# Hubs and Authorities

Introduction to Network Science

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Topic 09

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# Sources

- Networks, Crowds, and Markets Ch 14
- Fei Li's lecture on PageRank
- Evimaria Terzi's lecture on link analysis.
- C. Castillo: Link-based ranking slides 2016

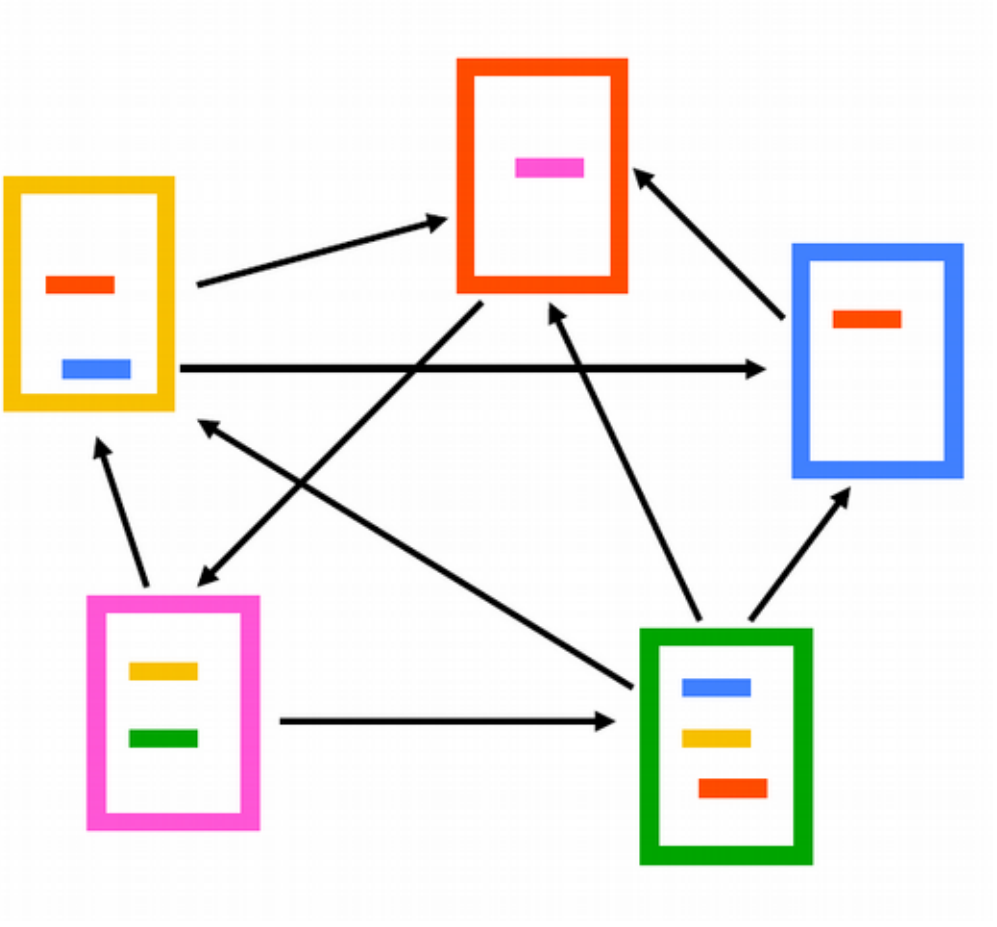
# Ranking on the web is hard

- Demand:
  - Information needs are unclear and evolving
- Supply
  - From scarcity to abundance: “filter failure”

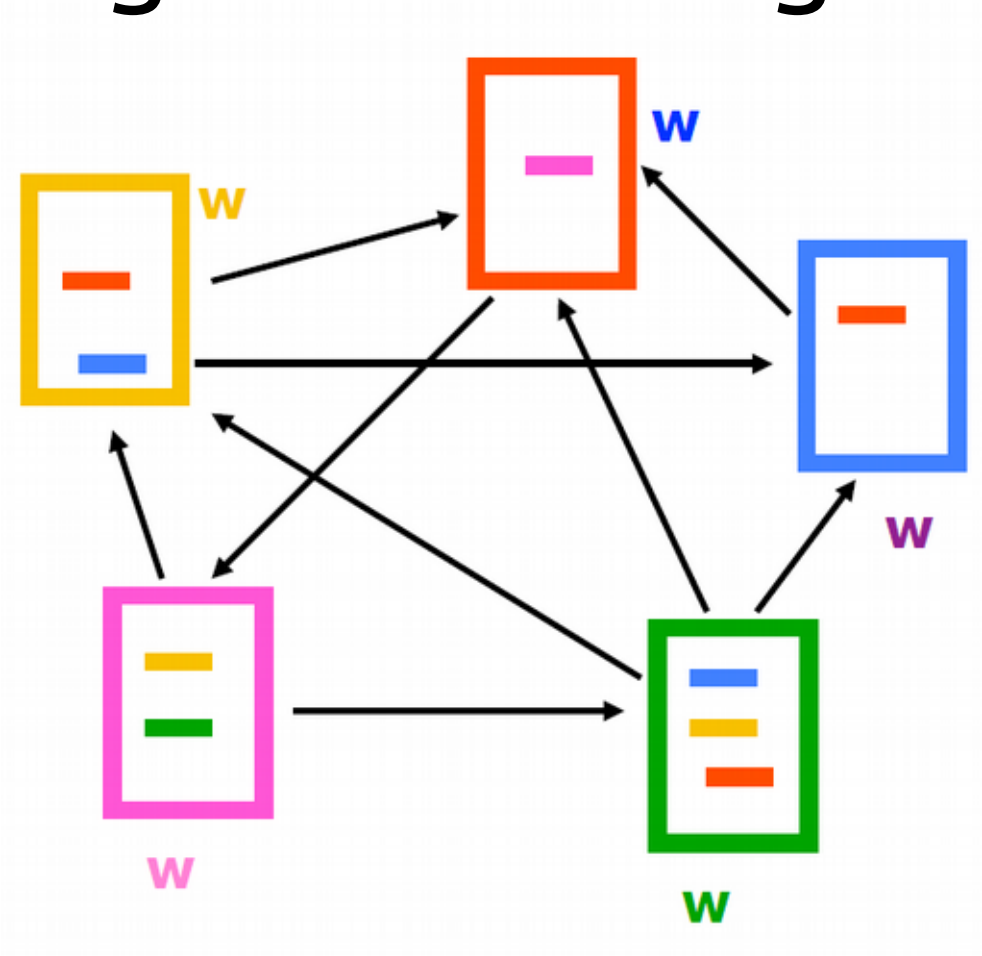
# Purpose of Link-Based Ranking

- **Static (query-independent)** ranking
- **Dynamic (query-dependent)** ranking
- Applications:
  - Search in social networks
  - Search on the web

