Home work:

Problem 3.1.

We have Y is a ~~decrease~~ discrete uniform random variable.

a) PMF of Y:

$$P_Y(y) = \begin{cases} \frac{1}{11}, & x = 5, 6, \ldots, 15 \\ 0, & \text{otherwise} \end{cases}$$

b) $P(Y < 10) = \sum_{i=5}^{9} P_Y(i) = \frac{5}{11}$

c) $P(Y > 12) = P_Y(13) + P_Y(14) + P_Y(15) = \frac{3}{11}$

d) $P(8 \leq Y \leq 12) = P_Y(8) + P_Y(9) + P_Y(10) + P_Y(11) + P_Y(12)$

$= \frac{5}{11}$

Problem 3.2

~~Assume~~

Let ~~K~~ be the number of times that a system transmits

the number of times the pager receives the same message

a) K is a number of times the pager

$$P_K(k) = \begin{cases} C_n^k \cdot p^k (1-p)^{n-k}, & k = 1, 2, \ldots, n \\ 0, & \text{otherwise} \end{cases}$$

b) We have:

$C_n^0 \cdot p^0 \cdot (1-p)^n = 1 - 0{,}95$

$\Leftrightarrow (1-0{,}8)^n = 0{,}05$

$\Leftrightarrow n = 2$

Problem 3.3.

the same message

a) N is the number of times it has to send

$$P_N(n) = \begin{cases} p(1-p)^{n-1}, & n = 1, 2, \ldots \\ 0, & \text{otherwise} \end{cases}$$

b)

Problem 3.4.

a) Two of microchips were defective

~~We have:~~ Let $\hat{X}$ is the number of defective chips found among the two chips inspected.

$X \sim B(2; 0,5)$

The PMF of X: $P_X(x) = \begin{cases} C_2^x \cdot 0,5^x \cdot 0,5^{2-x} & , x = 0,1,2 \\ 0 & \text{otherwise} \end{cases}$

b) similarly

$P_X(x) = \begin{cases} C_1^x \; 0,5^x \cdot 0,5^{1-x} & , x = 0,1 \\ 0 & \text{otherwise} \end{cases}$

c) $P_X(x) = \begin{cases} 0 & , x = 1,2 \\ 1 & , x = 0 \end{cases}$

Problem 3.5.

a) Let E is the event that the plane will fly safely

K is the number of engines work and $K \sim (4, 1-q)$

$P(E) = \cancel{P_K(k)} \; P(K \geq 2) = 1 - P_K(0) - P_K(1)$

$= 1 - C_4^0 \cdot q^4 - C_4^1 \cdot q^3(1-q)^1$

$= 1 - q^4 - 4q^3(1-q)^1$

$= 1 + 3q^4 - 4q^3$

b) Let D is the event that the plane will fly safely

H is the number of engines work and $H \sim (2, 1-q)$

(H is a binomial random variable)

$P(D) = P(H \geq 1) = 1 - P(0) = 1 - C_2^0 \cdot q^2 = 1 - q^2$

c) consider: $P(E) \geq P(D)$

$\Leftrightarrow 1 + 3q^4 - 4q^3 \geq 1 - q^2$

$\Leftrightarrow 3q^2 - 4q + 1 \geq 0$

$\Leftrightarrow \left[\begin{array}{l} q < \frac{1}{3} \\ q > 1 \text{ (eliminate)} \end{array} \right.$

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So, with $q < \frac{1}{3}$, 4-engine plane is safest

$q > \frac{1}{3}$, 2-engine plane is safest.

Problem 3.6. P ~~turn~~ turn right = P turn left = 0,5

a) Let X is the number of rats turn right

X is binomial random variable and $X \sim \beta(10; 0,5)$

PMF of X:

$$P_X(x) = \begin{cases} C_{10}^{x} \cdot 0,5^{10} & , x = 0,1,\dots,10 \\ 0 & , \text{otherwise} \end{cases}$$

b) probability at least 9 will turn right = turn left

=> " " the same way $= 2(P_X(9) + P_X(10))$

$= 2\left(C_{10}^{9} \cdot 0,5^{10} + C_{10}^{10} \cdot 0,5^{10}\right)$

$= 2 \cdot \frac{11}{1024} = \frac{11}{512}$

Problem 3.7

a) - If a student chooses topic A:

