Introduction to continuous-time movement modeling

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&
Smithsonian Conservation Biology Institute

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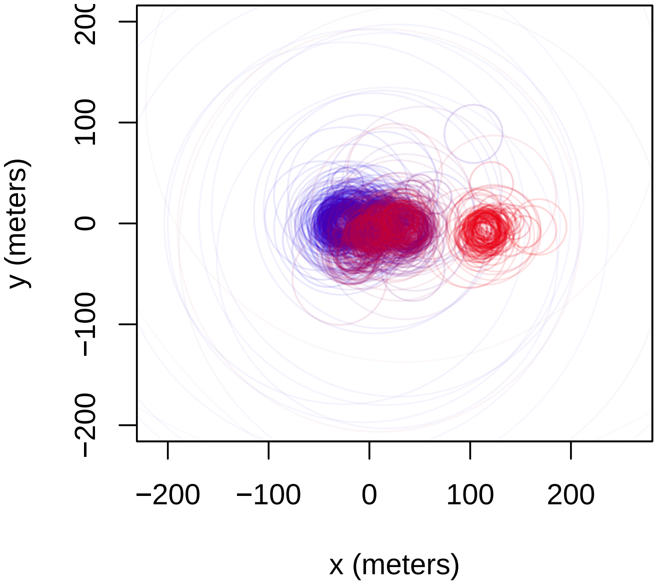
Animal tracking data are complex



Animal tracking data analysis goals in this workshop

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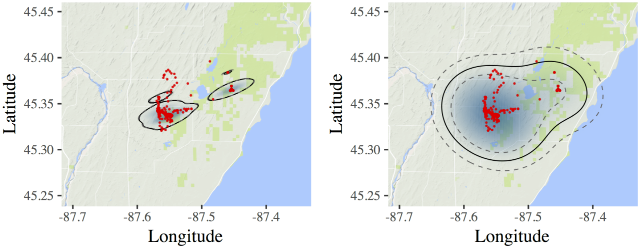
- Reducing bias and error as much as possible

Motivating example: Neglecting autocorrelation in home-range estimation



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(Noonan, Tucker, Fleming, et al., Ecological Monographs. 2019)

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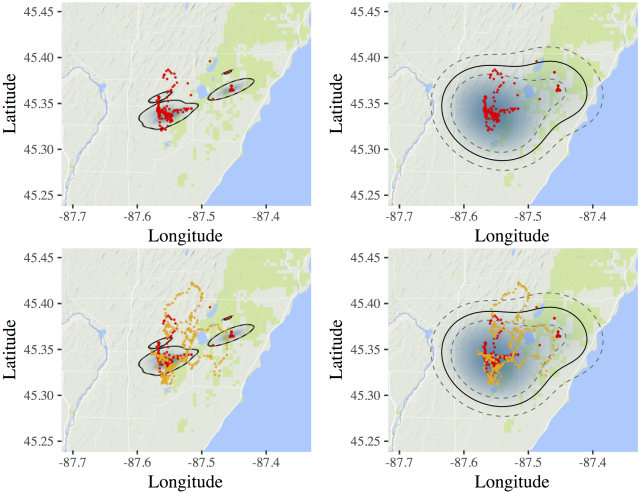

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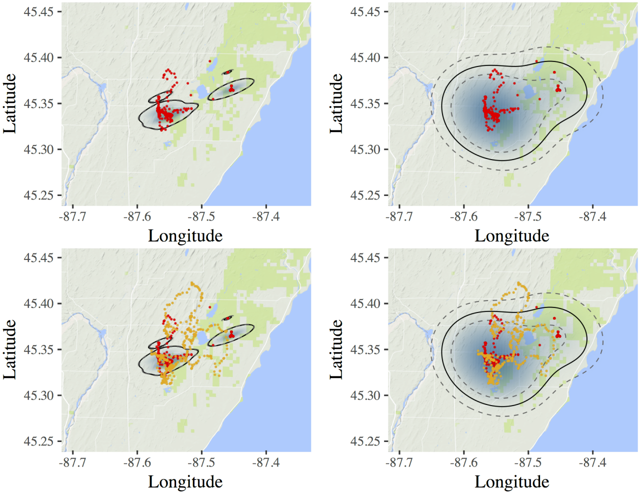

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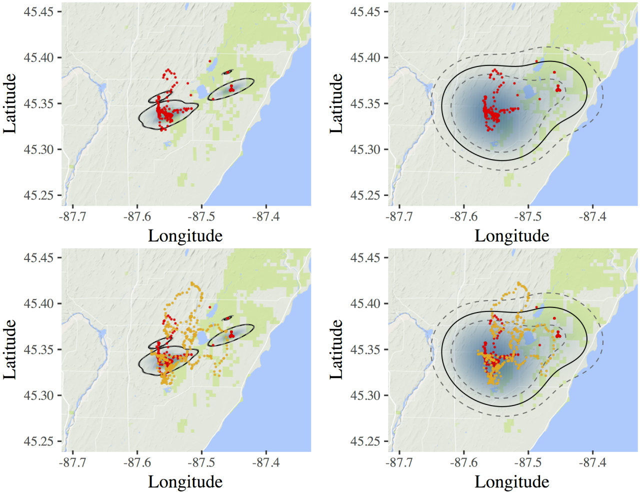

