MATHS ASSIGNMENT-1

DIFFERENTIAL EQ'S

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1st Q&A

i) $(y')^3 = \sqrt{y^2+1}$

$\left(\frac{dy}{dx}\right)^3 = \sqrt{y^2+1}$

$\left(\frac{dy}{dx}\right)^6 = y^2+1$

* It is ordinary, linear differential eqn

* order = 1
degree = 6

ii) $y'' + (y')^2 = \sin x$

$\frac{d^2y}{dx} + \left(\frac{dy}{dx}\right)^2 = \sin x$

* Non linear, ordinary

* order = 2
degree = 2

iii) $y^4 + 3(y')^3 + y = 0$

$\left(\frac{dy}{dx}\right)^4 + 3\left(\frac{dy}{dx}\right)^3 + y = 0$

* Non-linear, ordinary differential eqn

* order = 4
degree = 1

iv) $x\frac{d^2y}{dx^2} + u\frac{du}{dy} = 0$.

sol: * Nonlinear, partial differential eqn

* order = 2
degree = 1

v) $y' + xy = e^x$

sol: $\frac{dy}{dx} + xy = e^x$

* linear, ordinary differential eqn

* order = 1
* degree = 1

vi) $x^2dy + y^2dx = 0$

$x^2dy = -y^2dx \Rightarrow \frac{dy}{dx} + \frac{y^2}{x^2} = 0$

* It is linear, ordinary differential eqn

* order = 1
degree = 1

2nd Q|A

i) $y' = 1+y^2 \quad (\pi/4, 0)$

Sol: $\frac{dy}{dx} = 1+y^2$

$\Rightarrow \int \frac{1}{1+y^2}\,dy = \int dx$

$\Rightarrow \tan^{-1} y = x + c$

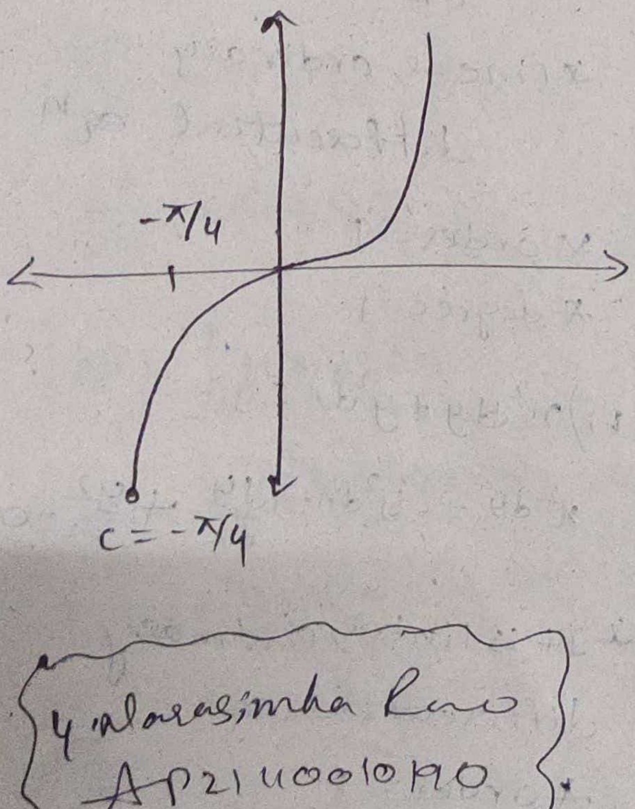
$y = \tan(x+c$

At $(\pi/4, 0)$

$\tan^{-1}(0) = \pi/4 + c$

$0 = \pi/4 + c$

$\boxed{c = -\pi/4}$



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ii) $yy' + 4x = 0$

$y\frac{dy}{dx} + 4x = 0$

$\int y\,dy = \int -4x\,dx$

$\frac{y^2}{2} = -2x^2 + c$

$y^2 = -4x^2 + c$

At $(1, 1)$

$(1)^2 = -4(1)^2 + c$

$1 + 4 = c \Rightarrow \boxed{c = 5}$

eq^n is

$y^2 = -4x^2 + 5$

$\cancel{\frac{x^2}{1^2} + \frac{y^2}{4} = 5}$

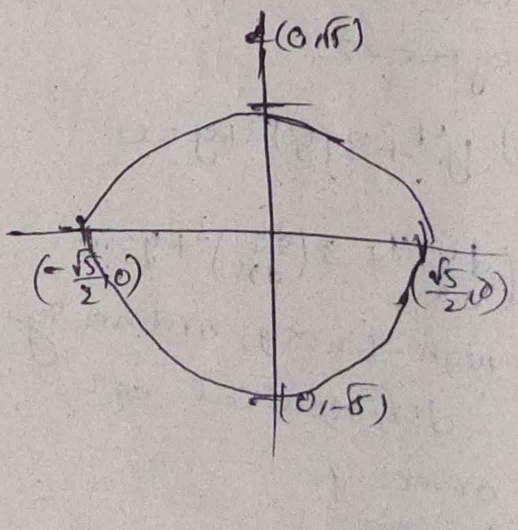
$\frac{x^2}{\left(\frac{5}{4}\right)} + \frac{y^2}{5} = 1$

we can write

$\Rightarrow \frac{x^2}{\left(\sqrt{\frac{5}{4}}\right)^2} + \frac{y^2}{(\sqrt{5})^2} = 1$

