与OPT表的不同点"

$$= 2 \cdot \#\text{ inversions}$$

note: $\phi(L_i) \ge 0, \forall i$

$\phi(L_0) = 0$ if MTF and OPT start with same list

how much does ϕ change from one transpose?

a transpose creates or destroys one inversion

so $\Delta\phi = \pm 2$

以下证明略(看不懂)