Studying the Optimal Control of a Non-Holonomic System using an Inverted Pendulum

Team 12

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PROBLEM STATEMENT

- An inverted pendulum on a cart is an essential optimal control problem because it is a classic example of a non-linear and unstable system that can be stabilized using control theory. It is widely used in engineering, robotics, and control systems as a benchmark for testing and evaluating control algorithms.

- The challenge lies in finding the right balance between keeping the pendulum upright while keeping the cart stable.

- The solution to this problem requires precise controls in real-time to maintain balance, which makes it a good project for future optimal control developments.

MATHEMATICAL MODELLING

- Nonlinear system of equations:

$$m\ddot{x}cos(\theta) + ml\ddot{\theta} = mgsin(\theta)$$

$$(M+m)\ddot{x} - mlsin(\theta)\dot{\theta}^2 + mlcos(\theta)\ddot{\theta} = u$$

- State-space model:

$$\frac{d}{dt}\delta x = A\delta x + B\delta u$$

$$A = J_x(x_0, u_0) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{(M+m)g}{Ml} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-mg}{M} & 0 & 0 & 0 \end{bmatrix}$$

$$B = J_u(x_0, u_0) = \begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \\ \frac{\partial f_3}{\partial u} \\ \frac{\partial f_4}{\partial u} \end{bmatrix}_{(x_0, u_0)} = \begin{bmatrix} 0 \\ \frac{-1}{Ml} \\ 0 \\ \frac{1}{M} \end{bmatrix}$$

$$y = \begin{bmatrix} \theta \\ x \end{bmatrix} = Cx = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix}$$

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MATHEMATICAL MODELLING

- Optimal Control using LQR is designed using the state space model defined in the previous slide to minimize the cost function,

$$J = \int (X^T Q X + u^T R u)\,dt$$

- The LQR gain is obtained by,

$$K = R^{-1} B^T P$$

- Here, P is found from the solution of the matrix algebraic Riccatii equation,

$$A^T P + PA - PBR^{-1} B^T P + Q = 0$$

SIMULATION

- The model depicted in Figure 1 determines the position and angle of the Inverted Pendulum, which is transmitted to the LQR function block in Figure 2 for further processing.

- Linear analysis of the model presented in Figure 1 demonstrates that the inverted pendulum is inherently unstable and relies on the cart for stabilization.

- The reference for the upward position of the inverted pendulum is established using constant blocks.

- The LQR output and PID controls are employed to keep the pendulum in an upright position.

