In [1]:
```
%matplotlib inline
```

In [2]:
```
#Set script to run optimization
run_fit = False
```

Importing Libraries

Importing libraries for script

In [3]:
```
import pynamics
from pynamics.frame import Frame
from pynamics.variable_types import Differentiable,Constant,Variable
from pynamics.system import System
from pynamics.body import Body
from pynamics.dyadic import Dyadic
from pynamics.output import Output,PointsOutput
from pynamics.output_points_3d import PointsOutput3D
from pynamics.constraint import AccelerationConstraint,KinematicConstraint
from pynamics.particle import Particle
import pynamics.integration
import numpy
import matplotlib.pyplot as plt
plt.ion()
from math import pi,sin
import sympy
from sympy import sqrt
import math
import logging
import scipy.optimize
import pynamics.integration
import pynamics.system
import numpy.random
import scipy.interpolate
import scipy.optimize
import cma
import pandas as pd
import idealab_tools.units
from matplotlib import animation, rc
from IPython.display import HTML


system = System()
pynamics.set_system(__name__,system)
```

Constants of System

In this block of code we are defining all the constants of our system that we will use for our simulation

In [4]:

```
#[0.0276 0.01]

#seg = segment, t = tail
seg_l = 0.0276
t_l = 0.0276
seg_h = 0.01
len_factor = 5.5

#Set segment lengths
l = Constant(seg_l,'l',system) #Segment Length, Formula:seg_len
lT = Constant(seg_l,'tail',system) #Tail Length, Formula:tail_len
lP = Constant(seg_l*len_factor,'lP',system) #Constrained length, Forumla:seg_len*constr

#Set masses, 666.7 is density of laminate structure
m = Constant(666.7*seg_l*seg_h*0.001,'m',system) #Segment Mass, Formula:666.7*seg_len*s
mT = Constant(666.7*t_l*seg_h*0.001,'mT',system) #Tail Mass, Formula:666.7*tail_len*seg

b = Constant(2.148e-6,'b',system)
k = Constant(1.599e-4,'k',system)
rho = Constant(998,'rho',system)
area_p = Constant(seg_l*seg_h,'area_p',system) #area of flat plates
area_f = Constant(seg_h*0.001,'area_f',system) #area of flat plates

freq = Constant(1,'freq',system) #frequency of head oscilation
amp = Constant(40*pi/180,'amp',system) #maximum pitch angle of servo

Ixx = Constant(1/12*(666.7*seg_l*seg_h*0.001)*(seg_h**2 + 0.001**2),'Ixx',system) #Form
Iyy = Constant(1/12*(666.7*seg_l*seg_h*0.001)*(seg_h**2 + seg_l**2),'Iyy',system) #Form
Izz = Constant(1/12*(666.7*seg_l*seg_h*0.001)*(seg_l**2 + 0.001**2),'Izz',system) #Form
Ixx_T = Constant(1/12*(666.7*t_l*seg_h*0.001)*(seg_h*2 + 0.001**2),'Ixx_T',system) #For
Iyy_T = Constant(1/12*(666.7*t_l*seg_h*0.001)*(seg_h**2 + t_l**2),'Iyy_T',system) #Form
Izz_T = Constant(1/12*(666.7*t_l*seg_h*0.001)*(t_l**2 + 0.001**2),'Izz_T',system) #Form
```

In [5]:

```
#Set integration tolerance
tol = 1e-12
```

In [6]:

```
#Set simulation run time
fps = 30
tinitial = 0
tfinal = 2
tstep = 1/fps
t = numpy.r_[tinitial:tfinal:tstep]
```

In [7]:

```
#Define derivatives of frames
qA,qA_d,qA_dd = Differentiable('qA',system)
qB,qB_d,qB_dd = Differentiable('qB',system)
qC,qC_d,qC_dd = Differentiable('qC',system)
qD,qD_d,qD_dd = Differentiable('qD',system)
qE,qE_d,qE_dd = Differentiable('qE',system)
qF,qF_d,qF_dd = Differentiable('qF',system)
qT,qT_d,qT_dd = Differentiable('qT',system)

x,x_d,x_dd = Differentiable('x',system)
y,y_d,y_dd = Differentiable('y',system)
```