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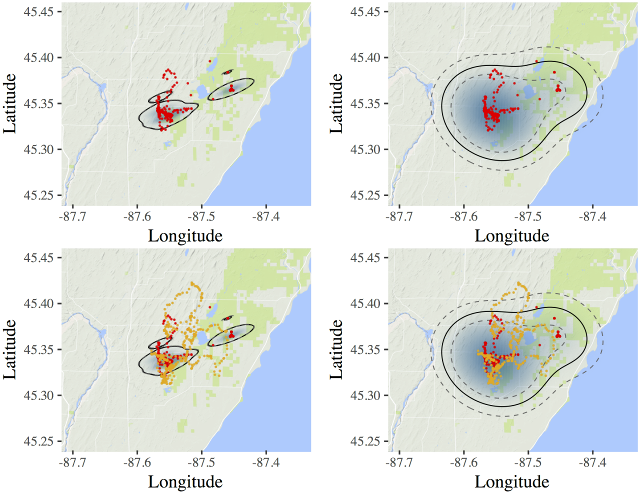

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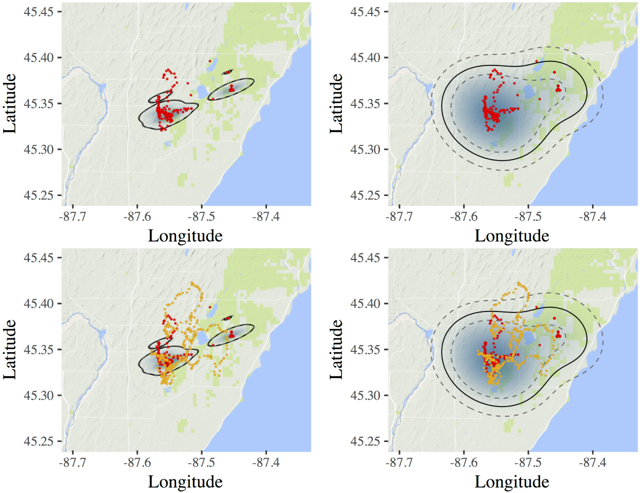

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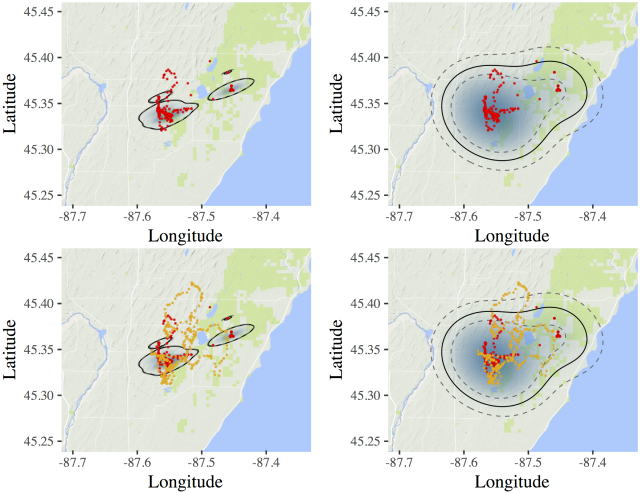

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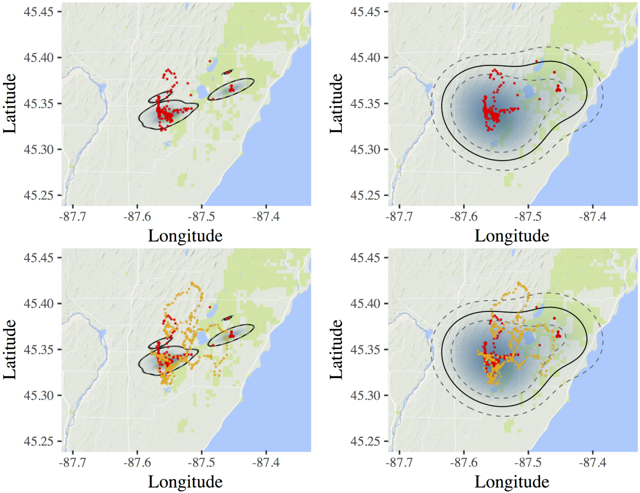