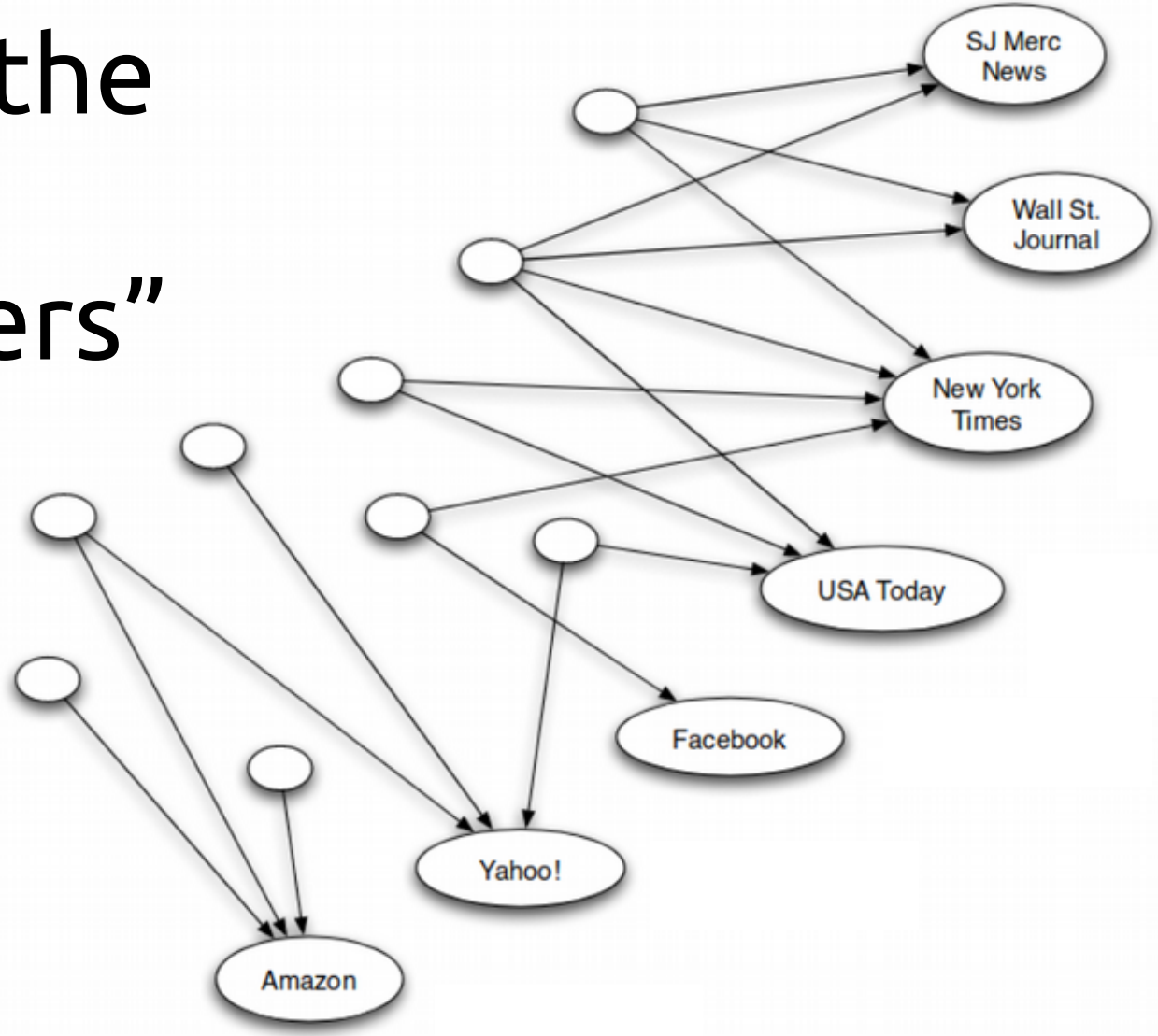
# Given a set of connected objects



# Assign some weights

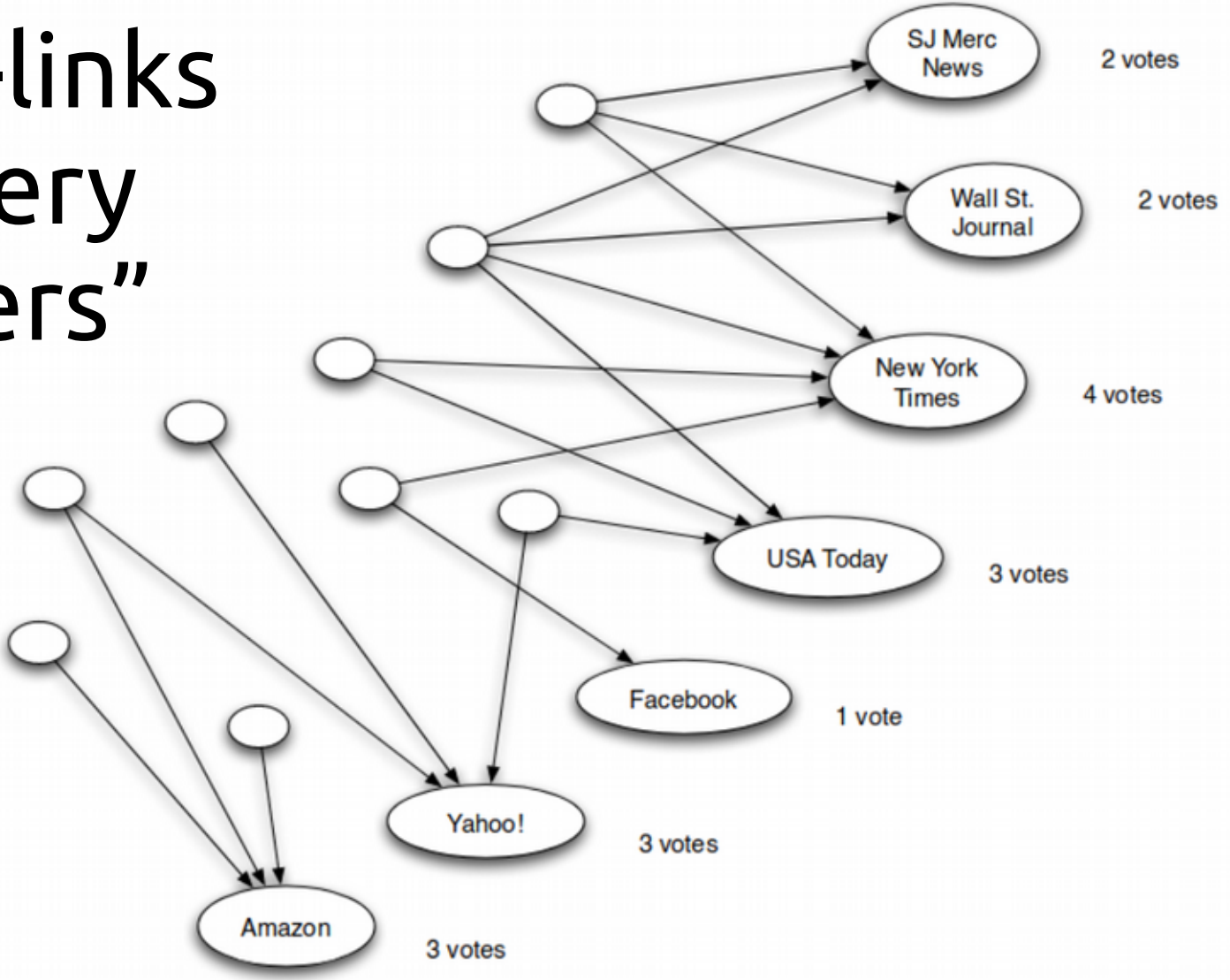


