Hubs and Authorities

Introduction to Network Science
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Topic 07

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Sources

- Networks, Crowds, and Markets Ch 14
- Fei Li's lecture on PageRank
- Evimaria Terzi's lecture on link analysis.
- C. Castillo: Link-based ranking slides 2016

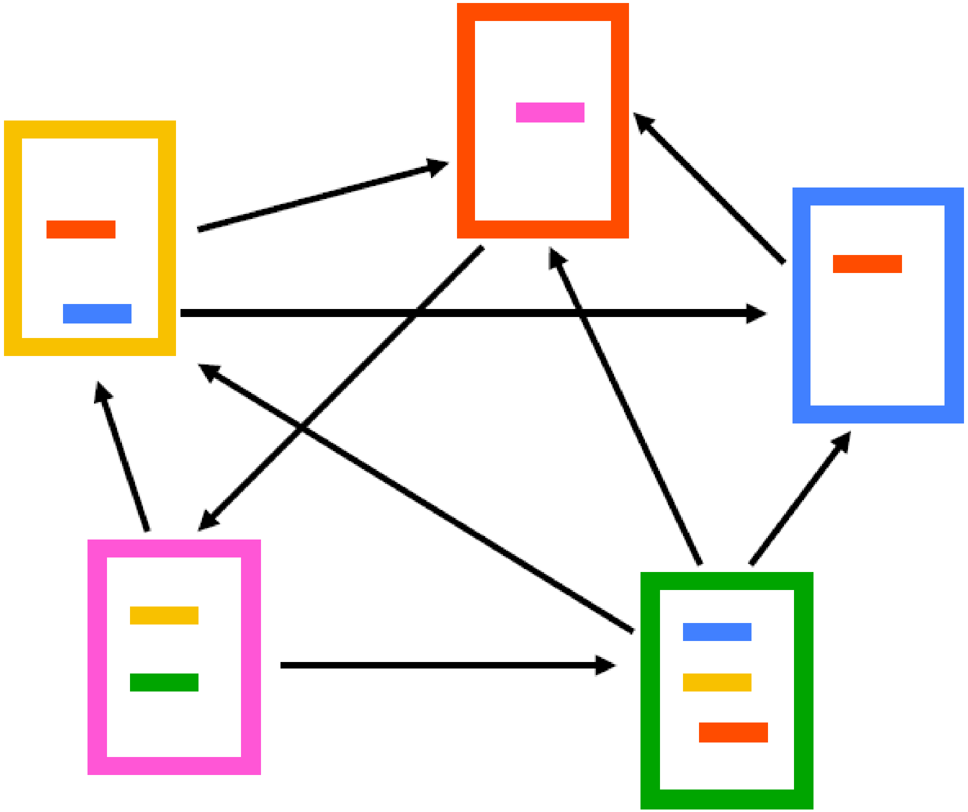
Ranking on the web is hard

- Demand
  - Information needs are unclear and evolving
- Supply
  - From scarcity to abundance: “filter failure”

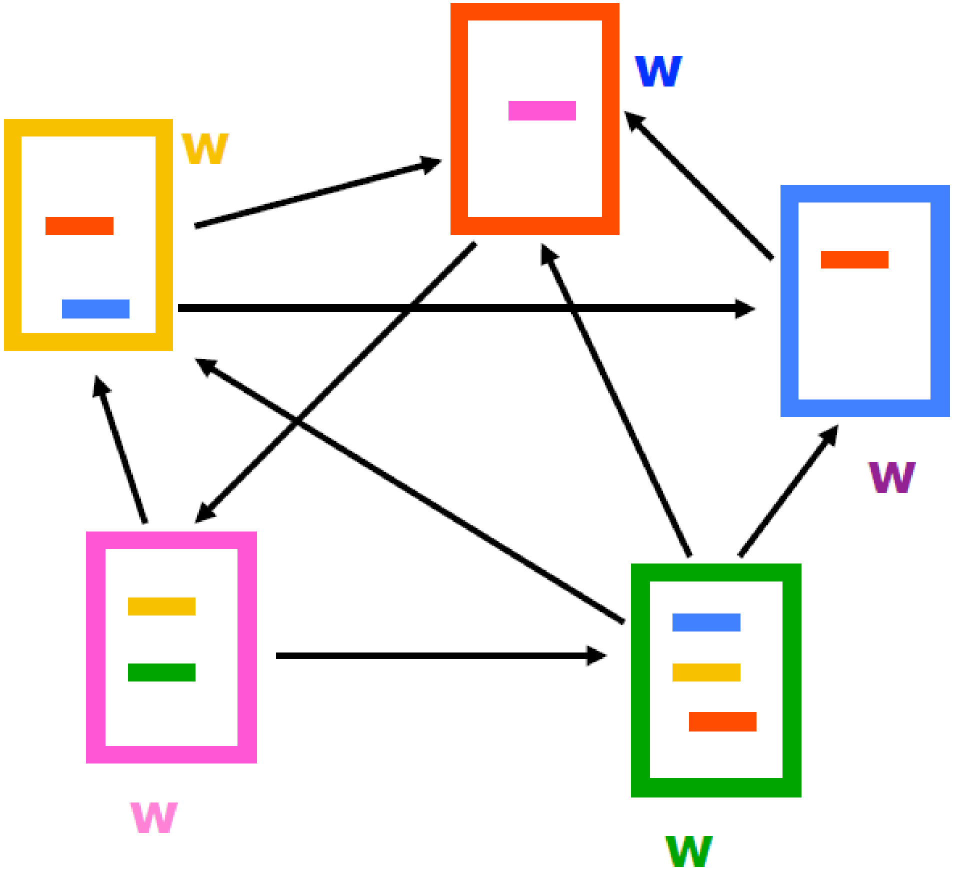
Purpose of Link-Based Ranking

- **Static (query-independent)** ranking
- **Dynamic (query-dependent)** ranking
- Applications:
  - Search in social networks
  - Search on the web