In [8]:
```
#set initial conditions
initialvalues = {}
initialvalues[qA]=40*pi/180
initialvalues[qA_d]=0*pi/180
initialvalues[qB]=20*pi/180
initialvalues[qB_d]=0*pi/180
initialvalues[qC]=10*pi/180
initialvalues[qC_d]=0*pi/180
initialvalues[qD]=0*pi/180
initialvalues[qD_d]=0*pi/180
initialvalues[qE]=-10*pi/180
initialvalues[qE_d]=0*pi/180
initialvalues[qF]=-40*pi/180
initialvalues[qF_d]=0*pi/180
initialvalues[qT]=0*pi/180
initialvalues[qT_d]=0*pi/180

initialvalues[x]=0*pi/180
initialvalues[x_d]=0*pi/180
initialvalues[y]=0*pi/180
initialvalues[y_d]=0*pi/180

statevariables = system.get_state_variables()
ini0 = [initialvalues[item] for item in statevariables]
```

In [9]:
```
#Frames
N = Frame('N',system)
A = Frame('A',system)
B = Frame('B',system)
C = Frame('C',system)
D = Frame('D',system)
E = Frame('E',system)
F = Frame('F',system)
T = Frame('T',system)

system.set_newtonian(N)

A.rotate_fixed_axis(N,[0,0,1],qA,system)
B.rotate_fixed_axis(N,[0,0,1],qB,system)
C.rotate_fixed_axis(N,[0,0,1],qC,system)
D.rotate_fixed_axis(N,[0,0,1],qD,system)
E.rotate_fixed_axis(N,[0,0,1],qE,system)
F.rotate_fixed_axis(N,[0,0,1],qF,system)
T.rotate_fixed_axis(N,[0,0,1],qT,system)
```

Defining Vectors

In this section of code we are defining all the position and center of mass vecotors. Additionally we are calculating angular velocity of each frame and the respective linear velocities at the center of mass. We also build each body of the system in this section.

In [10]:
```
#Vectors

pNA=x*N.x + y*N.y + 0*N.z
```

```
pP = lP*N.x + pNA

pAB= pNA + l*A.x
pBC = pAB + l*B.x
pCD = pBC + l*C.x
pDE = pCD + l*D.x
pEF = pDE + l*E.x
pFT = pEF + l*F.x
pTtip = pFT + lT*T.x

#Center of Mass
pAcm=pNA+l/2*A.x
pBcm=pAB+l/2*B.x
pCcm=pBC+l/2*C.x
pDcm=pCD+l/2*D.x
pEcm=pDE+l/2*E.x
pFcm=pEF+l/2*F.x
pTcm=pFT+lT/2*T.x

#Angular Velocity
wNA = N.get_w_to(A)
wAB = A.get_w_to(B)
wBC = B.get_w_to(C)
wCD = C.get_w_to(D)
wDE = D.get_w_to(E)
wEF = E.get_w_to(F)
wFT = F.get_w_to(T)

#Velocities
vA=pAcm.time_derivative()
vB=pBcm.time_derivative()
vC=pCcm.time_derivative()
vD=pDcm.time_derivative()
vE=pEcm.time_derivative()
vF=pFcm.time_derivative()
vTtip=pTtip.time_derivative()

#Interia and Bodys
IA = Dyadic.build(A,Ixx,Iyy,Izz)
IB = Dyadic.build(B,Ixx,Iyy,Izz)
IC = Dyadic.build(C,Ixx,Iyy,Izz)
ID = Dyadic.build(D,Ixx,Iyy,Izz)
IE = Dyadic.build(E,Ixx,Iyy,Izz)
IF = Dyadic.build(F,Ixx,Iyy,Izz)
IT = Dyadic.build(T,Ixx_T,Iyy_T,Izz_T)

BodyA = Body('BodyA',A,pAcm,m,IA,system)
BodyB = Body('BodyB',B,pBcm,m,IB,system)
BodyC = Body('BodyC',C,pCcm,m,IC,system)
BodyD = Body('BodyD',D,pDcm,m,ID,system)
BodyE = Body('BodyE',E,pEcm,m,IE,system)
BodyF = Body('BodyF',F,pFcm,m,IF,system)
BodyT = Body('BodyT',T,pTcm,mT,IT,system)
```

Adding Forces

In this section of code we are adding the aerodynamic, spring, and damping forces in the system. The damping and spring values will be calculated experimentally.

