Information cascades

Introduction to Network Science
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Topic 17

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Sources

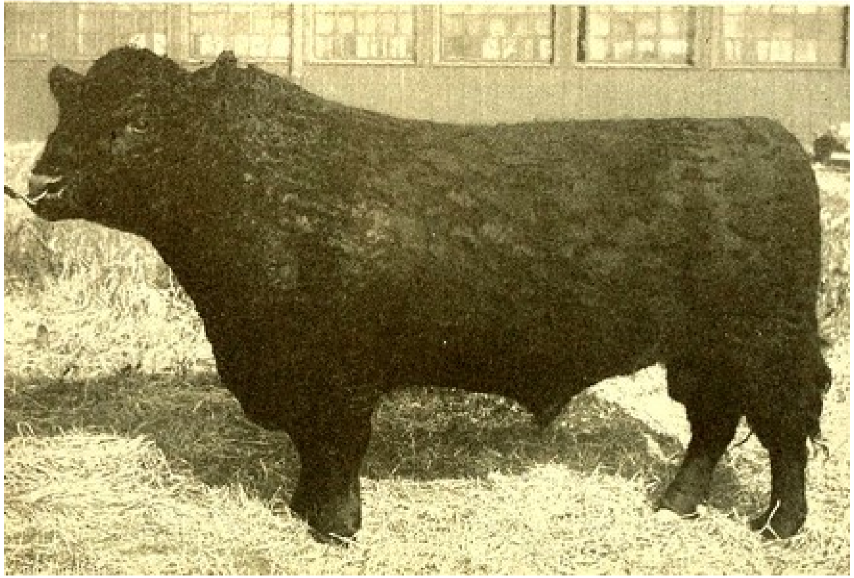
- Easley and Kleinberg (2010): Networks, Crowds, and Markets Ch 16
- Carlos Castillo (2017): Social influence slides

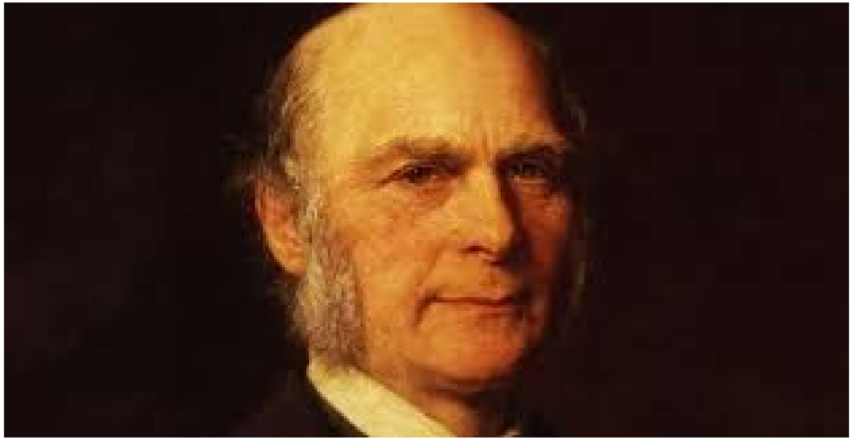
The following slides analyze cascades in crowds (think of them as cliques: everybody is influenced by everybody else)

Later we will see information cascades in networks

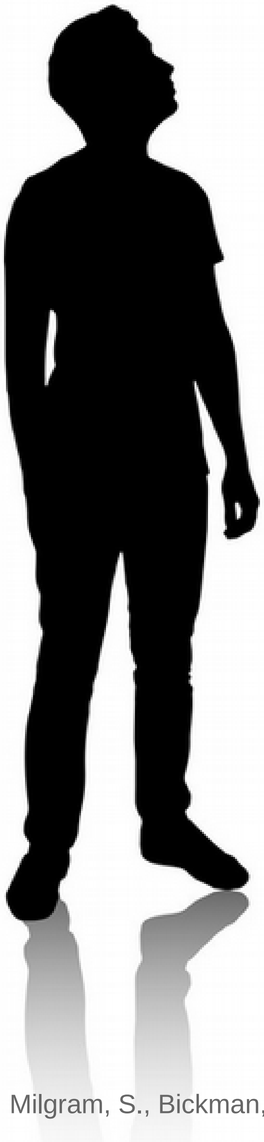
- Francis Galton attends a competition to guess the weight of an ox in 1906
- Observes ≃800 guesses
- Average: 542.9 kg
- Actual weight: 543.4 kg