# Pages for the query “newspapers”

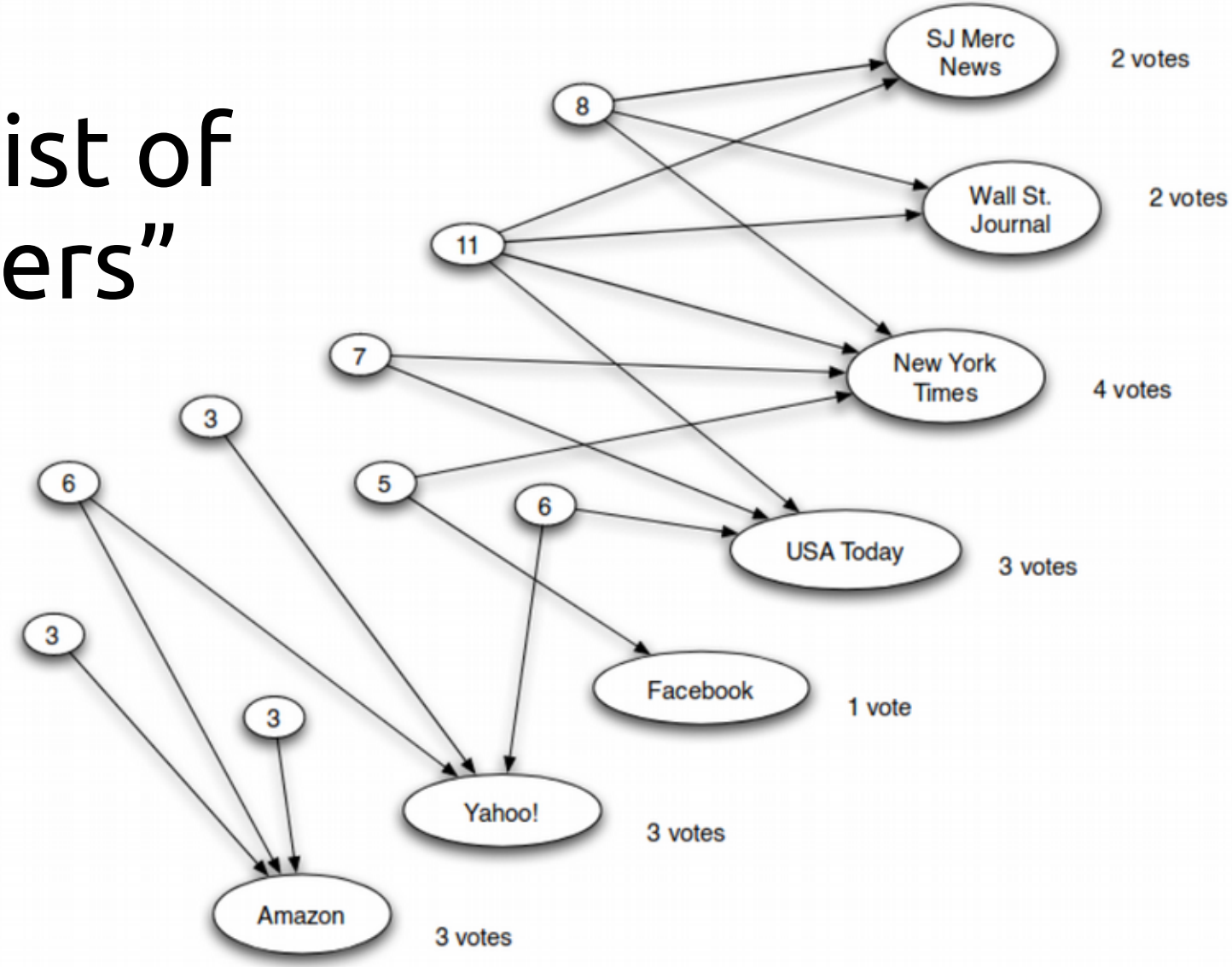



How would you rank these pages?

