Closeness and betweenness

Introduction to Network Science

Carlos Castillo

Topic 11

upf. Universitat Pompeu Fabra Barcelona

Sources

- Networks, Crowds, and Markets Ch 3.6B
- Barabási 2016 Section 9.3.2
- P. Boldi and S. Vigna: Axioms for Centrality in Internet Mathematics 2014.
- Esposito and Pesce: Survey of Centrality 2015.
- C. Castillo: Other centrality slides 2016

Types of centrality measure

- Spectral
  - HITS
  - PageRank
- **Non-spectral**
  - Degree
  - Closeness and harmonic closeness
  - Betweenness

Is u a well-connected person?

- Degree: u has many connections
- Eigenvector: u is connected to the well-connected
- **Closeness:** u is close to many people
  - Average distance from u is small
- **Betweenness**: many connections pass through u
  - Large number of shortest paths pass through u

Closeness

- Distance between two nodes is d(u,v)
- **Closeness** if the reciprocal of distances

$$\mathrm{closeness}(u) = \frac{1}{\sum_{v \in V, v \neq u} d(u,v)}$$

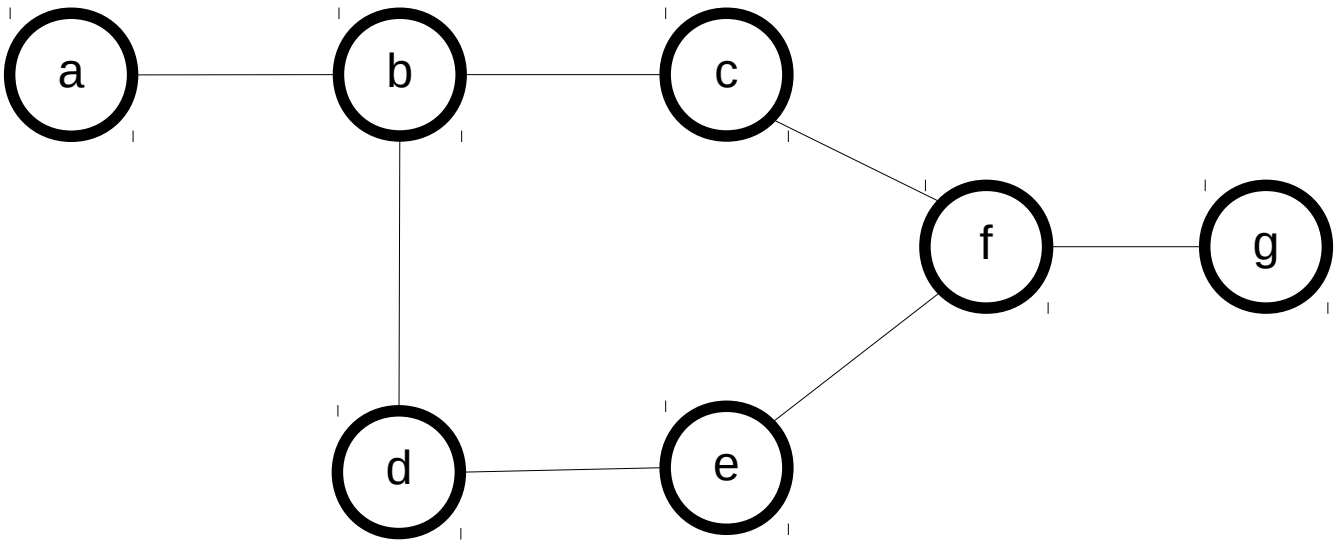
- Some graphs are not connected, in that case d(u,v) can be ∞; setting 1/∞ = 0 one can define the **harmonic closeness**:

$$\mathrm{hcloseness}(u) = \sum_{v \neq u} \frac{1}{d(u,v)}$$

Try it!