Let X is the number of books arrive on time

E is the ~~...~~ event that the student feels a good paper

We have: X is ~~the~~ discrete random variable has binomial distribution, $X \sim (2, 0,9)$

$P(E) = P(X \geq 1) = 1 - P(0) = 1 - 0,1^2 = 0,99$

- If student chooses topic B

~~Let Y is the number of books arrive on time~~

D is the event that the student feels a good paper

We have: Y is ~~...~~ binomial random variable

$Y \sim (4; 0,9)$

$P(D) = P(Y \geq 2) = 1 - P(0) - P(1) = 1 - C_4^0 \cdot 0,1^4 - C_4^1 \cdot 0,9 \dots$

$= 0,9963$

Problem 3.9.

Let E is the event that every one who appears for the departure of this flight will have a seat.

X is the number of purchases appears, $X \sim B(200; 0{,}99)$

$P(E) = P(X \leq 198) = 1 - P_X(199) - P_X(200)$

$= 1 - C_{200}^{199} \cdot 0{,}99^{199} \cdot 0{,}01 - C_{200}^{200} \cdot 0{,}99^{200}$

$= 1 - 0{,}405$

$= 0{,}595$

Problem 3.10.

a) Let X is the number of questions correct

X is binomial random variable, $X \sim B(4, \frac{1}{4})$

$P_X(2) = 4C2 \times \left(\frac{1}{4}\right)^2 \cdot \left(\frac{3}{4}\right)^2 = \frac{27}{128}$

b) X is the number of points

X	-10	-3	6	13	20
P	$\frac{81}{256}$	$\frac{27}{64}$	$\frac{27}{128}$	$\frac{3}{64}$	$\frac{1}{256}$

Problem 3.11

$$f(x) = \begin{cases} 5e^{-5x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

a) $E(X) = \int_0^{+\infty} x \cdot 5e^{-5x} dx = \cancel{-\int_0^{+\infty} e^{-5x} d{-5x}}$

$= \int_0^{+\infty} -x \, d e^{-5x}$

$= -x \cdot e^{-5x} \Big|_0^{+\infty} + \int_0^{+\infty} e^{-5x} dx$

$= \frac{-1}{5} e^{-5x} \Big|_0^{+\infty} = \frac{-1}{5}(0-1) = \frac{1}{5}$

~~Because~~ Since, $P(D) > P(E) \Rightarrow$ should chose topic B.

b) If the arrival probability is 0,5.
Then $P(E) = 1 - 0{,}5^2 = 0{,}75$
$P(D) = 1 - 0{,}5^4 - C_4^1 \cdot 0{,}5^4$
$= 0{,}6875$
$\Rightarrow$ Should choose topic A

Problem 3.8.

a) In an interval of 2 minutes, ~~$\lambda = 2 \times 2 = 4$~~ the number of calls H is poisson random variable with $\lambda = 4$ (2 calls/minutes) $\times$ (2 minutes) = 4 calls. The PMF of H is

$$P_H(h) = \begin{cases} \frac{4^h \cdot e^{-4}}{h!}, & h = 0, 1, 2 \ldots \\ 0, & \text{otherwise} \end{cases}$$

$$P_H(5) = \frac{4^5 \cdot e^{-4}}{5!} = 0{,}1563.$$

b) In an interval of 30s, the number of call B is poisson random variable with $\lambda = 2 \cdot \frac{1}{2} = 1$ calls. The PMF of B

$$P_B(b) = \begin{cases} \frac{e^{-1}}{b!}, & b = 0, 1, 2 \ldots \\ 0, & \text{otherwise} \end{cases}$$

$P_B(0) = e^{-1} = 0{,}3679$

c) In an interval of 10s, the number of call C is poisson random variable with $\lambda = 2 \cdot \frac{1}{6} = \frac{1}{3}$ calls. The PMF of C

$$P_C(c) = \begin{cases} \frac{\left(\frac{1}{3}\right)^k \cdot e^{-\frac{1}{3}}}{k!}, & k = 0, 1, 2 \ldots \\ 0 & \text{otherwise} \end{cases}$$