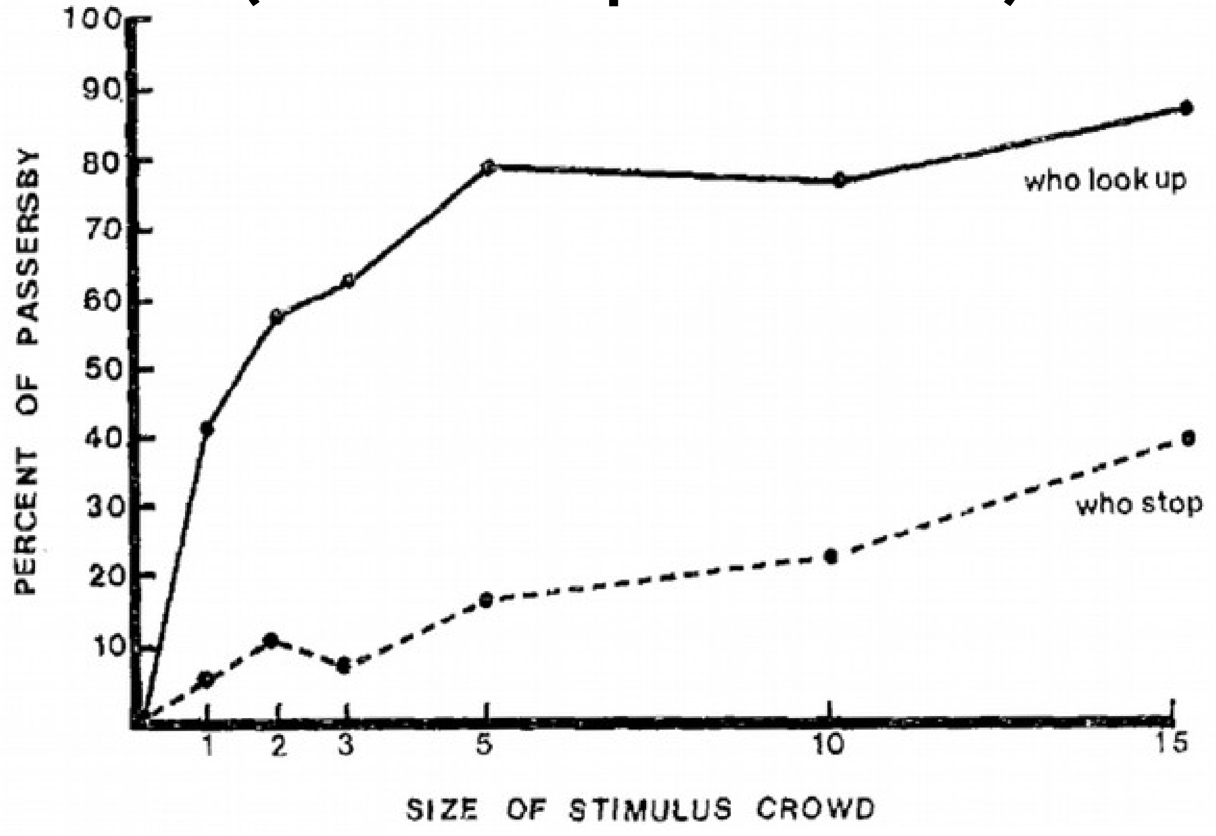
- This works well if guesses are **INDEPENDENT**