In [11]:

```
#Forces
#system.addforce(-torque*sympy.sin(freq*2*pi*system.t)*A.z,wNA) #setting motor paramete

#Aerodynamic Forces orthogonal to flat plates
f_aero_Ay = 998 * vA.length()*(vA.dot(A.y)) * area_p * A.y
f_aero_By = 998 * vB.length()*(vB.dot(B.y)) * area_p * B.y
f_aero_Cy = 998 * vC.length()*(vC.dot(C.y)) * area_p * C.y
f_aero_Dy = 998 * vD.length()*(vD.dot(D.y)) * area_p * D.y
f_aero_Ey = 998 * vE.length()*(vE.dot(E.y)) * area_p * E.y
f_aero_Fy = 998 * vF.length()*(vF.dot(F.y)) * area_p * F.y
f_aero_Ty = 998 * vTtip.length()*(vTtip.dot(T.y)) * area_p * T.y

system.addforce(-f_aero_Ay,vA)
system.addforce(-f_aero_By,vB)
system.addforce(-f_aero_Cy,vC)
system.addforce(-f_aero_Dy,vD)
system.addforce(-f_aero_Ey,vE)
system.addforce(-f_aero_Fy,vF)
system.addforce(-f_aero_Ty,vTtip)

#Aerodynamic Forces against front of device
f_aero_Ax = 998 * vA.length()*(vA.dot(A.x)) * area_f * A.x
system.addforce(-f_aero_Ax,vA)

#Damping Forces
system.addforce(-b*wAB,wAB)
system.addforce(-b*wBC,wBC)
system.addforce(-b*wCD,wCD)
system.addforce(-b*wDE,wDE)
system.addforce(-b*wEF,wEF)
system.addforce(-b*wFT,wFT)

#Spring Force (Torsion)
system.add_spring_force1(k,(qB-qA)*N.z,wAB)
system.add_spring_force1(k,(qC-qB)*N.z,wBC)
system.add_spring_force1(k,(qD-qC)*N.z,wCD)
system.add_spring_force1(k,(qE-qD)*N.z,wDE)
system.add_spring_force1(k,(qF-qE)*N.z,wEF)
system.add_spring_force1(k,(qT-qF)*N.z,wFT)
```

Out[11]:

```
(<pynamics.force.Force at 0x201a9de4e50>,
 <pynamics.spring.Spring at 0x201a9f5d580>)
```

Initial Condition

Solving for initial condition constraints and using scipy to solve for initial states and setting initial states to system initial states.

In [12]:

```
#Constraints for initial condition

eq = []
```

```
eq.append(pFT-pP)

eq_scalar = []
eq_scalar.append(eq[0].dot(N.x))
eq_scalar.append(eq[0].dot(N.y))
```

In [13]:

```
#Solve for Intial Conditions

qi = [qA,x,y]

qd = [qB,qC,qD,qE,qF,qT]

eq_scalar_c = [item.subs(system.constant_values) for item in eq_scalar]
defined = dict([(item,initialvalues[item]) for item in qi])
eq_scalar_c = [item.subs(defined) for item in eq_scalar_c]

error = (numpy.array(eq_scalar_c)**2).sum()

f = sympy.lambdify(qd,error)

def function(args):
    return f(*args)

guess = [initialvalues[item] for item in qd]

result = scipy.optimize.minimize(function,guess)
if result.fun>1e-3:
    raise(Exception("out of tolerance"))

ini = []
for item in system.get_state_variables():
    if item in qd:
        ini.append(result.x[qd.index(item)])
    else:
        ini.append(initialvalues[item])
```

Setting Dynamic Constraints

Solving for dynamic constraints of system to run simulation.

In [14]:

```
#Adding Dynamic Constraints

#Position of motor limits
pos = amp*sympy.cos(freq*2*pi*system.t)

eq = []

eq.append(pFT-pP)
eq.append(pos*N.z-qA*A.z)

eq_d = []
eq_d = [item.time_derivative() for item in eq]

eq_dd = []
eq_dd = [item.time_derivative() for item in eq_d]
```

```
eq_dd_scalar = []
eq_dd_scalar.append(eq_dd[0].dot(N.x))
eq_dd_scalar.append(eq_dd[0].dot(N.y))
eq_dd_scalar.append(eq_dd[1].dot(N.z))

system.add_constraint(AccelerationConstraint(eq_dd_scalar))
```

Solving for Simulation

Code to run simulation and plot motion, states, and total energy in system.