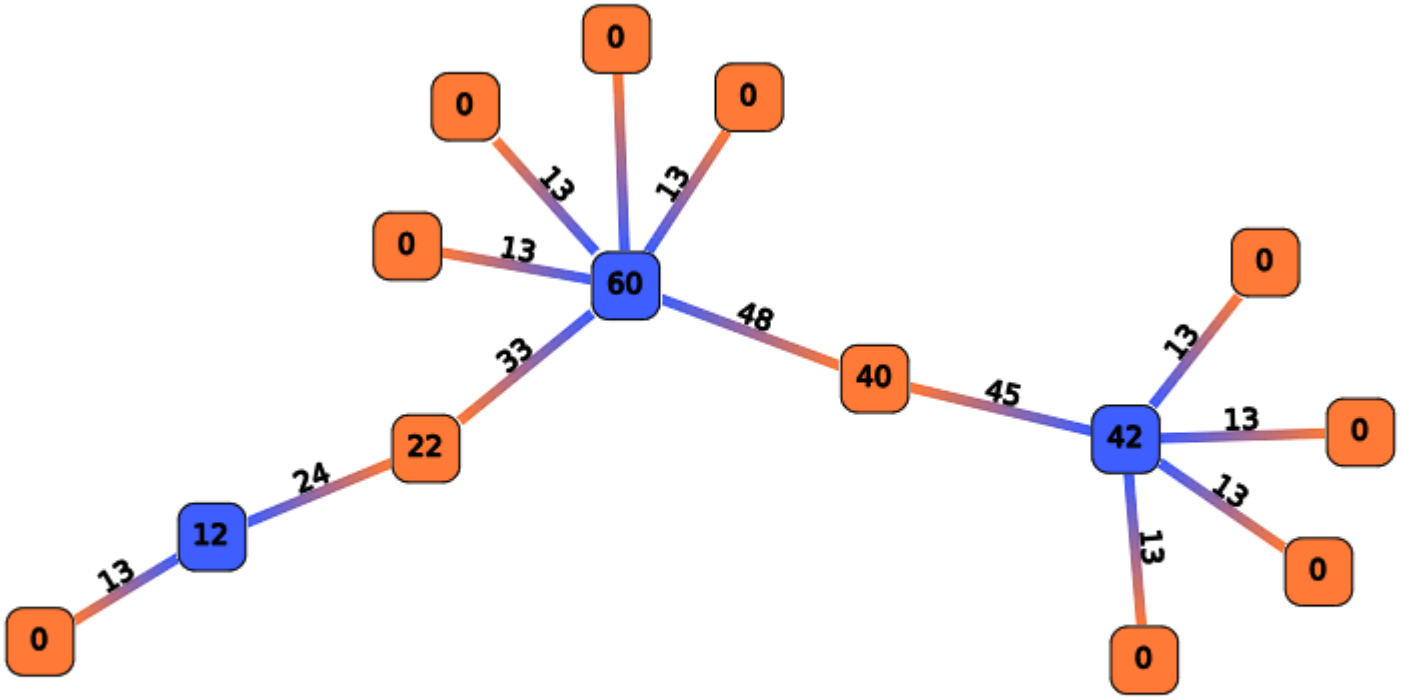
Compute closeness and harmonic closeness for all the nodes