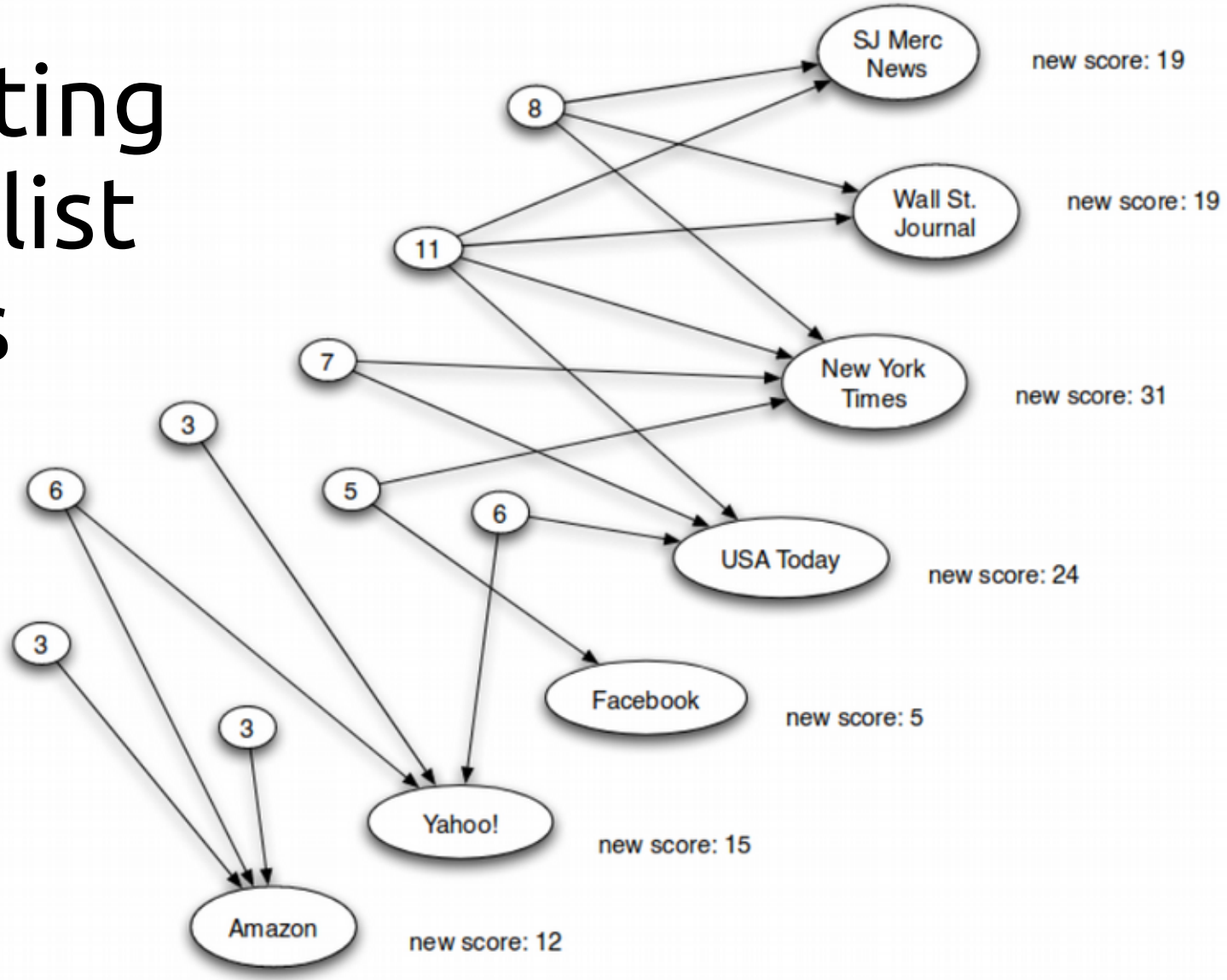
# Counting in-links for the query “newspapers”



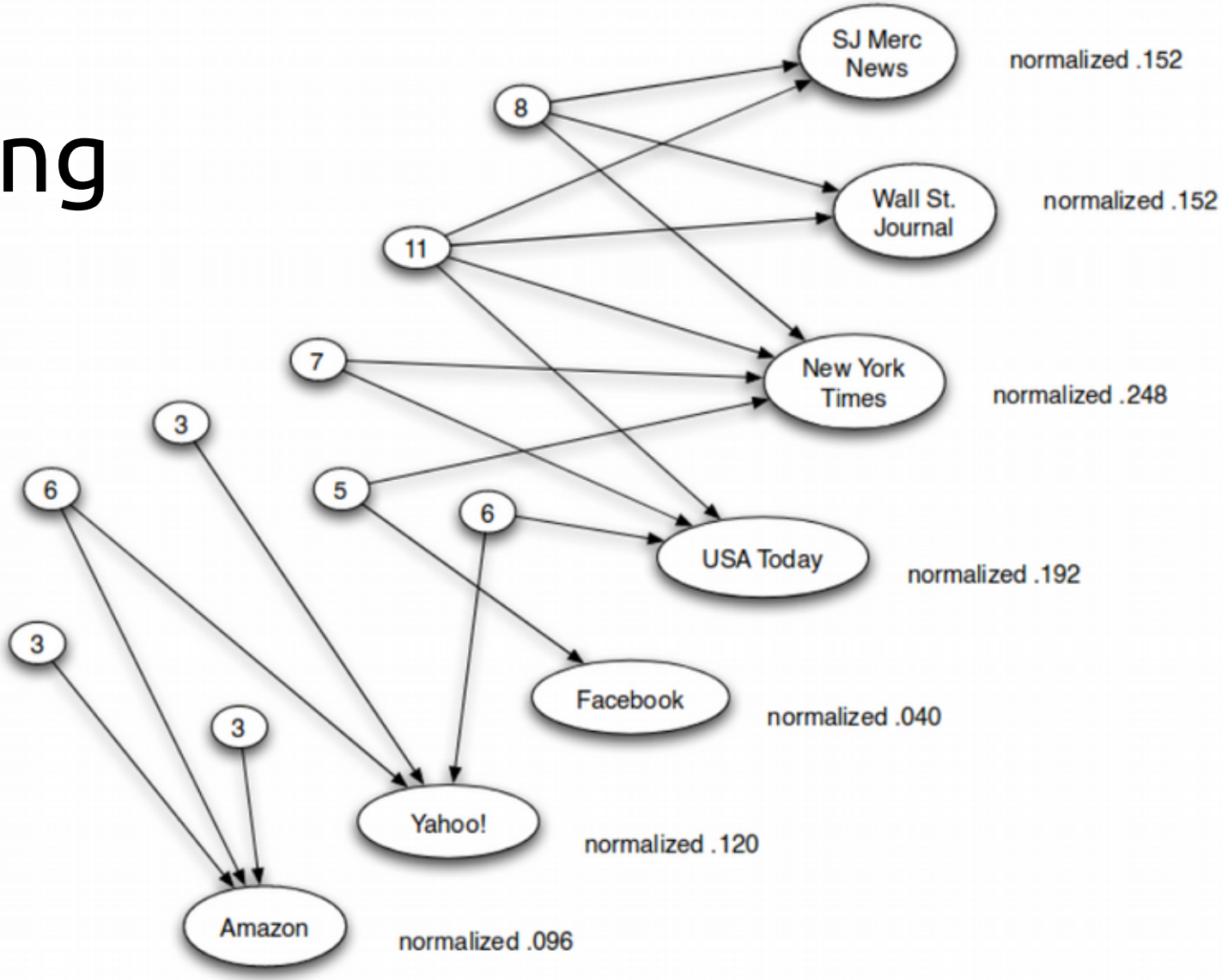
# Value of a list of “newspapers”