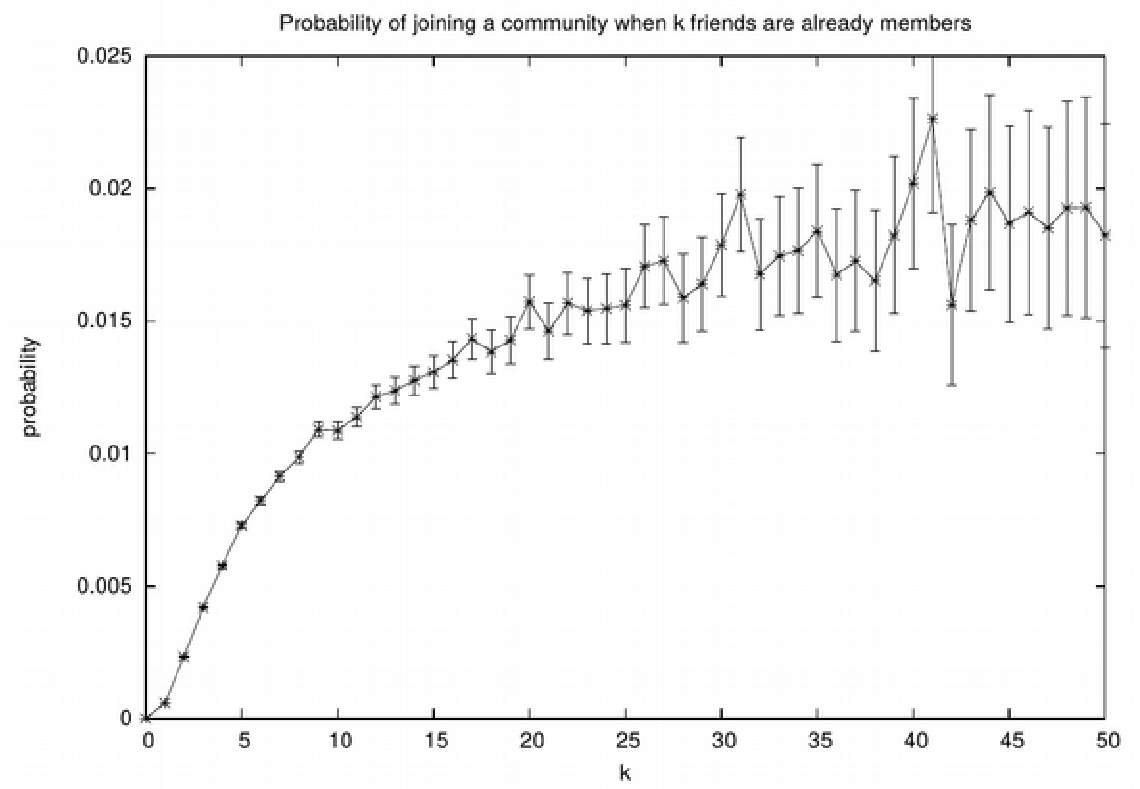
A crowd looking up on a busy street (1969 experiment)





Milgram, S., Bickman, L., Berkowitz, L. (1969). Note on the drawing power of crowds of different sizes. Journal of Personality and Social Psychology, 13, 79-82