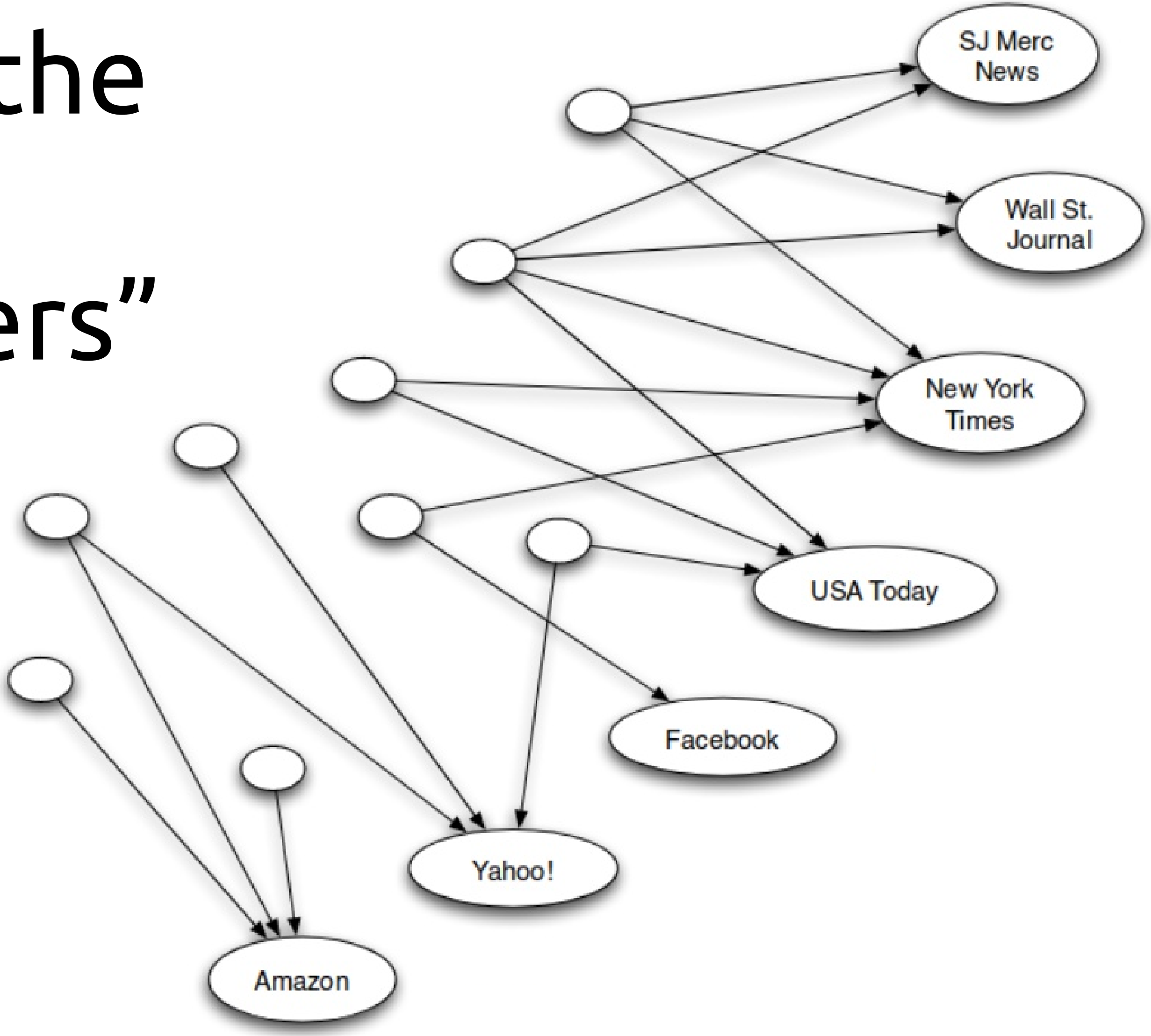
Given a set of connected objects



Assign some weights

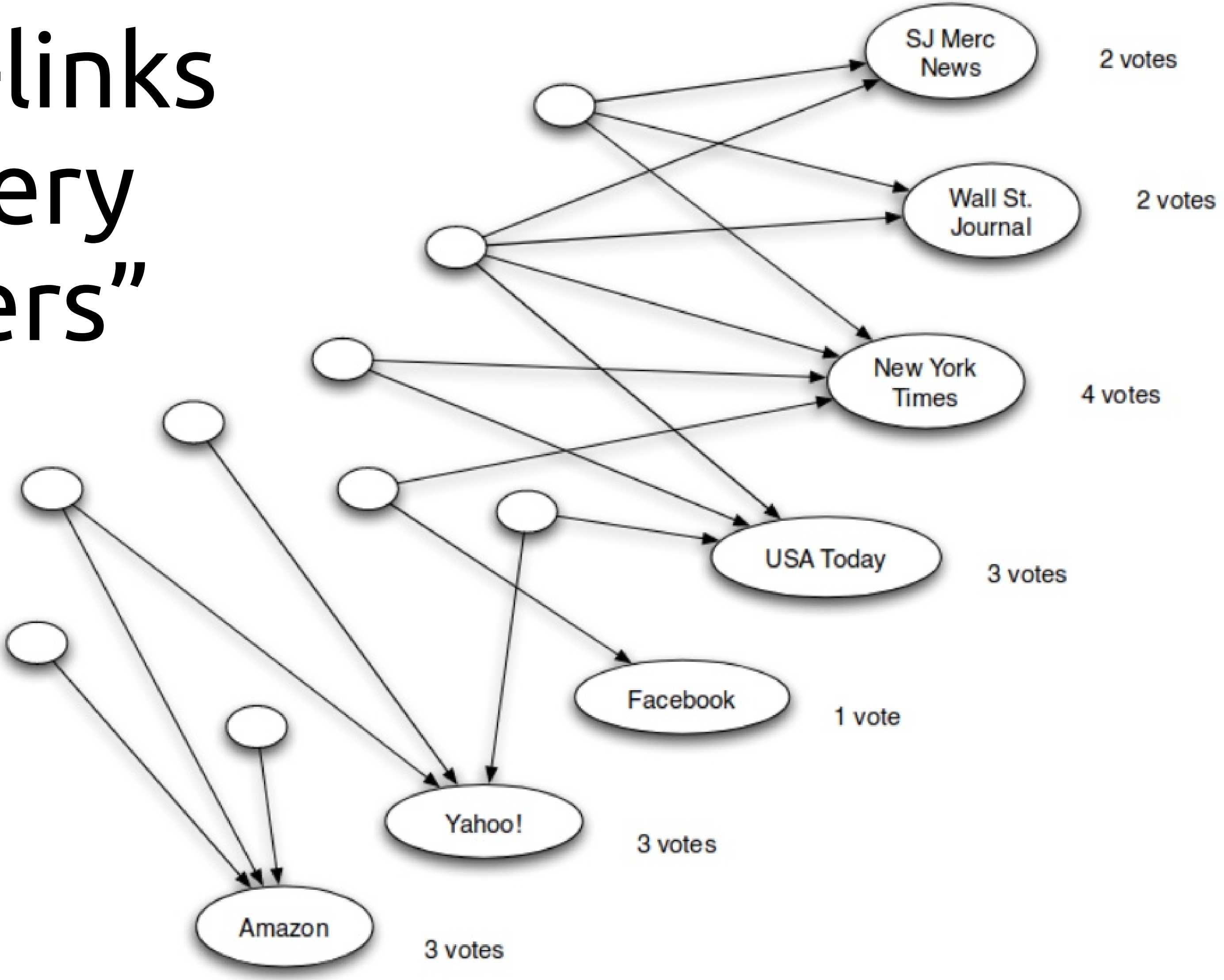


Pages for the query "newspapers"

