The Object Unknown Parameters Estimation for the “Inverted Pendulum - Cart” System in the Steady State

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***Abstract* – The “inverted pendulum - cart” system, consisting of object and two regulators, is described by the discrete equations in the state space. The case when unknown parameters are in the state matrix and the man-agement matrix is considered. The calculation technique of Fischer in-formation matrix in the steady state is used for the object parameters specification. The offered approach is actual and useful in engineering calculations.**

***Index Terms* – Inverted pendulum, Fisher information matrix, steady state, Kalman filter.**

I. Introduction

BY MEANS of the system “inverted pendulum – cart” it is possible to simulate various real systems, first of all, it is transport systems. Here it is possible to consider the robots behavior also. In modern literature the “inverted pendulum - cart” model is considered rather widely [1], [2]. It means that yet the management method which would be equally good in all possible cases still isn’t developed. In this work it is necessary to estimate the dynamic system condition on measurements which are functions of a state and stochastic noises. Various methods are offered for the solution of the object parameters estimation problem [3] – [10]. In many works the preference is given to use of Fischer information matrix when experiments are planning. Research of the steady state for discrete systems allows to receive the calculation ratios having practical value.

II. Problem Definition

Generally the task consists in that by means of the cart to hold a pendulum in vertical situation. The pendulum is attached to the cart and it is affected by attraction force. The cart moves in the horizontal direction. In Fig. 1 the system «inverted pendulum – cart» is shown. The object is unstable. In this figure m – the mass of the core on the platform, M – the platform mass, s – the platform coordinate, u – the input signal on the platform, θ – the core deviation corner from the balance position, l – the middle of the core. Output sizes are the corner of the pendulum and the position of the cart [1], [2], [11].

It is supposed that the regulators used to stabilize the pendulum position in vertical situation ($\theta = 0$) in case of the cart movement on distance $S = S_1$ are already calculated. It is required to receive mathematical model which gives rather complete description of physical process and thus isn’t too difficult, that is could be used for computing calculations.

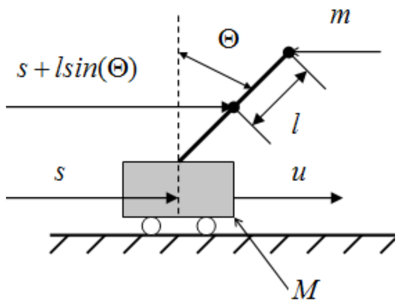

Fig. 1. The inverted pendulum on the cart.

We will write down the differential equations as follows [11]:

$$(1-\frac{ml}{M_t L})\ddot{S} + \frac{ml}{M_t L} g\theta = \frac{1}{M_t} u ,$$

$$(1-\frac{ml}{M_t L})\ddot{\theta} + \frac{1}{L} g\theta = -\frac{1}{M_t L} u ,$$

where $M_t = M + m$, $L = \dfrac{I + ml^2}{ml}$.

The object and two regulators are the parts of the system. One of regulators (the regulator of the cart position) can be presented in the form:

$$[v(s) - S(s)] \cdot \left(\frac{\delta + \varepsilon \cdot s}{s}\right) - v \cdot (S^*(s) + w) = u_s(s) ,$$

where $S^*(s) = s \cdot S(s)$. Other regulator (the cart deviation corner regulator from vertical situation) is described in the following form:

$$(\theta^*(s) + v) \cdot \chi + \theta(s) \cdot \left(\frac{\alpha + \beta \cdot s}{s}\right) = u_\theta(s) ,$$

where $\theta^*(s) = s \cdot \theta(s)$.