d(u,v) = 1 if v is a neighbor of u



Node Betweenness

A node has high betweenness if it participates in many shortest-paths



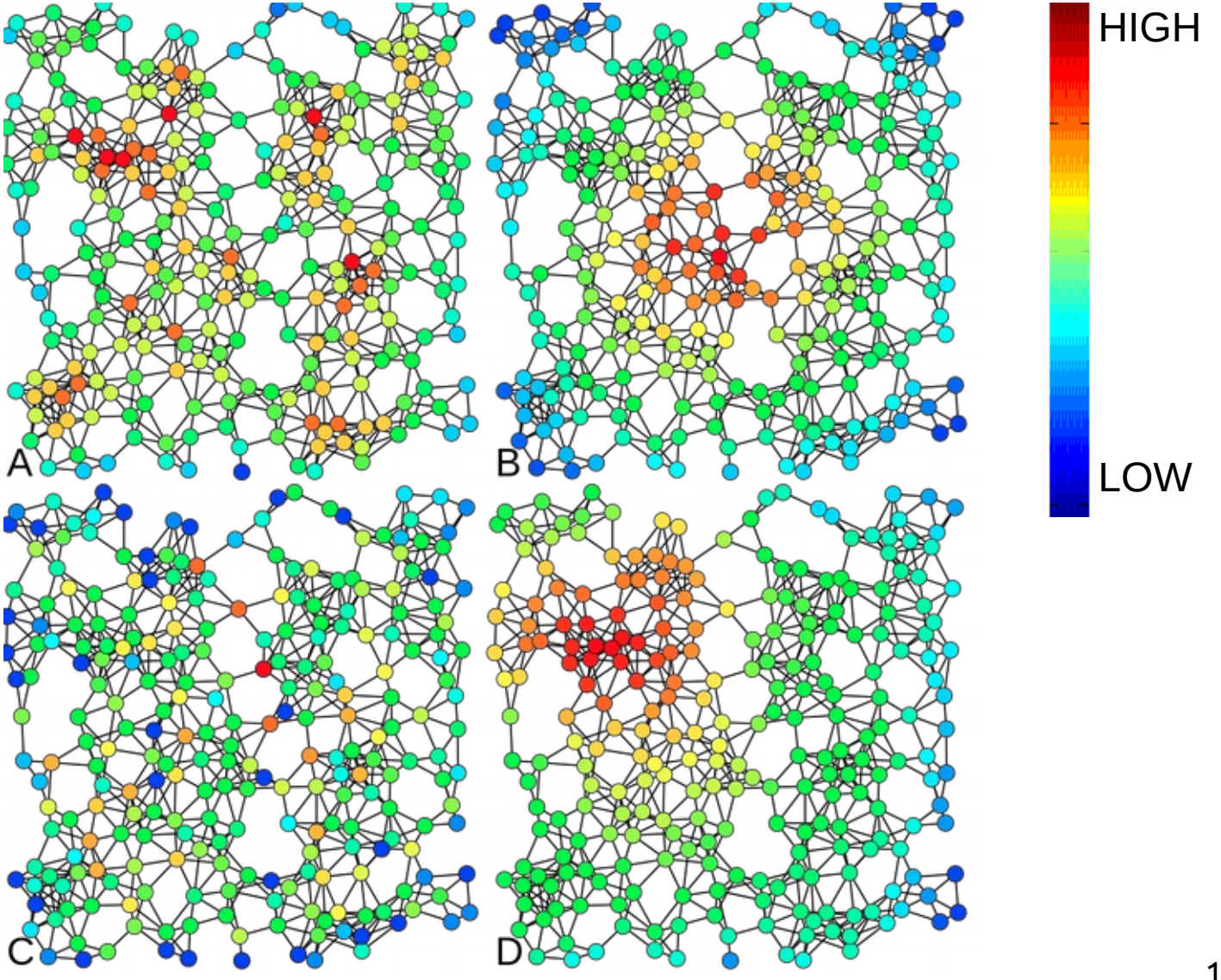
http://rnav.labri.fr/rNAV2.0_public_version/online_doc/_images/ComputeBetweennessCentrality.png