For my optimization, the goal was to optimize the link surface area for our system. In order to accomplish this, the desired result of the system was the linear displacement of the system relative to the link length. Initially, I minimzed the total linear x distance. The issue with this is that as the link length increases, random movements that generate forward thrust. An example of this is, if the link length is 500mm, then the system moves 290mm. However, if the link length is the optimized value of 27.6mm, the system moves 234mm. These distances are very similar, but the relative size of the system is much smaller for the latter system. This means that the propulsion generated by the second system is much more substantial and additionally repeatable. It was for this reason that the 'perf' value that was optimized was the relative system displI was trying to maximize this value using a minimizing algorithm.

Additionally, the method that was used for optimization was scipy.minimize. The reason that I decided to use this algorithm was that it generated the best results of the different oprimizations given by Scipy. I also added bounds to the system of a minimum value of 10mm and a maximum value of 70mm for both the height and length. The reason that these values were selected were the constraints of the material using and the laser cutter. 10mm would result in very small segmennts and would risk burning the jonts of segments and 70mm was the maximum dimensions supported by the laser cutter for the bed size that we would be using. Additionally, the material we were using for our prototype was fiberglass and we only had 2 sheets for all group members. This meant that our overall dxf files could only be 12 inches long. This is how the 70mm bound was generated.

In [15]:

```
#Solve model and plot angles

#Constraints and Plots

f,ma = system.getdynamics();

tol = 1e-12
points = [pNA,pAB,pBC,pCD,pDE,pEF,pFT,pTtip]

def run_sim(args):
    new_l = args[0] #Set to variables that optimizing
    new_h = args[1] #Set to variables that optimizing

    #updating constant values affected by changing optimized values
    new_lT = new_l
    new_lP = new_l*5.5
```

```
    new_m = 666.7*new_l*new_h*0.001
    new_mT = 666.7*new_l*new_h*0.001
    new_area_p = new_l*new_h
    new_area_f = new_h*0.001
    new_Ixx = 1/12*(666.7*new_l*new_h*0.001)*(new_h**2 + 0.001**2)
    new_Iyy = 1/12*(666.7*new_l*new_h*0.001)*(new_h**2 + new_l**2)
    new_Izz = 1/12*(666.7*new_l*new_h*0.001)*(new_l**2 + 0.001**2)
    new_Ixx_T = 1/12*(666.7*new_l*new_h*0.001)*(new_h*2 + 0.001**2)
    new_Iyy_T = 1/12*(666.7*new_l*new_h*0.001)*(new_h**2 + new_l**2)
    new_Izz_T = 1/12*(666.7*new_l*new_h*0.001)*(new_l**2 + 0.001**2)

    #Populate constants with new values
    constants = system.constant_values.copy()
    constants[l] = new_l
    constants[lT] = new_lT
    constants[lP] = new_lP
    constants[m] = new_m
    constants[mT] = new_mT
    constants[area_p] = new_area_p
    constants[area_f] = new_area_f
    constants[Ixx] = new_Ixx
    constants[Iyy] = new_Iyy
    constants[Izz] = new_Izz
    constants[Ixx_T] = new_Ixx_T
    constants[Iyy_T] = new_Iyy_T
    constants[Izz_T] = new_Izz_T

    states=pynamics.integration.integrate(func1,ini,t,rtol=tol,atol=tol,hmin=tol, args=

    return states

def measured_perf(args):
    print(args)
    try:
        states = run_sim(args)
        linear_disp = abs(states[-1,7])/args[0] #linear displacement relative to segmen
        perf = (1/linear_disp)**2
        return perf
    except scipy.linalg.LinAlgError:
        return 1000


pynamics.system.logger.setLevel(logging.ERROR)

if run_fit:

    func1 = system.state_space_post_invert(f,ma)

    guess = [0.0276,0.01] #Change depending on what factor you are optimizing

    pynamics.system.logger.setLevel(logging.ERROR)

    sol = scipy.optimize.minimize(measured_perf,guess,bounds=[(0.01,0.07),(0.01,0.07)])
    result = sol.x
    print(result)
```

