

Introduction to continuous-time movement modeling

Christen H. Fleming, Inês Silva

University of Maryland College Park
&
Smithsonian Conservation Biology Institute

`hfleming@umd.edu`

2022-09-16

① Home-range underestimation

—a motivating problem

- ① Home-range underestimation
- ② Autocorrelation

—a motivating problem
—problem or feature

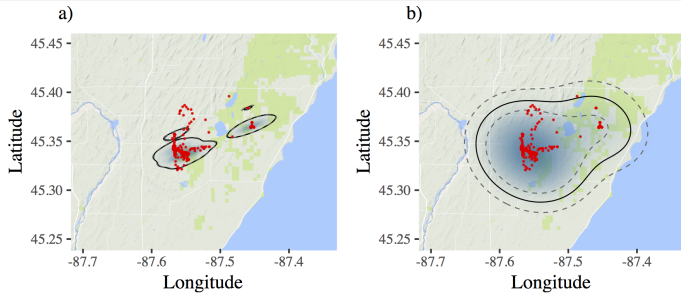
- ① Home-range underestimation
- ② Autocorrelation
- ③ Continuous time versus discrete time

—a motivating problem
—problem or feature

- ① Home-range underestimation
- ② Autocorrelation
- ③ Continuous time versus discrete time
- ④ Continuous-time movement models

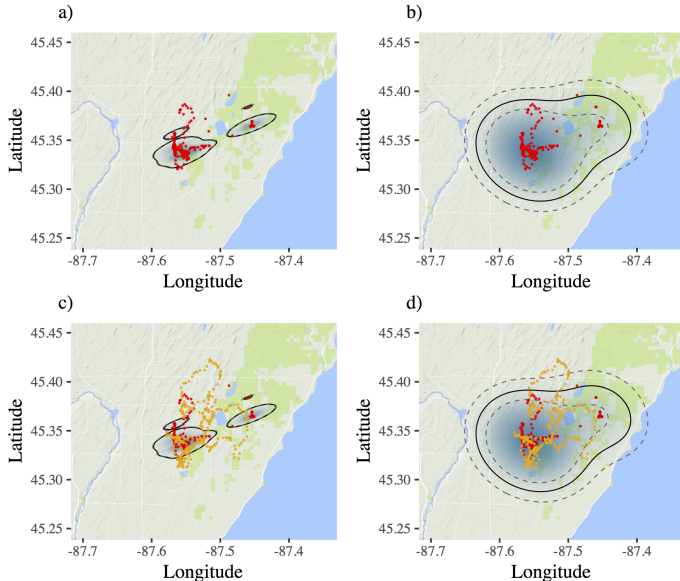
—a motivating problem
—problem or feature

The underestimation of animal space use



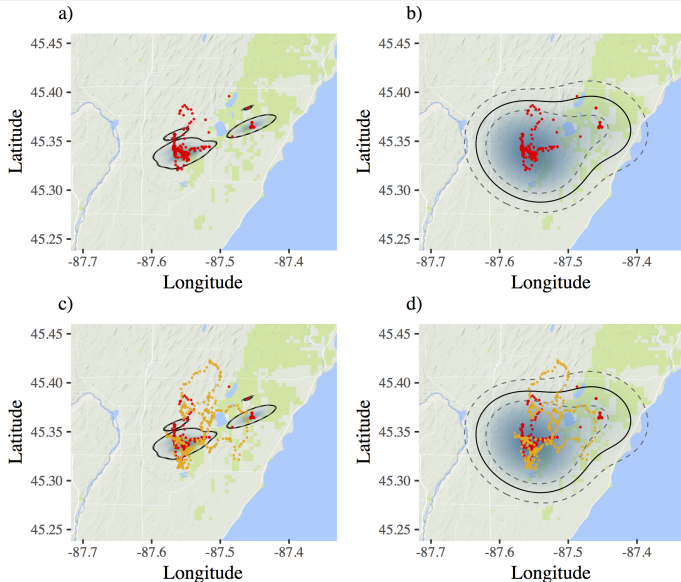
- GPS-tracked black bear

The underestimation of animal space use



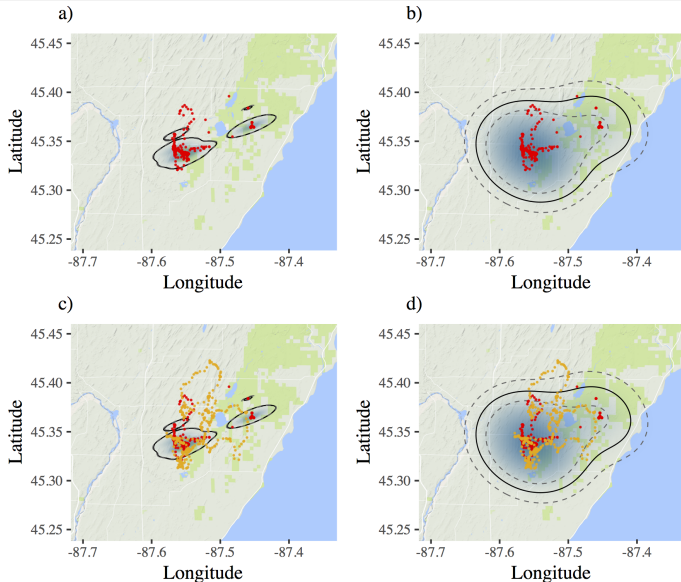
● GPS-tracked black bear

The underestimation of animal space use



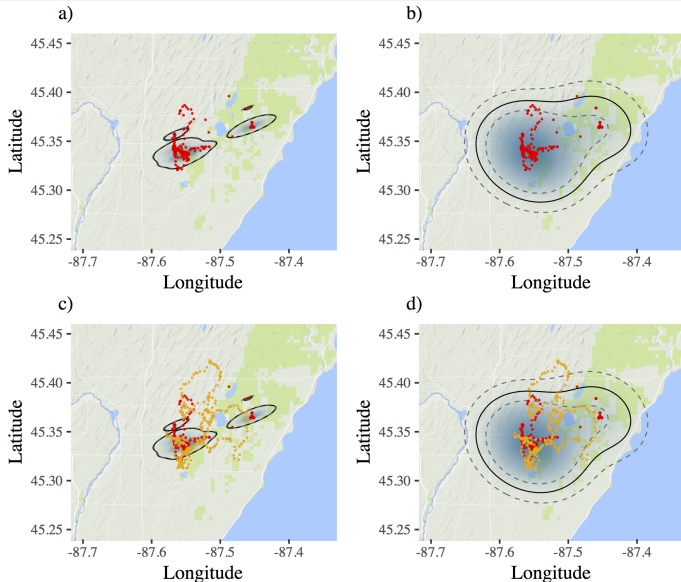
- GPS-tracked black bear
- Ignoring non-independence leads to overconfidence

