Studying the Optimal Control of a Non-Holonomic System using an Inverted Pendulum

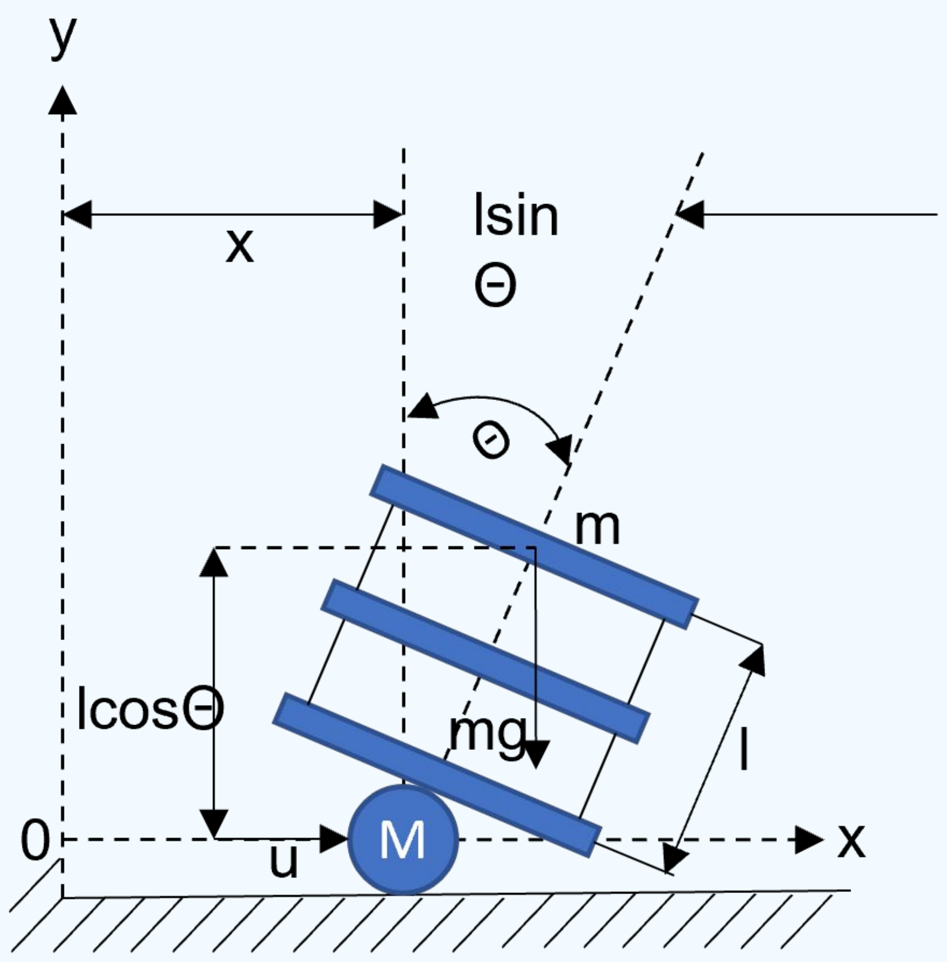
Team 12

Aniruddha Anand Damle 1222585013

Vishal Rajesh Vasisht 1222546390

Prakriti Biswas 1222851266

PROBLEM STATEMENT



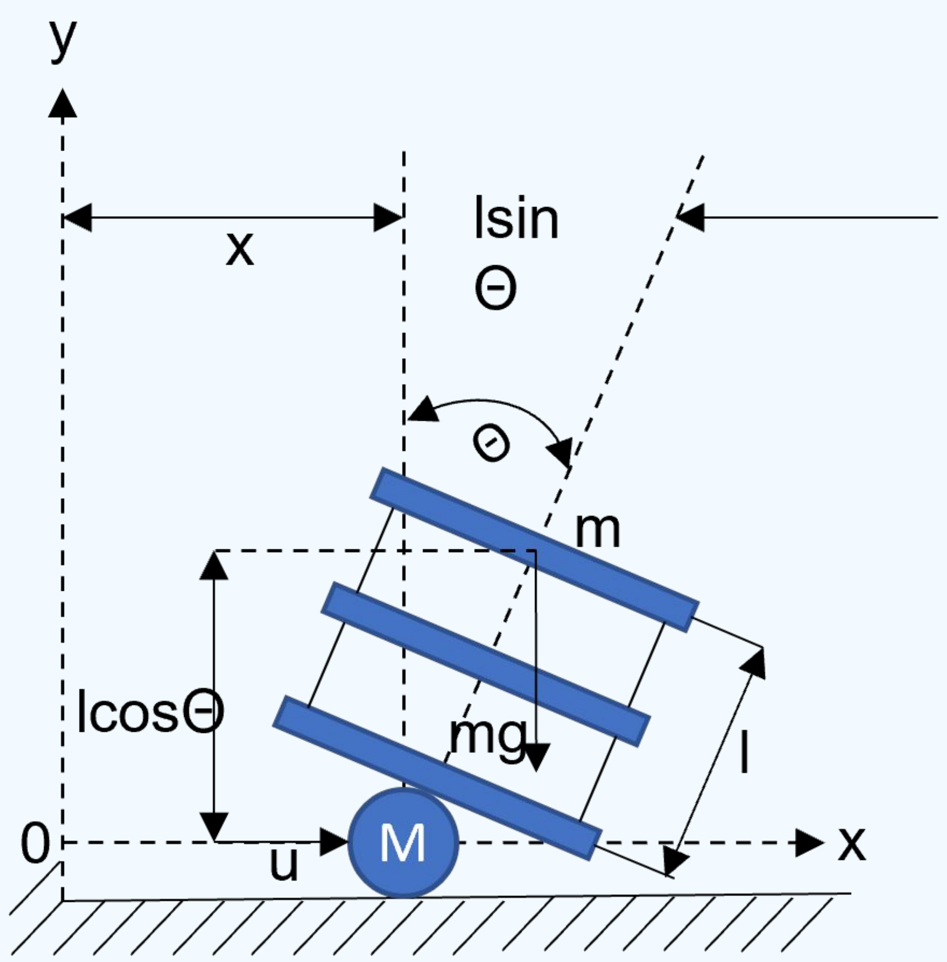
- An inverted pendulum on a cart is an essential optimal control problem because it is a classic example of a non-linear and unstable system that can be stabilized using control theory
- It is widely used in engineering, robotics, and control systems as a benchmark for testing and evaluating control algorithms
- The challenge lies in finding the right balance between keeping the pendulum upright and keeping the cart stable
- The solution to this problem requires precise controls in real-time to maintain balance, which makes it a good project for future optimal control developments

MATHEMATICAL MODELLING

- Nonlinear system of equations:

$$m\ddot{x}cos(\theta) + ml\ddot{\theta} = mgsin(\theta)$$

$$(M+m)\ddot{x} - mlsin(\theta)\dot{\theta}^2 + mlcos(\theta)\ddot{\theta} = u$$



- State-space model:

$$\frac{d}{dt}\delta\mathbf{x} = \mathbf{J}_x(\mathbf{x}_0, u_0)\delta\mathbf{x} + \mathbf{J}_u(\mathbf{x}_0, u_0)\delta\mathrm{u}$$