$P(C \geq 1) = 1 - P(0) = 1 - e^{-\frac{1}{3}} = 0{,}2835$

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c) Distribution function of Y:

$$F_Y(y) = \begin{cases} 0 & , y \le 0 \\ 1-e^{-\lambda} & , 0 < y \le 1 \\ 1-e^{-\lambda}+e^{-\lambda}-e^{-2\lambda} & , 1 < y \le 2 \\ \dots & \\ 1-e^{-n\lambda} & , n-1 < y \le n \\ \dots & \\ 1 & , \end{cases}$$

d)

Let A is the event that the machine still working until at the end of 10^{th} period

B is the event that it does not fail before 6th pe[riod]

~~P(A) =~~ P

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(A)}{P(B)}$$

$$P(A) = P_Y(10) = e^{-10\lambda}(1-e^{-\lambda})$$

$$P(B) = P(Y \ge 6) = \cancel{P\ 1 - P(0) - P(1) - P(2) - P(3) - P(4) - P(5)}$$

$$= 1 - P(Y < 6)$$

$$= 1 - F(6)$$

$$= 1 - 1 + e^{-6\lambda}$$

$$= e^{-6\lambda}$$

Problem 3.14

Problem 3.15. $E(X) = 0$ and $P[|X| \le 10] = 0,1$

$$P[|X| \le 10] = P[-10 \le X \le 10] = 0,1$$

$$\Leftrightarrow -\Phi\left(\frac{-10}{\sigma}\right) + \Phi\left(\frac{10}{\sigma}\right) = 0,1$$

$$\Leftrightarrow 2\Phi\left(\frac{10}{\sigma}\right) = 1,1$$

$$\Leftrightarrow \Phi\left(\frac{10}{\sigma}\right) = 0,55 \Rightarrow \sigma = 83,34$$

b) $P(0,4 < X < 1) = \int_{0,4}^{1} 5e^{-5x}\,dx = -e^{-5x}\Big|_{0,4}^{1}$

$= -e^{-5} + e^{-2}$

Problem 3.12. $E(X) = 0$ and $\sigma = 0,4$

a) The CDF of X

$F_X(x) = \Phi\left(\frac{x}{0,4}\right) = \Phi(2,5x)$

$P(X > 3) = 1 - F_X(3) = 1 - \Phi(7,5) =$

b) $P(3-c < X < 3+c) = 0,9$

$\Rightarrow -\Phi\left(\frac{3-c}{0,4}\right) + \Phi\left(\frac{3+c}{0,4}\right) = 0,9$

$\Rightarrow$

Problem 3.13. X be an exponential random variable

$\Rightarrow f_X(x) = \begin{cases} \lambda \cdot e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$

a) We have, $Y = [X]$

$\Rightarrow P(Y = y) = P(y \leq X < y+1)$

$= \int_{y}^{y+1} f_X(x)\,dx$

$= -e^{-\lambda x}\Big|_{y}^{y+1}$

$= -e^{-\lambda(y+1)} + e^{-\lambda y}$

$= e^{-\lambda y}(1 - e^{-\lambda})$

b) Let $e^{-\lambda} = p \Rightarrow \cancel{P(Y=y)} = P_Y(y) = p^y(1-p)$

$E(Y) = \sum_{0}^{\infty} y \cdot p^y (1-p)$

$= \sum_{0}^{\infty} \cancel{y \cdot p^y - y\,p^{y+1}}$ $(1-p) \cdot \sum_{0}^{\infty} y \cdot p^y = (1-p) \cdot \frac{p}{(1-p)^2}$

$= \frac{p}{1-p}$

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Problem 3.14.

Let X is time (minutes) from 8:45 to 9:45, we have $X \sim U([0,60])$

- She waits at most 10 minutes if she arrives at airport ~~at~~ from 8:50 to 9 or ~~9h~~ 9:20 to 9:30

$\Rightarrow$ The probability need to calculus $P = \frac{10}{60} + \frac{10}{60} = \frac{1}{3}$

- She waits at least 15 minutes if she arrives at airport from ~~8:45~~ ~~to~~ ~~8h~~ 9:00 to 9:15

$\Rightarrow P = \frac{15}{60} = \frac{1}{4}$