The underestimation of animal space use



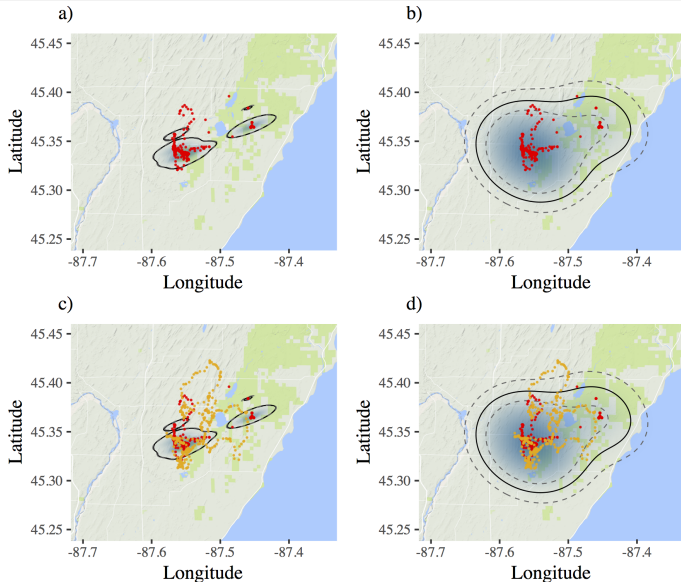
- GPS-tracked black bear
- Ignoring non-independence leads to overconfidence
- Space use is underestimated

The underestimation of animal space use



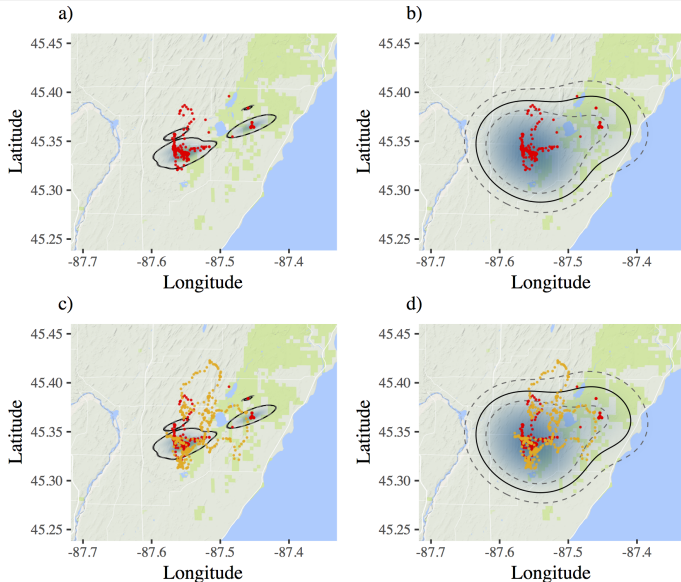
- GPS-tracked black bear
- Ignoring non-independence leads to overconfidence
- Space use is underestimated
- Q1:
- Q2:
- Q3:

The underestimation of animal space use



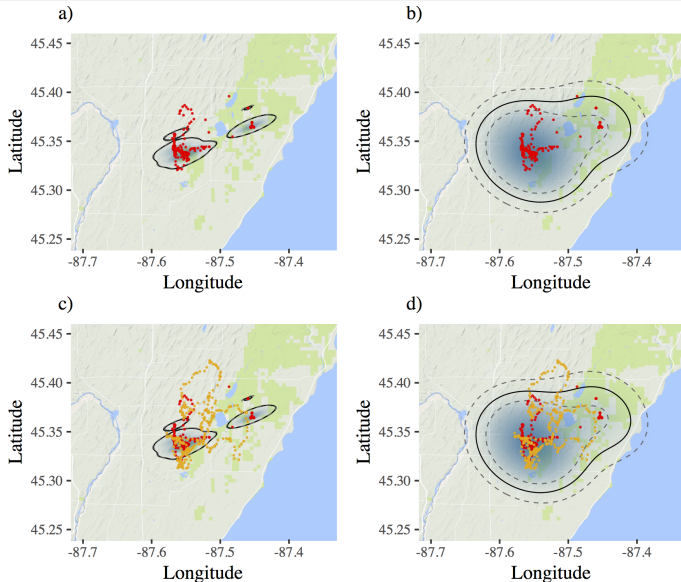
- GPS-tracked black bear
- Ignoring non-independence leads to overconfidence
- Space use is underestimated
- Q1: Why does this happen?
- Q2:
- Q3:

The underestimation of animal space use



- GPS-tracked black bear
- Ignoring non-independence leads to overconfidence
- Space use is underestimated
- Q1: Why does this happen?
- Q2: Does this happen in practice?
- Q3:

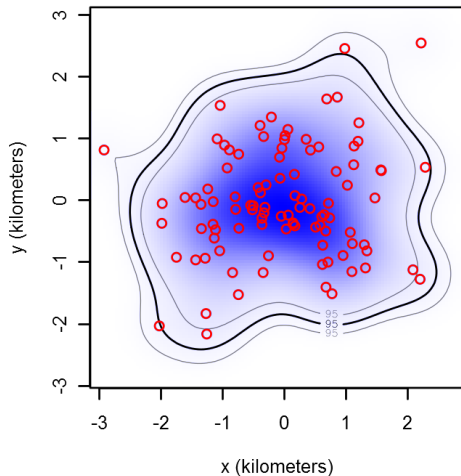
The underestimation of animal space use



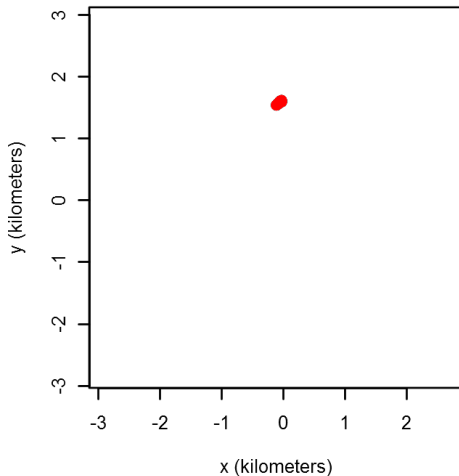
- GPS-tracked black bear
- Ignoring non-independence leads to overconfidence
- Space use is underestimated
- Q1: Why does this happen?
- Q2: Does this happen in practice?
- Q3: How did we fix this?

Q1: Why does this happen?

1 year of data (n=100)












1 hour of data (n=100)



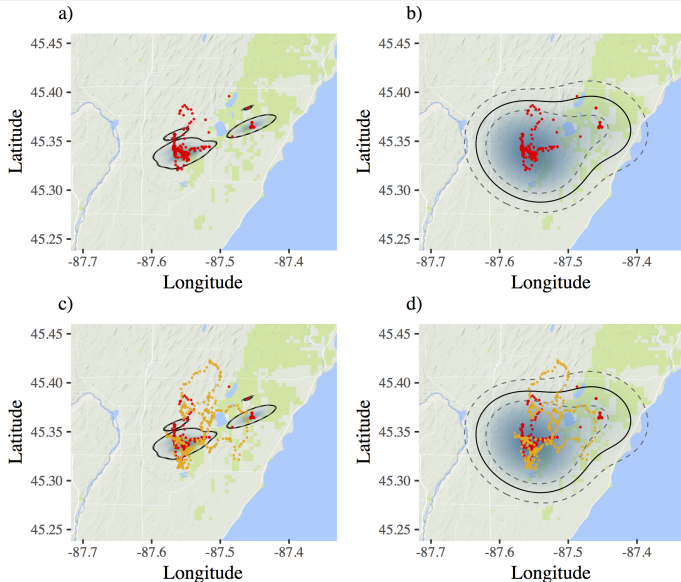
Q2: Does this happen in practice?