ellipse with

$\boxed{a = \pm\frac{\sqrt{5}}{2},\ b = \pm\sqrt{5}}$



3rd QIA

i) $y = y$, $y(0) = 1$, $h = 0.1$

Sol: Formula,

$$y' = F(x, y)$$

$$y_n = y_{n-1} + hF(x_{n-1}, y_{n-1})$$

n-1	x_{n-1}	y_{n-1}	$F(x_{n-1}, y_{n-1})$	y_n
0	0	1	1	1.1
1	0.1	1.1	1.1	1.21
2	0.2	1.21	1.21	1.331
3	0.3	1.332	1.331	1.4641
4	0.4	1.4641	1.4641	1.61051
5	0.5	1.61051	1.64051	1.771561

ii) $y' = |y - x|^2$,

$y(0) = 0$, $h = 0.1$

n-1	x-1	y_{n-1}	$F(x_{n-1}, y_{n-1})$	y_n
0	0	0	0	0
1	0.1	0	0.01	0.001
2	0.2	0.001	0.039601	0.00496
3	0.3	0.00496	0.08704	0.92008
4	0.4	0.092008	0.94858	0.101493
5	0.5	0.101493	0.158807	0.1173789

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{4th Q/A}

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4. i) $\frac{dy}{dx} = \frac{3x^2}{1-e^{-y}}$

$(1-e^{-y})\,dy = 3x^2\,dx$

$\int(1-e^{-y})\,dy = \int 3x^2\,dx$

$\int 1\,dy - \int e^{-y}\,dy = \cancel{3}\left(\frac{x^3}{\cancel{3}}\right)+c$

$y - (-e^{-y}) + c = x^3 + c$

$y + e^{-y} + c = x^3 + c$

$y + e^{-y} = x^3 + c$ is sol^n of d.E.

ii) $\frac{dy}{dx} = \frac{x^3+y^3}{x^2y+xy^2}$

Sol: divide with x^3 $\Rightarrow \frac{dy}{dx} = \frac{1+\frac{y^3}{x^3}}{\left(\frac{y}{x}\right)+\left(\frac{y}{x}\right)^2}$

let $u = \frac{y}{x} \Rightarrow \frac{du}{dx} = \frac{x\frac{dy}{dx} - y}{x^2}$

$\Rightarrow \frac{du}{dx} = \frac{1}{x}\frac{dy}{dx} - \frac{y}{x^2}$

$\Rightarrow \frac{du}{dx} = \frac{1}{x}\left(\frac{dy}{dx} - \frac{y}{x}\right) \Rightarrow x\frac{du}{dx} = \frac{dy}{dx} - u$

$\Rightarrow x\frac{du}{dx} + u = \frac{dy}{dx}$ sub in eq 1

$\Rightarrow u + \frac{du}{dx}x = \frac{1+u^3}{u+u^2} \Rightarrow x\frac{du}{dx} = \frac{1+u^3}{u+u^2} - u$

$x\frac{du}{dx} = \frac{1+\cancel{u^3}-u^2-\cancel{u^3}}{u+u^2} \Rightarrow \frac{1-u^2}{u(1+u)} \Rightarrow \frac{(1+u)(1-u)}{u(1+u)}$

$x\frac{du}{dx} = \frac{1-u}{u} \Rightarrow \int \frac{u}{1-u} du = \int \frac{1}{x} dx$

$\Rightarrow \int \frac{1+u-1}{1-u} du = \int \frac{1}{x} dx \Rightarrow \int \frac{1}{1-u} du + \int \frac{-(1-u)}{1-u} du$

$= \int \frac{1}{x} dx$

$\Rightarrow -\log(1-u) - u = \log x + c$

$\Rightarrow -\log(1-u) - u = \log x + \log c$

$\Rightarrow -\log(1-u) - u = \log xc$

$u = -(\log xc + \log(1-u))$

$u = -\log(xc(1-u))$

we know

$u = \frac{y}{x}$

$\frac{y}{x} = -\log(xc(1-\frac{y}{x})) \Rightarrow y = -x\log(xc(\frac{x-y}{x}))$

$\Rightarrow \boxed{y = -x\log(c(x-y))}$

iii) $\frac{dy}{dx} = \frac{x+2y+3}{2x+y+3}$

let us assume $a_1 = 1, b_1 = 2, c_1 = 3$
$a_2 = 2, b_2 = 1, c_2 = 3$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

let $x = X+h$	$y = Y+k$
$dx = dX$	$dy = dY$

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Sub in eq ①

$\frac{dY}{dX} = \frac{X+2Y+(h+2k+3)}{2X+Y+(2h+k+3)} \rightarrow$ eq ②

For it to be homogenous, $h+2t+3=0 \to (1)$

$2h+t+3=0 \to (2)$

solving (1), (2) we get

$h=t=-1$

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(eq) is $\frac{dy}{dx} = \frac{x+2y}{2x+y}$

case-i: $\frac{dy}{dx} = \frac{x+2y}{2x+y}$

$\int 2x\,dy + \int y\,dy = \int x\,dx + \int 2y\,dx$

$\cancel{2xy} + \frac{y^2}{\cancel{2}} = \frac{x^2}{\cancel{2}} + \cancel{2yx}$

$\boxed{x=y}$

Now,

$\cancel{x-h=y}-k$

$\cancel{x}+\cancel{1}=y+\cancel{1}$

$\boxed{x=y}$

$u + x\frac{du}{dx} = \frac{1+2u}{2+u}$

$x\frac{du}{dx} = \frac{1+2u}{u+2} - u$

$x\frac{du}{dx} = \frac{1+\cancel{2u}-u^2-\cancel{2u}}{u+2}$

$x\frac{du}{dx} = \frac{u^2-1}{-(u+2)}$

$\Rightarrow \int \frac{-u-2}{u^2-1}\,du = \int \frac{1}{x}\,du \to (3)$

LHS $\int \frac{(-u-2)}{u^2-1}\,du$

case-ii: $\frac{dx}{dy} = \frac{x+2y}{2x+y}$

let, $y=ux$

$dy = u\,dx + x\,du$

we get,

$\frac{u\,dx + \cancel{x}du}{dx} = \frac{\cancel{x}(1+2\frac{y}{x})}{\cancel{x}(2+\frac{y}{x})}$