# Re-weighting votes by list values



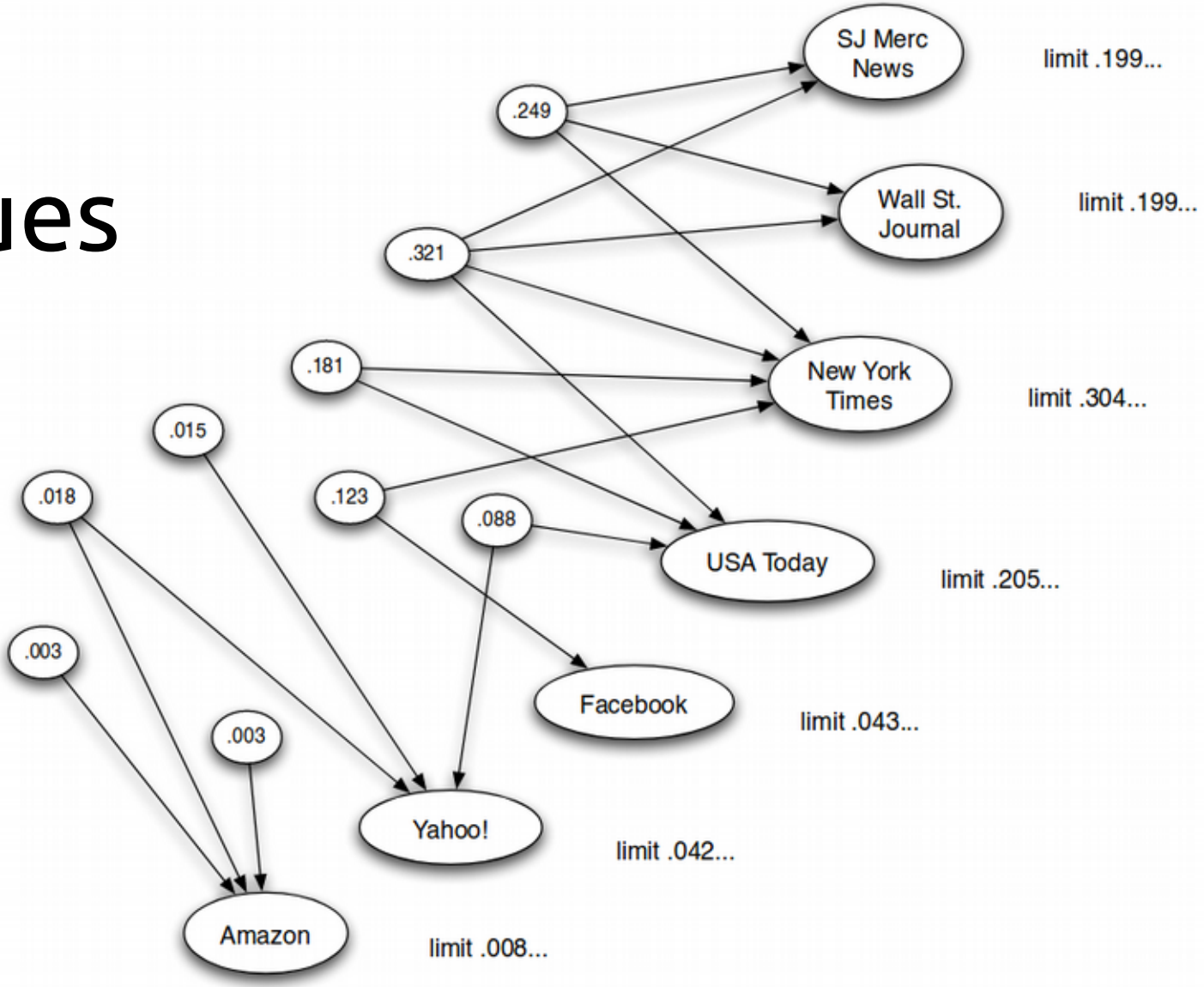
# Normalizing scores



# The idea behind Hubs and Authorities [Kleinberg 1999]

- Highly-recommended items
  appear in high-value lists
- High-value lists
  contain highly-recommended items
- **Repeated improvement**
  - Re-calculate scores several times