Ecological Monographs, 89(2), 2019, e01344
© 2019 by the Ecological Society of America

A comprehensive analysis of autocorrelation and bias in home range estimation

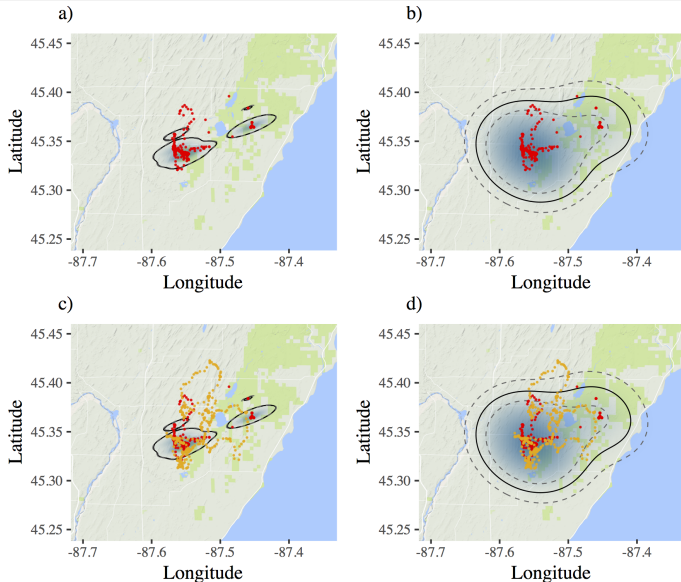
MICHAEL J. NOONAN ^{1,2} MARLEE A. TUCKER ^{3,4} CHRISTEN H. FLEMING ^{1,2} THOMAS S. AKRE,¹ SUSAN C. ALBERTS,⁵ ABDULLAHI H. ALI,⁶ JEANNE ALTMANN,⁷ PAMELA CASTRO ANTUNES,⁸ JERROLD L. BELANT,⁹ DEAN BEYER,¹⁰ NIELS BLAUM,¹¹ KATRIN BÖHNING-GAESE,^{3,4} LAURY CULLEN JR.,¹² ROGERIO CUNHA DE PAULA,¹³ JASJA DEKKER,¹⁴ JONATHAN DRESCHER-LEHMAN,^{1,15} NINA FARWIG,¹⁶ CLAUDIA FICHTEL,¹⁷ CHRISTINA FISCHER,¹⁸ ADAM T. FORD,¹⁹ JACOB R. GOHEEN,²⁰ RENÉ JANSSEN,²¹ FLORIAN JELTSCH ¹¹ MATTHEW KAUFFMAN,²² PETER M. KAPPELER,¹⁷ FLÁVIA KOCH,¹⁷ SCOTT LAPPOINT ^{23,24} A. CATHERINE MARKHAM,²⁵ EMILIA PATRICIA MEDICI,²⁶ RONALDO G. MORATO ^{13,27} RAN NATHAN,²⁸ LUIZ GUSTAVO R. OLIVEIRA-SANTOS,⁸ KIRK A. OLSON,^{1,29} BRUCE D. PATTERSON,³⁰ AGUSTIN PAVIOLO ³¹ EMILIANO ESTERCI RAMALHO,^{27,32} SASCHA RÖSNER,¹⁶ DANA G. SCHABO,¹⁶ NURIA SELVA,³³ AGNIESZKA SERGIEL,³³ MARINA XAVIER DA SILVA,³⁴ ORR SPIEGEL ³⁵ PETER THOMPSON,² WIEBKE ULLMANN,¹¹ FILIP ZIĘBA,³⁶ TOMASZ ZWIJACZ-KOZICA,³⁶ WILLIAM F. FAGAN,² THOMAS MUELLER,^{3,4} AND JUSTIN M. CALABRESE ^{1,2,37}

The underestimation of animal space use, the solution



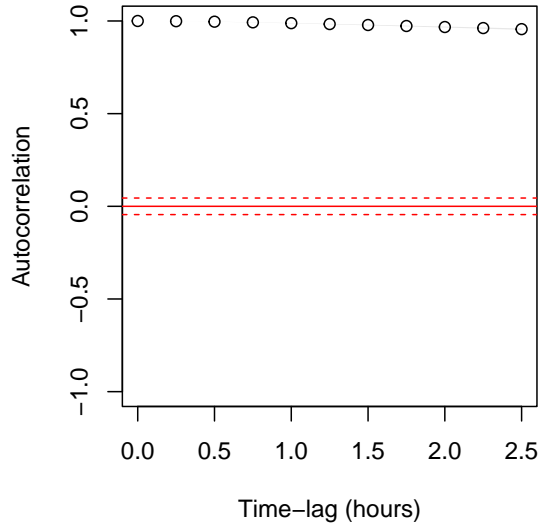
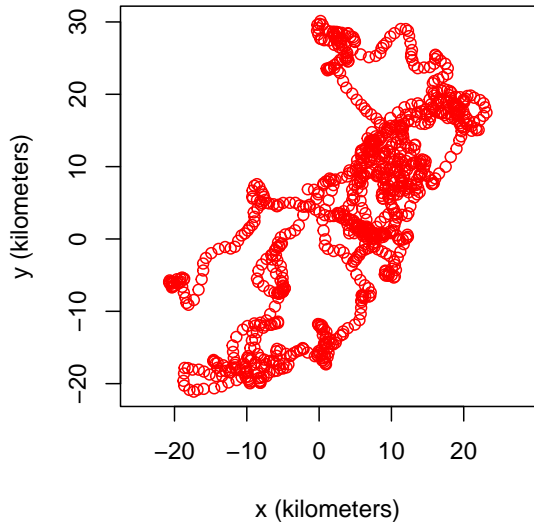
- GPS-tracked black bear
- Ignoring non-independence leads to overconfidence
- Space use is underestimated
- Q1: Why does this happen?
- Q2: Does this happen in practice?
- Q3: How did we fix this?