$\frac{-u-2}{u^2-1} = \frac{A}{u+1} + \frac{B}{u-1}$

$= \frac{(B+A)u + B-A}{(u+1)(u-1)}$

we get $B-A=-2$, $B+A=-1$

Solving we get $A=\frac{1}{2}, B=-\frac{3}{2}$

Then,

$\int \frac{-u-2}{u^2-1}du = \int \frac{1}{2(u+1)}du - \int \frac{3}{2(u-1)}du$

$\Rightarrow \frac{1}{2}\int \frac{1}{u+1} du - \frac{3}{2}\int \frac{1}{u-1} du$

$\Rightarrow \log(u+1)^{1/2} - \log(u-1)^{3/2}$

Sub in eq 3

$\log(u+1)^{1/2} - \log(u-1)^{3/2} = \log x + \log e^c$

$\log \frac{(1+u)^{1/2}}{(u-1)^{3/2}} = \log x e^c$

$\frac{(1+u)^{1/2}}{(u-1)^{3/2}} = x e^c$

$u = y/x$

$\frac{\left(1+\frac{y}{x}\right)^{1/2}}{\left(\frac{y}{x}-1\right)^{3/2}} = x e^c$

$y/x = \frac{y-k}{x-h} = \frac{y+1}{x+1}$

$\Rightarrow \frac{\left(\frac{y+1}{x+1}+1\right)^{1/2}}{\left(\left(\frac{y+1}{x+1}\right)-1\right)^{3/2}} = x e^c$

solving this we get,

$\frac{x+y+2}{(y-x)^3} = c$

sol^n $\frac{x+y+2}{(y-x)^3} = c$, $x = y$

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iv) $\sec^2 x \tan y \, dx +$
$\sec^2 y \tan x \, dy = 0$

Sol: divide $\tan x \tan y$ B.OS.

$\frac{\sec^2 x}{\tan x} dx + \frac{\sec^2 y}{\tan y} dy = 0$

$\Rightarrow \int \frac{\sec^2 x}{\tan x} dx = \int -\frac{\sec^2 y}{\tan y} dy$ — (1)

LHS

let,
$\tan x = t$
$\sec^2 x \, dx = dt$

then $\int \frac{1}{t} dt \Rightarrow \log t$

$\Rightarrow \log(\tan x)$

Similary RHS

$-\int \frac{\sec^2 y}{\tan y} y \Rightarrow -\log(\tan y)$.

sub in eq (1)

$\log(\tan x) = -\log(\tan y)$

$\log(\tan x \tan y) = \log c$

$\tan x \tan y = c$

v) $\sin^{-1}\frac{dy}{dx} = x+y$

let $x+y = u$

$1+\frac{dy}{dx} = \frac{du}{dx}$

$\frac{du}{dx} - 1 = \frac{dy}{dx}$

$\frac{du}{dx} - 1 = \sin u$

$\frac{du}{dx} = \sin u + 1$

$\int \frac{1}{\sin u+1}\, du = \int dx$

$1-\sin u$ Multiply & Divide LHS

$\int \frac{1-\sin u}{(1+\sin^2 u)}\, du = \int dx$

$\downarrow \cos^2 u$

$\Rightarrow \int \cancel{\sin u}$

$\Rightarrow \int (\sec^2 u - \tan u \sec u)\, du = x + c$

$\Rightarrow \int \sec^2 u\, du - \int \tan u \sec u\, du = x+c$

$\tan u - \sec u = x + c$

$\boxed{\tan(x+y) - \sec(x+y) = x + c}$

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vi) $\cancel{x^2 dy + y^2 dx = 0}$

$(\cos x + y\sin x)\,dx = \cos x\, dy;$

$y(\pi) = 0.$

Sol: Divide $\cos x$ on B.S.

$(1+y\tan x)\,dx = dy$

$\frac{dy}{dx} = 1 + y\tan x$

$\frac{dy}{dx} - y\tan x = 1$

Here, $P(x) = -\tan x$, $Q(x) = 1$

$I.F = e^{\int P(x)\, dx}$

$\Rightarrow e^{\int -\tan x\, dx} \Rightarrow e^{\cancel{-\sec x}}$

$\Rightarrow e^{\log_e|\sec x|^{-1}} \Rightarrow \sec^{-1} x$

$\Rightarrow \cos x$

Sol^n $\Rightarrow y = \frac{1}{\cos x}\int \cos x \cdot Q(x)\, dx$

$\Rightarrow y = \frac{1}{\cos x}\left(\int \cos x \cdot 1 \cdot dx\right.$

$\Rightarrow y = \frac{1}{\cos x}(-\sin x) + c$

$\Rightarrow y = -\tan x + c$

Given $y(\pi) = 0 \Rightarrow 0 = -\tan\pi + c$

$\boxed{c = 0}$

$\boxed{y = -\tan x}$

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ii) 5th Q1A

~~theorem~~

$(x^2-4xy-2y^2)dx + (y^2-4xy-2x^2)dy = 0.$

~~Here~~

$M(x,y) = x^2 - 4xy - 2y^2$

$N(x,y) = y^2 - 4xy - 2x^2$

$\frac{\partial M}{\partial y} = -4x - 4y$

$\frac{\partial N}{\partial x} = -4y - 4x$

~~Here~~ $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ so it is exact.