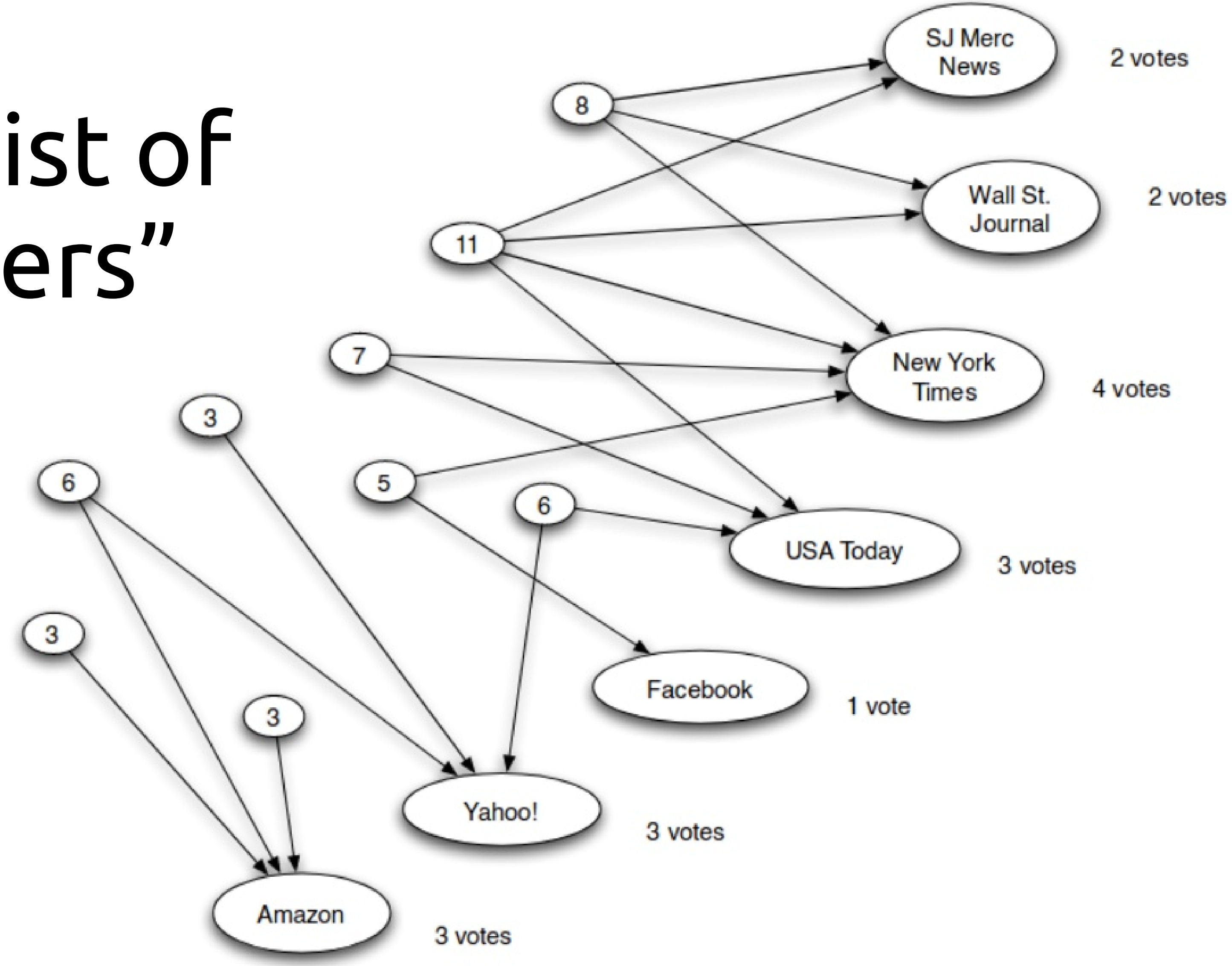


How would you rank these pages?

