Universal approximation theorem

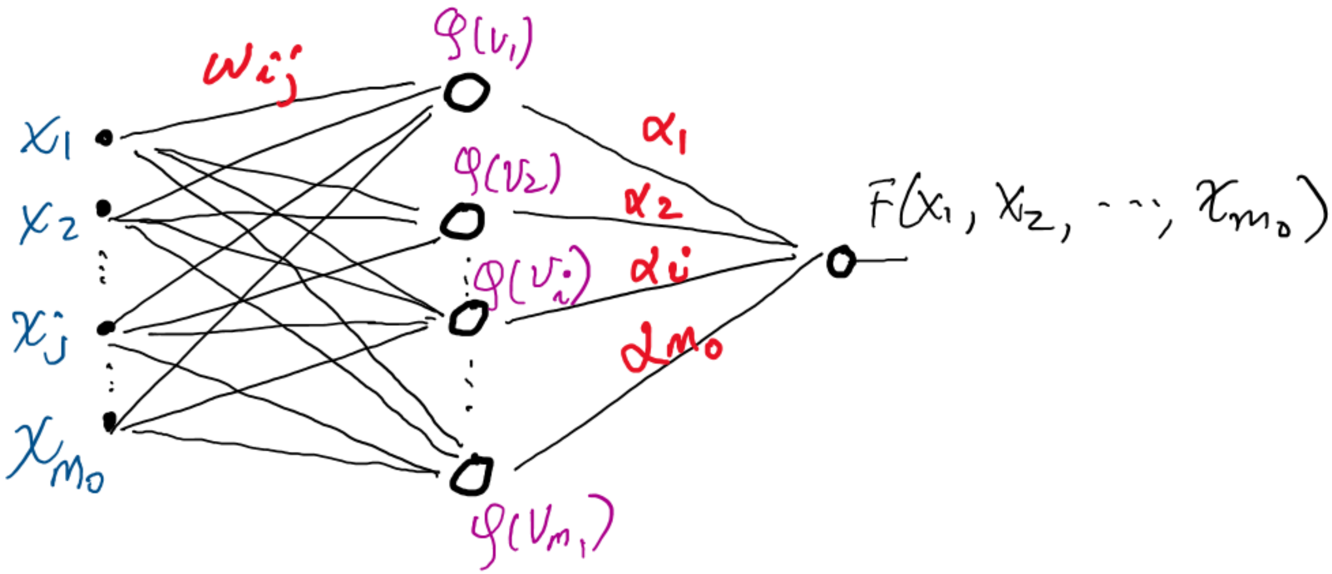
Let $\varphi(.)$ be a non-constant, bounded, and monotonically increasing function. Let I_{m_0} denote the m_0 -dimension unit hypercube $[0, 1]^{m_0}$. The space of continuous functions on I_{m_0} is denoted by $C(I_{m_0})$. Then given any function $f \in C(I_{m_0})$, and $\varepsilon > 0$, there exists an integer m_1, and sets of real constants α_i, b_i, and w_{ij} , where $i = 1, \cdots, m_1$, and $j = 1, 2, \cdots, m_0$ such that we may define

$$F(x_1, \cdots, x_{m_0}) = \sum_{i=1}^{m_1} \alpha_i \, \varphi\left(\sum_{j=1}^{m_0} w_{ij} x_j + b_i\right)$$

as an approximate realization of the function $f(.)$; that is

$$\left|F(x_1, \cdots, x_{m_0}) - f(x_1, \cdots, x_{m_0})\right| < \varepsilon$$

for all $x_1, x_2, \cdots, x_{m_0}$ lie in the input space.



$$F\left(x_1, \cdots, x_{m_0}\right) = \alpha_1 \varphi(v_1) + \alpha_2 \varphi(v_2) + \cdots + \alpha_i \varphi(v_i) + \cdots + \alpha_{m_1} \varphi\left(v_{m_1}\right)$$

$$= \sum_{i=1}^{m_1} \alpha_i \cdot \varphi(v_i) = \sum_{i=1}^{m_1} \alpha_i \cdot \varphi\left(\sum_{j=1}^{m_0} w_{ij} x_j + b_i\right)$$