A: Degree

B: Closeness

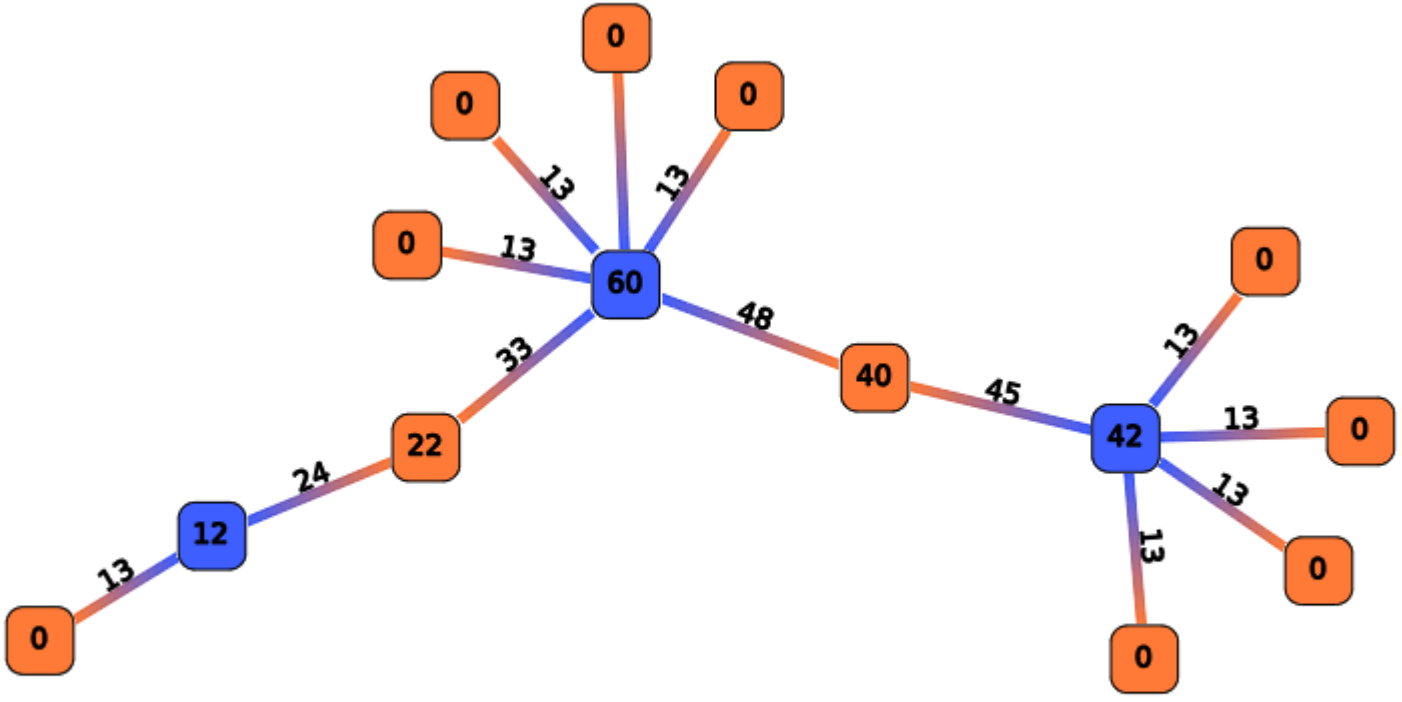
C: Betweenness

D: PageRank



Edge Betweenness

An **edge** has high betweenness if it is part of many shortest-paths … how to compute this efficiently?