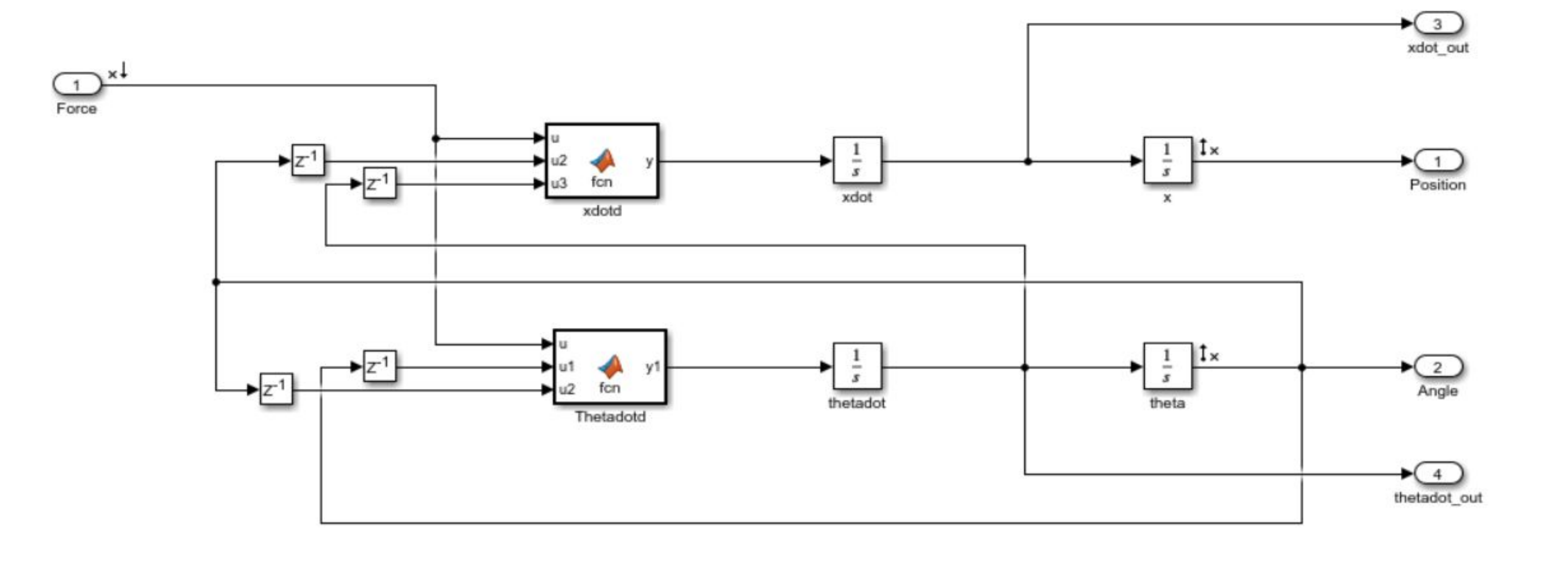

Figure 1: Simulink Model of the Inverted Pendulum

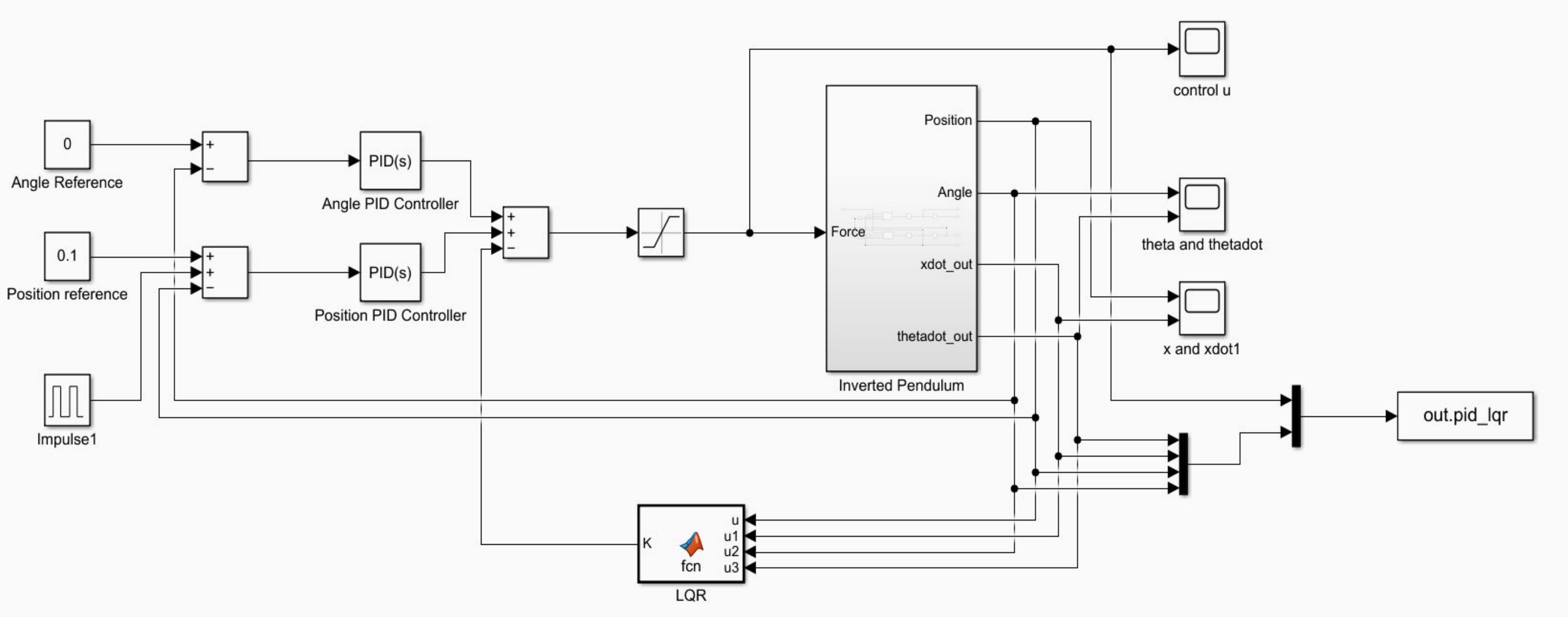

Figure 2: Simulink Model for the Inverted Pendulum-Cart System with PID and LQR

FUTURE WORK

- Simulation:
  - Fix bugs to get values of Q and R to generate the K vector
  - The K vector will serve as the feedback for the two PID controllers in our robot.
  - A dynamic simulation of the robot can be created to fine-tune the PID error coefficients.

- Implementation:
  - Manufacturing of the physical cart is completed, the electronics and firmware are in progress.
  - The hardware components include a controller, motor driver, and IMU sensor, and if time permits, we plan to add an ultrasonic sensor to prevent collisions.
  - We will use a timer-based scheduler on an Arduino platform to sample the sensors at a constant time interval $\Delta(t)$.