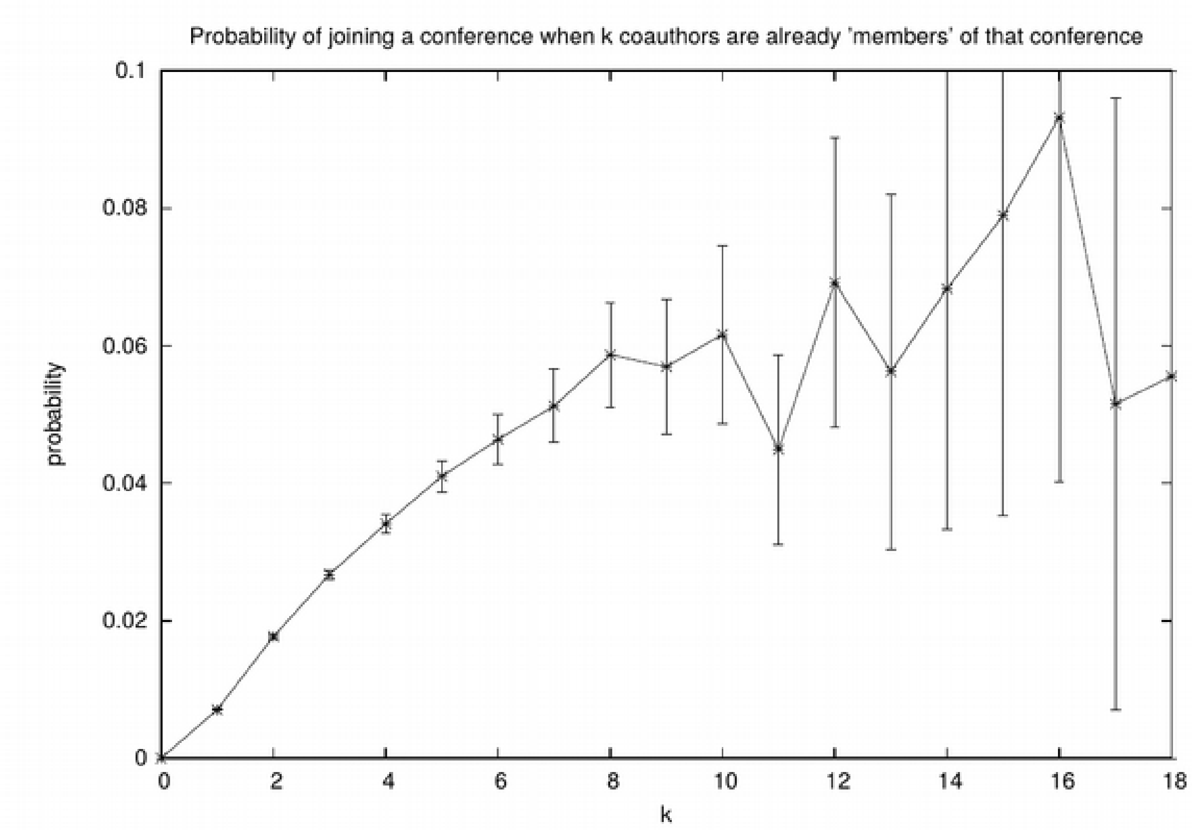
Probability of joining a community
given k friends are already members



L. Backstrom, D. Huttenlocher, J. Kleinberg, and X. Lan. Group formation in large social networks. In Proc. KDD 2006.

Prob. of publishing on a conference

given k co-authors published there



L. Backstrom, D. Huttenlocher, J. Kleinberg, and X. Lan. Group formation in large social networks. In Proc. KDD 2006.

“Herding” can be rational

The actions of others …

… tell us about what they find valuable …

… which tell us about what we might value.

A framework to study herding

1. Assume a decision must be made
2. Assume people make decisions in sequence, you see what others decided before you
3. Assume each person has some private information that helps them decide
4. Assume you can only observe their decisions, not their private information

Task: guess if an urn is “majority-blue” or “majority-red”

You can only draw one ball

Herding experiment

- Each person from 1 … N
  - Draws a ball
  - Looks at the ball in secret
  - Places the ball back in the urn
  - Announces his/her guess publicly
- People who guess correctly win € / $ / ¥ / £

Person #1

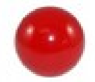
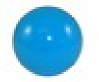
- Case 1: 

  Guess: majority-blue

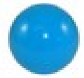
- Case 2: 

  Guess: majority-red

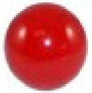
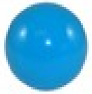
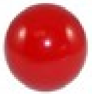
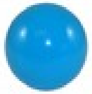
Person #2

- If person#1 said “majority-red” and person#2 draw 
  - Guess 
- If person#1 said “majority-blue” and person#2 draw 
  - Guess 

Person #2 (cont.)

- If person #1 said "majority-blue" and person #2 draw 
  – Guess at random
- If person #1 said "majority-red" and person #2 draw 
  – Guess at random

Person #3

- If the first two guessed differently (“majority-red” and “majority-blue”)
  - Guess red if draw 
  - Guess blue if draw 
- If the first two people guessed red
  - Guess red if draw 
  - **Guess red if draw**  Ignore your eyes, follow the herd!