$$\frac{d}{dt}\delta\mathbf{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{(M+m)g}{Ml} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{mg}{M} & 0 & 0 & 0 \end{bmatrix}\delta\mathbf{x} + \begin{bmatrix} 0 \\ \frac{-1}{Ml} \\ 0 \\ \frac{1}{M} \end{bmatrix}\delta u$$

$$y = \begin{bmatrix} \theta \\ x \end{bmatrix} = Cx = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix}$$

OPTIMAL CONTROL USING PID

- The PID controller calculates the error between the measured output and the desired value and attempts to minimize this error
- As P increases, the response of the system becomes faster. I is used to stop oscillations and steady state error and D decreases overshoot and yields higher gain and stability of the system
- Tuning of the PID controller is the finding of the optimal gains for P, I and D
- Optimality of the controller is obtained by minimizing the steady state error, rise time, settling time and overshoot
- Two PID Controllers have been developed to control this system
- The equations for the PID control are given as below:

$$u_p = K_{pp} e_\theta(t) + K_{ip} \int e_\theta(t) + K_{dp} \frac{de_\theta(t)}{dt}$$

$$u_c = K_{pc} e_x(t) + K_{ic} \int e_x(t) + K_{dc} \frac{de_x(t)}{dt}$$

OPTIMAL CONTROL USING LQR

- Optimal Control using LQR is designed using the state space model defined in the previous slide to minimize the performance measure,