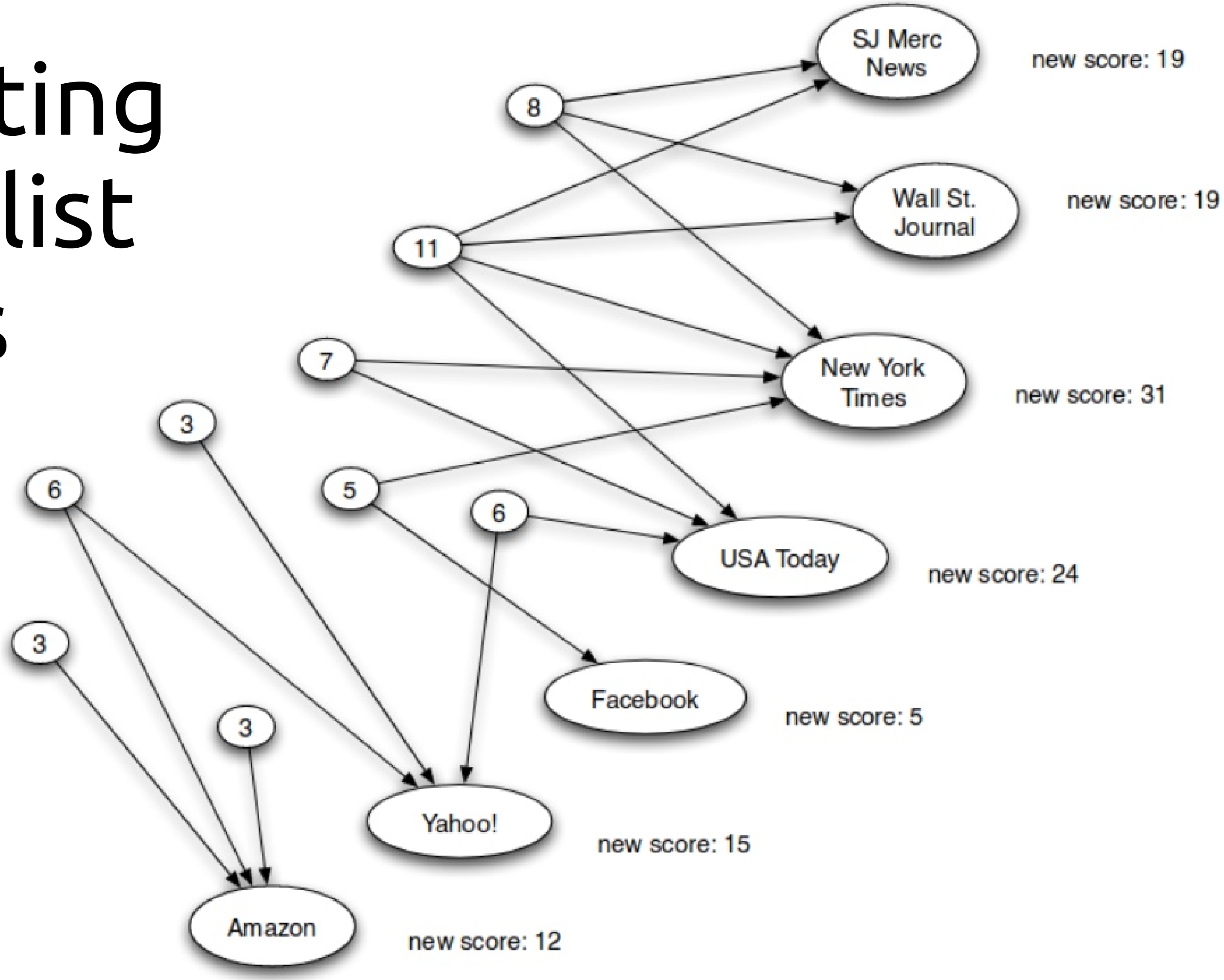
Counting in-links for the query "newspapers"



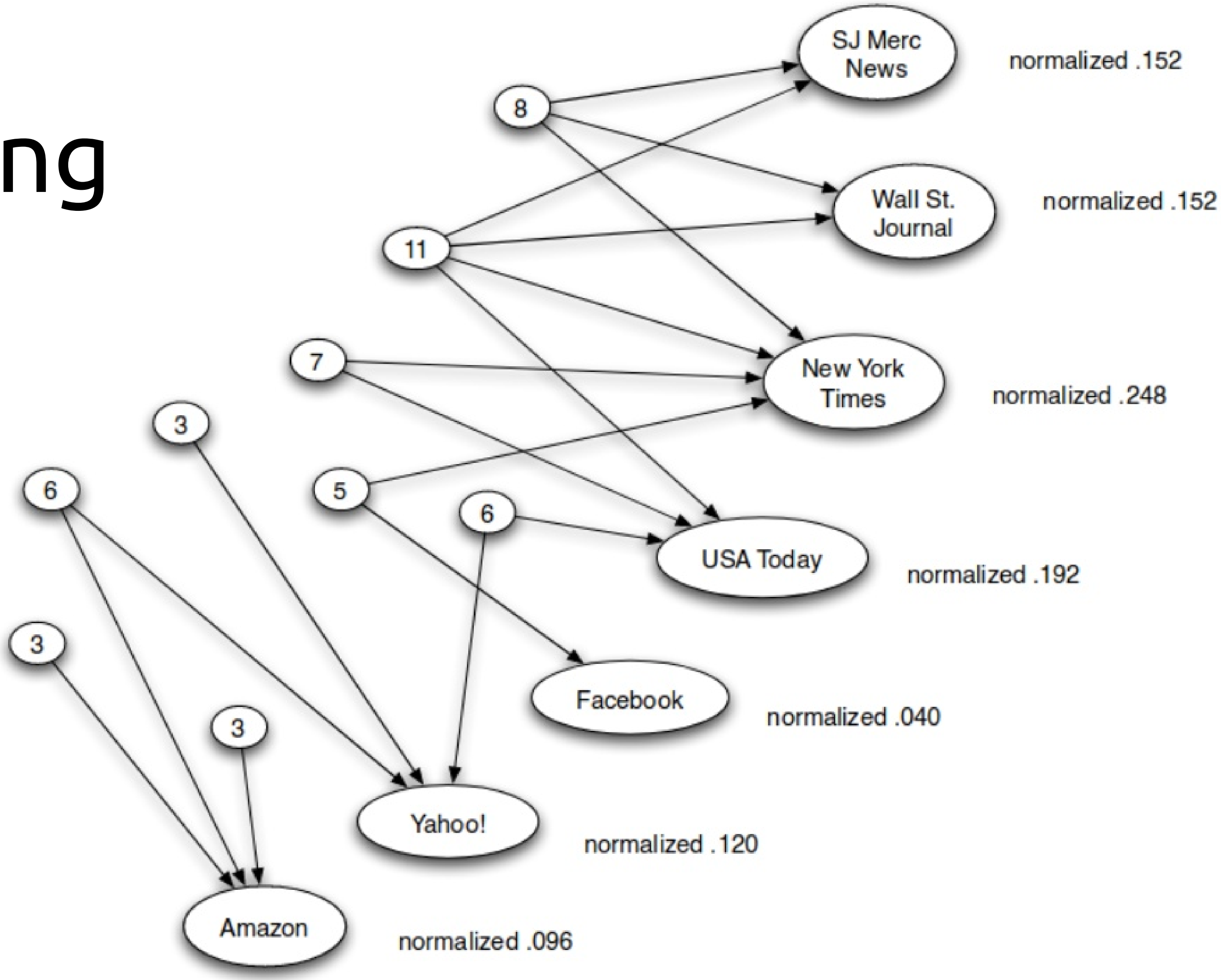
Value of a list of "newspapers"