The underestimation of animal space use, the solution

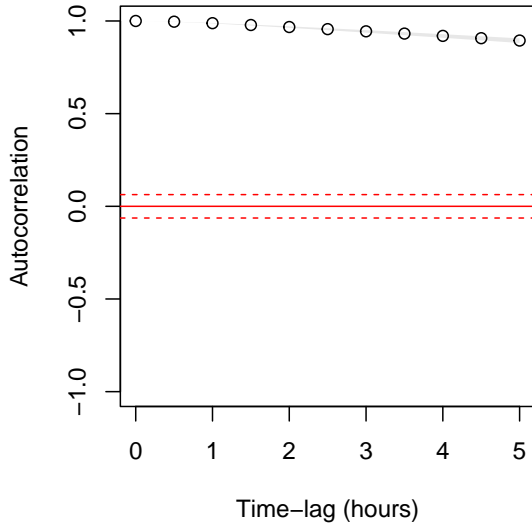
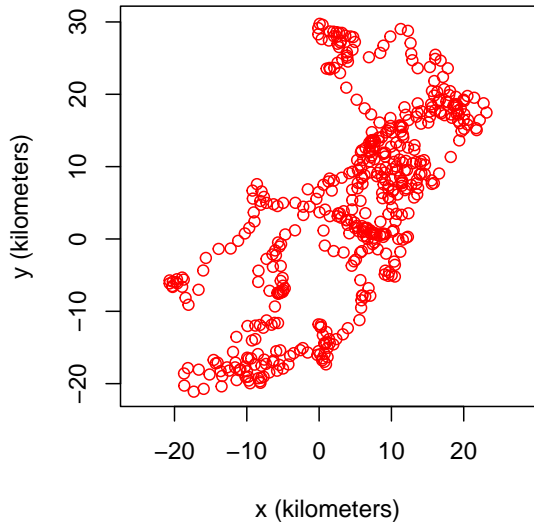


- GPS-tracked black bear
- Ignoring non-independence leads to overconfidence
- Space use is underestimated
- Q1: Why does this happen?
- Q2: Does this happen in practice?
- Q3: How did we fix this?
- A3: Continuous-time stochastic process models of the *autocorrelation*

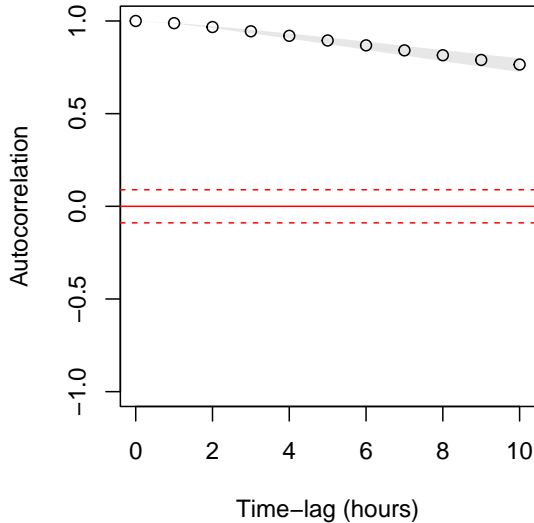
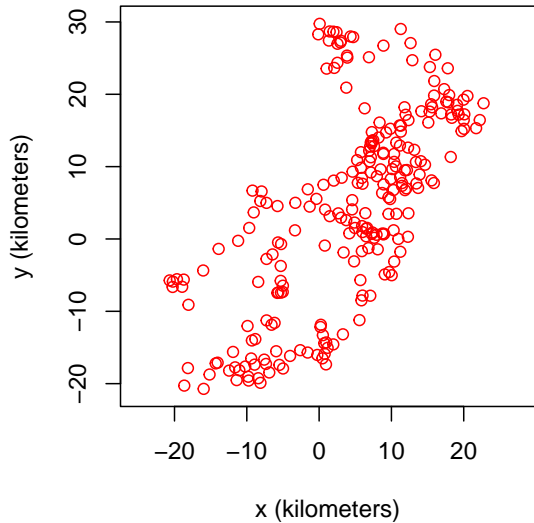
autocorrelation is



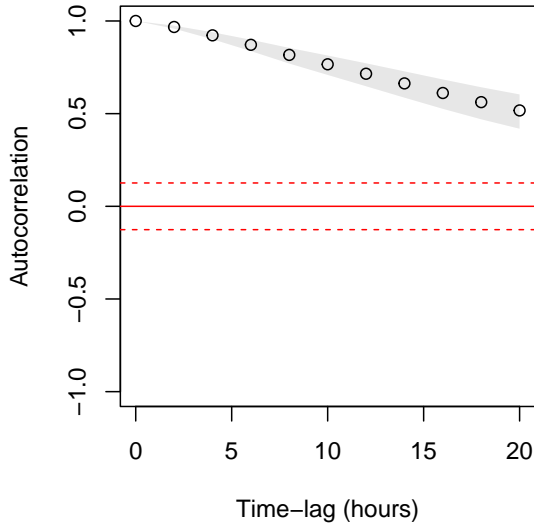
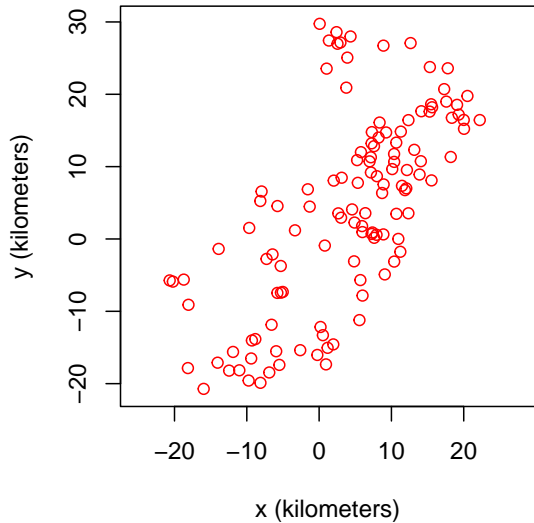
If autocorrelation is the culprit...



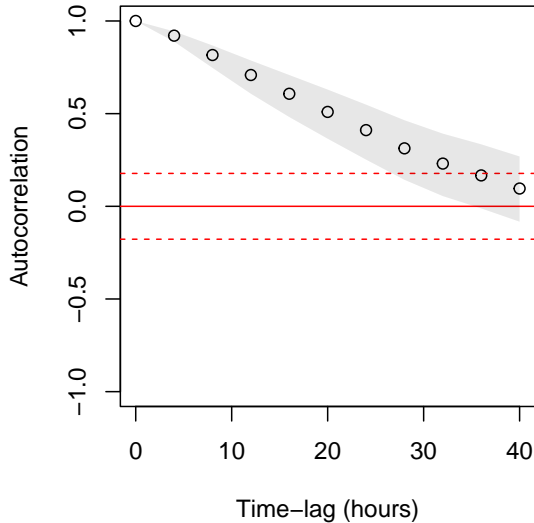
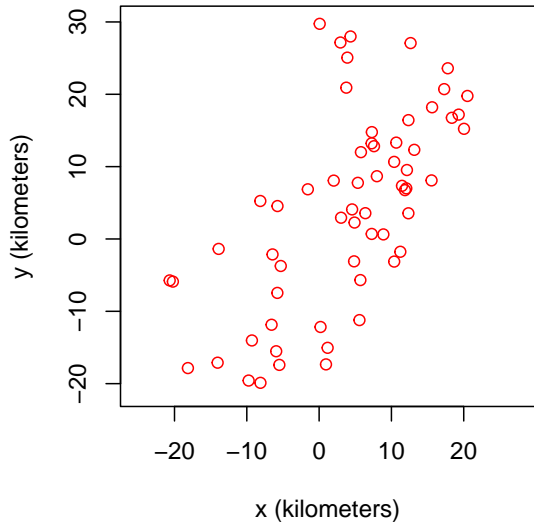
If autocorrelation is the culprit...