$$J = \int (X^T Q X + u^T R u)dt$$

- The LQR gain is obtained by,

$$K = R^{-1} B^T P$$

- Here, P is found from the solution of the matrix algebraic Riccatii equation,

$$A^T P + PA - PBR^{-1} B^T P + Q = 0$$

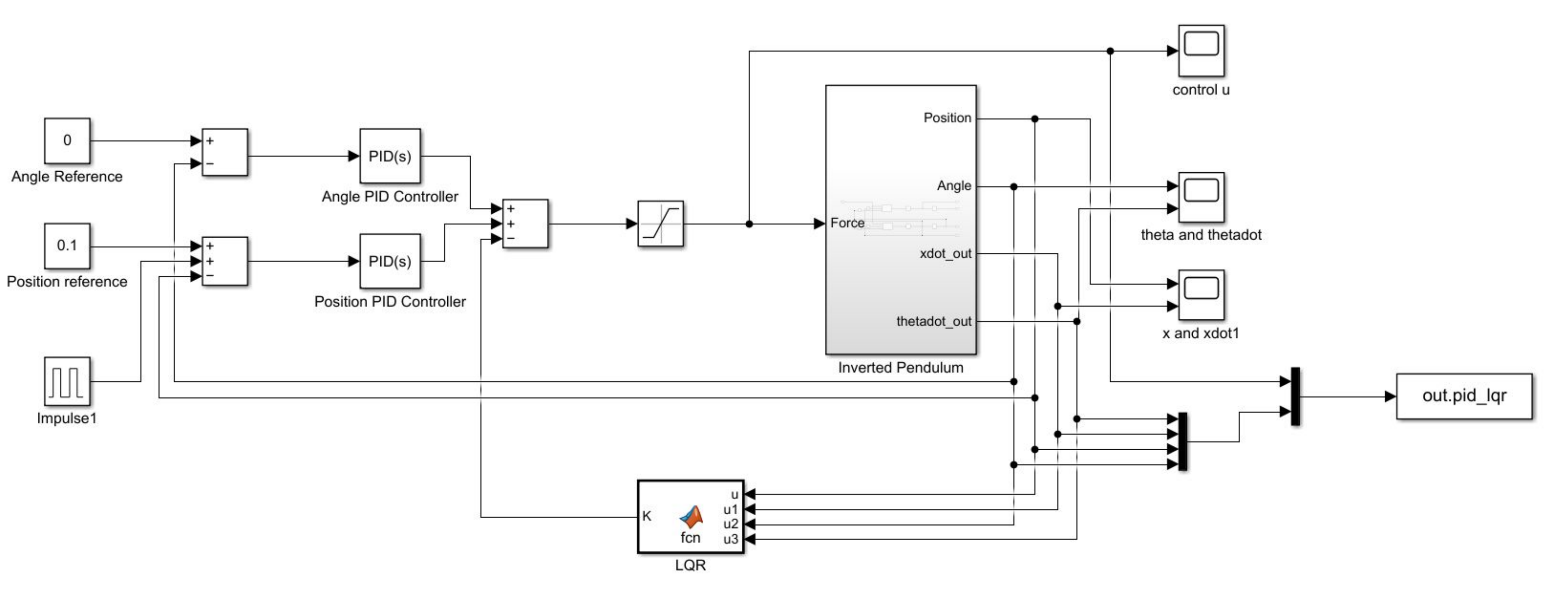
PID + LQR CONTROLLER

- An LQR controller is a type of state feedback controller
- It provides closed loop, stable and high performance outputs for control systems

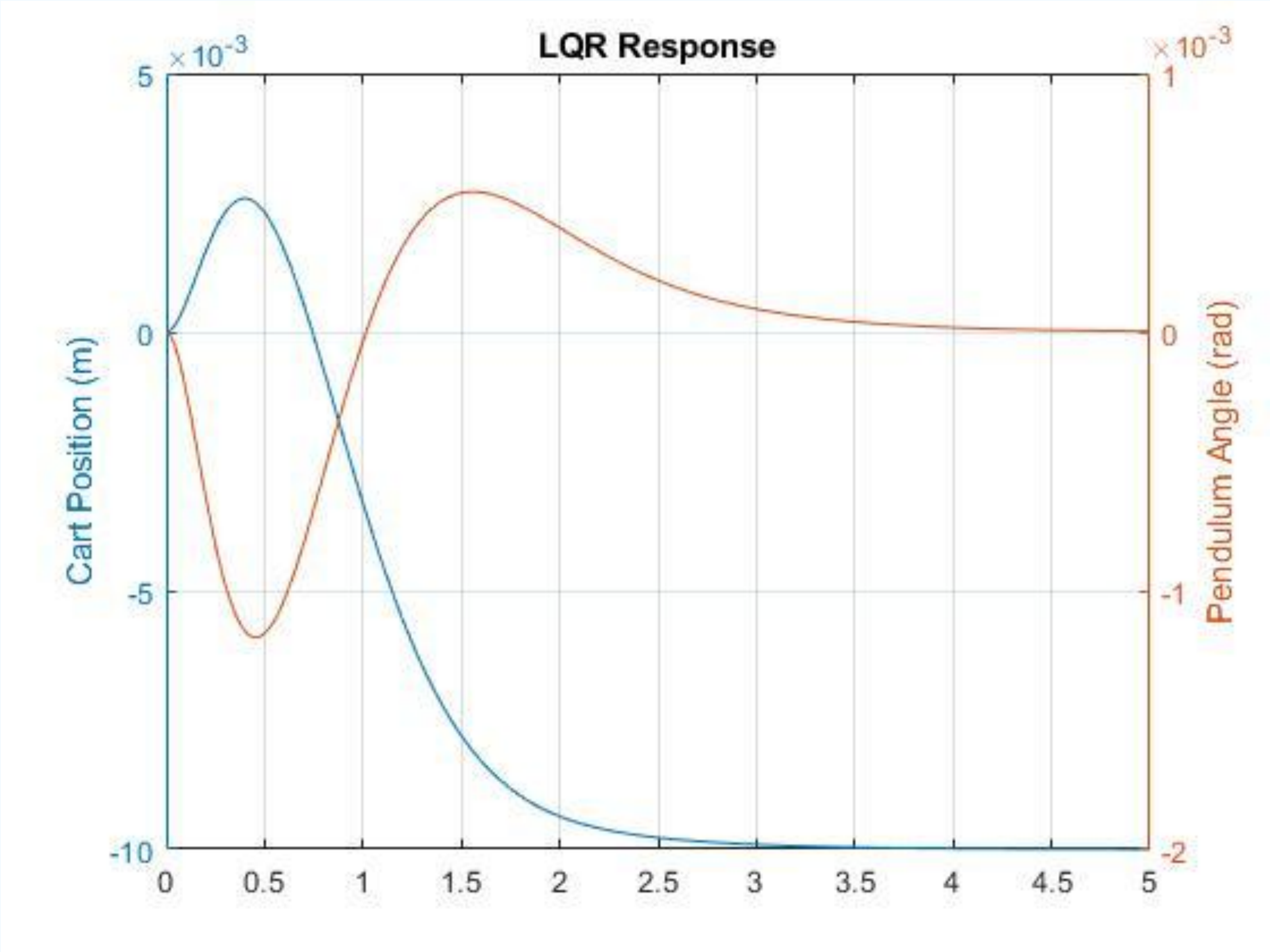
- Combining a PID and LQR controller helps both controllers overcome setbacks in their respective designs
- An LQR controller provides stability and a high performance index but struggles at providing a desired steady state output
- PID controllers come with the ability to tune the gains to achieve the optimized steady state behavior but cannot provide the robust stability found in the LQR controller