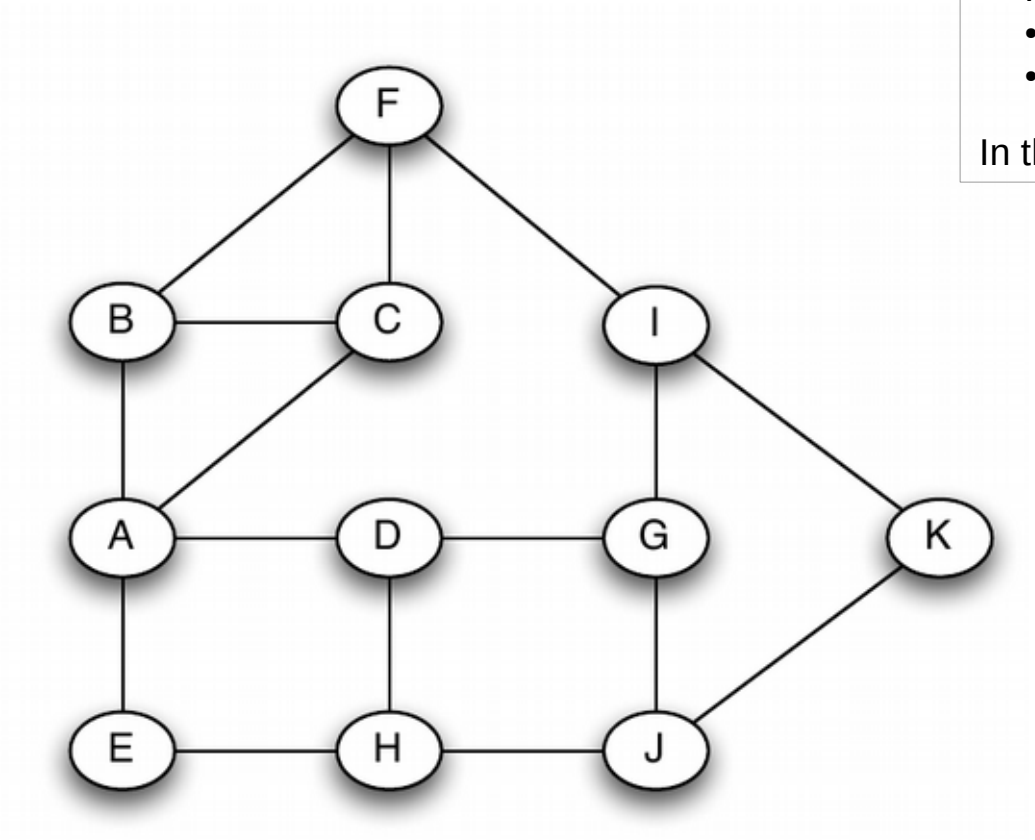
http://rnav.labri.fr/rNAV2.0_public_version/online_doc/_images/ComputeBetweennessCentrality.png

Algorithm [Brandes, Newman]

- For every node u in V
  - Layer the graph performing a BFS from u
  - For every node v in V, v≠u, sorted by layer
    - Assign to v a number s(v) indicating how many shortest paths from u arrive to v
  - For every node v in V, v≠u, sorted by reverse layer
    - Score to distribute = 1 + score from children
    - Add score to parent edges in proportion to s(v)
- In the end divide all edge scores by two

Ulrich Brandes. A faster algorithm for betweenness centrality. Journal of Mathematical Sociology, 25:163–177, 2001.
Mark E. J. Newman. Scientific collaboration networks: II. Shortest paths, weighted networks, and centrality. Physical Review E, 64:016132, 2001.