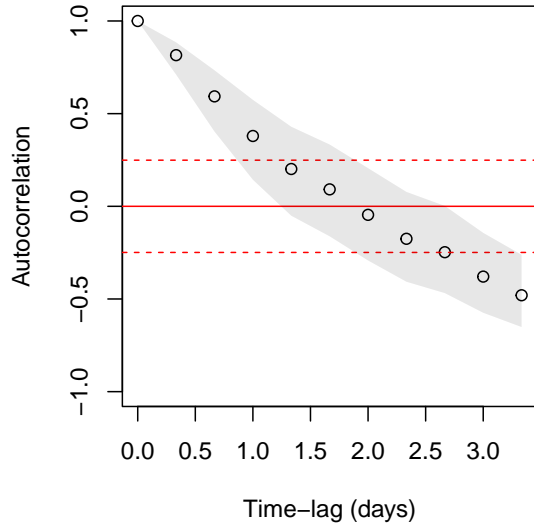
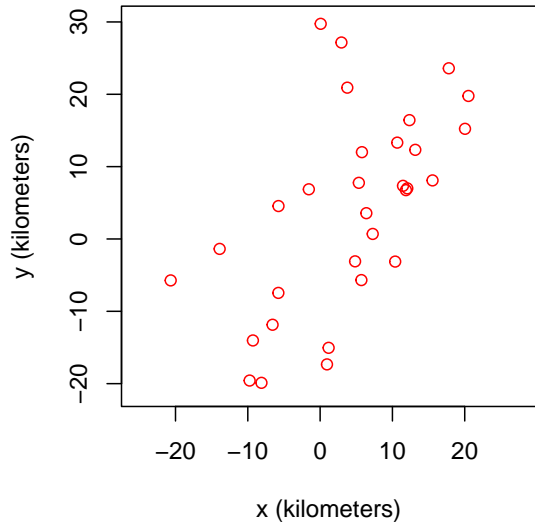
If autocorrelation is the culprit...



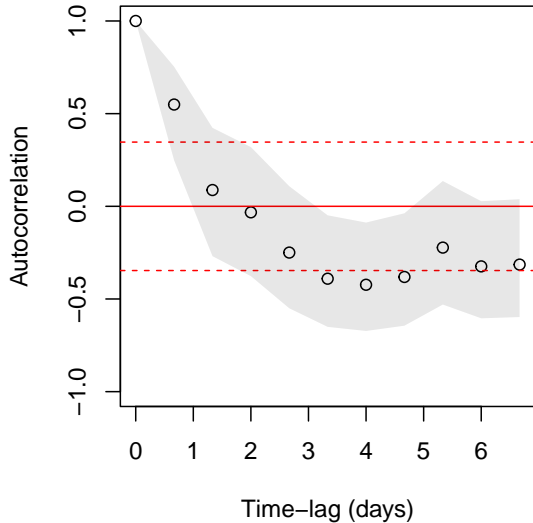
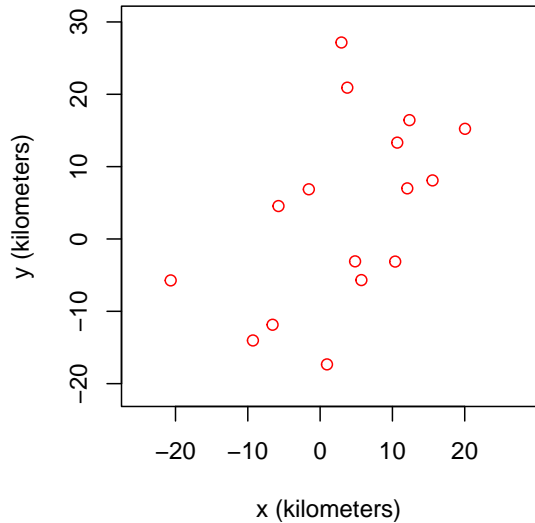
If autocorrelation is the culprit...



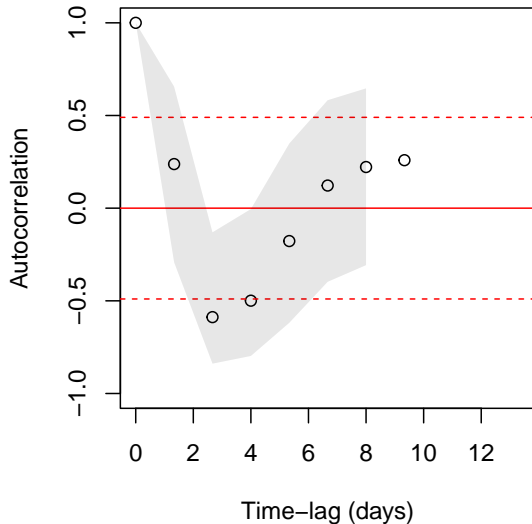
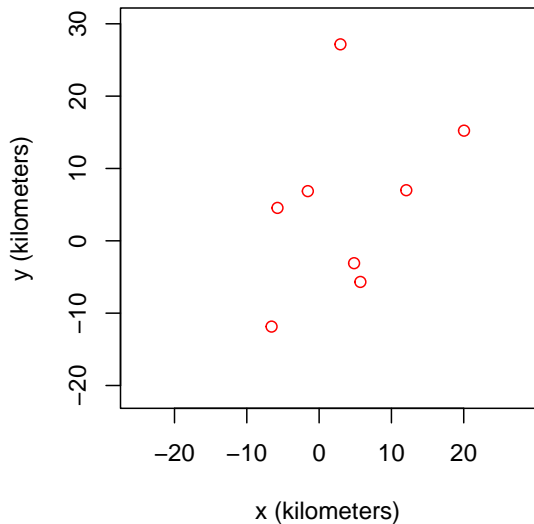
If autocorrelation is the culprit...



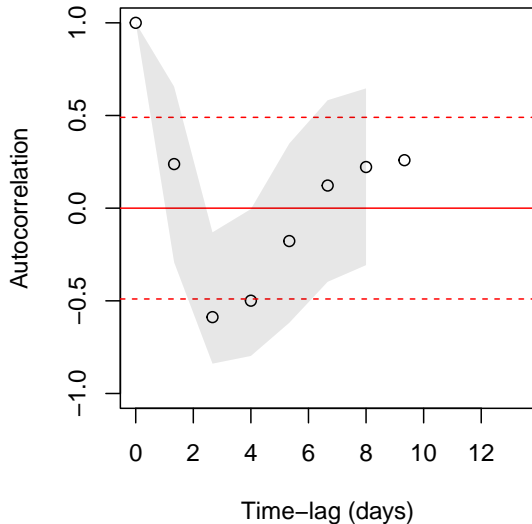
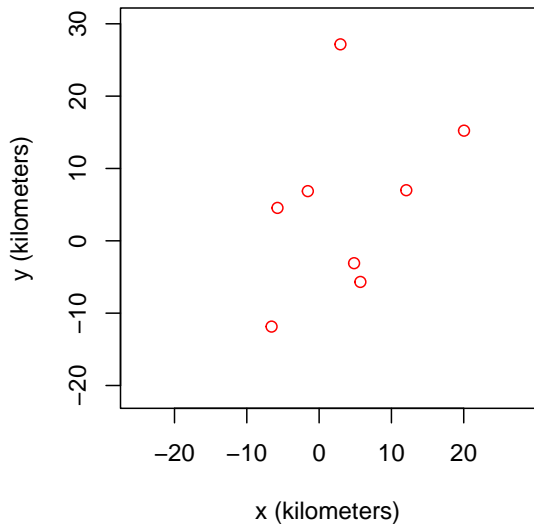
If autocorrelation is the culprit...



