Q ⇒ If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2,000 individuals more than two will get a bad reaction.

Sol^n ⇒ Since the probability of occurrence is very small, it follows a Poisson distribution.

$$\text{Mean } m = np = 2000(0.001) = 2$$

∴ Probability that more than 2 will get a bad reaction

= 1 − [Prob. that no one gets a bad reaction
+ Prob. that one gets a bad reaction]
+ Prob. that two get bad reaction

$$= 1 - \left[\frac{e^{-m} m^0}{\lfloor 0} + \frac{e^{-m} m^1}{\lfloor 1} + \frac{e^{-m} m^2}{\lfloor 2}\right]$$

$$= 1 - \left[e^{-2} + e^{-2}.2 + e^{-2}.\frac{2\times\cancel{2}}{\cancel{2}\times 1}\right] \quad [\because m=2]$$

$$= 1 - e^{-2}[1 + 2 + 2]$$

$$= 1 - \frac{5}{e^2} \qquad [\because e = 2.718]$$

$$= 1 - \frac{5}{(2.718)^2}$$

$$= 0.32$$

Q→2 Fit a Poisson distribution to the following and calculate theoretical frequencies ($e^{-0.5} = 0.61$):

Deaths:	0	1	2	3	4
Frequency:	122	60	15	2	1

Solution:

Deaths x	Frequency f	CF. fx
0	122	0
1	60	60
2	15	30
3	2	6
4	1	4
	$N = \Sigma f = 200$	$\Sigma fx = 100$

$$\text{mean } m = \frac{\Sigma fx}{\Sigma f} = \frac{100}{200} = \frac{1}{2} = 0.5$$

$$\text{Now, } e^{-m} = e^{-0.5} = 1 - 0.5 + \frac{(0.5)^2}{2} - \frac{(0.5)^3}{6} + \cdots$$

$$= 1 - 0.5 + 0.125 - 0.0208 + \cdots$$

$$= 0.61 \text{ (approximately)}$$

$\therefore$ Therefore frequencies of r deaths

$$= N \frac{e^{-m} m^r}{r!}$$

$$= 200 (0.61) \frac{(0.5)^r}{r!}$$

it gives frequencies 122, 61, 15, 2 and 4 respectively for $r = 0, 1, 2, 3$ and 4.

Q→ Find the probability that at most 5 defective fuses will be found in a box of 200 fuses, if experience shows that 2 percent of such fuses are defective.

Solⁿ→ Here $p = 2\% = \frac{2}{100} = \frac{1}{50}$, $n = 200$

$\therefore np = \frac{200 \cdot 1}{50} = 4$

Maximum number of defective fuses $= 5$

$\therefore r \le 5$

Also $e^{-4} = 1 - 4 + \frac{4^2}{2} - \frac{1}{3!}4^3 + \frac{1}{4!}4^4 - \dots$

$= 0.0183$

$\therefore$ Required probability $(r \le 5)$

$$= \sum_{r=0}^{5} \frac{e^{-4} 4^r}{r!} = e^{-4}\left[1 + \frac{4}{1!} + \frac{4^2}{2!} + \frac{4^3}{3!} + \frac{4^4}{4!} + \frac{4^5}{5!}\right]$$

$$= 0.0183[1 + 4 + 8 + 10.6667 + 10.6667 + 8.5333]$$

$$= 0.0183(42.8667)$$

$$= 0.7845$$

Q→ For Poisson distribution prove that $M\sigma r_1 r_2 = 1$ where symbols have their usual meanings.

Solⁿ→ Here $M = m$, $\sigma = \sqrt{m}$, $r_1 = \frac{1}{\sqrt{m}}$, $r_2 = \frac{1}{m}$

$\therefore M\sigma r_1 r_2 = m\sqrt{m}\left(\frac{1}{\sqrt{m}}\right)\left(\frac{1}{m}\right)$

$= 1$

Proved

Q.1) In a Poisson distribution with unity mean, Show that the mean deviation from mean is $2/e$ times the standard deviation.

Solⁿ: We know that the probability of x successes in Poisson distribution $= e^{-m} m^{x}/x!$

Here mean $= m = 1$

$\therefore$ S.D $= \sqrt{m} = \sqrt{1} = 1$

$p(x) =$ Prob of x successes $= \dfrac{e^{-1}}{x!}$ $[\because m=1]$

Now mean deviation (M.D.) from mean

$$= \sum_{x=0}^{\infty} |x-m| P(x), \quad \text{where } m=1$$

$$= \sum_{x=0}^{\infty} |x-1| \frac{e^{-1}}{x!}$$

$$= \frac{1}{e} \sum_{x=0}^{\infty} \frac{|x-1|}{x!} = \frac{1}{e}\left[1 + \frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \frac{4}{5!} + \cdots\right] \quad \text{— (1)}$$

General term of the series in (1)

$$= \frac{n}{(n+1)!} = \frac{(n+1)-1}{(n+1)!} = \frac{1}{n!} - \frac{1}{(n+1)!} \quad \text{— (2)}$$

Substituting $n = 1, 2, 3, \cdots$ etc. in (1) and using (2) for different terms, the M.D. from mean

$$= \frac{1}{e}\left[1 + \left(\frac{1}{1!} - \frac{1}{2!}\right) + \left(\frac{1}{2!} - \frac{1}{3!}\right) + \cdots\right]$$

$$= \frac{1}{e}[1 + 1]$$

$$= \frac{2}{e} = \left(\frac{2}{e}\right) \times (1) = \frac{2}{e} \times \text{S.D.}$$

Proved