we know,

$\frac{\partial F}{\partial x} = M(x,y)$

$\frac{\partial F}{\partial y} = N(x,y)$

$\int \partial F = \int M(x,y)\,dx$

$F = \int (x^2 - 4xy - 2y^2)\,dx$

$F = \frac{x^3}{3} - 2x^2y - 2y^2x + \phi(y) \rightarrow (1)$

$\frac{\partial F}{\partial y} = -2x^2 - 4xy + \phi'(y)$

$\cancel{\int \partial F = \int (-4x - 4xy + \phi'(y))\,dy}$

$y^2 - 4xy - 2x^2 = -2x^2 - 4xy + \phi'(y)$

$\phi'(y) = y^2$

$\int \phi'(y) = \int y^2\,dy$

$\phi(y) = \frac{y^3}{3}$

Sub eq (1)

$$\boxed{F = \frac{x^3}{3} - \frac{y^3}{3} - 2xy(x+y)}$$

ii) $4e^{xy}dx + e^{x/y}(1+\frac{x}{y})dy = 0$

Sol: For the above question we can't write the question in $M(x,y)dx + N(x,y)dy = 0$.

So it is Not exact on ODE.

iii) $(r+\sin\theta - \cos\theta)dr + r(\sin\theta + \cos\theta)d\theta = 0$.

Sol: Here

$dx = dr,\ dy = d\theta$

$M(x,y) = r + \sin\theta - \cos\theta$

$N(x,y) = r(\sin\theta + \cos\theta)$

$\frac{\partial M}{\partial\theta} = \frac{d}{d\theta}(r + \sin\theta\ \cos\theta)$

$\Rightarrow \cos\theta + \sin\theta$

$\frac{\partial N}{\partial r} = \frac{d}{dr}(r(\sin\theta + \cos\theta)$

$\Rightarrow \cos\theta + \sin\theta$

$\frac{\partial M}{\partial\theta} = \frac{\partial N}{\partial r}$ so it is an exact ~~soln~~.

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~~Sol:~~ so For sol^n

$\frac{\partial F}{\partial r} = M(x,y)$

$\frac{\partial F}{\partial\theta} = N(x,y)$

$\int \partial F = \int(r + \sin\theta - \cos\theta)\, dr$

$F = \frac{r^2}{2} + r(\sin\theta - \cos\theta) + \phi(\theta)$ ...(1)

$\frac{\partial F}{\partial\theta} = r\cos\theta + r\sin\theta + \phi'(\theta)$

$r\sin\theta + r\cos\theta = r\sin\theta + r\cos\theta + \phi'(\theta)$

$\phi'(\theta) = 0 \Rightarrow \int\phi'(\theta) = \int 0$

$\phi(\theta) = c$

sub in eq (1)

$\boxed{F = \frac{r^2}{2} + r(\sin\theta - \cos\theta) + c}$

iv) $(e^y+1)\cos x\,dx + (e^y\sin x)dy = 0$

Sol:

$M(x,y) = (e^y+1)\cos x$

$N(x,y) = e^y \sin x$

$\frac{\partial M(x,y)}{dy} = e^y\cos x + \cancel{0}$

$\frac{\partial N(x,y)}{dx} = e^y\cos x$

$\frac{\partial M(x,y)}{dy} = \frac{\partial N(x,y)}{dx}$

It is exact

For sol^n

$\frac{\partial F}{dx} = M(x,y)$

$\frac{\partial F}{dy} = N(x,y)$

$\int \partial F = \int M(x,y)dx$

$\int \partial F = \int (e^y+1)\cos x\,dx$ — 1

$F = (e^y+1)\sin x + \phi(y)$

$\frac{\partial F}{dy} = \sin x e^y + \phi'(y)$

$e^y\sin x = \cancel{\sin x} e^y + \phi'(y)$

$\phi'(y) = 0 \Rightarrow \phi(y) = c$

Sub in eq 1

$F = (e^y+1)\sin x + c.$

$\boxed{F = \sin x + e^y\sin x + c}$

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6th Q/A.

i) $y\,dx - x\,dy + (1+x^2)\,dx + x^2 \sin y\,dy = 0$

Sol: Divide by x^2, we get

$\frac{y\,dx - x\,dy}{x^2} + \left(\frac{1}{x^2}+1\right)dx + \sin y\,dy = 0.$

$\Rightarrow -\frac{(x\,dy - y\,dx)}{x^2} + \left(\frac{1}{x^2}+1\right)dx + \sin y\,dy = 0$

here, $\left(\frac{x\,dy - y\,dx}{x^2}\right)$ is a derivative of $\left(\frac{y}{x}\right)$

$\Rightarrow -d\left(\frac{y}{x}\right) + \left(\frac{1}{x^2}+1\right)dx + \sin y\,dy = 0.$

integrate

$\Rightarrow -\int d\left(\frac{y}{x}\right) + \int\left(\frac{1}{x^2}+1\right)dx + \int \sin y\,dy = 0$

$\Rightarrow -\frac{y}{x} + \left(-\frac{1}{x}\right) + x - \cos y + c = 0$

$$\boxed{\frac{y}{x} + \frac{1}{x} - x + \cos y + c = 0}$$

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ii) $(x^2+y^2+x)\,dx + xy\,dy = 0$