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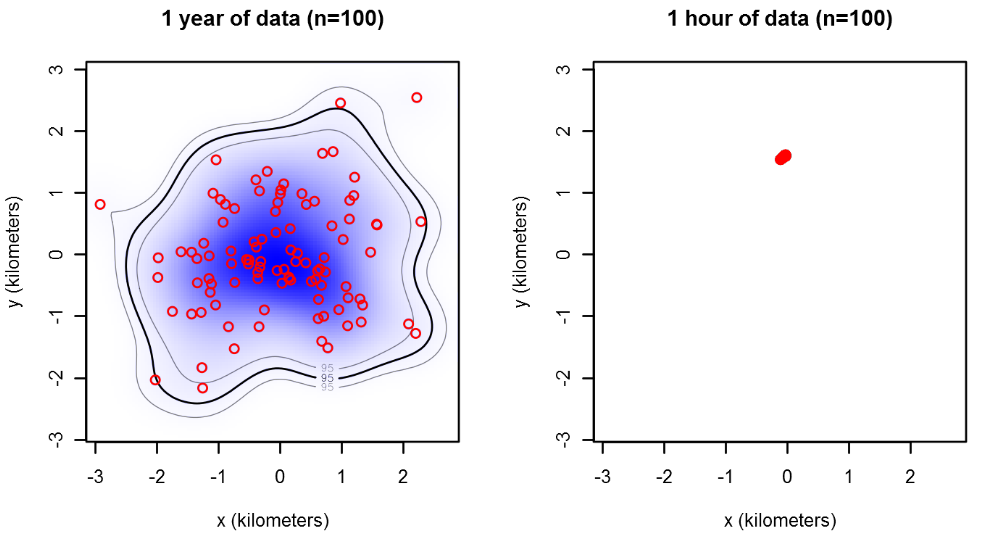

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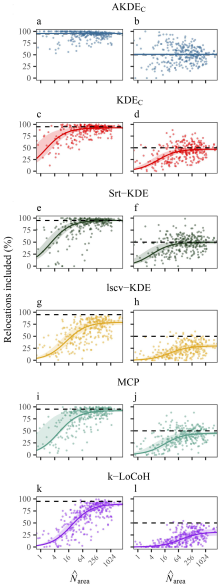

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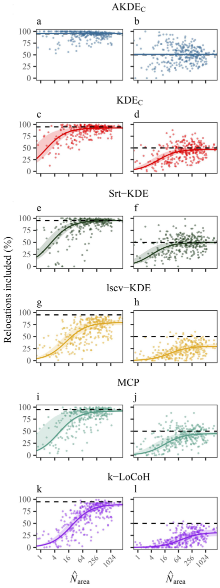

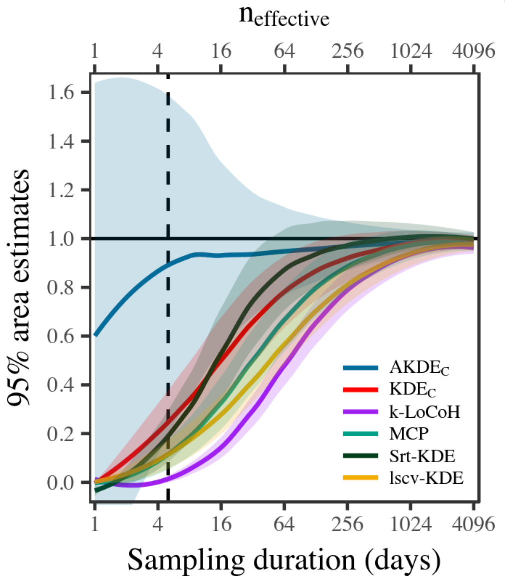

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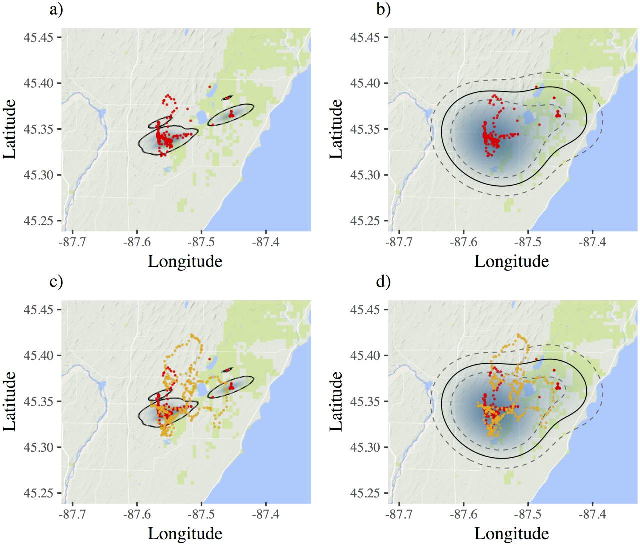
- Conventional estimates are 2–20 times too small, on average
- Bias is worse for larger species (Noonan, Fleming, et al., Conservation Biology 2020)

The underestimation of animal space use, the solution



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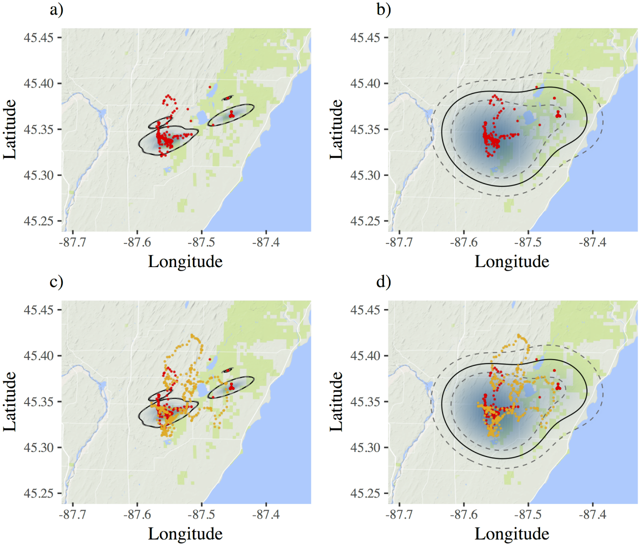