SIMULATION

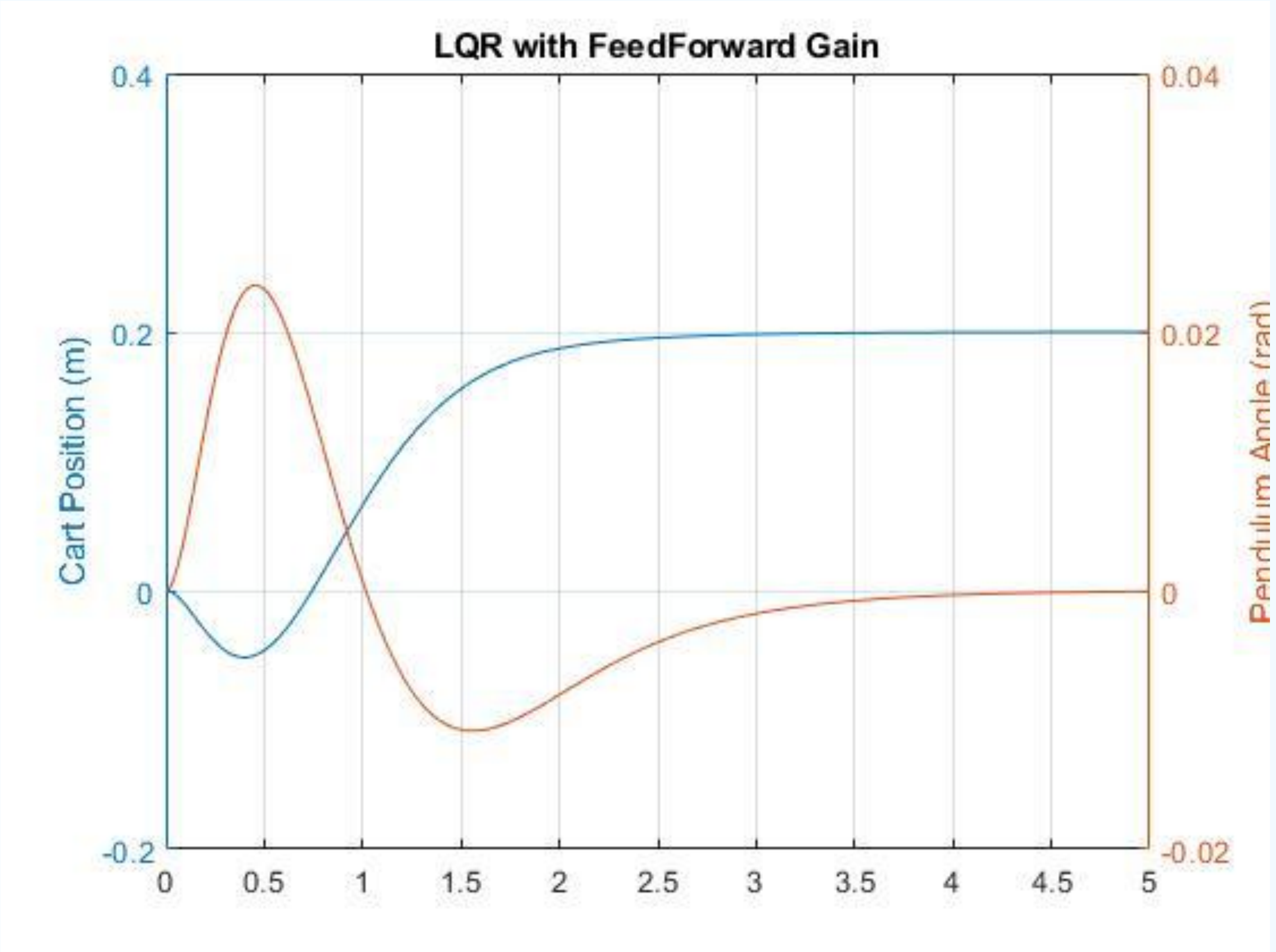
- The model depicted in the free body diagram of the inverted pendulum-cart system determines the position and angle of the Inverted Pendulum, which is transmitted to the LQR function block for further processing
- The reference for the upward position of the inverted pendulum is established using constant blocks
- The LQR output and PID controls are employed to keep the pendulum in an upright position