...you might have to thin a lot to reach IID

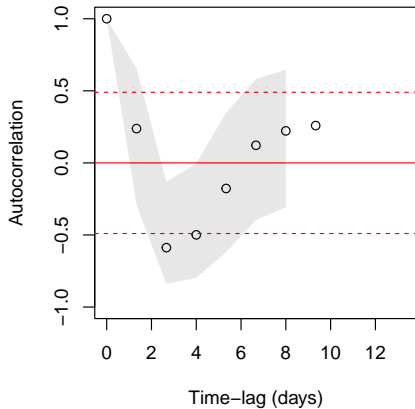
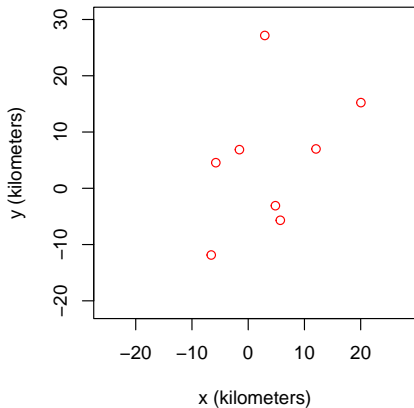


...you might have to thin a lot to reach IID



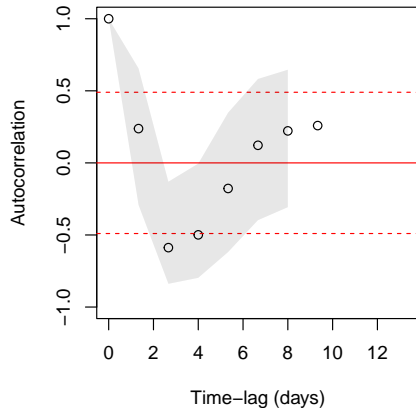
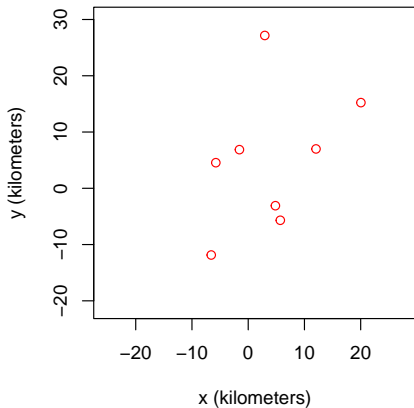
What about speed estimation with these data?

Autocorrelation is information



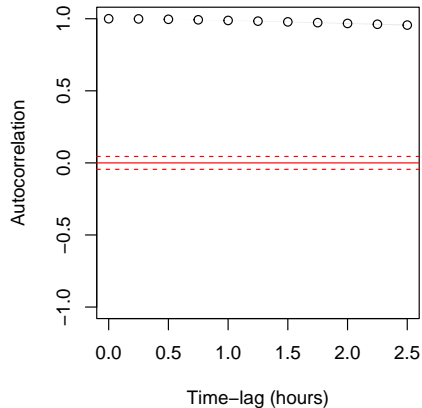
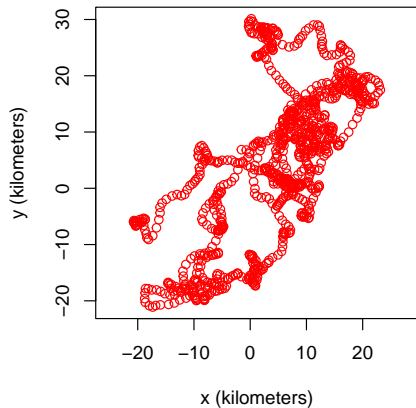
- Convenient for home-range analysis

Autocorrelation is information



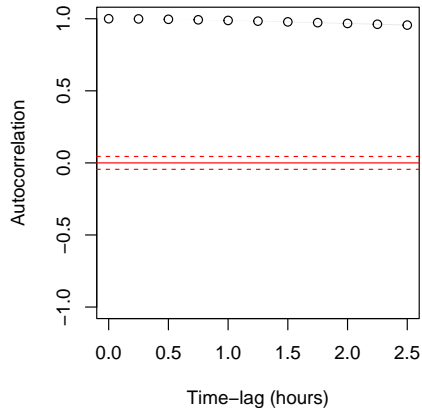
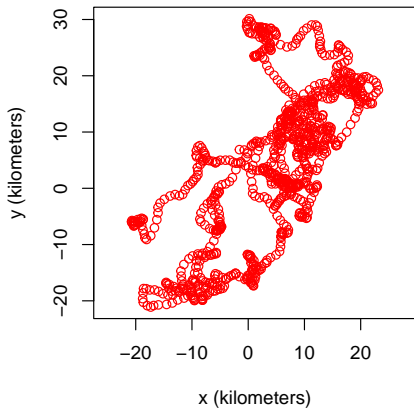
- Convenient for home-range analysis
- Worthless for speed/distance estimation

Autocorrelation is information



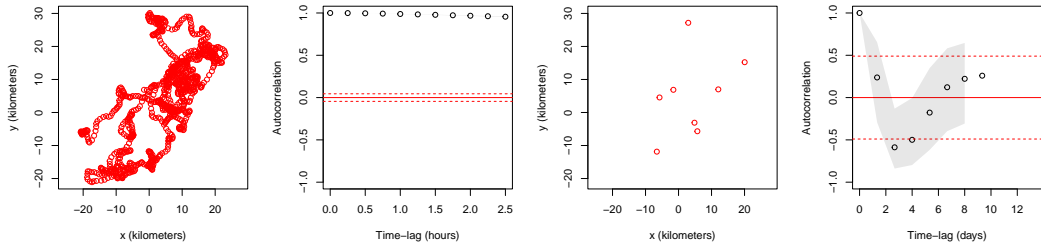
- Inconvenient for home-range analysis

Autocorrelation is information



- Inconvenient for home-range analysis
- Great for speed/distance estimation

Objective



We want methods that can handle whatever autocorrelation is present in the data

Why continuous time?