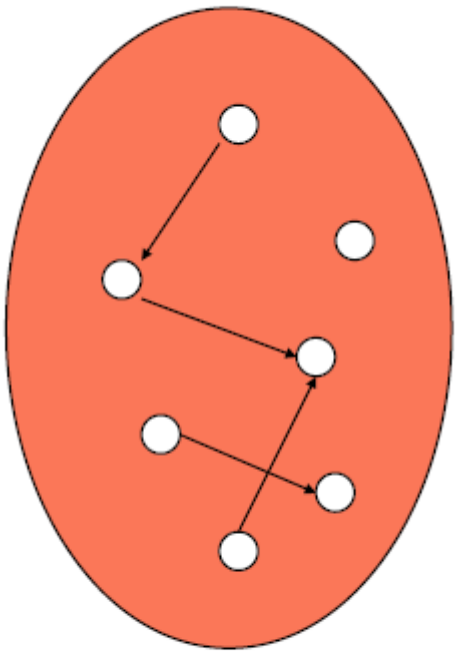
# Limit values



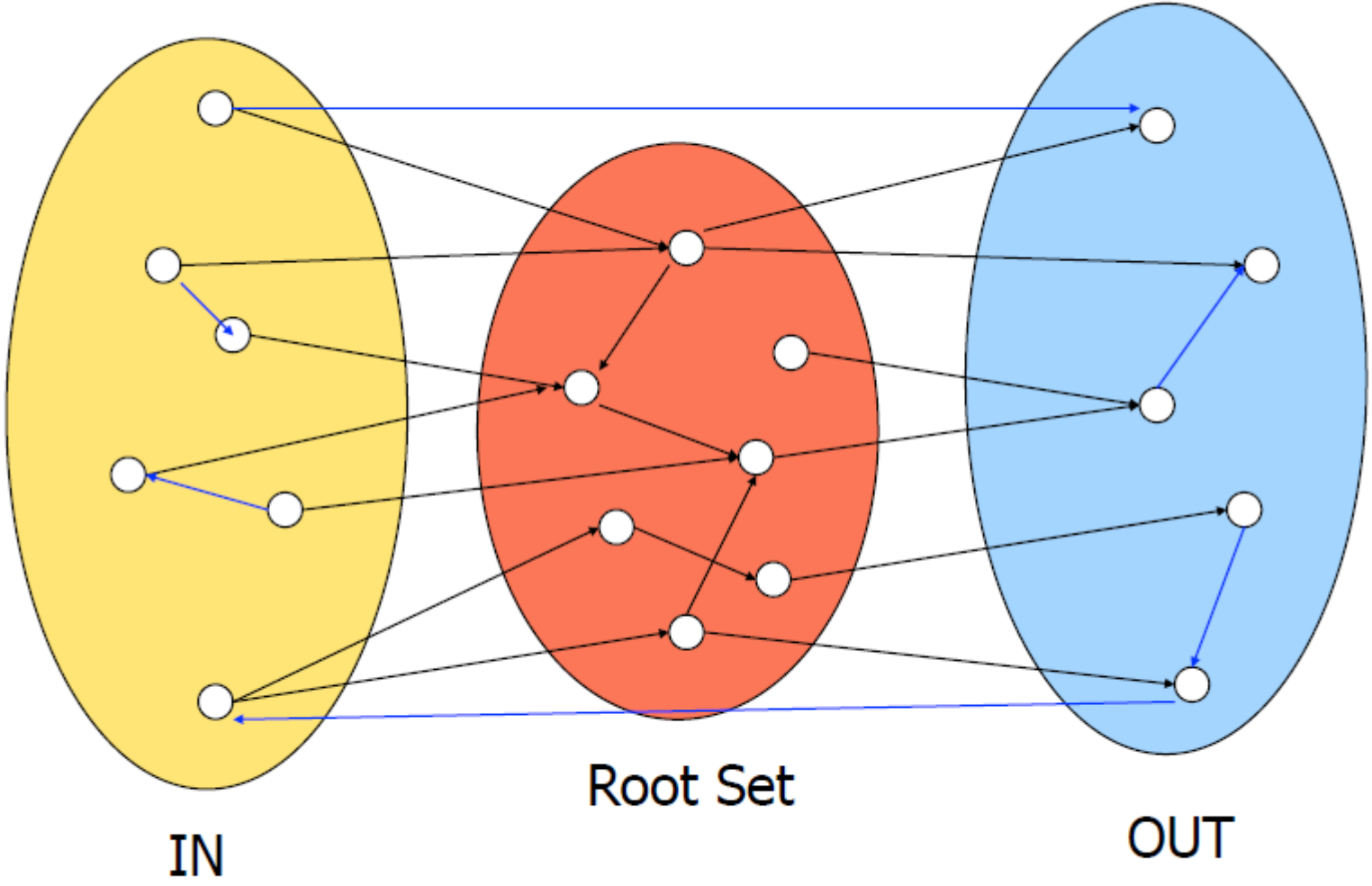
# This algorithm is called “HITS”

- *Jon M. Kleinberg. 1999. Authoritative sources in a hyperlinked environment. J. ACM 46, 5 (September 1999), 604-632. [DOI]*
- Query-dependent algorithm
  - Get pages matching the query
  - Expand to 1-hop neighborhood
  - Find pages with good out-links (“hubs”)
  - Find pages with good in-links (“authorities”)

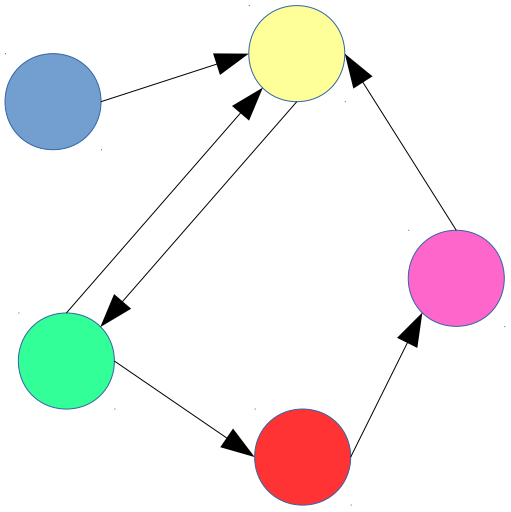
# Root set = matches the query



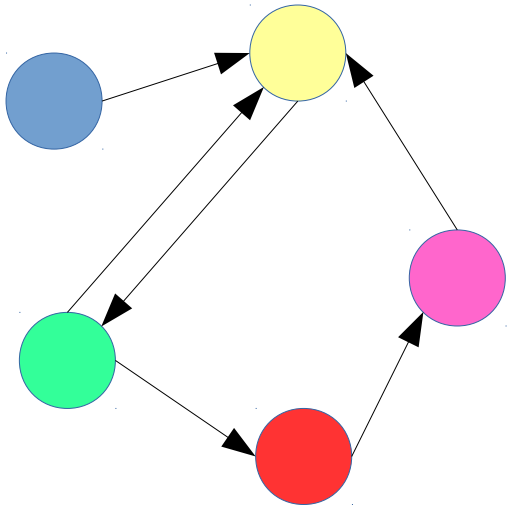
# Base set S = root set plus 1-hop neighbors

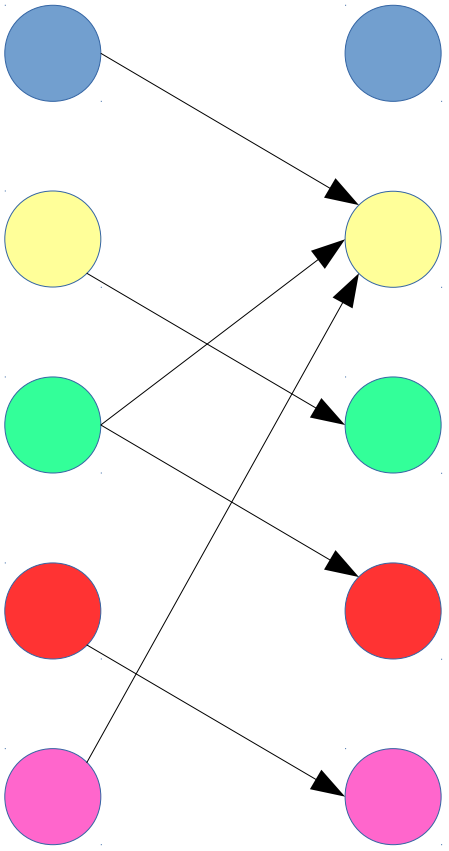


# Base graph *S* of *n* nodes



# Bipartite graph of *2n* nodes





# Bipartite graph of *2n* nodes

0) Initialization:

$$\mathrm{h}_1 = h_2 = h_3 = h_4 = h_5 = 1$$

1) Iteration:

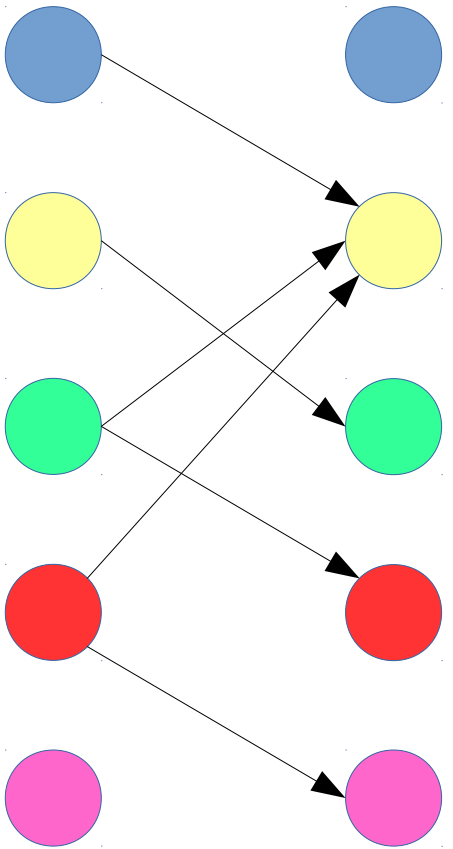
$$a_i = \sum_{j \to i} h_j$$

$$h_i = \sum_{i \to j} a_j$$

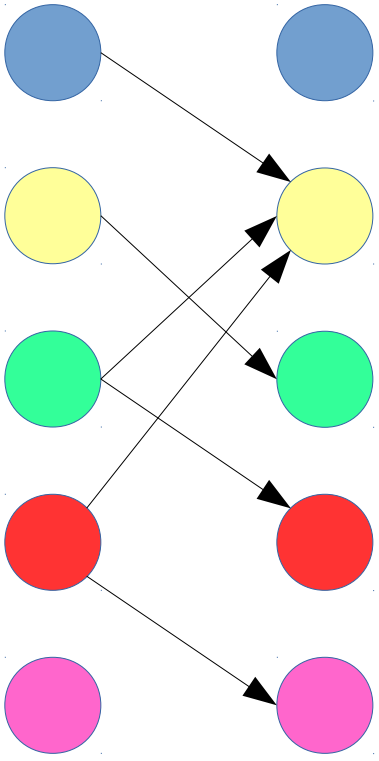
2) Normalization:

$$a_i = \frac{a_i}{\sum_j a_j}$$

$$h_i = \frac{h_i}{\sum_j h_j}$$



# Try it!



| H(1) | A(1) | Â(1) | H(2) | **Ĥ(2)** | A(2) | **Â(2)** |
|---|---|---|---|---|---|---|
| 1 | 0 | | | | | |
| 1 | 3 | | | | | |
| 1 | 1 | | | | | |
| 1 | 1 | | | | | |
| 1 | 1 | | | | | |

*Complete the table. Which one is the biggest hub? Which the biggest authority? Does it differ from ranking by degree?*

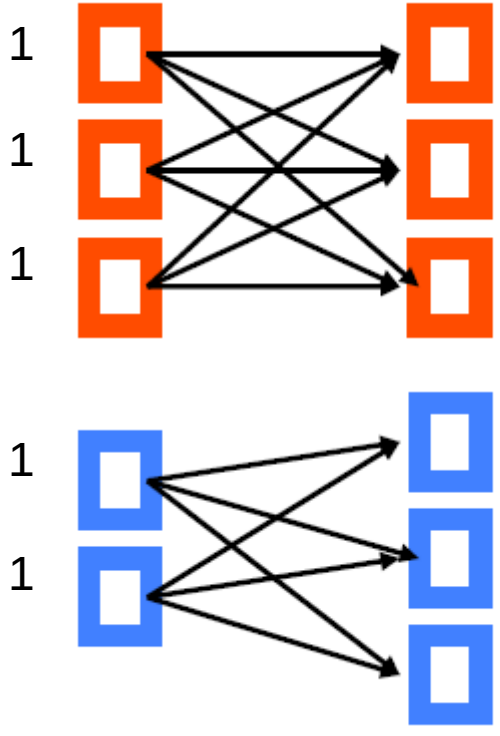
# What are we computing?

$$
\begin{aligned}
a^t &= A^T h^{t-1} \\
h^t &= A a^{t-1} \\
\text{replacing}: \quad a^t &= A^T A a^{t-1} \\
\text{after convergence}: \quad a &= A^T A a
\end{aligned}
$$

- Vector *a* is an eigenvector of $A^TA$
- Conversely, vector *h* is an eigenvector of $AA^T$

# Problem: tightly-knit communities

- Example: a graph made of a (3,3)-clique and a (2,3)-clique



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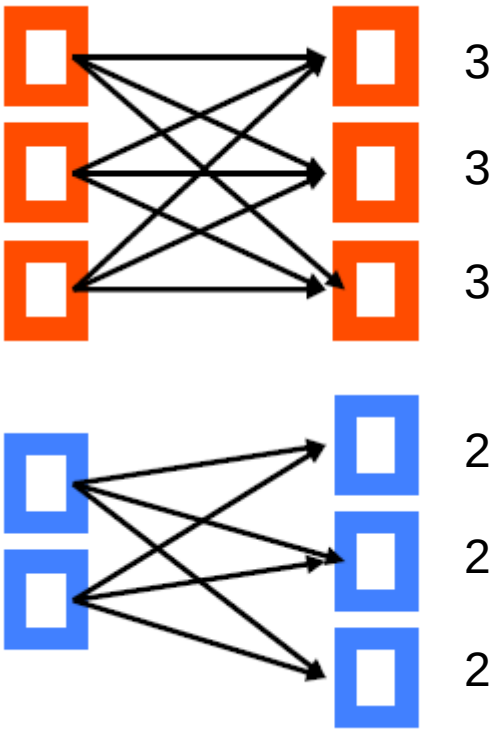