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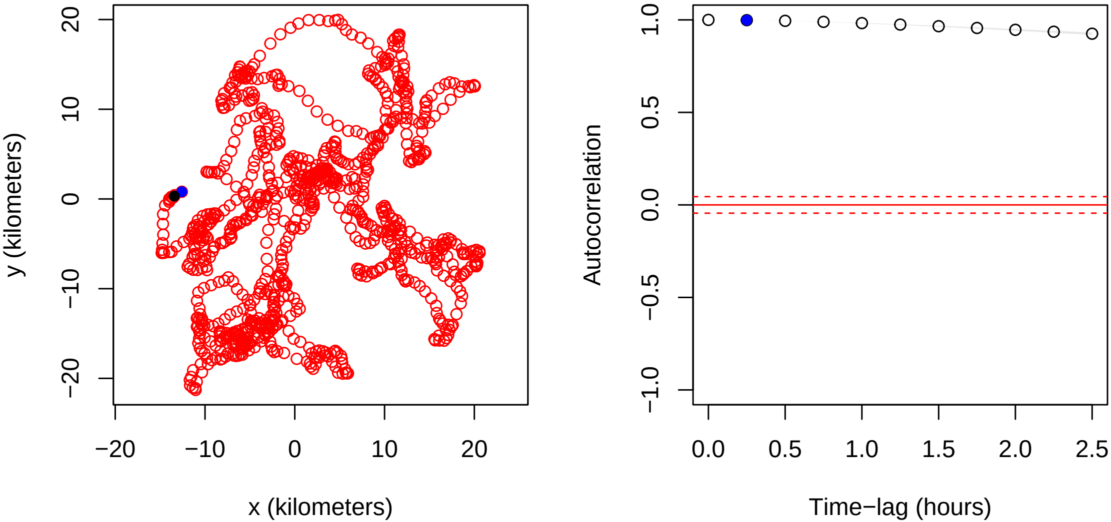

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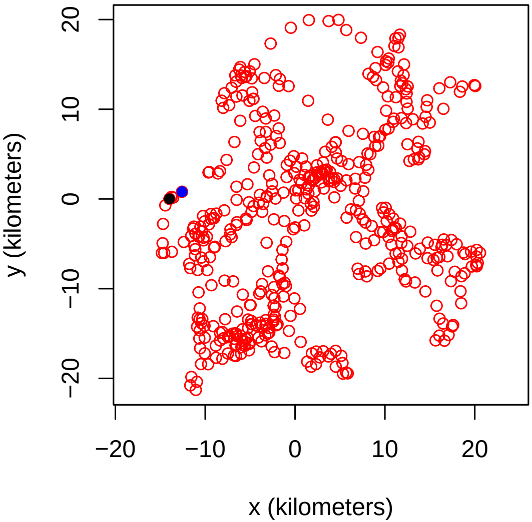