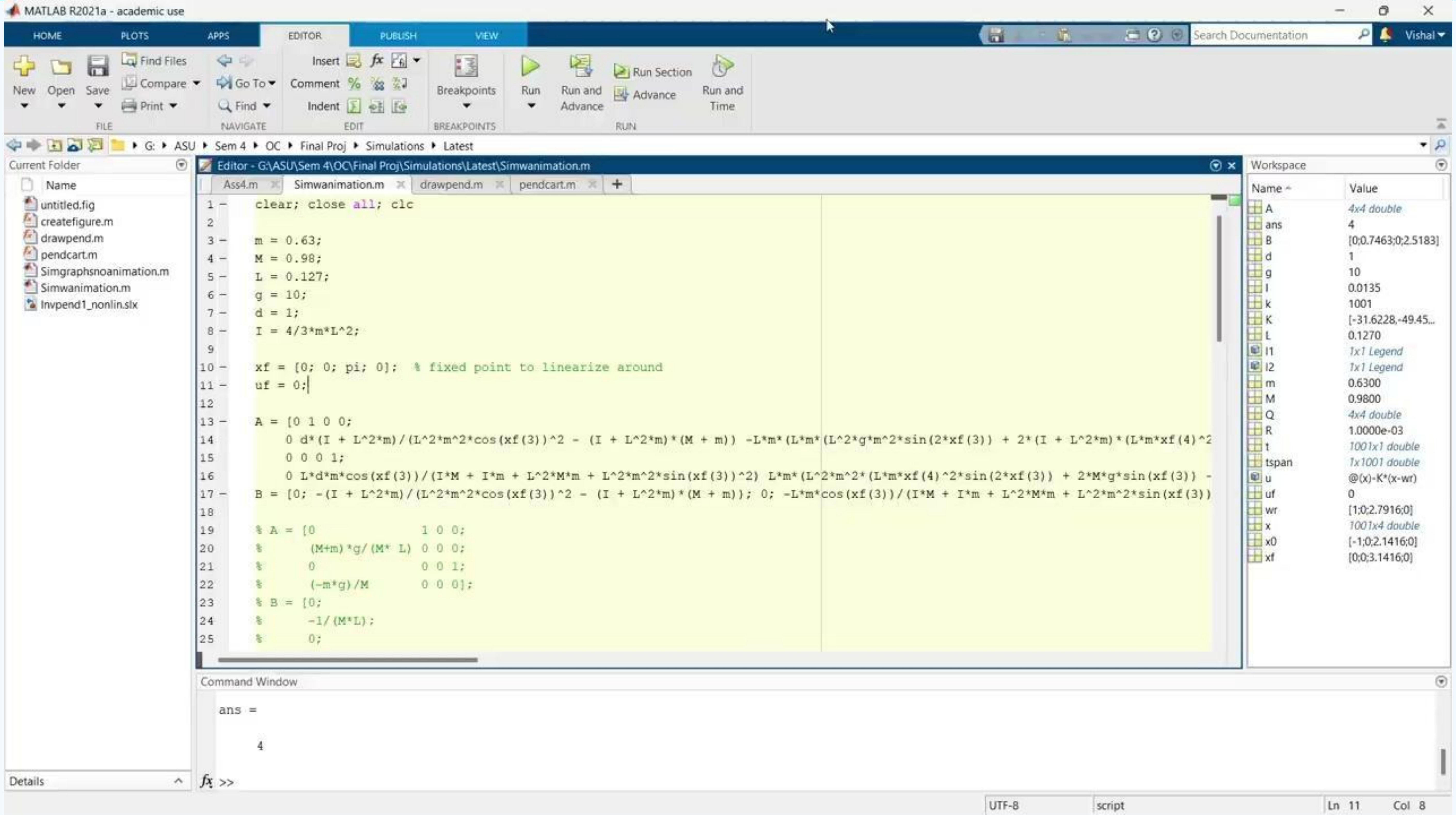
SIMULATION



LQR Response showing the pendulum angle stabilizing at 0 with the movement of the cart compensating for the change in angle of the pendulum