Re-weighting votes by list values



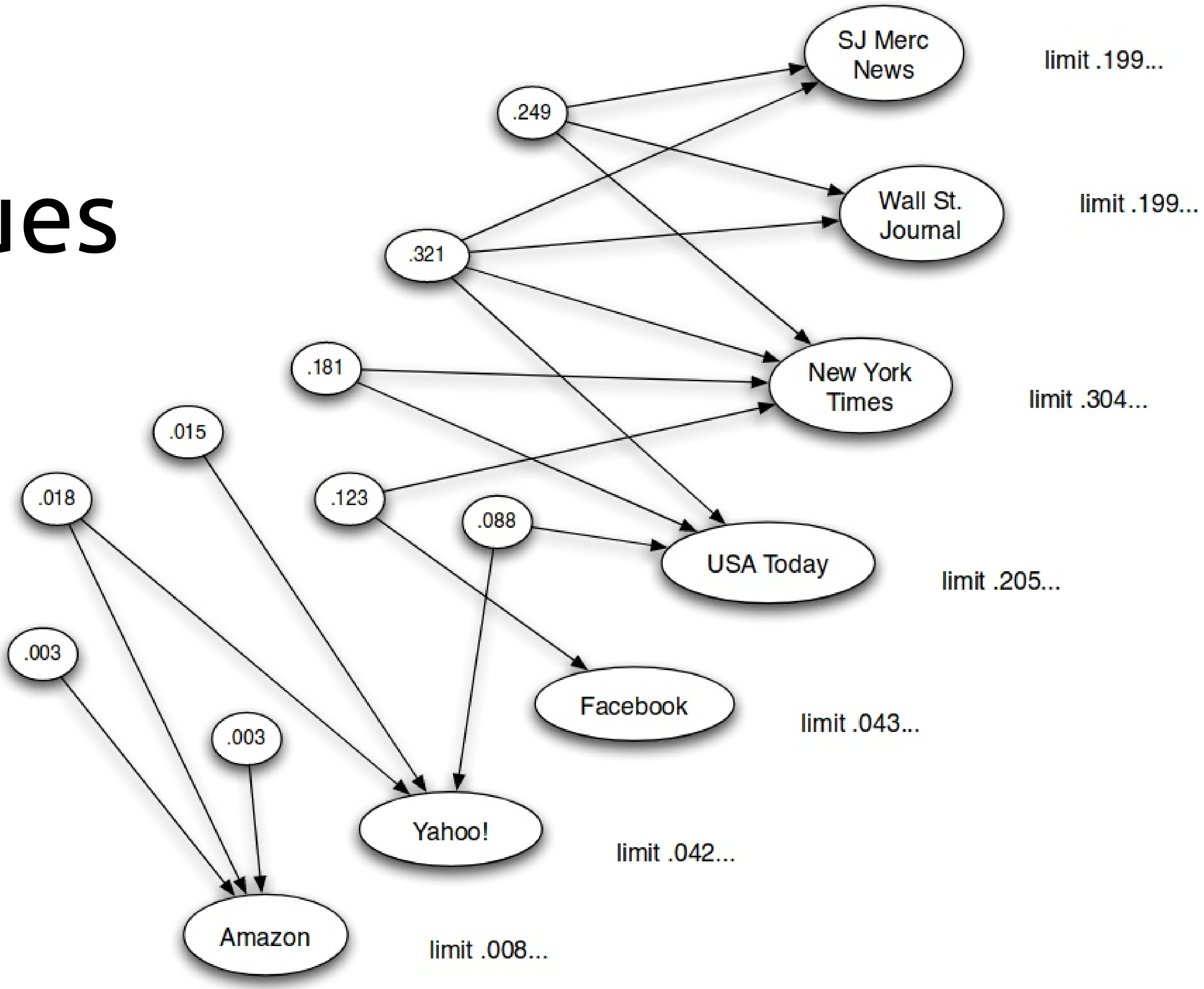
Normalizing scores



The idea behind Hubs and Authorities [Kleinberg 1999]

- Highly-recommended items
  appear in high-value lists
- High-value lists
  contain highly-recommended items
- **Repeated improvement**
  - Re-calculate scores several times

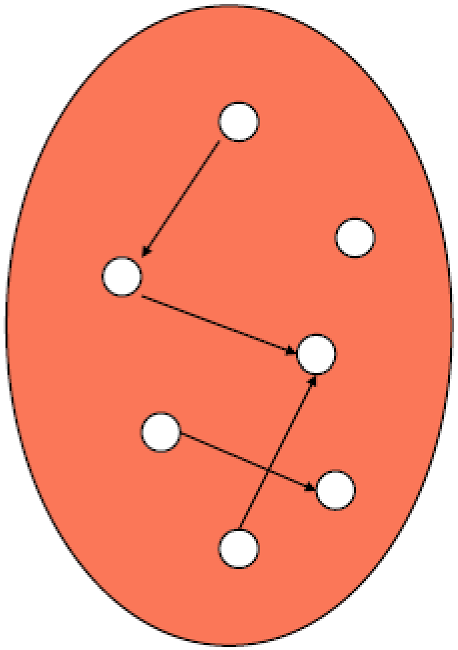
Limit values



This algorithm is called “HITS”