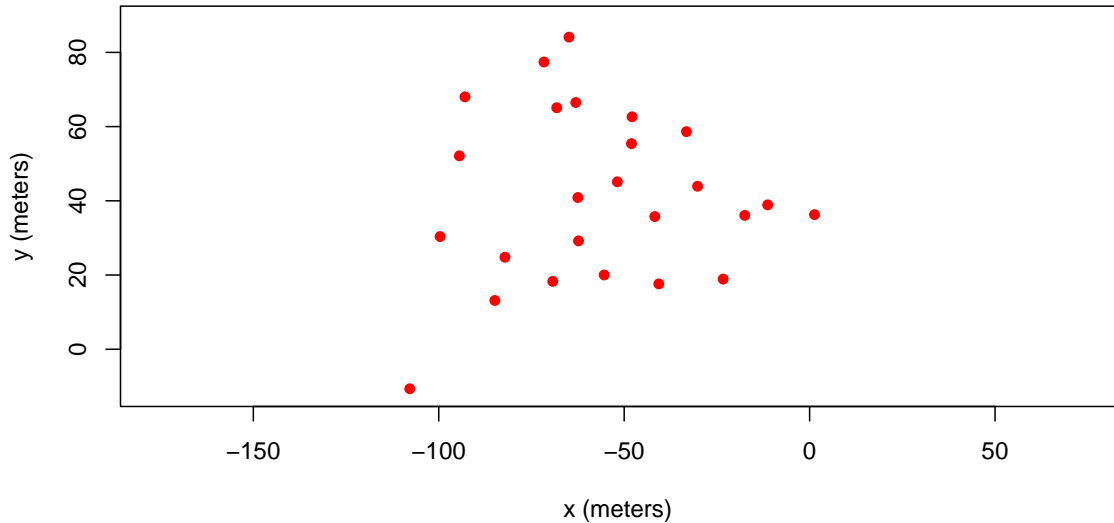
Why continuous time?

- I've convinced you that we want to address autocorrelation head on

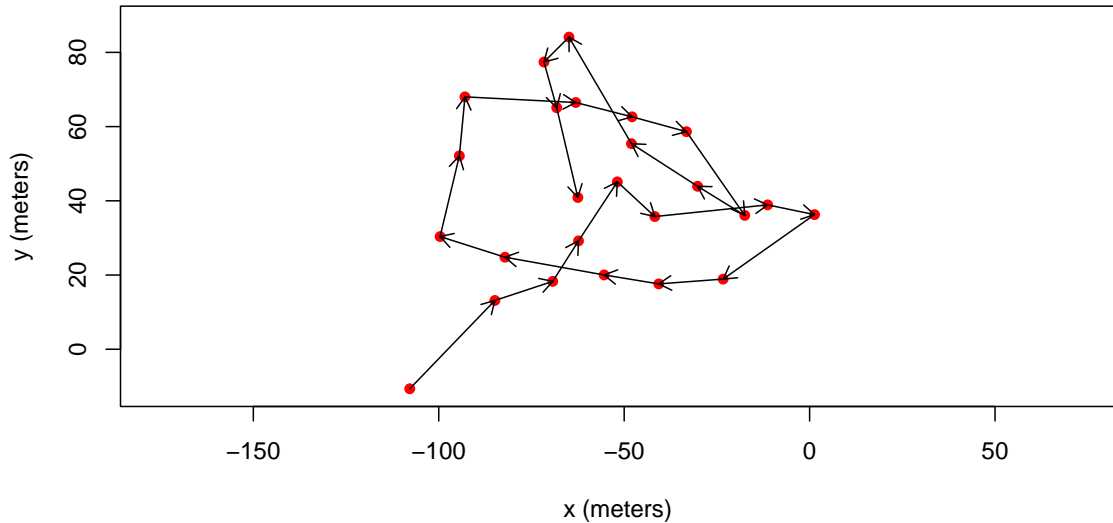
Why continuous time?

- I've convinced you that we want to address autocorrelation head on
- But why continuous time? Why not discrete time?

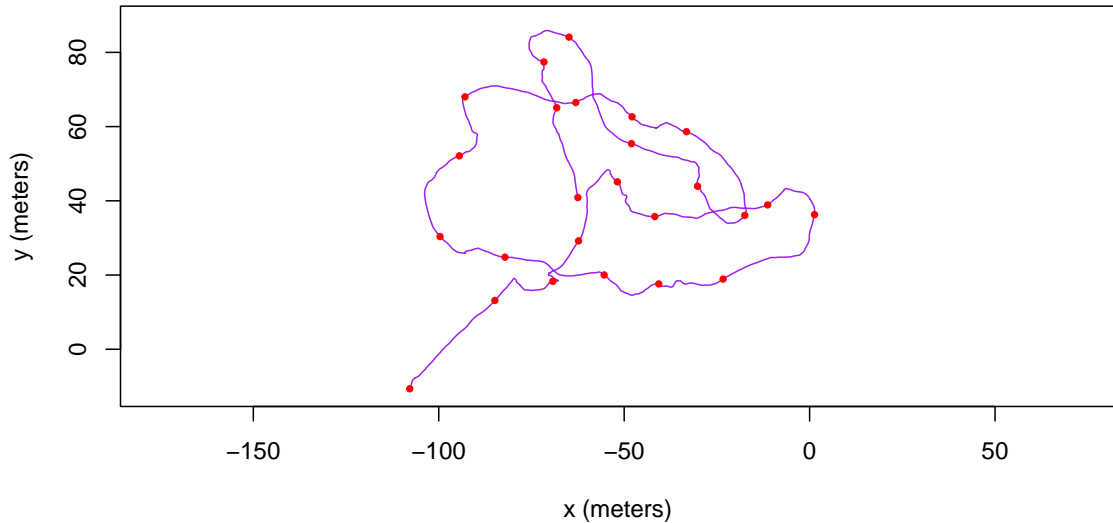
What do I mean by continuous time versus discrete time?



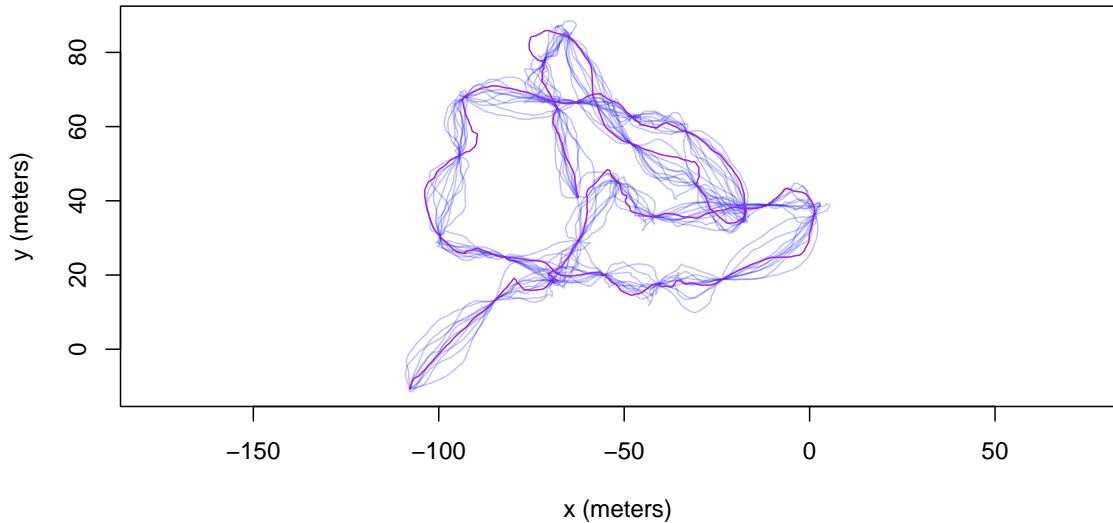
What do I mean by continuous time versus discrete time?