LQR Response with feedforward gain showing the pendulum angle stabilizing at 0 with the movement of the cart compensating for the change in angle of the pendulum



This video shows an animation of the simulation of our code. It can be seen that the inverted pendulum system is quickly stabilized.

SIMULATION
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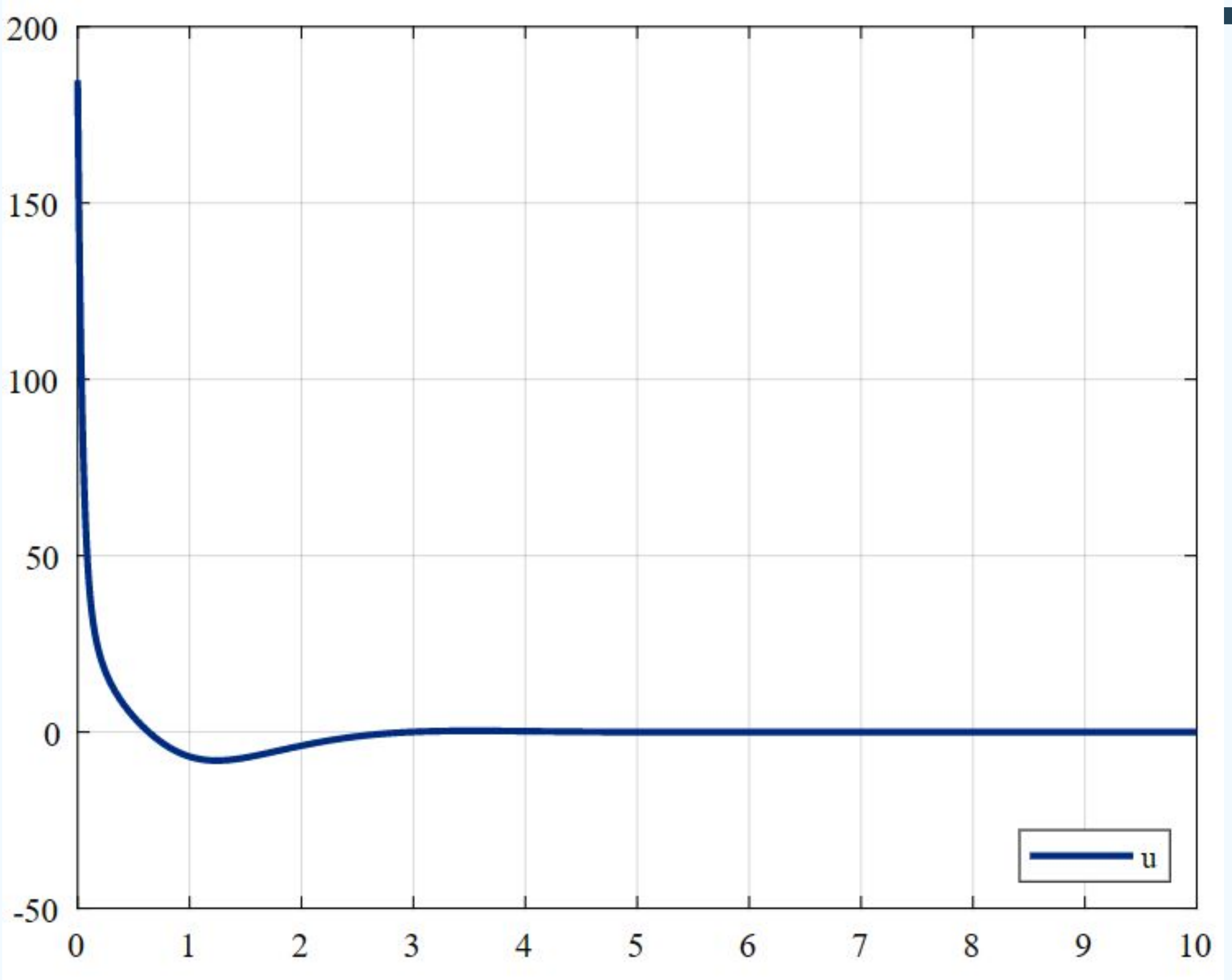
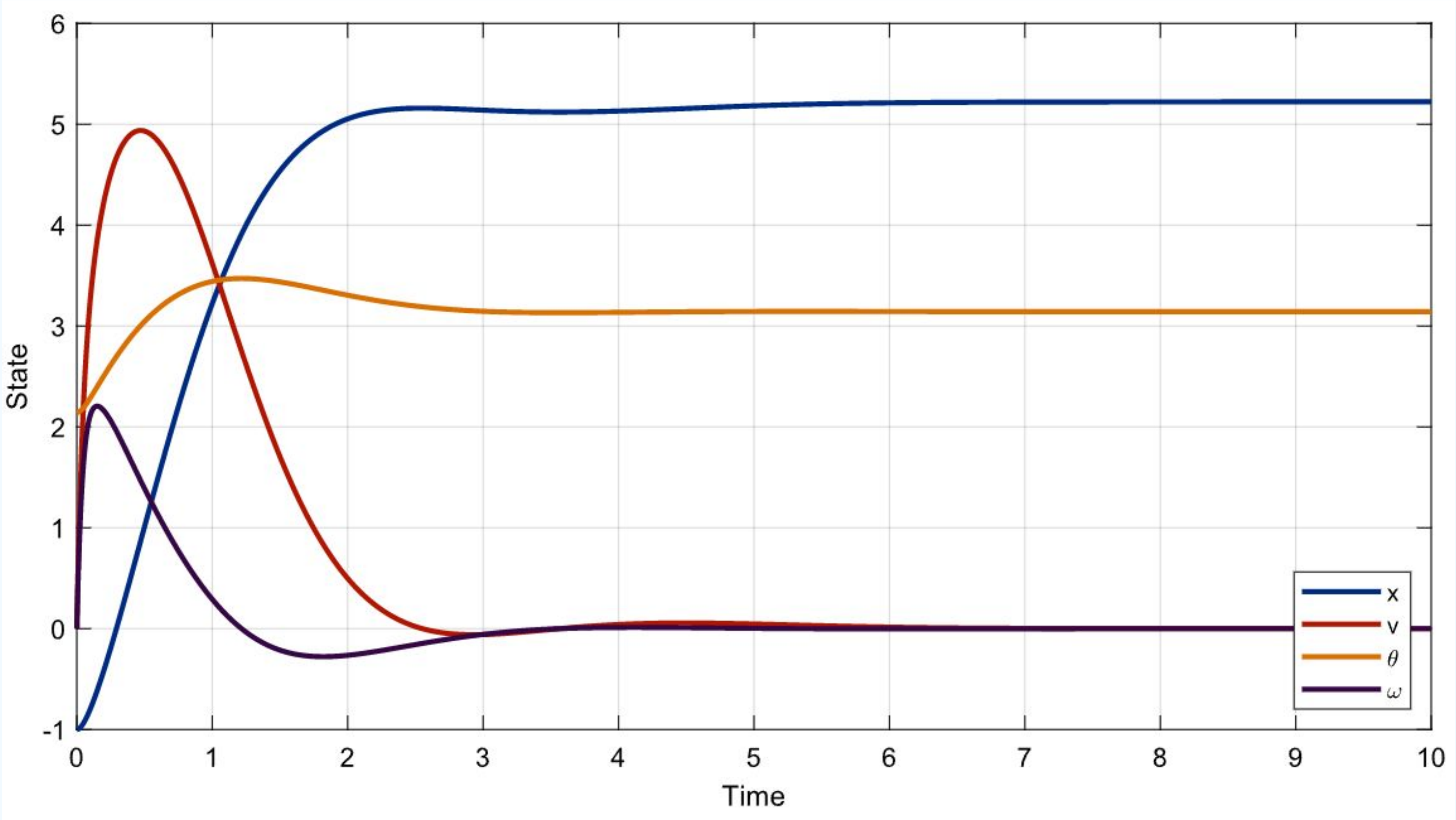


