Thứ ngày

Problem 6.1:

$\sigma = 40$ hours

$n = 30$ bulbs , $\bar{x} = 780$ hours

$96\% = 100(1-\alpha)\%$

$\Leftrightarrow 1-\alpha = 0,96$

$\Rightarrow \alpha = 1 - 0,96$

$\Rightarrow \alpha = 0,04$

$$\Rightarrow \left(780 - z_{0,02} \cdot \frac{40}{\sqrt{30}} < z < 780 + z_{0,02} \cdot \frac{40}{\sqrt{30}}\right)$$

Use the table $\Rightarrow z_{0,02} = 2,05$

$$\Rightarrow \left(780 - 2,05 \cdot \frac{40}{\sqrt{30}} < z < 780 + 2,05 \cdot \frac{40}{\sqrt{30}}\right)$$

$$\Rightarrow 765 < z < 795$$

Problem 6.2:

$n = 50$, $\bar{x} = 174,5$

$\sigma = 6,9$ cm

$z_{0,01} = 2,33$

a, A 98% confidence interval for the population mean is:

$$174,5 - (2,33) \cdot \frac{6,9}{\sqrt{50}} < \mu < 174,5 + (2,33) \cdot \frac{6,9}{\sqrt{50}}$$

$$\Leftrightarrow 172,23 < \mu < 176,77$$

b, $e < (2,33) \cdot \dfrac{6,9}{\sqrt{50}} = 2,27$

Problem 6.3:

The sample size is $n = 9$.

$$\bar{x} = \frac{1}{n} \sum x_i = 1,0056$$

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$= \frac{(1,01 - 1,0056)^2 + (0,97 - 1,056)^2 + \dots + (1,03 - 1,0056)^2}{9-1}$$

$$= 0,000603$$

$$s = \sqrt{s^2} = 0,02455$$

$99\% = 100(1-\alpha)\%$

$\Rightarrow \alpha = 0,01$

$$\Rightarrow 1,0056 - t_{0,005} \cdot \frac{0,02455}{\sqrt{9}} < \mu < 1,0056 + t_{0,005} \frac{0,02455}{\sqrt{9}}$$

$$\Leftrightarrow 1,0056 - 3,355 \cdot \frac{0,02455}{\sqrt{9}} < \mu < 1,0056 + 3,355 \cdot \frac{0,02455}{\sqrt{9}}$$

$$\Leftrightarrow 0,9781 < \mu < 1,0331$$

Problem 6.4.

The sample size $n = 15$.

The sample mean, $\bar{x}$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i = \frac{3,4 + 2,5 + 4,8 + 2,9 + \dots + 3,0 + 4,8}{15}$$

$$= \frac{56,8}{15} = 3,8$$

$$s^2 = \frac{1}{(n-1)} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$= \frac{1}{14} \left((3,4-3,787)^2 + (2,5-3,787)^2 + \dots + (4,8-3,787)^2 \right)$$

$$= 0,9427$$

$$s = \sqrt{s^2} = \sqrt{0,9427} = 0,97$$

$$95\% = 100(1-\alpha)\%$$
$$\Rightarrow \alpha = 0,05$$

$$\Rightarrow 3,8 - t_{0,025} \cdot 0,97 \cdot \sqrt{1+\frac{1}{15}} < x_0 < 3,8 + t_{0,025} \cdot 0,97\sqrt{1+\frac{1}{15}}$$

$$\Leftrightarrow 3,8 - 2,145 \cdot 0,97 \cdot \sqrt{1+\frac{1}{15}} < x_0 < 3,8 + 2,145 \cdot 0,97\sqrt{1+\frac{1}{15}}$$

$$\Leftrightarrow 1,65 < x_0 < 5,95$$

Problem 6.5

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

The sample size: $n = 1000$

$$\hat{p} = \frac{228}{1000} \Rightarrow \hat{q} = 1 - \hat{p} = 1 - \frac{228}{1000}$$

$$\Rightarrow \frac{228}{1000} - 2,576 \cdot \sqrt{\frac{228/1000 \cdot (1 - 228/1000)}{1000}} < p < \frac{228}{1000} + 2,576\sqrt{\frac{\frac{228}{1000} \cdot (1 - \frac{228}{1000})}{1000}}$$

$$\Leftrightarrow 0,194 < p < 0,262$$

Thứ ngày

where $100(1-\alpha)\% = 99\%$

$\alpha = 0{,}01$

Problem 6.6

We have

$n = 200$

$x = 114$

$\Rightarrow \hat{p} = \frac{x}{n} = \frac{114}{200} = 0{,}57$

$\hat{q} = 1 - \hat{p} = 0{,}43$

$96\% = 100\%(1-\alpha)\%$

$\Rightarrow \alpha = 0{,}04$

$\Rightarrow 0{,}57 - 2{,}055\sqrt{\frac{(0{,}56 \cdot 0{,}43)}{200}} < p < 0{,}57 + 2{,}055\sqrt{\frac{(0{,}57 \cdot 0{,}43)}{200}}$

$\Leftrightarrow 0{,}4981 < p < 0{,}6419$

b,

Using data from part a, we conclude that we can be 96% sure the error of estimator $\hat{p} = 0{,}57$ will be exceed.

$2.055\sqrt{\frac{0{,}57 \cdot 0{,}43}{200}} = 0{,}0719$