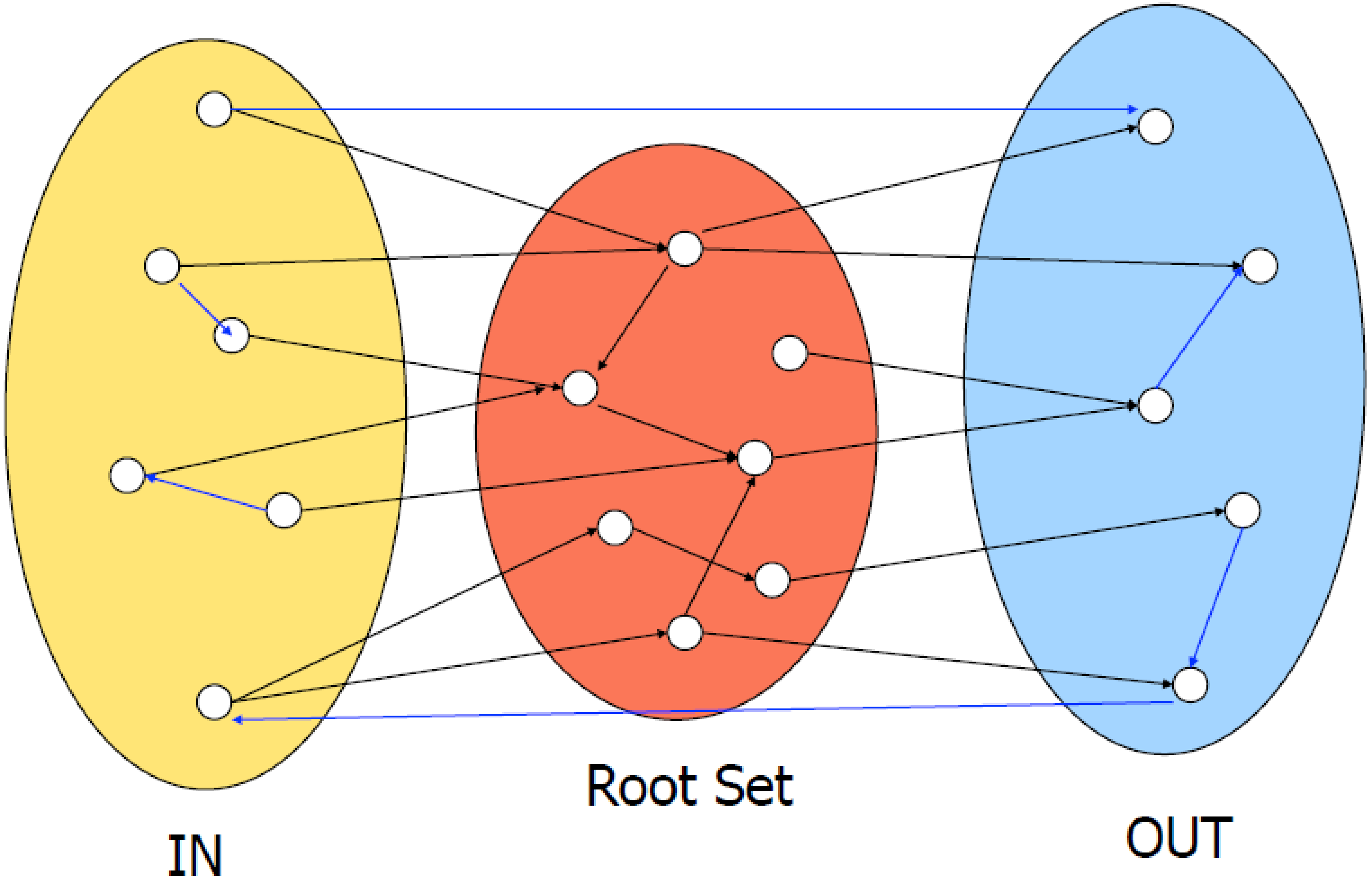
- *Jon M. Kleinberg. 1999. Authoritative sources in a hyperlinked environment. J. ACM 46, 5 (September 1999), 604-632. [DOI]*
- Query-dependent algorithm
  - Get pages matching the query
  - Expand to 1-hop neighborhood
  - Find pages with good out-links (“hubs”)
  - Find pages with good in-links (“authorities”)

Root set = matches the query

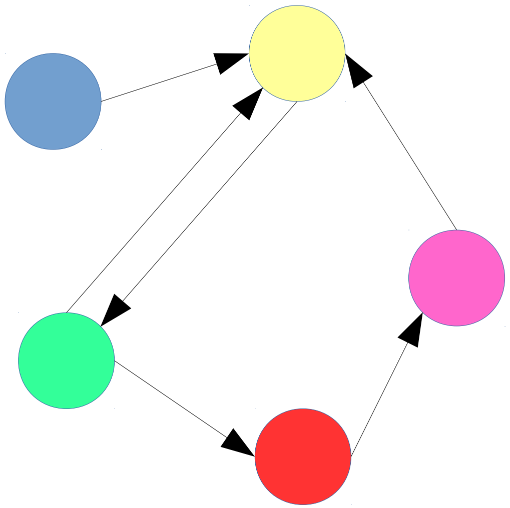


Root Set

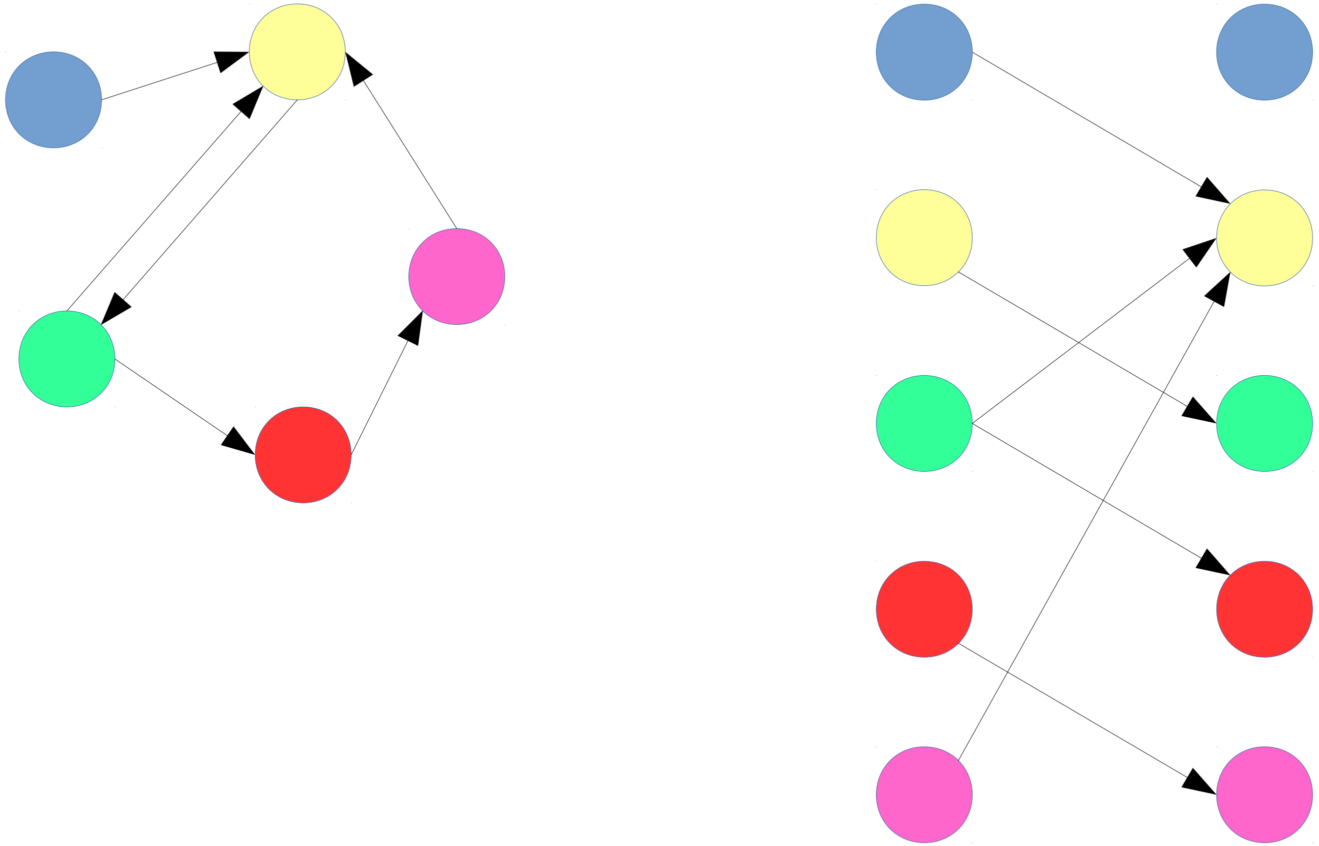
Base set S = root set plus 1-hop neighbors