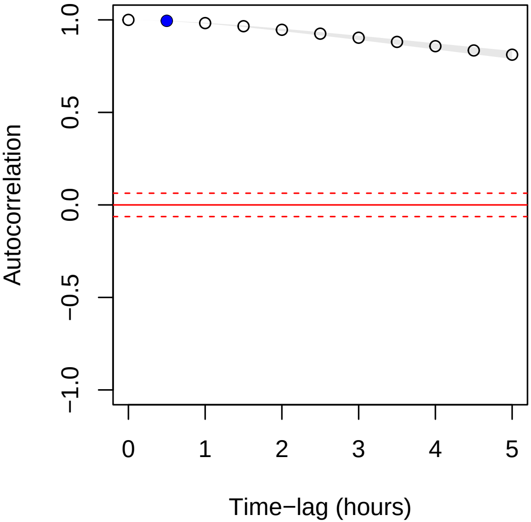

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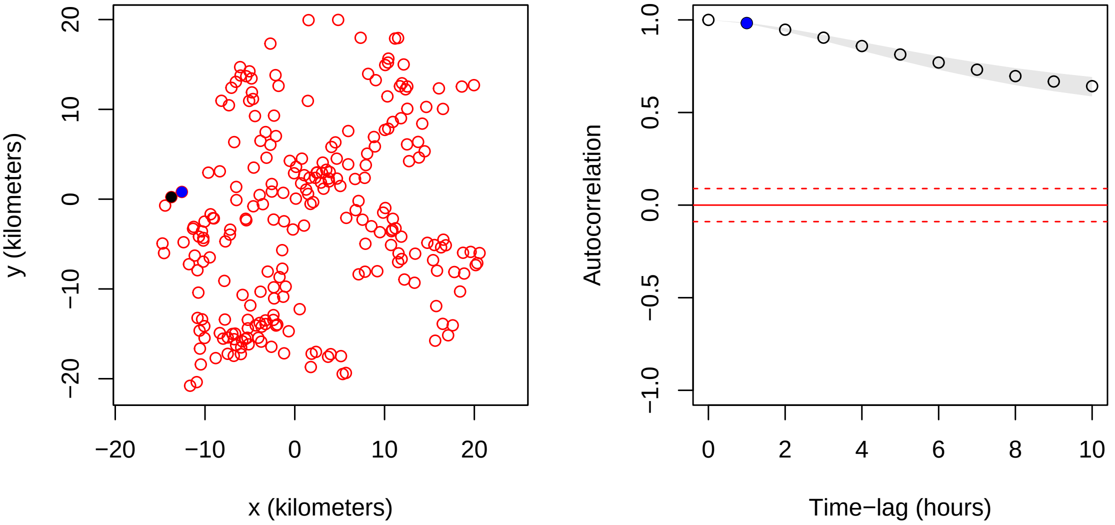

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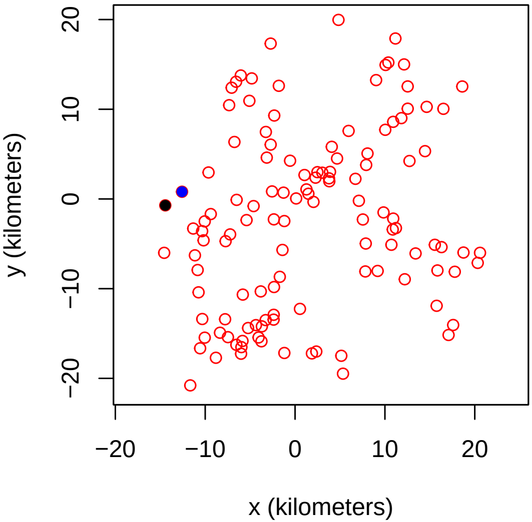

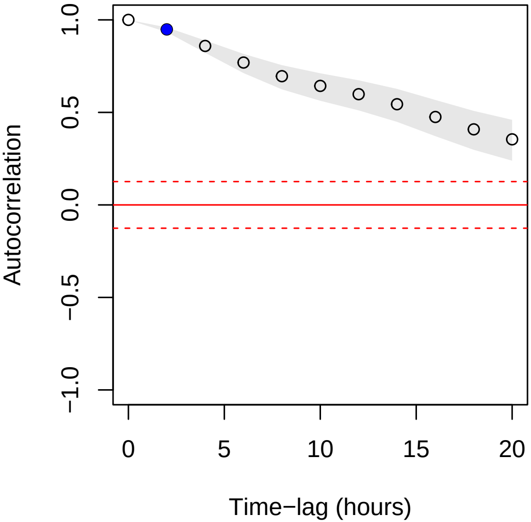

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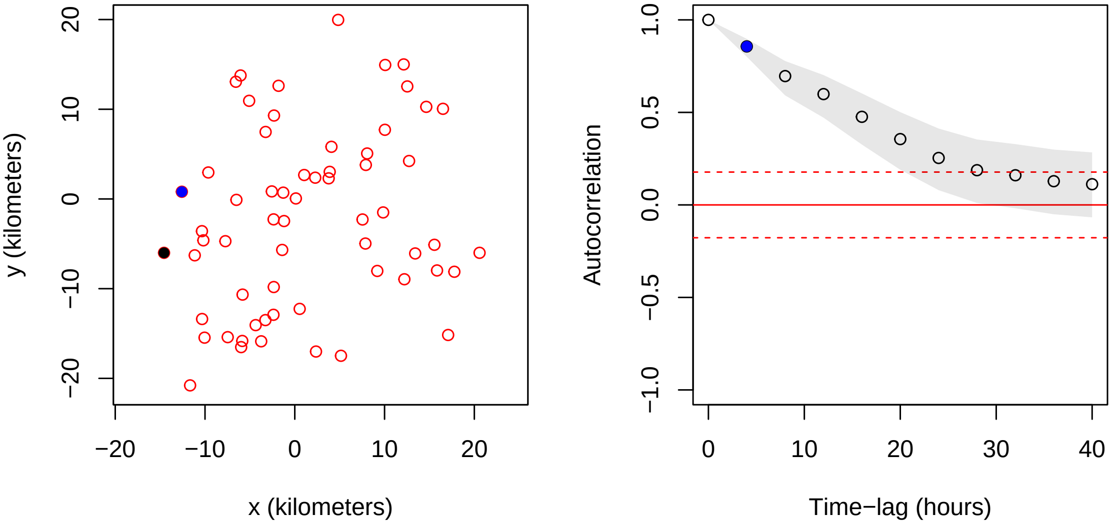