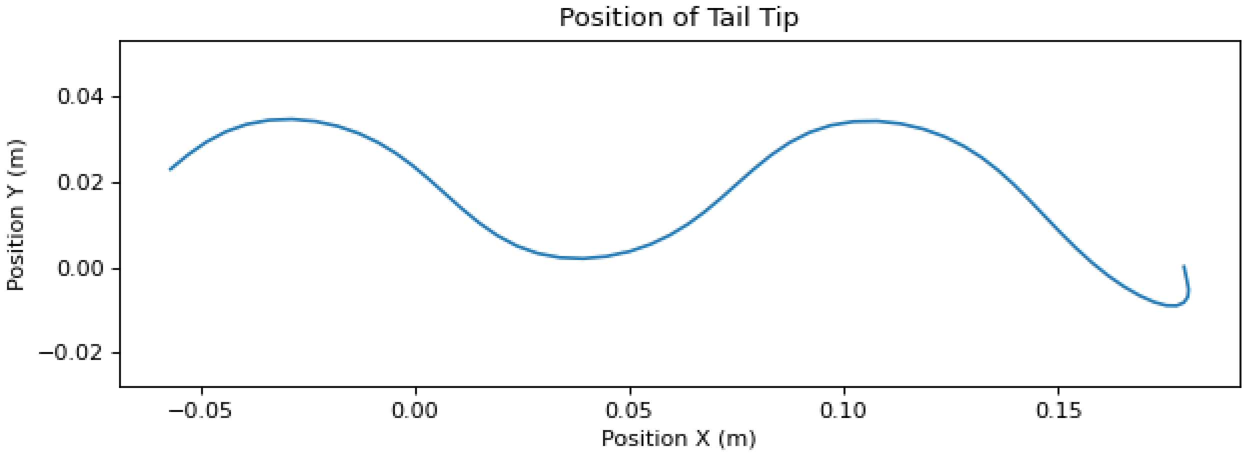
```
2022-04-26 18:30:29,040 - pynamics.system - INFO - getting dynamic equations
```

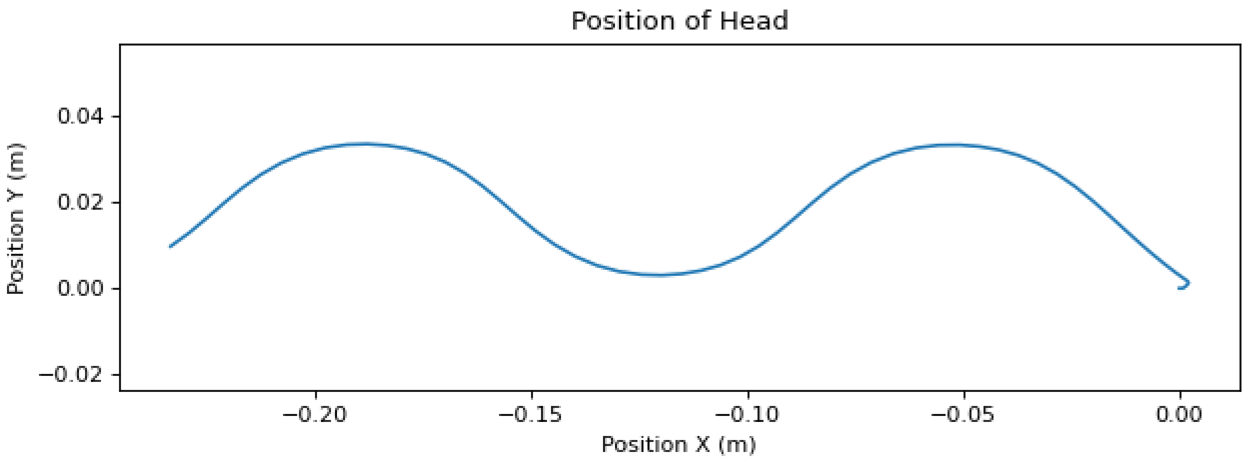
In [16]:

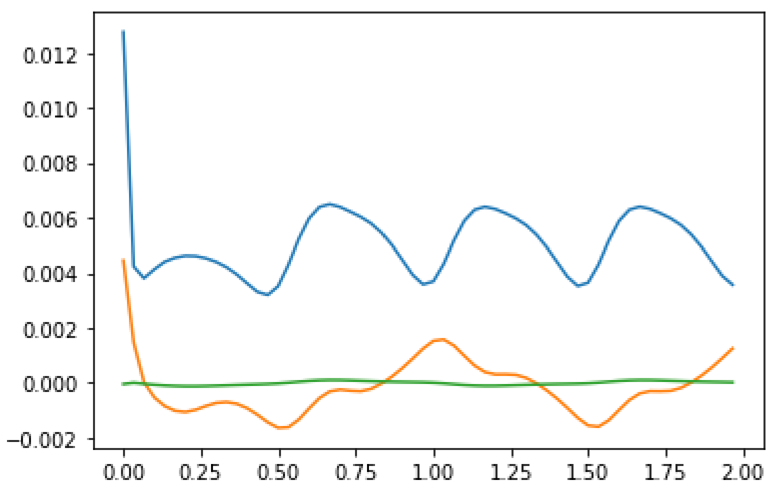
```
#Constraint Forces
```

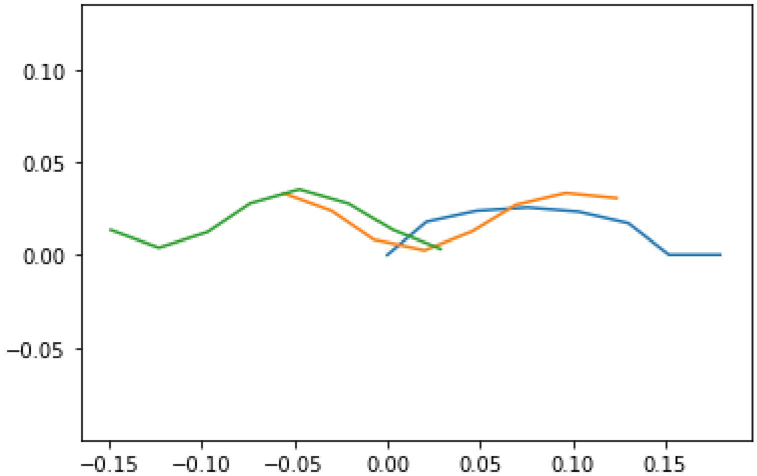
```
if run_fit:
    states2 = run_sim(result)
    points_output = PointsOutput(points,system)
    y2 = points_output.calc(states2,t)

    fig = plt.figure()
    ax1 = plt.subplot(2,1,2)
    ax1.plot(t,states2[:,:7])
    ax1.legend(['qA','qB','qC','qD','qE','qF','qT'])
    ax1.set_title('State Positions')
    ax1.set_xlabel('Time (s)')
    ax1.set_ylabel('Position (mm)')

    ax2 = plt.subplot(2,1,1)
    ax2.plot(y2[:,0,0],y2[:,0,1])
    ax2.axis('equal')
    ax2.set_title('Position of Head')
    ax2.set_xlabel('Position X (m)')
    ax2.set_ylabel('Position Y (m)')

    fig.tight_layout()

    print(result)

else:

    func1,lambda1 = system.state_space_post_invert(f,ma,return_lambda = True)
    constants = system.constant_values.copy()
    states=pynamics.integration.integrate_odeint(func1,ini,t, args=({'constants':consta


    points_output = PointsOutput(points,system)
    y = points_output.calc(states,t)

    fig = plt.figure(figsize=(8, 6), dpi=80)

    ax1 = plt.subplot(2,1,1)
    ax1.plot(y[:,7,0],y[:,7,1])
    ax1.axis('equal')
    ax1.set_title('Position of Tail Tip')
    ax1.set_xlabel('Position X (m)')
    ax1.set_ylabel('Position Y (m)')

    ax2 = plt.subplot(2,1,2)
    ax2.plot(y[:,0,0],y[:,0,1])
    ax2.axis('equal')
    ax2.set_title('Position of Head')
    ax2.set_xlabel('Position X (m)')
    ax2.set_ylabel('Position Y (m)')


    fig.tight_layout()

    lambda2 = numpy.array([lambda1(item1,item2,system.constant_values) for item1,item2
    plt.figure()
    plt.plot(t, lambda2)

    points_output = PointsOutput(points,system)
    y = points_output.calc(states,t)
    points_output.plot_time(20)
```

```
2022-04-26 18:30:40,682 - pynamics.integration - INFO - beginning integration
2022-04-26 18:31:18,892 - pynamics.integration - INFO - finished integration
2022-04-26 18:31:18,932 - pynamics.output - INFO - calculating outputs
2022-04-26 18:31:18,932 - pynamics.output - INFO - done calculating outputs
2022-04-26 18:31:19,618 - pynamics.output - INFO - calculating outputs
2022-04-26 18:31:19,626 - pynamics.output - INFO - done calculating outputs
```