Base graph S of n nodes



Bipartite graph of *2n* nodes



Bipartite graph of *2n* nodes

0) Initialization:

$$\mathrm{h}_i = \hat{h}_i = 1$$

1) Iteration:

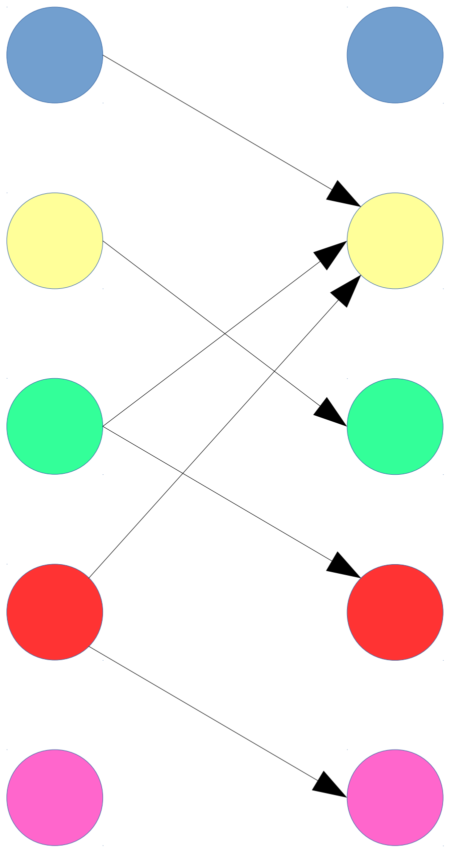
$$a_i = \sum_{j \to i} \hat{h}_j$$

$$h_i = \sum_{i \to j} \hat{a}_j$$

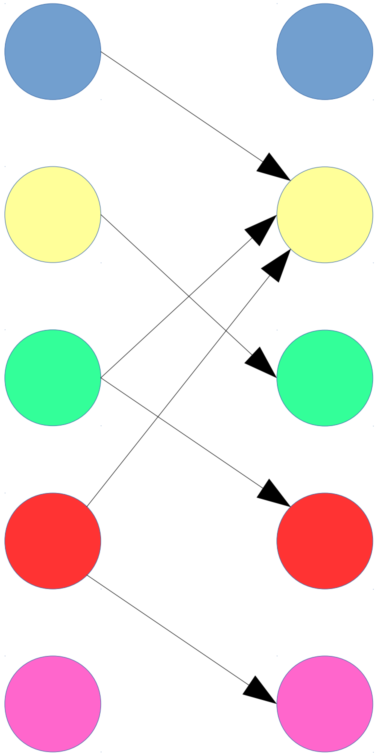
2) Normalization:

$$\hat{a}_i = \frac{a_i}{\sum_j a_j}$$

$$\hat{h}_i = \frac{h_i}{\sum_j h_j}$$



Try it!



$\hat{H}(1)$	$A(1)$	$\hat{A}(1)$	$H(2)$	$\boldsymbol{\hat{H}(2)}$	$A(2)$	$\boldsymbol{\hat{A}(2)}$
1	0					
1	3					
1	1					
1	1					
1	1					

Complete the table. Which one is the biggest hub? Which the biggest authority? Does it differ from ranking by degree?

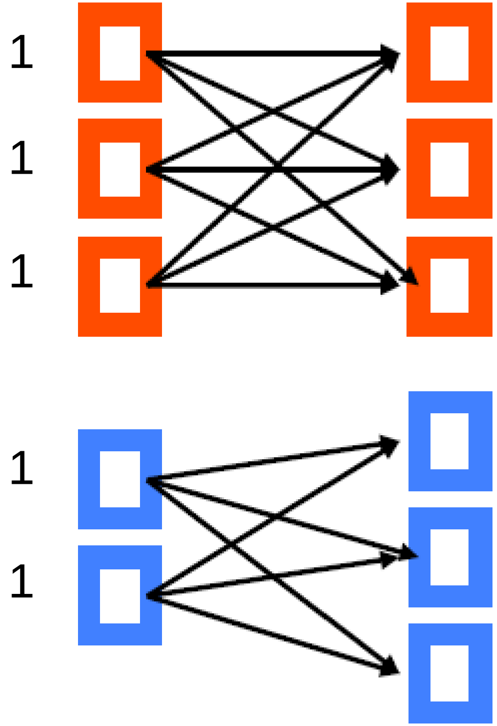
What are we computing?

$$\begin{aligned}
a^t &= A^T h^{t-1} \\
h^t &= A a^t \\
\text{replacing}: \; a^t &= A^T A a^{t-1} \\
\text{after convergence}: \; a &= A^T A a
\end{aligned}$$

- Vector *a* is an eigenvector of $A^T A$
- Conversely, vector *h* is an eigenvector of $A A^T$

Problem: tightly-knit communities

- Example: a graph made of a (3,3)-clique and a (2,3)-clique



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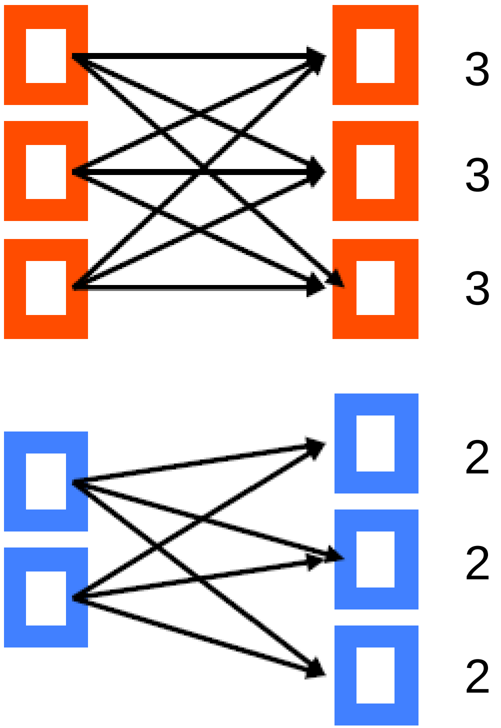

