Fundamentals of optimization

Backtracking Search

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Backtracking

- Solve enumeration/optimization of combinatorial problems
- $X = \{(x_1, x_2, \ldots, x_n) \mid x_i \in A_i, \forall\, i = 1, \ldots, n\}$
- List all $x \in X$ satisfying some properties P
- procedure try(k):
  - Try all values v that can be assigned to x_k without violating P
  - For each legal value v:
    - Assign v to x_k
    - If $k < n$: call recursively try(k+1) to try all values for x_{k+1}
    - If $k = n$: accept a solution

Backtracking

```
TRY(k)
  Begin
    Foreach v of A_k
     if check(v,k) /* check if v is legal */
       Begin
         x_k = v;
         [Update incremental data structure D]
         if(k = n) solution();
         else TRY(k+1);
        [Recover incremental data structure D]
       End
  End
Main()
Begin
  TRY(1);
End
```

EXERCISES

- Binary sequence generation
- Permutation generation
- Positive integer linear equations
- Sudoku
- Traveling Salesman Problem
- Capacitated Bus Routing
- Balanced Course Teacher Assignment

Sudoku

- Given a grid 9×9, the goal is to assign digits (from 1 to 9) to the cells so that every row, column, and subgrid of size 3×3 contains exactly one instance of the digits from 1 to 9

1	2	3	4	5	6	7	8	9
4	5	6	7	8	9	1	2	3
7	8	9	1	2	3	4	5	6
2	1	4	3	6	5	8	9	7
3	6	5	8	9	7	2	1	4
8	9	7	2	1	4	3	6	5
5	3	1	6	4	2	9	7	8
6	4	2	9	7	8	5	3	1
9	7	8	5	3	1	6	4	2

Travelling Salesman Problem

- A person departs from point 0. He want to visit points 1, 2, …, n, once and come back to 0. Given $c(i,j)$ which is the traveling distance from point i to point j (i, j = 0,1,…,n), help that person to compute the shortest route

Capacitated Bus Routing

- There are n passengers 1, 2, …, n. The passenger i want to travel from point i to point $i + n$ (i = 1,2,…,n). There is a bus located at point 0 and has k places for transporting the passengers (it means at any time, there are at most k passengers on the bus). You are given the distance matrix c in which $c(i,j)$ is the traveling distance from point i to point j (i, j = 0,1,…, 2n). Compute the shortest route for the bus, serving n passengers and coming back to point 0 (the route visits each point 1, 2, …, 2n exactly once)

Balanced Course Teacher Assignment

- At the beginning of the semester, the head of a computer science department have to assign courses to teachers in a balanced way.
- The department has *m* teachers $T=\{1, 2, ..., m\}$ and *n* courses $C=\{1, 2,..., n\}$.
- Each course $c \in C$ has a duration h_c.
- Each teacher $t \in T$ has a preference list which is a list of courses he/she can teach depending on his/her specialization.
- We know a list of pairs of conflicting two courses that cannot be assigned to the same teacher as these courses have been already scheduled in the same slot of the timetable. This conflict information is represented by a conflict matrix *A* in which $A(i,j)=1$ indicates that course *i* and *j* are conflict.
- The load of a teacher is the total duration of courses assigned to her/him.
- How to assign *n* courses to *m* teachers such that each course assigned to a teacher is in his/her preference list, no two conflicting courses are assigned to the same teacher, and the maximal load for all teachers is minimal