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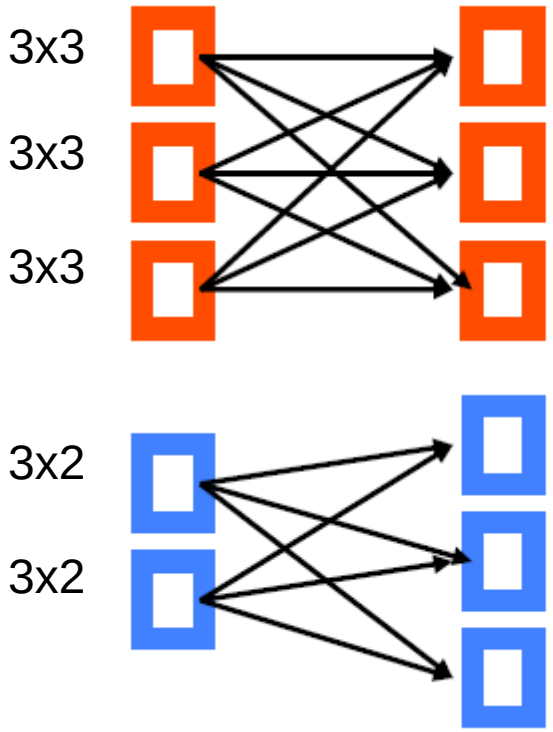

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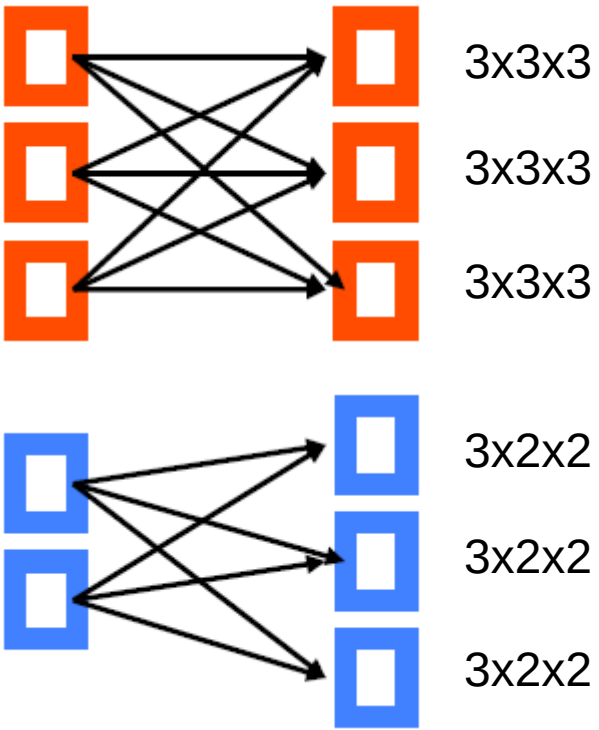
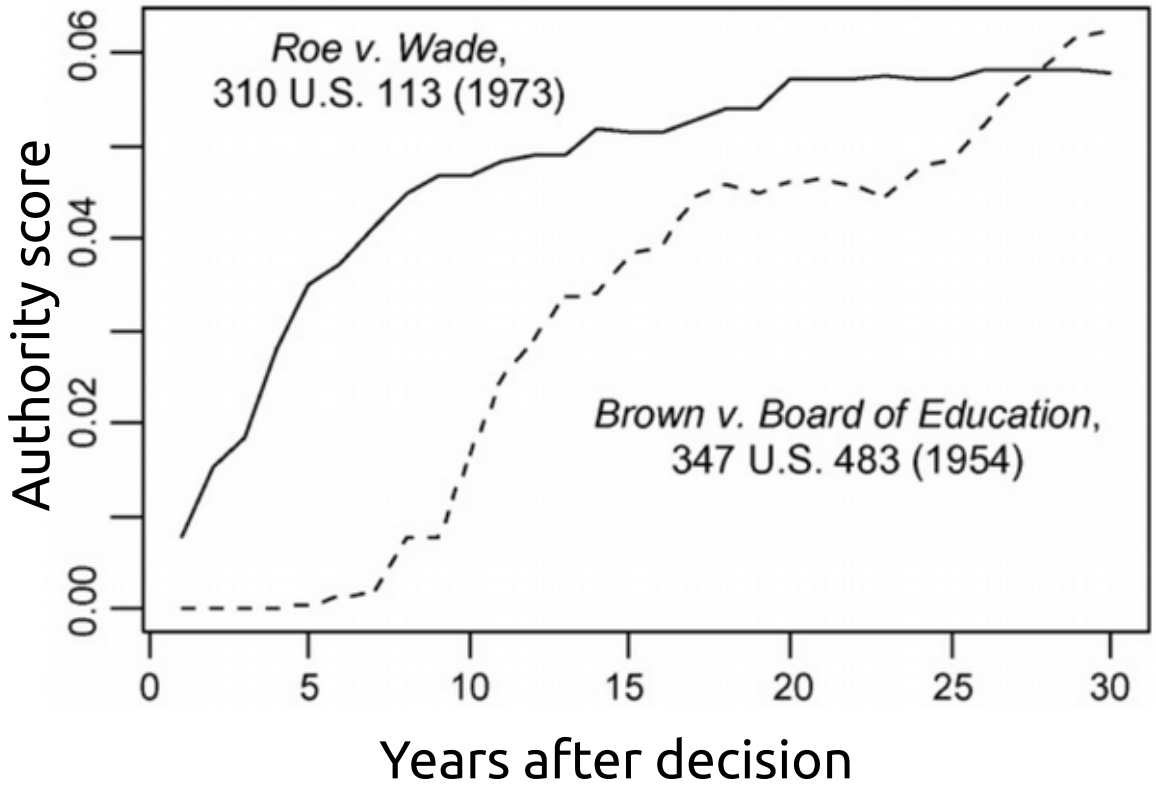



What happens after n iterations? Which community ”wins”?

# Hubs and authorities: not just for the web

- Citations in US Supreme Court Cases
- Different cases acquired authority at different speeds

(Roe v Wade legalized abortion, Brown v Board of Education declared race-segregated schools unconstitutional)




Fowler, J. H., & Jeon, S. (2008). The authority of Supreme Court precedent. Social networks, 30(1), 16-30.