Control Law Stability: We can see that the control law u is stabilized at 0 over time which shows that the system is stable and no more adjustments are necessary. This figure shows an instance of control of the inverted pendulum where the instability after coming to the required position is not considered.

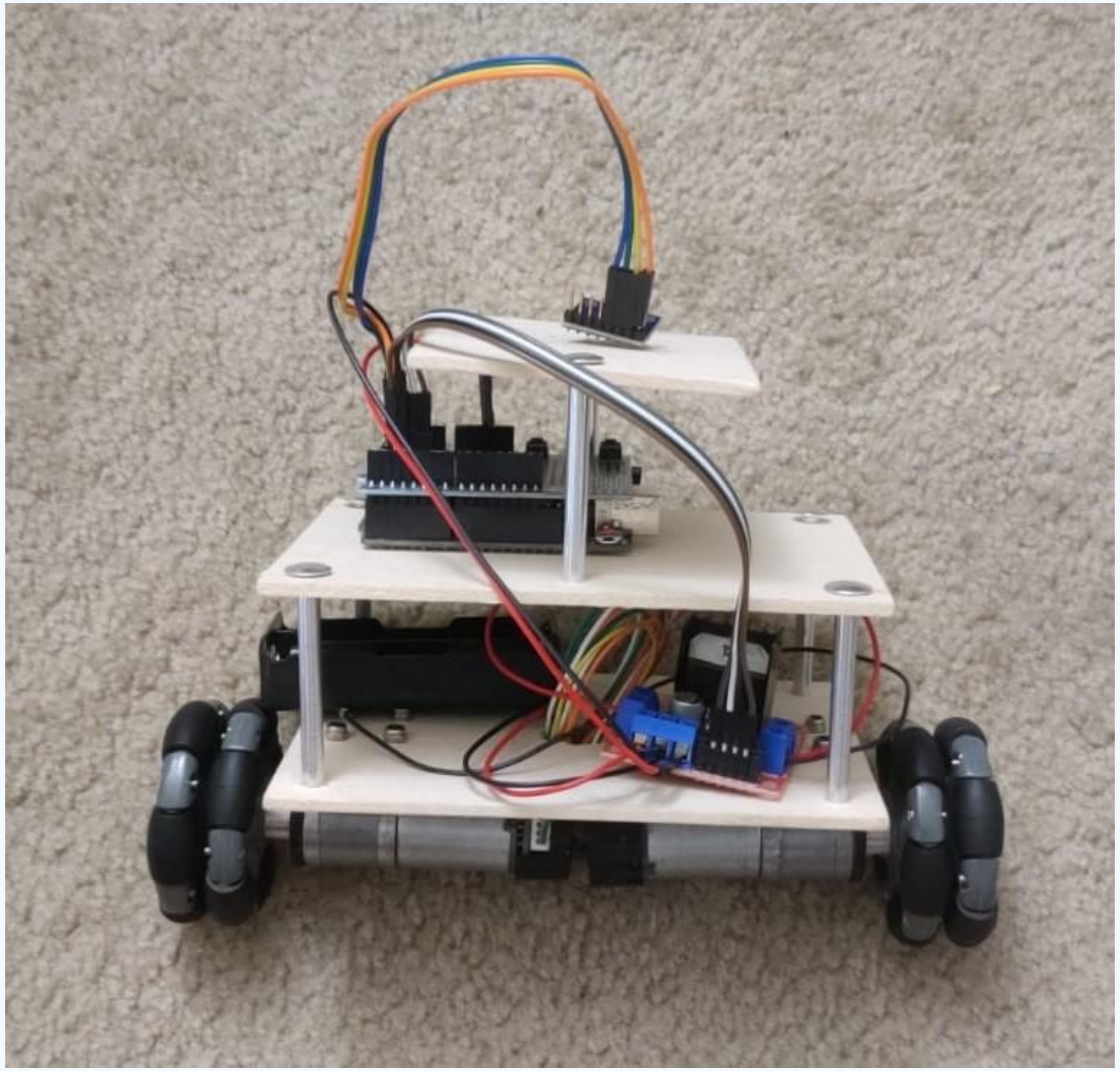


Graph showing the change in state of the position of the cart, x, angle of the pendulum, θ, velocity and angular velocity of the cart, v and ω respectively.

This graph shows that over time the angle of the inverted pendulum is stabilized and that the cart accelerates and moves to stabilize the system.

PHYSICAL MODEL OF THE CART

- The physical model of the cart has been designed and built
- The physical model has the following parameters:
    - Mass of the Cart = 98g
    - Mass of the Pendulum = 63g
    - Length of the Pendulum = 0.127m
    - Height of the cart = 0.05m
    - Length of the track = 1m



HARDWARE SETUP

- The hardware components include a controller, motor driver, and IMU sensor
- An Arduino Uno board was used as the brain of the system
- L298 motor driver was used to control the speed and direction of the motors
- Each motor's speed and direction were managed by an integrated optical encoder, which also managed the robot's orientation.
- The robot's roll, pitch, yaw, and orientation in the x, y, and z directions were measured using the MPU9250.
- The simulation results for kp, ki, and kd are utilized as the system's control parameters.
- As the simulation does not account for the actual system with all of its manufacturing flaws, these numbers were once again adjusted in accordance with the actual implementation.

FUTURE WORK

- Simulation:
    - Fine-tune results
    - Further tune Kp, Ki, Kd values of the angle and cart PIDs
    - Make the cart return to its original position after stabilizing the system inverted pendulum
    - Correct the angle in which the inverted pendulum is deemed to be stable and make sure that angle is 0

- Implementation:
    - Fine-tuning the working of the physical cart to quickly and efficiently stabilize the inverted pendulum

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QUESTIONS?