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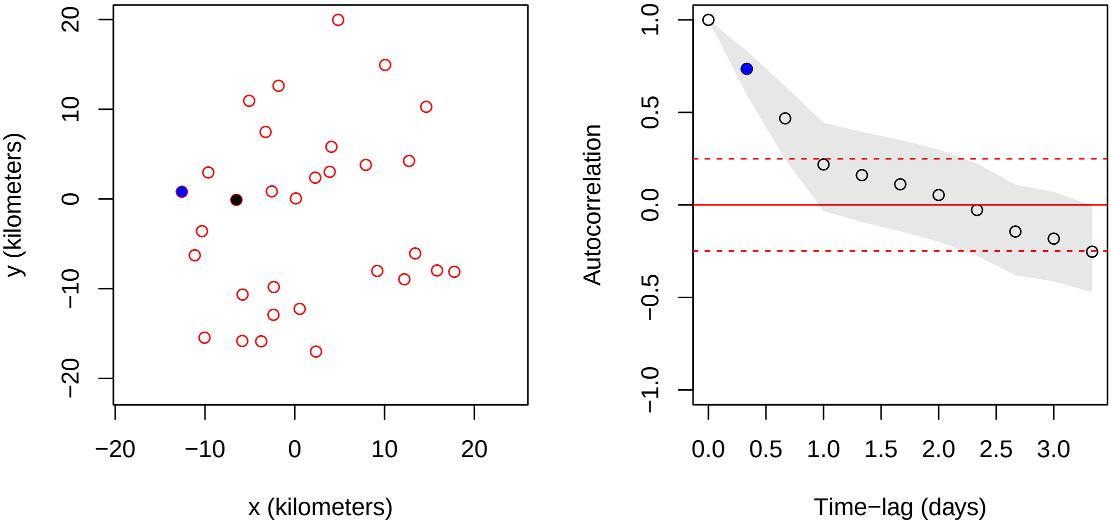

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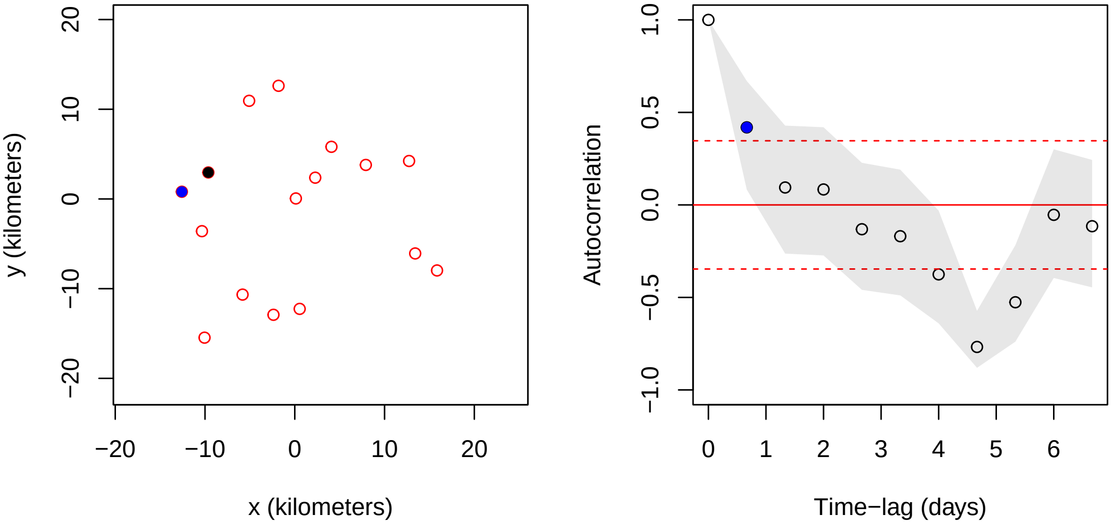
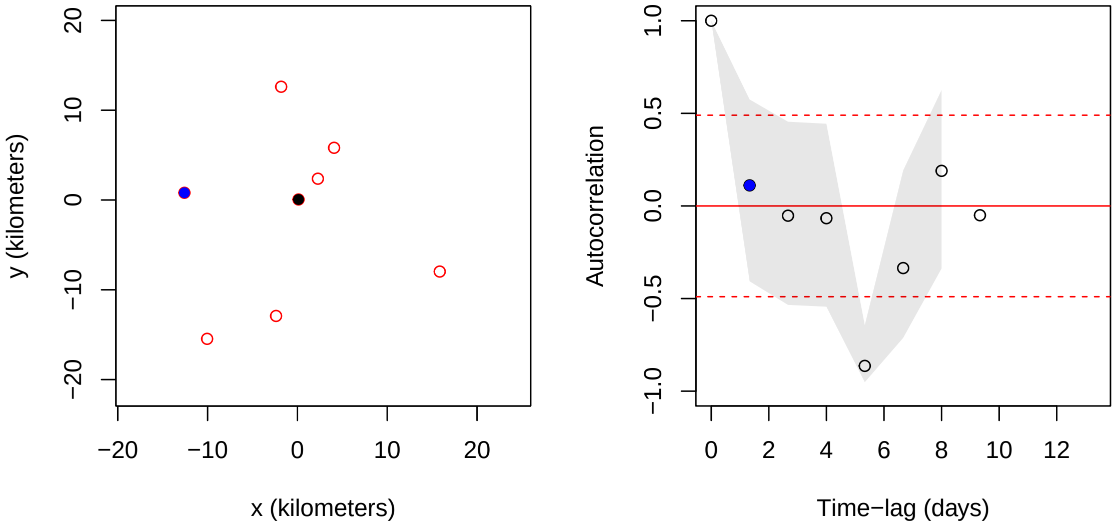
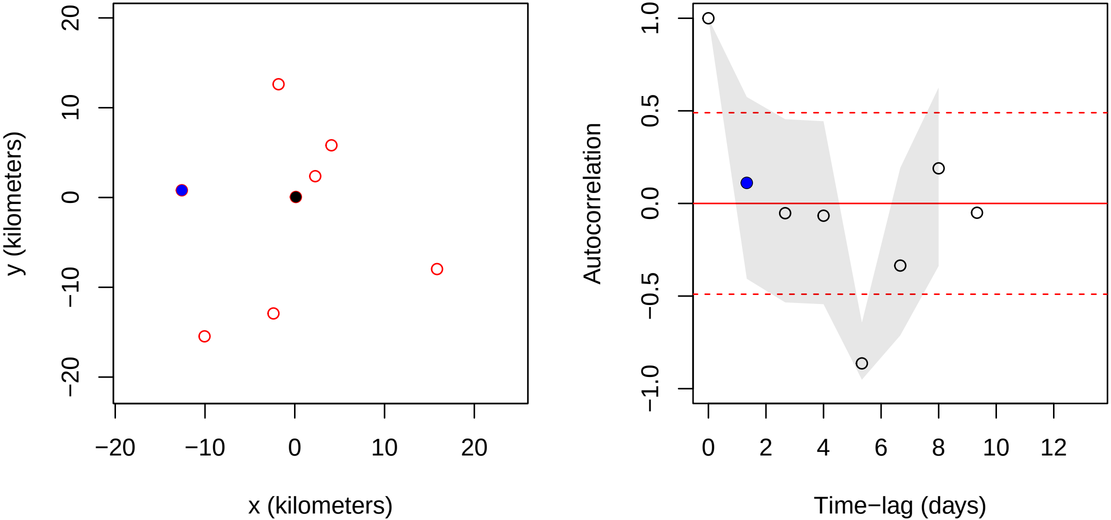