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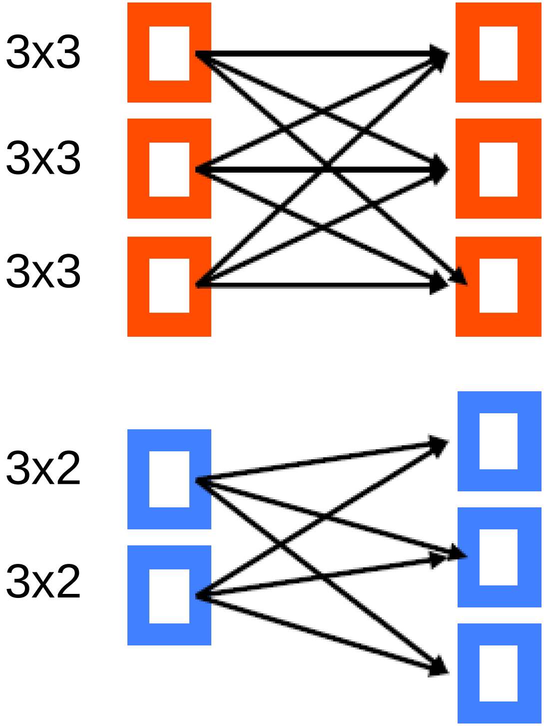

Problem: tightly-knit communities

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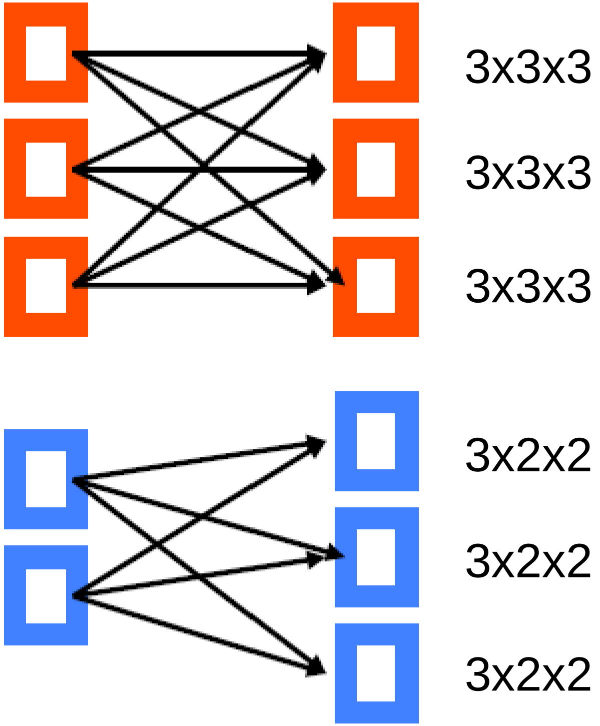
What happens after n iterations?

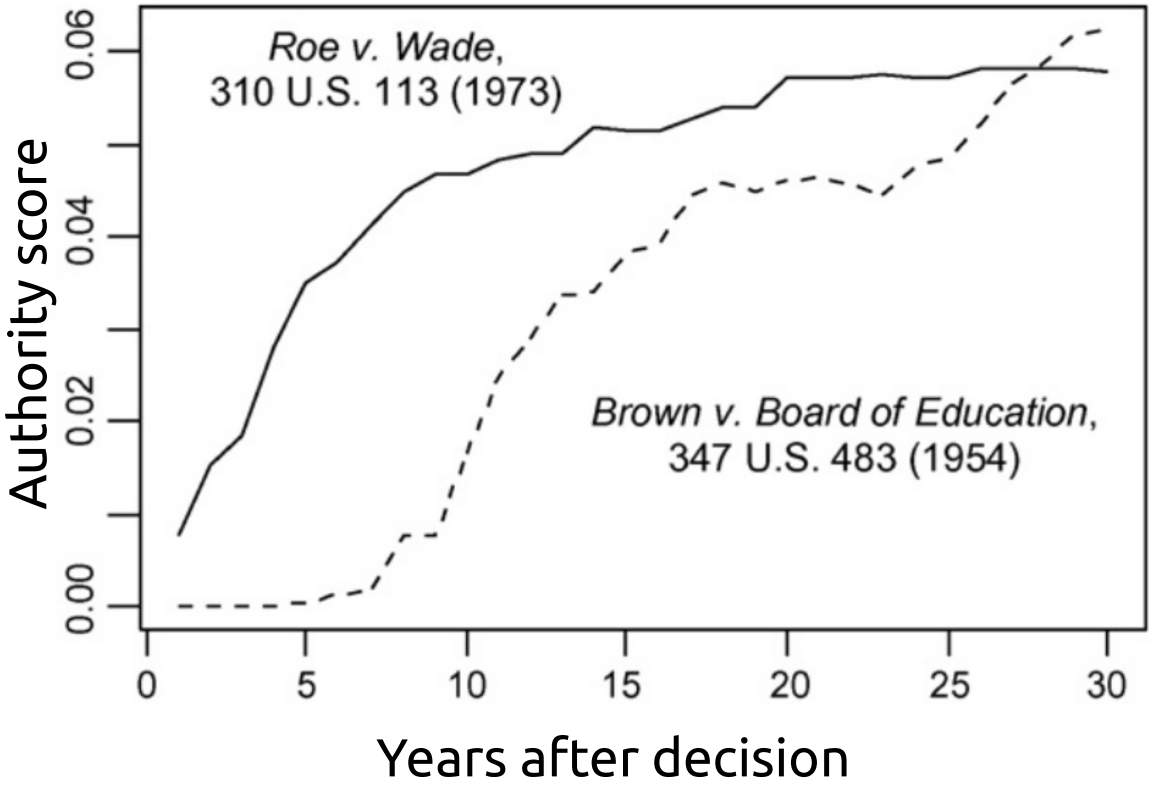
Which community "wins" (i.e., has the largest sum of scores)?



Hubs and authorities: not just for the web

- Citations in US Supreme Court Cases
- Different cases acquired authority at different speeds

(Roe v Wade legalized abortion, Brown v Board of Education declared race-segregated schools unconstitutional)



Fowler, J. H., & Jeon, S. (2008). The authority of Supreme Court precedent. Social networks, 30(1), 16-30.

Summary

Things to remember

- What is the hubs and authority algorithm
- How to execute it step by step
- Practice with graphs on your own