Sol:

here,

$M(x,y) = x^2+y^2+x$

$\frac{\partial M}{\partial y} = 2y$

$N(x,y) = xy$

$\frac{\partial N}{\partial x} = y$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

It is not an exact ODE

To make is exact we find integrating factor, $e^{\int P(x)dx}$

$P(x) = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}$

$= \frac{2y-y}{xy} \Rightarrow \frac{y}{xy} = \frac{1}{x}.$

$I.F = e^{\int P(x)dx} = e^{\log x} = x$

$$\boxed{I.F = x}$$

Multiply I.F to ODE, it becomes

$$(x^3+xy^2+x^2)dx + x^2y\,dy = 0$$

Now,

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial y} = 2xy. \text{ It}$$

become exact :

so,

$$\frac{\partial F}{\partial x} = M(x,y)$$

$$\frac{\partial F}{\partial y} = N(x,y)$$

$$\frac{\partial F}{\partial x} = x^3 + xy^2 + x^2$$

$$\int \partial F = \int (x^3+xy^2+x^2)dx$$

$$F = \frac{x^4}{4} + y^2\frac{x^2}{2} + \frac{x^3}{3} + \phi(y) \quad \text{(i)}$$

$$\frac{\partial F}{\partial y} = y\frac{x^2}{2} + \phi'(y)$$

$$xy = x^2y + \phi'(y)$$

$$xy - x^2y = \phi'(y)$$

$$\int \phi'(y) = \int (xy - x^2y)\,dy$$

$$\phi(y) = \frac{xy^2}{2} - \frac{x^2y^2}{2}$$

$$\phi(y) = \frac{1}{2}xy^2(1-x)$$

$$F = \frac{x^4}{4} + \frac{yx^2}{2} + \frac{x^3}{3} + \frac{xy^2}{2}(1-x)$$

$$F = \frac{x^4}{4} + \cancel{\frac{x^2y^2}{2}} + \frac{x^3}{3} + \frac{xy^2}{2} - \cancel{\frac{x^2y^2}{2}}$$

$$\boxed{F = \frac{x^4}{4} + \frac{x^3}{3} + \frac{xy^2}{2}}$$

iii)

$$(x^3y^3+y)dx + 2(x^2y^2+x+y^4)dy = 0$$

Sol:-

$$M(x,y) = x^3y^3 + y$$

$$N(x,y) = 2(x^2y^2+x+y^4)$$

$$\frac{\partial M}{\partial y} = 3xy^2 + 1$$

$$\frac{\partial M}{\partial x} = 2\cdot 2xy^2 + 2 + 0$$

$$= 4xy^2 + 2$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial M}{\partial x} \text{ isn't exact ODE}$$

$$P(y) = \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M}$$

$$\Rightarrow \frac{4xy^2+2-3xy^2-1}{y(xy^2+1)}$$

$$\Rightarrow \frac{xy^2+1}{y(xy^2+1)} = 1/y$$

Now,

$I.F = e^{\int P(y)dy}$

$= e^{\int 1/y\, dy}$

$\Rightarrow e^{\log y}$

$\boxed{I.F \Rightarrow y}$

Multiply I.F to given ODE

$(xy^4 + y^2)dx + 2(x^2y^3 + xy + y^5)dy = 0$

here,

$\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$, It become exact

so,

$\frac{\partial F}{\partial x} = M(x,y)$

$\frac{\partial F}{\partial y} = N(x,y)$

$\int \partial F = \int M(x,y)dx$

$F = \frac{x^2y^4}{2} + xy^2 + \phi(y)$

$\frac{\partial F}{\partial y} = \frac{x^2}{\cancel{2}}(\cancel{4}y^3) + x(2y) + \phi'(y)$

$\cancel{2x^2y^3} + \cancel{2xy} + y^5 = \cancel{\frac{2x^2y^3}{2}} + \cancel{2xy} + \phi'(y)$

$y^5 = \phi'(y)$

$\phi(y) = \frac{y^6}{6}$

$$\boxed{F = \frac{x^2y^4}{2} + xy^2 + \frac{y^6}{6}}$$

in the soln

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iv)

$(y + \frac{y^3}{3} + \frac{x^2}{2})dx + \frac{1}{4}(x + xy^2)dy = 0$

Sol: $M(x,y) = y + \frac{y^3}{3} + \frac{x^2}{2}$

$N(x,y) = \frac{1}{4}(x + xy^2)$

$\frac{\partial M}{\partial y} = 1 + \frac{\cancel{3}y^2}{\cancel{3}} = 1 + y^2$

$\frac{\partial N}{\partial x} = \frac{1}{4}(1+y^2)$

here, $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

Now,

$P(x) = \dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}$

$\Rightarrow \frac{4}{x(1+y^2)}\left(1 + y^2 - \frac{1}{4} - \frac{y^2}{4}\right)$

$\Rightarrow \frac{\cancel{4}}{x\cancel{(1+y^2)}}\left(\frac{3}{\cancel{4}}\cancel{(1+y^2)}\right)$

$P(x) = \frac{3}{x}$

$I.F = e^{\int P(x)dx}$

$= e^{3\int \frac{1}{x}dx}$

$\Rightarrow e^{3\log x}$

$\Rightarrow x^3$

multiply I.F to given ODE

$\left(x^4y + \frac{x^3y^2}{3} + \frac{x^5}{2}\right)dx +$

$\frac{1}{4}\left(x^4 + x^4y^2\right)dy = 0$

now, it is exact.

