

Feedback — Problem Set-4

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You submitted this quiz on **Sun 22 Feb 2015 6:35 PM IST**. You got a score of **5.00** out of **5.00**.

Question 1

Given an adjacency-list representation of a directed graph, where each vertex maintains an array of its outgoing edges (but *not* its incoming edges), how long does it take, in the worst case, to compute the in-degree of a given vertex? As usual, we use n and m to denote the number of vertices and edges, respectively, of the given graph. Also, let k denote the maximum in-degree of a vertex. (Recall that the in-degree of a vertex is the number of edges that enter it.)

Your Answer	Score	Explanation
<input type="radio"/> Cannot determine from the given information		
<input type="radio"/> $\theta(n)$		
<input type="radio"/> $\theta(k)$		
<input checked="" type="radio"/> $\theta(m)$	✓ 1.00	Without explicitly maintaining a list of incoming edges, you might have to scan all the edges to identify the incoming arcs.
Total	1.00 / 1.00	

Question 2

Consider the following problem: given an undirected graph G with n vertices and m edges, and two vertices s and t , does there exist at least one s - t path?

If G is given in its adjacency list representation, then the above problem can be solved in

$O(m + n)$ time, using BFS or DFS. (Make sure you see why this is true.)

Suppose instead that G is given in its adjacency *matrix* representation. What running time is required, in the worst case, to solve the computational problem stated above? (Assume that G has no parallel edges.)

Your Answer	Score	Explanation
<input checked="" type="radio"/> $\theta(n^2)$	✓ 1.00	For the lower bound, observe that you might need to look at every entry of the adjacency matrix (e.g., if it has only one "1" and the rest are zeroes). One easy way to prove the upper bound is to first build an adjacency list representation (in $\Theta(n^2)$ time, with a single scan over the given adjacency matrix) and then run BFS or DFS as in the video lectures.
<input type="radio"/> $\theta(n * m)$		
<input type="radio"/> $\theta(m + n)$		
<input type="radio"/> $\theta(m + n \log n)$		
Total	1.00 / 1.00	

Question 3

This problem explores the relationship between two definitions about graph distances. In this problem, we consider only graphs that are undirected and connected. The *diameter* of a graph is the maximum, over all choices of vertices s and t , of the shortest-path distance between s and t . (Recall the shortest-path distance between s and t is the fewest number of edges in an s - t path.)

Next, for a vertex s , let $l(s)$ denote the maximum, over all vertices t , of the shortest-path distance between s and t . The *radius* of a graph is the minimum of $l(s)$ over all choices of the vertex s .

Which of the following inequalities always hold (i.e., in every undirected connected graph) for the radius r and the diameter d ? [Select all that apply.]

Your	Score	Explanation
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Answer

<input type="checkbox"/>	✓	0.25	The triangle is a counterexample.
$r \leq d/2$			
<input type="checkbox"/>	✓	0.25	A path is a counterexample.
$r \geq d$			
<input checked="" type="checkbox"/>	✓	0.25	Let c minimize $l(s)$ over all vertices s . Since every pair of vertices s and t have paths to c with at most r edges, stitching these paths together yields an s - t with only $2r$ edges; of course, the shortest s - t path is only shorter.
$r \geq d/2$			
<input checked="" type="checkbox"/>	✓	0.25	By the definitions, $l(s) \leq d$ for every single choice of s .
$r \leq d$			
Total		1.00 /	
		1.00	

Question 4

Consider our algorithm for computing a topological ordering that is based on depth-first search (i.e., NOT the "straightforward solution"). Suppose we run this algorithm on a graph G that is NOT directed acyclic. Obviously it won't compute a topological order (since none exist). Does it compute an ordering that minimizes the number of edges that go backward? For example, consider the four-node graph with the six directed edges $(s, v), (s, w), (v, w), (v, t), (w, t), (t, s)$. Suppose the vertices are ordered s, v, w, t . Then there is one backwards arc, the (t, s) arc. No ordering of the vertices has zero backwards arcs, and some have more than one.

Your Answer	Score	Explanation
<input type="radio"/> Never		
<input type="radio"/> If and only if the graph is a directed cycle		
<input checked="" type="radio"/> Sometimes yes, sometimes no	✓ 1.00	
<input type="radio"/> Always		
Total	1.00 / 1.00	

Question 5

On adding one extra edge to a directed graph G , the number of strongly connected components...?

Your Answer	Score	Explanation
<input type="radio"/> ...never decreases (no matter what G is)		
<input checked="" type="radio"/> ...could remain the same (for some graphs G)	✓ 1.00	For example, the graph might already be strongly connected.
<input type="radio"/> ...never decreases by more than 1 (no matter what G is)		

☐ ...will definitely not change (no matter what G is)

Total	1.00 /
	1.00