Example



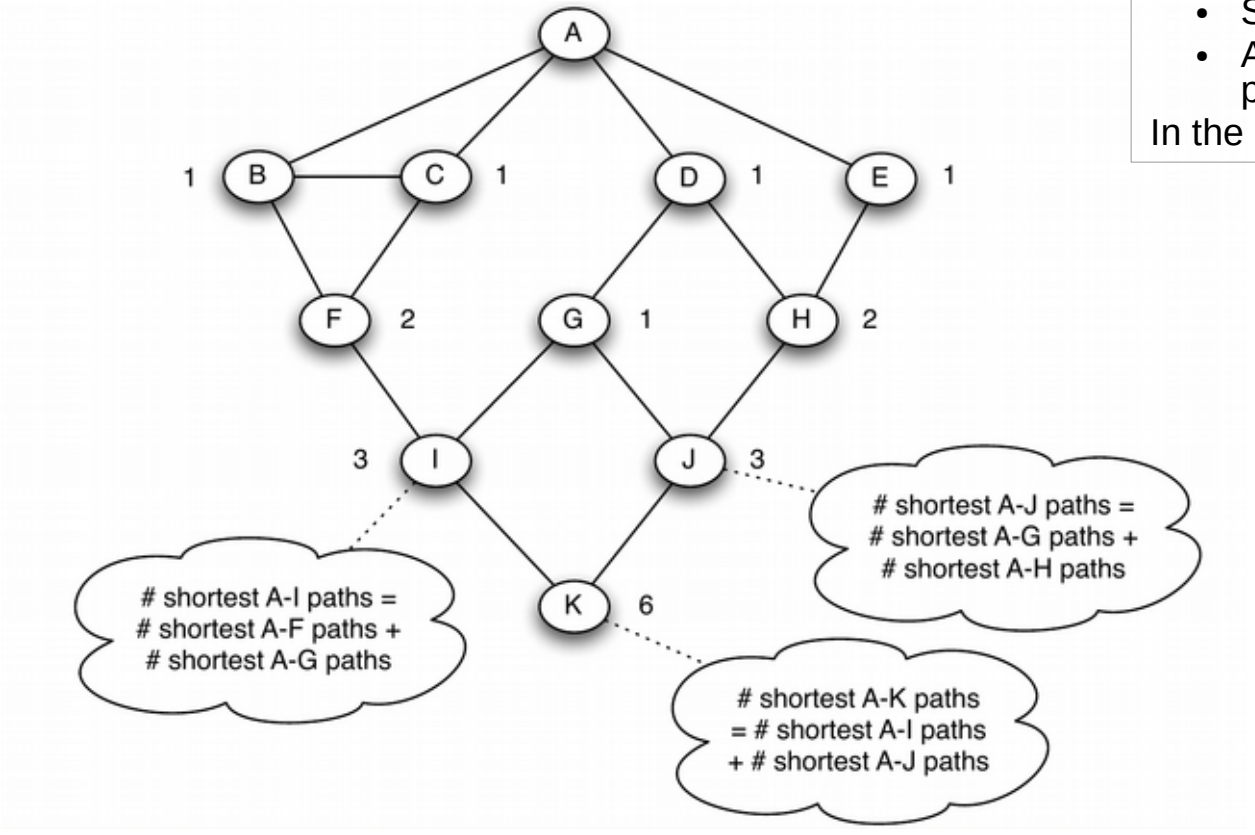
For every node u in V
- Layer the graph performing a BFS from u
- For every node v in V, v≠u, sorted by layer
  - Assign to v a number s(v) indicating how many shortest paths from u arrive to v
- For every node v in V, v≠u, sorted by reverse layer
  - Score to distribute = 1 + score from children
  - Add score to distribute to parent edges in proportion to s(v)

In the end divide all edge scores by two

Example

For every node u in V
- Layer the graph performing a BFS from u
- **For every node v in V, v≠u, sorted by layer**
  - **Assign to v a number s(v) indicating how many shortest paths from u arrive to v**
- For every node v in V, v≠u, sorted by reverse layer
  - Score to distribute = 1 + score from children
  - Add score to distribute to parent edges in proportion to s(v)