$\frac{\partial F}{\partial x} = x^4y + \frac{x^3y^2}{3} + \frac{x^5}{2}$

$\int \partial F = \int \left(x^4y + \frac{x^3y^2}{3} + \frac{x^5}{2}\right)dx$

$F = \frac{x^5y}{5} + \frac{y^2}{3}\frac{x^4}{4} + \frac{x^6}{12} + \phi(y)$

$\frac{\partial F}{\partial y} = \frac{x^5}{5} + \frac{x^4y^2}{4} + \phi'(y)$

$\frac{\partial F}{\partial y} = \frac{x^5}{5} + \frac{x^4y^2}{4} + \phi'(y)$

$\frac{x}{4} + \frac{xy^2}{4} - \frac{x^5}{5} - \frac{x^4y^2}{4} = \phi'(y)$

Integrate w.r.t y

$\frac{xy}{4} + \frac{xy^3}{12} - \frac{x^5y}{5} - \frac{x^4y^3}{12} = \phi(y)$

Sub in ①

$F = \cancel{\frac{x^5y}{5}} + \cancel{\frac{x^4y^3}{12}} + \frac{x^6}{12} + \frac{xy}{4} + \frac{xy^3}{12}$

$\cancel{-\frac{x^5y}{5}} \cancel{-\frac{x^4y^3}{12}}$

$F = \frac{x^6}{12} + \frac{xy^3}{12} + \frac{xy}{4}$

$$\boxed{F = \frac{x^6 + xy^3 + 3xy}{12}}$$

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AP21110010190
CSE - C

$\{7^{th}$ Q1A$\}$

i) $x\frac{dy}{dx} - (x+1)dy = x^2 - x^3$

Sol:

divide by x on both Sides.

$\frac{dy}{dx} - \frac{(1+x)}{x}dy = x - x^2$

$P(x) = -\left(\frac{1+x}{x}\right)$, $Q(x) = x - x^2$

$I.F = e^{-\int(1+\frac{1}{x})dx}$

$\Rightarrow e^{-(x+\log x)}$

$\Rightarrow \frac{1}{e^{x+\log x}} \Rightarrow \frac{1}{e^x . e^{\log x}}$

$\Rightarrow \frac{1}{x.e^x}$

Sol^n is,

$y = xe^x \int \frac{1}{xe^x}(x-x^2)dx + c$

$y = xe^x \int e^{-x}(1-x)dx + c$

$y = xe^x \int (\overset{u}{1-x}) \overset{v}{e^{-x}} dx + c$

$y = xe^x \left[(1-x)\int e^{-x}dx - \int(-1)\left(\int e^{-x}dx\right)dx\right] + c$

$y = xe^x\left[-(1-x)e^{-x} + \int(-e^{-x})dx\right] + c$

$y = xe^x\left[(x-1)e^{-x} - \int e^{-x}dx\right]$

$y = xe^x\left[(x-1)e^{-x} + e^{-x}\right] + c$

$y = xe^x.(x-1)e^{-x} + xe^x.e^{-x} + c$

$y = x(x-1) + x + c$

$y = x^2 \cancel{-x} \cancel{+x} + c$

$\boxed{y = x^2 + c}$

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ii)

$\frac{dy}{dx} + y\tan x = \sin 2x$, $y(0) = 1$

Sol: Given,

$\frac{dy}{dx} + y\tan x = \sin 2x$

$P(x) = \tan x$, $Q(x) = \sin 2x$

$I.F = e^{\int p(x)dx} = e^{\int \tan x dx}$

$\Rightarrow e^{\log|\sec x|} \Rightarrow \sec x \Rightarrow \frac{1}{\cos x}$

The sol^n is

$y = \cos x \int \frac{1}{\cos x}(\sin 2x) + c$

$y = \cos x \int \frac{1}{\cos x}(2\sin x\cos x) + c$

$y = 2\cos x \int \sin x + c$

$y = -2\cos^2 x + c$

Given $y(0) = 1$

$1 = -2\cos^2(1)^2 + c$

$1+2 = c \Rightarrow \boxed{c = 3}$

sol^n is $\boxed{y = -2\cos^2 x + 3}$

iii) $x\frac{dy}{dx} + 2y = 4x^2$

Sol: divide by x

$\frac{dy}{dx} + \frac{2}{x}(y) = 4x$

$P(x) = 2/x,\ Q(x) = 4x$

$I.F = e^{\int P(x)dx}$

$\Rightarrow e^{2\int 1/x\, dx} \Rightarrow e^{\log x^2}$

$\Rightarrow x^2$

Sol^n:

$y = \frac{1}{x^2}\int x^2 \cdot 4x\, dx$

$y = \frac{4}{x^2}\int x^3 dx \Rightarrow \frac{4}{x^2} \times \frac{x^4}{4}$

$\boxed{y = x^2 + c}$

iv) $y\,dx + (xy^2 + x - y)dy = 0$

Sol: Divide by dy

$y\frac{dx}{dy} + xy^2 + x - y = 0$

divide by y

$\frac{dx}{dy} + x\left(y + \frac{1}{y}\right) = 1$

$P(y) = y + \frac{1}{y},\ Q(y) = 1$

$e^{\int P(y)dy} = e^{\int y + \frac{1}{y}\, dy}$

$\Rightarrow e^{y^2/2 + \log y}$

The sol^n is

$x = \frac{1}{e^{y^2/2 + \log y}}\int e^{y^2/2 + \log y}\, dy$

$x = \frac{1}{\cancel{e^{y^2/2 + \log y}}}\left(\frac{\cancel{e^{y^2/2 + \log y}}}{\frac{y^3}{6} + y\log y - y}\right) + c$