Equivalently, if first two guessed blue, guess blue

Person #4 and following

- If person#1 and person#2 guessed the same, then we know person#3 will guess the same (his/her own draw is worthless)
- Hence in this case everybody will continue guessing the same

  ⇒ **Information cascade**

Information cascades

- Can appear easily
  - E.g., in this stylized setting
- Can be broken easily
  - Suppose after 49 draws, persons #50 and #51 break the protocol by showing publicly the balls they draw
  - Fresh information enters the decisions, and the cascade stops (it might reappear, though)

Analyzing the herding experiment

$$Pr[A|B] = \frac{Pr[A]Pr[B|A]}{Pr[B]}$$

- Person #1

$$Pr[\text{maj-blue} \mid \text{blue}] = \frac{Pr[\text{maj-blue}] \cdot Pr[\text{blue} \mid \text{maj-blue}]}{Pr[\text{blue}]}$$

$$= \frac{\frac{1}{2} \cdot \frac{2}{3}}{\frac{1}{2}} = \frac{2}{3}$$

- This justifies guessing majority-blue if drawing blue

Analyzing the herding experiment

$$Pr[A|B] = \frac{Pr[A]Pr[B|A]}{Pr[B]}$$

- Person #2

$$Pr[\text{maj-blue} \,|\, \text{blue, blue}] = \frac{Pr[\text{maj-blue}] \cdot Pr[\text{blue, blue} \,|\, \text{maj-blue}]}{Pr[\text{blue, blue}]}$$

$$= \frac{\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{2}{3}}{\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{2}{3} + \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{3}} = \frac{4}{5}$$

- This justifies guessing majority-blue if drawing blue

Compute for person #3

$$Pr[\text{maj-blue} \,|\, \text{blue, blue, red}] = ?$$

$$Pr[A|B] = \frac{Pr[A]Pr[B|A]}{Pr[B]}$$

A general cascade model

- Sequential decisions of people 1, 2, 3, …
- The world is in a hidden state G or B
  - The world is G with probability p, B with probability 1-p
- People must decide whether to accept or reject an option
  - If they accept and the world is G, they win
  - Otherwise they lose
- Nobody knows in advance the hidden state

Example

- You have a 
- “Accepting” means exchanging it with what’s behind the door
- If the world is G, that’s a car, if the world is B, that’s a kick scooter

Each person receives in advance a signal (L or H) about the outcome

		States: B	States: G
Signals	L	q	$1-q$
	H	$1-q$	q

$$Pr(H|G) = q > \frac{1}{2}$$

- E.g., you can see the audience and by their reaction gather L or H, we will assume this information is better than random (q > 1/2)

Person #1

$$
\begin{aligned}
Pr(G|H) &= \frac{Pr(H|G)Pr(G)}{Pr(H)} \\
&= \frac{qp}{Pr(H|G)Pr(G) + Pr(H|B)Pr(B)} \\
&= \frac{qp}{qp + (1-q)(1-p)} \\
&> \frac{qp}{qp + q(1-p)} \qquad (1-q<q) \\
&= p
\end{aligned}
$$

$Pr(G|H) > p$ Person must chose “G” if s/he received signal “H”

In general

- Suppose you receive a signal S containing:

  a signals of type "H"

  b signals of type "L"

- Then:

$$Pr(G|S) > Pr(G) \text{ if } a > b$$

$$Pr(G|S) < Pr(G) \text{ if } a < b$$

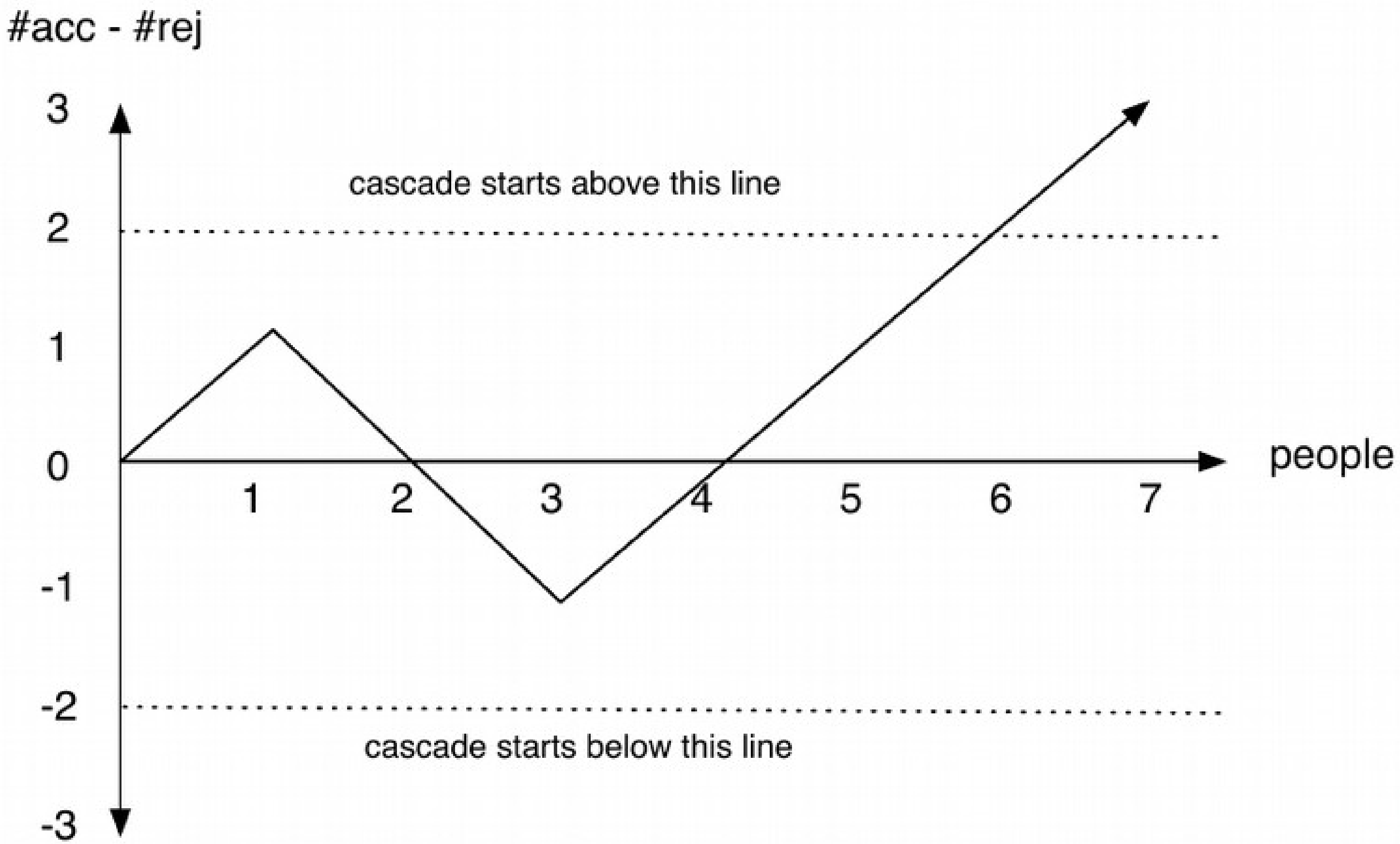
$$Pr(G|S) = Pr(G) \text{ if } a = b$$

Proof in Networks, Crowds, and Markets Section 16.5

Cascade situation

- Person i has a signal composed of:
  - Previous i-1 signals (accepts/rejects by previous)
  - His/her own signal
- If the previous i-1 signals have $a = b + 1$
  - Then signal i receives can be L, in which case we have a=b and i chooses at random
  - The signal I receives can be H, in which case we have $a > b$ and i should chose G

In general, cascades happen if $a>b+1$ or $a<b-1$



Summary

- Cascades can be wrong ($1-q > 0$)
- Cascades are inevitable in this model as $N \to \infty$
- Cascades can be stopped in a sequential model if people have access to private information