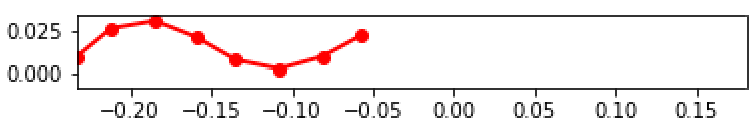


In [17]:

```
points_output.animate(fps = fps,movie_name = 'dynamics_free_swimming_opt_plate_area.mp4

HTML(points_output.anim.to_html5_video())
```

Out[17]:

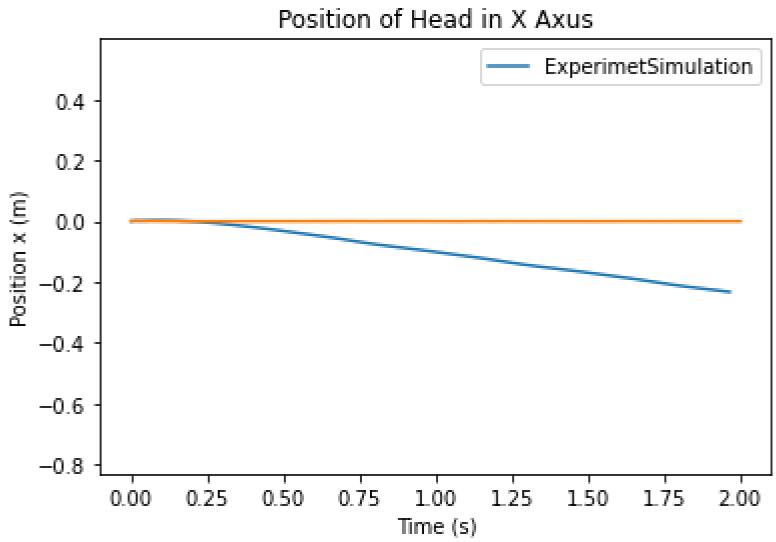
0:00 / 0:02



In [19]:

```
#Plots of Simulation Vs Data

#import test data and convert to numPy arrays
data = pd.read_excel(r'Link_Data.xlsx')
df = data.to_numpy()

fig = plt.figure()
plt.plot(t,y[:,0,0],df[:,0],df[:,1])
plt.axis('equal')
plt.title('Position of Head in X Axus')
plt.xlabel('Time (s)')
plt.ylabel('Position x (m)')
plt.legend(['Experimet' 'Simulation'])
```

Out[19]: <matplotlib.legend.Legend at 0x201ae5a94c0>



Prototype

After optimizing the link dimensions with a height of 10mm and length of 27.6mm. I then made a laminate structure using 0.010 inch fiberglass for the rigid material and 0.005 inch polyesther as the flexible material. The following pictures are of that prototype and the test set up for the system to swim in water.

I ran the experiment with the same motor parameters and length constrained distnace as the simulation. However, the system did not move at all (Video attached in sumbission). The likely reason for this is that the servo motor and carriage that were used to support the device was substantially heavier than the device itself. Therefore the inertia of the carriage required a thrust that the device could not generate. If an additional version of the system was to be made, using a stiffer flexible material would be necessary. The plate geometry was optimized for the given stiffness of the 0.005 inch polyesther plastic. This was a very soft material and therefore required a very small system to match the stiffness. If a stiffer material could be used for the flexible layer, then the whole system could get scaled to match the inertia and mass of the carriage.

Based off the simulation, the design is feasible but requires tuning based off the method in which it is tested. Because the device did not move at all, there was no reason to plot the experimental displacement and the simulated experiment. The experiment had a zero displacement and would not provide any correlation between the experiment and the simulation.