$x = \frac{1}{\frac{y^3}{6} + y\log y - y} + c$

v) $\frac{dy}{dx} + \frac{y}{x} = \frac{\sin x}{x}$; $y(0) = 0$

Sol:- $P(x) = 1/x$, $Q(x) = \frac{\sin x}{x}$

$I.F = e^{\int P(x)dx} = e^{\int 1/x\, dx}$

$\Rightarrow e^{\log x} \Rightarrow x$

The general solⁿ is,

$y = \frac{1}{x}\int \cancel{x}.\frac{\sin x}{\cancel{x}}\, dx + c$

$y = \frac{1}{x}\int \sin x\, dx + c$

$y = -\frac{\cos x}{x} + c$

$xy + \cos x = c$

$y(0) = 0$

$0 + 1 = c \Rightarrow \boxed{c = 1}$

$$\boxed{xy + \cos x = 1}$$

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CSE-C

vi) $x^2\frac{dy}{dx} + 2xy - x + 1 = 0$;

$y(1) = 0$

Sol:-

$x^2\frac{dy}{dx} + 2xy = x - 1$

Divide by x^2

$\frac{dy}{dx} + \frac{2y}{x} = \frac{x-1}{x^2}$

Now,

$P(x) = 2/x$, $Q(x) = \frac{x-1}{x^2}$

$I.F = e^{\int P(x)dx} \Rightarrow e^{2\int 1/x\, dx}$

$\rightarrow 2^{\log x^2} \Rightarrow x^2$.

solⁿ:-

$y = \frac{1}{x^2}\int\left(\frac{1}{x} - \frac{1}{x^2}\right)x^2 dx + c$

$y = \frac{1}{x^2}\int\left(\frac{x-1}{\cancel{x^2}}\right)\cancel{x^2}dx + c$

$y = \frac{1}{x^2}\left(\frac{x^2}{2} - x\right) + c$

$y = \frac{1}{2} - x + c \rightarrow (1)$

$y(1) = 0 \Rightarrow 0 = \frac{1}{2} - 1 + c$

$\boxed{c = 1/2}$

Solⁿ is $y = \frac{1}{2} - \frac{1}{x} + \frac{1}{2}$

$$\boxed{y = 1 - \frac{1}{x}}$$

8th Q1A

$$\frac{dy}{dx}+\frac{y}{x}=\frac{y^2}{x}\log x$$

Sol: Divide by y^2

$$y^{-2}\frac{dy}{dx}+\frac{1}{x}y^{-1}=\frac{1}{x}\log x \quad (1)$$

let, $u=y^{-1}$

$$\frac{du}{dx}=-y^{-2}\frac{dy}{dx}$$

$$-\frac{du}{dx}=y^{-2}\frac{dy}{dx}$$

eq (1) gets modified as,

$$-\frac{du}{dx}+\frac{1}{x}(u)=\frac{1}{x}\log x$$

here,

$P(x)=\frac{1}{x}$, $Q(x)=\frac{1}{x}\log x$

$$I.F=e^{\int 1/x\,dx}\Rightarrow e^{\log x}\Rightarrow x$$

Sol^n is,

$$u=\frac{1}{x}\int \cancel{x}\cdot\frac{1}{\cancel{x}}\log x+c$$

$$u=\frac{1}{x}\int\log x+c$$

$$u=\frac{1}{x}(x\log x-x+c)$$

$$\frac{1}{y}=\log x-1+c$$

$$\boxed{y=\frac{1}{\log x-1+c}}$$

ii)

$$\frac{dy}{dx}+\frac{4}{x}y=x^3y^2;$$

$y(2)=-1;\ x>0$

Sol:

Divide by y^2

$$y^{-2}\frac{dy}{dx}+\frac{4}{x}y^{-1}=x^3$$

let $u=y^{-1}$

$$\frac{du}{dx}=-y^{-2}\frac{dy}{dx}$$

$$-\frac{du}{dx}=y^{-2}\frac{dy}{dx}$$

eq (1) gets modified as,

$$-\frac{du}{dx}+\frac{4}{x}u=x^3$$

$$\Rightarrow\frac{du}{dx}-\frac{4}{x}u=-x^3$$

Here,

$P(x)=-4/x$, $Q(x)=-x^3$

Now,

$$I.F=e^{\int P(x)dx}=e^{-\int 4/x\,dx}\Rightarrow e^{-4\log x}\Rightarrow e^{\log_e x^{-4}}\Rightarrow x^{-4}$$

$\longrightarrow$

Y. Narasimha Rao
AP21110010190

Sol^n: s

$u = x^4 \left(-\int x^{+3} \cdot \frac{1}{x^4} dx\right.$

$u = x^4 \left(-\frac{1}{x} dx\right) + c$

$u = x^4(-\log x + c)$

$\frac{1}{y} = x^4(-\log x + c)$

$y = \frac{1}{x^4(-\log x + c)} \rightarrow (2)$

$y(2) = -1$

$-1 = \frac{1}{(2)^4(-\log 2 + c)}$

$-\log 2 + c = \frac{-1}{16}$

$c = -\frac{1}{16} + \log 2$

sub in eq (2)

$y = \frac{1}{x^4(-\log x + \log 2 - \frac{1}{16})}$

$$\boxed{y = \frac{1}{x^4\left(\log\left(\frac{2}{x}\right) - \frac{1}{16}\right)}}$$