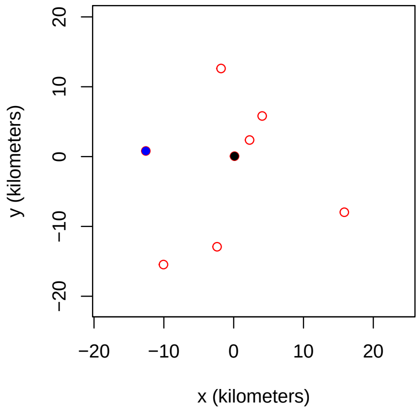
...you might have to thin a lot to reach IID

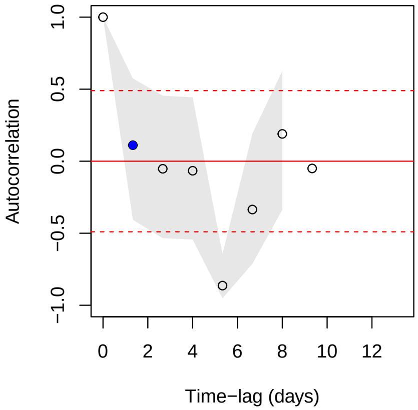




What about speed estimation with these data?

Autocorrelation is information





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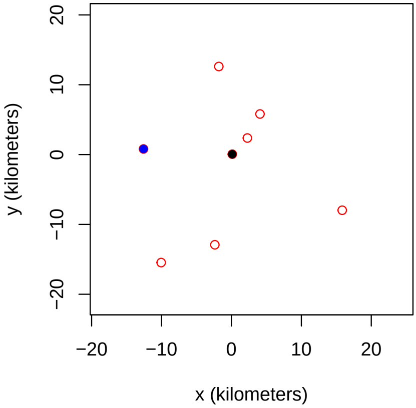

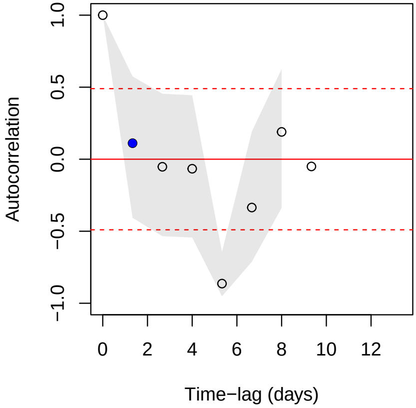

- Convenient for home-range analysis
- Worthless for speed/distance estimation

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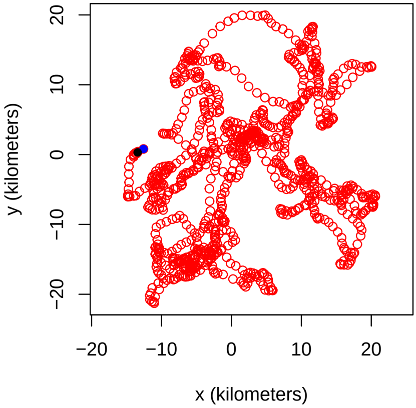