What do I mean by continuous time versus discrete time?



What do I mean by continuous time versus discrete time?



Why continuous time?

- I've convinced you that we want to address autocorrelation head on
- But why continuous time? Why not discrete time?

Why continuous time?

- I've convinced you that we want to address autocorrelation head on
- But why continuous time? Why not discrete time?
 - Sampling may be irregular in time

Why continuous time?

- I've convinced you that we want to address autocorrelation head on
- But why continuous time? Why not discrete time?
 - Sampling may be irregular in time
 - Sampling schedules may differ across individuals

Why continuous time?

- I've convinced you that we want to address autocorrelation head on
- But why continuous time? Why not discrete time?
 - Sampling may be irregular in time
 - Sampling schedules may differ across individuals
 - model parameters are biological, not sampling dependent

Why continuous time?

- I've convinced you that we want to address autocorrelation head on
- But why continuous time? Why not discrete time?
 - Sampling may be irregular in time
 - Sampling schedules may differ across individuals
 - model parameters are biological, not sampling dependent
 - Discrete-time models can be intrinsically scale dependent (e.g., SSFs)

Why continuous time?

- I've convinced you that we want to address autocorrelation head on
- But why continuous time? Why not discrete time?
 - Sampling may be irregular in time
 - Sampling schedules may differ across individuals
 - model parameters are biological, not sampling dependent
 - Discrete-time models can be intrinsically scale dependent (e.g., SSFs)
 - Continuous-time models are more realistic and have a wider scope of inference

Why continuous time?

- I've convinced you that we want to address autocorrelation head on
- But why continuous time? Why not discrete time?
 - Sampling may be irregular in time
 - Sampling schedules may differ across individuals
 - model parameters are biological, not sampling dependent
 - Discrete-time models can be intrinsically scale dependent (e.g., SSFs)
 - Continuous-time models are more realistic and have a wider scope of inference
 - Can easily model temporal and spatial scales that span orders of magnitude

Why continuous time?

- I've convinced you that we want to address autocorrelation head on
- But why continuous time? Why not discrete time?
 - Sampling may be irregular in time
 - Sampling schedules may differ across individuals
 - model parameters are biological, not sampling dependent
 - Discrete-time models can be intrinsically scale dependent (e.g., SSFs)
 - Continuous-time models are more realistic and have a wider scope of inference
 - Can easily model temporal and spatial scales that span orders of magnitude
 - Can accommodate speed, distance, acceleration, power and energy
 - Location error is easy to model (versus CRWs)

Building-block continuous-time stochastic process models

Building-block continuous-time stochastic process models

- Independent locations



Building-block continuous-time stochastic process models



- Independent locations
- Brownian motion



Building-block continuous-time stochastic process models



- Independent locations
- Brownian motion (milk fat © Paul Baker)

Accounting for non-independence: Brownian home range?

 "brownian bridge" "home range" 

 Scholar About 1.220 results (0,05 sec) YEAR 

[HTML] Using dynamic **Brownian Bridge** Movement Models to identify **home range** size and movement patterns in king cobras [HTML] plos.org
Full View
[I Silva](#), M Crane, [P Suwanwaree](#), [C Stringe](#), M Goode - PloS one, 2018 - journals.plos.org
Home range estimators are a critical component for understanding animal spatial ecology. The choice of **home range** estimator in spatial ecology studies can significantly influence ...
☆ Save  Cite Cited by 39 Related articles All 14 versions 

[HTML] Is there a single best estimator? Selection of **home range** estimators using area-under-the-curve [HTML] biomedcentral.com
[WD Walter](#), [DP Onorato](#)... - Movement ..., 2015 - movementecologyjournal ...
... based or **Brownian bridge**-based algorithms were used and in comparison to first- and second-generation estimators. We defined third-generation estimators of **home range** as any ...
☆ Save  Cite Cited by 93 Related articles All 21 versions 

What is the proper method to delineate **home range** of an animal using today's advanced GPS telemetry systems: the initial step [PDF] unl.edu
[WD Walter](#), [JW Fischer](#), [S Baruch-Mordo](#)... - Modern ..., 2011 - books.google.com
... 1393–1405 Bullard, F.(1999). Estimating the **home range** of an animal: a **Brownian Bridge** approach. Thesis, University of North Carolina at Chapel Hill, Chapel Hill Burt, WH (1943) ...

Accounting for non-independence: Brownian *home range*?



Building-block continuous-time stochastic process models

- Independent locations
- Brownian motion

Building-block continuous-time stochastic process models

- Independent locations
- Brownian motion
- Ornstein-Uhlenbeck motion

Building-block continuous-time stochastic process models

- Independent locations
- Brownian motion
- Ornstein-Uhlenbeck motion
- Integrated Ornstein-Uhlenbeck motion

Building-block continuous-time stochastic process models

- Independent locations
- Brownian motion
- Ornstein-Uhlenbeck motion
- Integrated Ornstein-Uhlenbeck motion
- ...

PHYSICAL REVIEW E **91**, 032107 (2015)

Maximum-entropy description of animal movement

Chris H. Fleming,^{1,2} Yiğit Subaşı,³ and Justin M. Calabrese¹

¹*Conservation Ecology Center, Smithsonian Conservation Biology Institute, National Zoological Park, 1500 Remount Rd., Front Royal, Virginia 22630, USA*

²*Department of Biology, University of Maryland, College Park, College Park, Maryland 20742, USA*

³*Department of Chemistry and Biochemistry, University of Maryland, College Park, College Park, Maryland 20742, USA*

(Received 7 October 2014; published 4 March 2015)

We introduce a class of maximum-entropy states that naturally includes within it all of the major continuous-time stochastic processes that have been applied to animal movement, including Brownian motion, Ornstein-Uhlenbeck motion, integrated Ornstein-Uhlenbeck motion, a recently discovered hybrid of the previous models, and a new model that describes central-place foraging. We are also able to predict a further hierarchy of new models that will emerge as data quality improves to better resolve the underlying continuity of animal movement. Finally, we also show that Langevin equations must obey a fluctuation-dissipation theorem to generate processes that fall from this class of maximum-entropy distributions when the constraints are purely kinematic.

DOI: [10.1103/PhysRevE.91.032107](https://doi.org/10.1103/PhysRevE.91.032107)

PACS number(s): 05.40.Jc, 05.10.Gg, 02.50.Ey, 87.10.Mn

Building-block continuous-time stochastic process models

- Independent locations
- Brownian motion
- Ornstein-Uhlenbeck motion
- Integrated Ornstein-Uhlenbeck motion
- Missing maximum-entropy model

Building-block continuous-time stochastic process models

- Independent locations
- Brownian motion
- Ornstein-Uhlenbeck motion
- Integrated Ornstein-Uhlenbeck motion
- Missing maximum-entropy model

Don't assume a model, select a model

Building-block continuous-time stochastic process models

- Independent locations (KDE,MCP,RSF,...)
- Brownian motion (Brownian bridge)
- Ornstein-Uhlenbeck motion
- Integrated Ornstein-Uhlenbeck motion (crawl)
- Missing maximum-entropy model

Don't assume a model, select a model