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AP21110010190
CSE - C

iii) $\frac{dy}{dx} + \frac{1}{x} y = y^{1/2}$

Sol: divide with $y^{1/2}$

$y^{-1/2} \frac{dy}{dx} + \frac{y^{1/2}}{x} = 1$

let $u = y^{1/2}$

$\frac{du}{dx} = \frac{1}{2\sqrt{y}} \frac{dy}{dx}$

$2\frac{du}{dx} = y^{-1/2} \frac{dy}{dx}$

Then (1), gets modified as

$2\frac{du}{dx} + \frac{1}{x} u = 1$

$\frac{du}{dx} + \frac{u}{2x} = \frac{1}{2}$

Here $P(x) = \frac{1}{2x}$, $Q(x) = 1/2$

$I.F = e^{\int P(x)dx} \Rightarrow e^{1/2 \int \frac{1}{x} dx}$

$\Rightarrow e^{\log x^{1/2}} \Rightarrow x^{1/2}$

sol^n:

$u = \frac{1}{x^{1/2}}\left(\int \frac{1}{2} x^{1/2}\right) + c$

$u = \frac{1}{x^{1/2}} \frac{1}{2}\left(\frac{2}{3} x^{3/2}\right) + c$

$u = \frac{1}{3} \frac{x^{3/2}}{x^{1/2}} + c \Rightarrow u = \frac{x}{3} + c$

$y^{1/2} = \frac{x}{3} + c$

$\sqrt{y} = \frac{x}{3} + c \Rightarrow \boxed{y = \left(\frac{x}{3} + c\right)^2}$

iv) $\frac{dy}{dx} = 5y + e^{-2x} y^{-2}$

Sol: Divide by y^{-2}.

$$\frac{dy}{dx} y^2 = 5y^{-1} + e^{-2x} \quad \text{---(1)}$$

Let, $u = y^3$

$$\frac{du}{dx} = 3y^2 \frac{dy}{dx}$$

$$\frac{1}{3}\frac{du}{dx} = y^2 \frac{dy}{dx}$$

Then eq(1) gets modified as,

$$\frac{1}{3}\frac{du}{dx} - 15u = 3e^{-2x}$$

$P(x) = -15$, $Q(x) = 3e^{-2x}$

$$I.F = e^{\int P(x)dx} \Rightarrow e^{-15x}$$

Solⁿ:

$$u = e^{15x}\int 3e^{-2x} \cdot e^{-15x} + c$$

$$u = 3e^{15x}\int e^{-17x} + c$$

$$u = 3e^{15x} \cdot \frac{e^{-17x}}{-17} + c$$

$$\boxed{u = \frac{-3}{17} e^{-2x} + c}$$

$$\boxed{y^3 = \frac{-3}{17} e^{-2x} + c}$$

v) $6\frac{dy}{dx} - 2y = xy^4$

Sol: Divide by y^4

$$6y^{-4}\frac{dy}{dx} - 2y^{-3} = x \quad \text{---(1)}$$

let $u = y^{-3}$

$$\frac{du}{dx} = -3y^{-4}\frac{dy}{dx}$$

$$-\frac{du}{dx} = 3y^{-4}\frac{dy}{dx}$$

$$-2\frac{du}{dx} = 6y^{-4}\frac{dy}{dx}$$

$\Rightarrow$

$$-2\frac{du}{dx} - 2u = x$$

$$\Rightarrow \frac{du}{dx} + u = -\frac{x}{2}$$

here $P(x) = 1$, $Q(x) = -x/2$

$$I.F = e^{\int P(x)dx} \Rightarrow e^{\int 1 dx}$$

$$\Rightarrow e^x$$

Solⁿ: $y = \frac{1}{e^x}\int e^x(-x/2)\,dx + c$

$$\Rightarrow y = \frac{-1}{2e^x}\left(\int xe^x dx + c\right)$$

$$\Rightarrow y = \frac{-1}{2e^x}\left(x\int e^x dx - \int 1 \int e^x dx)dx + c\right)$$

$$\Rightarrow y = \frac{-1}{2e^x}\left(xe^x - \int e^x + c\right)$$

$$y = \frac{-1}{2e^x}\left(xe^x - e^x + c\right)$$

Y. Narasimha Rao
AP21110010190
CSE-C

$$y = \frac{e^x}{2e^x}\left((x-1)+c\right)$$

$$y = \frac{(x-1)}{2} + \frac{c}{2}$$

$$\boxed{y = \frac{c+x-1}{2}}$$

9th Q|A

i) $y' = y^3 + 4$, $y(0) = 1$

Sol: $f(x) = y^3 + 1$, $\frac{\delta F}{\delta y} = 3y^2$ Both are continous at point $(0,1)$ so There exits a unique sol^n.

ii) $y' = \sqrt{|y|}$, $y(0) = 0$.

Sol: $f(x)y) = \sqrt{|y|}$ is Not continous at $(0,0)$.
the existence & uniqueness theorem doest apply to $y' = \sqrt{|y|}$, $y(0) = 0$.

iii). $y' = \sqrt{|y|}$, $y(1) = 1$.

Sol: $f(x,y) = \sqrt{|y|}$, $\frac{\delta F}{\delta y} = \frac{1}{2\sqrt{y}}$ are continous at $(1,1)$
There exist a unique sol^n to $y^2 = \sqrt{|y|}$, $y(1) = 1$.

iv) $y' = x\tan y$, $y(0) = 0$.

Sol: $f(x,y) = x\tan y$ & $\frac{\delta F}{\delta y} = x\sec^2 y$ are both continous at $(0,0)$
There exist a unique sol^n to $y' = x\tan y$, $y(0) = 0$.