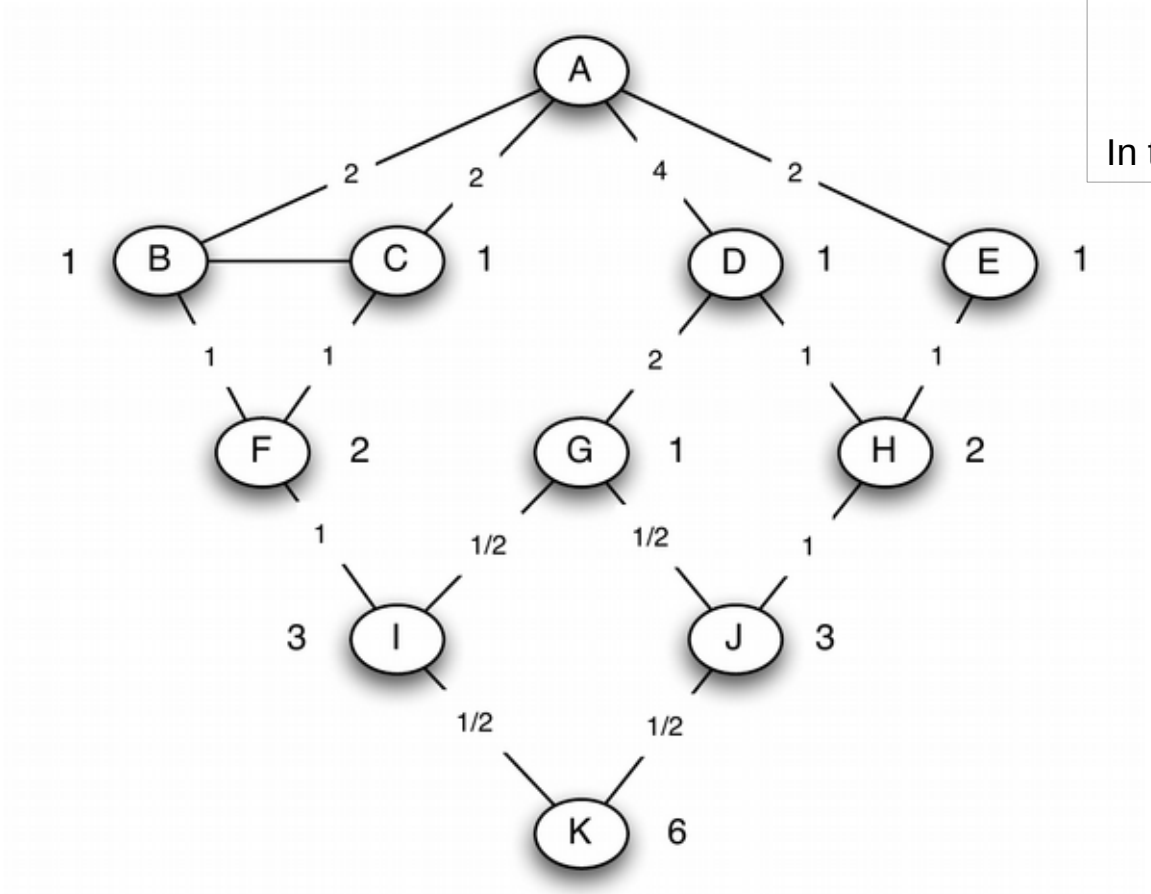
In the end divide all edge scores by two



All nodes in layer 1 get s(v)=1

Remaining nodes: simply add s(.) of their parents

Example



For every node u in V
- Layer the graph performing a BFS from u
- For every node v in V, v≠u, sorted by layer
  - Assign to v a number s(v) indicating how many shortest paths from u arrive to v
- **For every node v in V, v≠u, sorted by rev. layer**
  - **Score to distribute = 1 + score from children**
  - **Add score to distribute to parent edges in proportion to s(v)**

In the end divide all edge scores by two

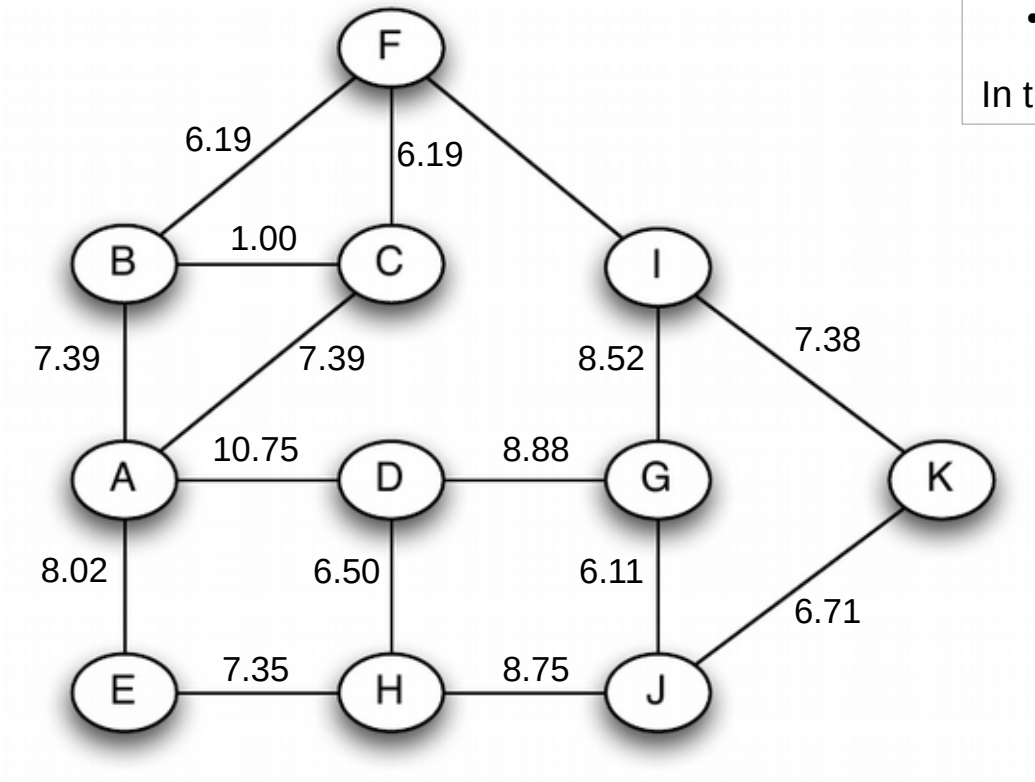
Nodes without children distribute a score of 1