For receiving some results sometimes it is useful to carry out sampling of the system equations, and then to consider

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the limit behavior of these algorithms when the interval between discrete timepoints aspires to zero. The system discrete model is given below:

$$x_1^{k+1} = x_1^k + \alpha x_4^k \Delta t,\ x_2^{k+1} = x_2^k + \delta V^k \Delta t - \delta x_6^k \Delta t,$$

$$x_3^{k+1} = x_3^k + \theta_1 x_4^k \Delta t - \theta_3 \cdot V2 \cdot \Delta t,\ x_4^{k+1} = x_4^k + x_3^k \Delta t,$$

$$x_5^{k+1} = x_5^k - \theta_2 x_4^k \Delta t + \theta_3 \cdot V2 \cdot \Delta t, x_6^{k+1} = x_6^k + x_5^k \Delta t,$$

where $V2 = \varepsilon V^k - \varepsilon x_6^k + x_2^k - \nu x_5^k + x_1^k + \beta x_4^k + \chi x_3^k$.

The linear discrete dynamic system briefly can be described in a look [12] – [20]:

$$x_{k+1} = \Phi x_k + \Psi u_k + \Gamma w_k,\ y_{k+1} = H x_{k+1} + v_{k+1},$$

where x - state vector of dimension six, u - control, w - process noise, y – measurement noise vector of dimension two, v – measurement noise, Φ – state matrix 6x6, Γ – process noise vector, Ψ – control vector 6x1, H – observation matrix 2x6. Further the main stages of the calculation technique of Fischer information matrix when unknown parameters $\theta = (\theta_1, \theta_2)$ are in the matrix $\Phi(\theta)$ and the matrix $\Psi(\theta)$ are considered. We assume that all transition processes ended, that the steady state is considered.

III. THE FISHER INFORMATION MATRIX CALCULATION WITH UNKNOWN PARAMETERS IN THE STATE AND THE CONTROL MATRIXES

The Kalman filter with the updated sequence is used for the system condition estimation. The Kalman filter equations in the steady state can be written down as follows:

$$P_1 = \Phi P_0 \Phi^T + \Gamma Q \Gamma^T,\ \Sigma_\infty = (HP_1H^T + R)^{1/2},$$

$$K1_\infty = P_1 H^T \Sigma_\infty^{-1},\ P_0 = (I - K1_\infty \Sigma_\infty^{-1} H) P_1.$$

The object parameters estimate are used in formulas of the Kalman filter with errors because exact values of parameters are unknown. For further calculations requires knowledge of the following matrixes:

$$\frac{\partial P_1}{\partial \theta_1}, \frac{\partial P_0}{\partial \theta_1}, \frac{\partial P_1}{\partial \theta_2}, \frac{\partial P_0}{\partial \theta_2}, \frac{\partial \Sigma_\infty}{\partial \theta_1}, \frac{\partial \Sigma_\infty}{\partial \theta_2}, \frac{\partial K1_\infty}{\partial \theta_1}, \frac{\partial K1_\infty}{\partial \theta_2},$$

$$\frac{\partial \Psi}{\partial \theta_1} = 0, \frac{\partial \Psi}{\partial \theta_2} = \begin{bmatrix} 0 \\ 0 \\ -\varepsilon \\ 0 \\ \varepsilon \\ 0 \end{bmatrix}, \frac{\partial \Phi}{\partial \theta_1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{\partial \Phi}{\partial \theta_2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & -\chi & -\beta & \nu & \varepsilon \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & \chi & \beta & -\nu & -\varepsilon \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The received expressions are used for the Fischer information matrix calculation. The Fischer matrix calculation algorithm is more in detail considered in works [9], [10]. In many cases it is required to improve apriory information with the help of the measurements received in the system work process. It is necessary when the input signals sequence is formed for the system behavior optimum control. By change of the input signal we find the maximum value of the Fischer matrix determinant. Further we calculate the object parameters estimates by method of the smallest squares. The received formulas are approved on test examples. All calculations were executed in the MATLAB.

IV. CONCLUSION

The procedure of object parameters estimation for the “inverted pendulum - cart” model which is rather simple for realization on the computer and gives the possibility of the system identification in real time is result of research. Thus the parameters estimation problem can be solved with the state estimation problem.

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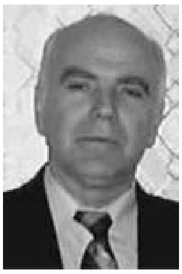

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