Other nodes distribute 1 + whatever they receive from their children

Example

For every node u in V
- **Layer the graph performing a BFS from u**
- For every node v in V, v≠u, sorted by layer
  - Assign to v a number s(v) indicating how many shortest paths from u arrive to v
- For every node v in V, v≠u, sorted by reverse layer
  - Score to distribute = 1 + score from children
  - Add score to distribute to parent edges in proportion to s(v)

In the end divide all edge scores by two



Computed using NetworkX (edge betweenness)

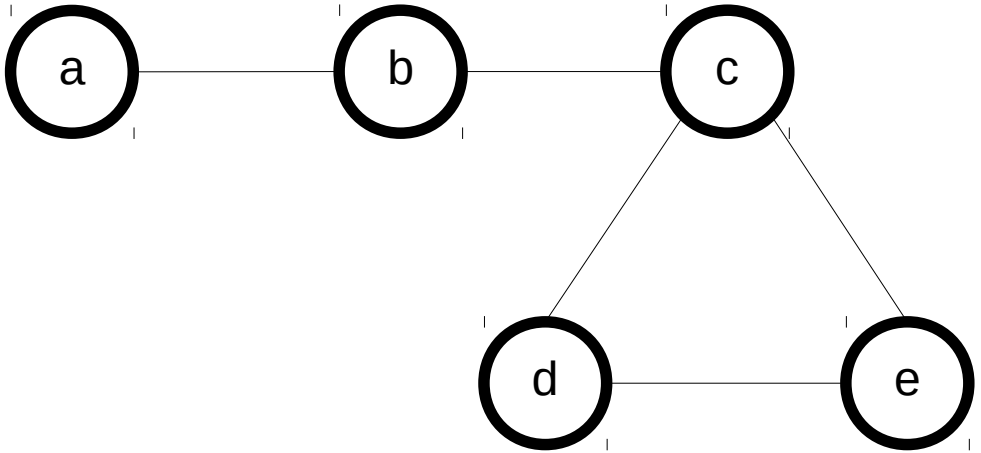
Try it!

Try to compute it by inspection first

Then use the algorithm;
you should get the same results

For every node u in V
- Layer the graph performing a BFS from u
- For every node v in V, v≠u, sorted by layer
  - Assign to v a number s(v) indicating how many shortest paths from u arrive to v
- For every node v in V, v≠u, sorted by reverse layer
  - Score to distribute = 1 + score from children
  - Add score to distribute to parent edges in proportion to s(v)

In the end divide all edge scores by two



Application: the Girvan-Newman algorithm (more on this later)

- Repeat:
  - Compute edge betweenness
  - Remove edge with larger betweenness