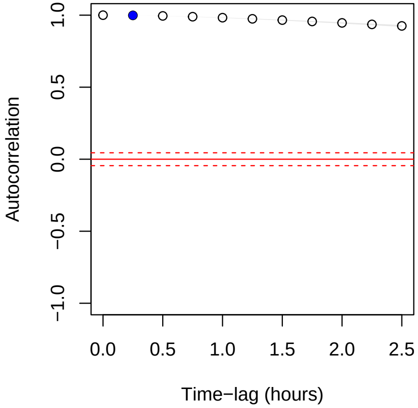

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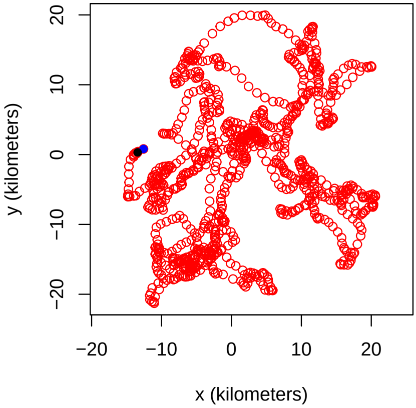
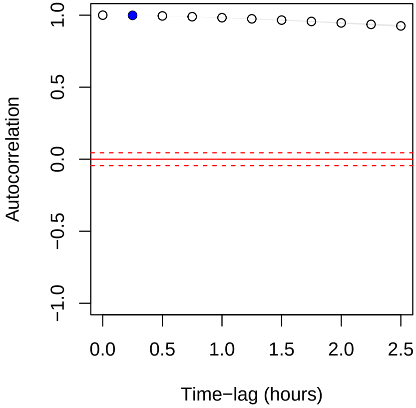
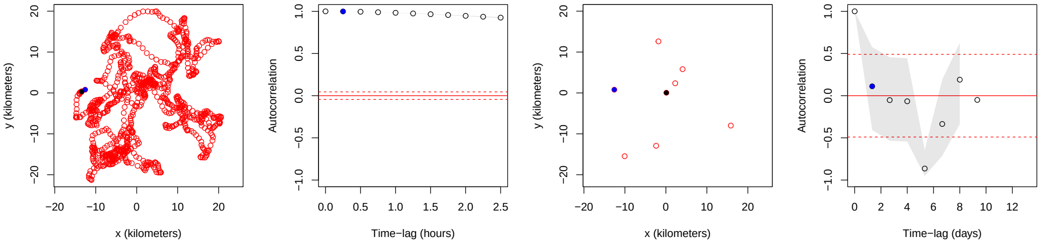




- Inconvenient for home-range analysis
- Great for speed/distance estimation

Objective



We want methods that can handle whatever autocorrelation is present in the data

Why continuous time?

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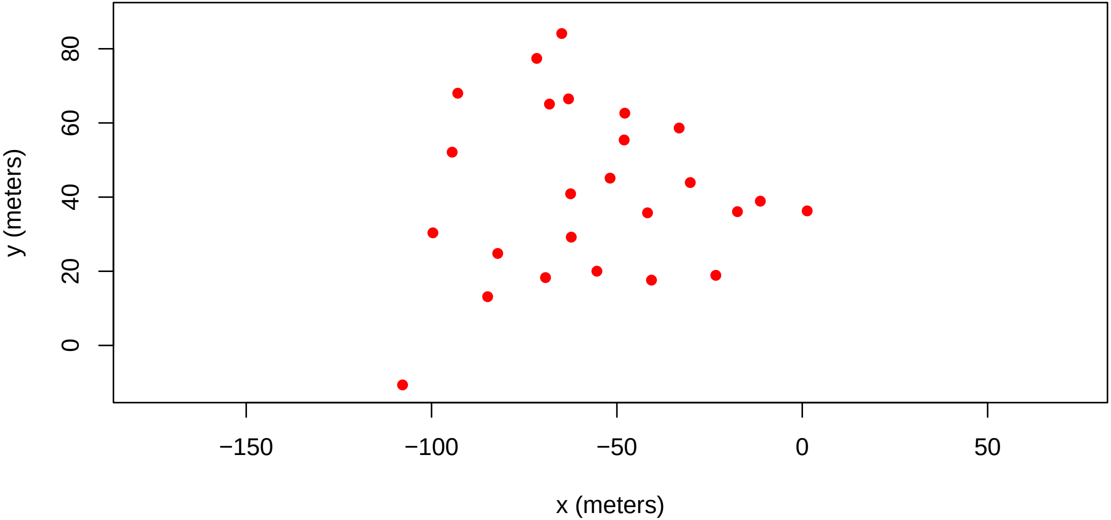
What do I mean by continuous time versus discrete time?



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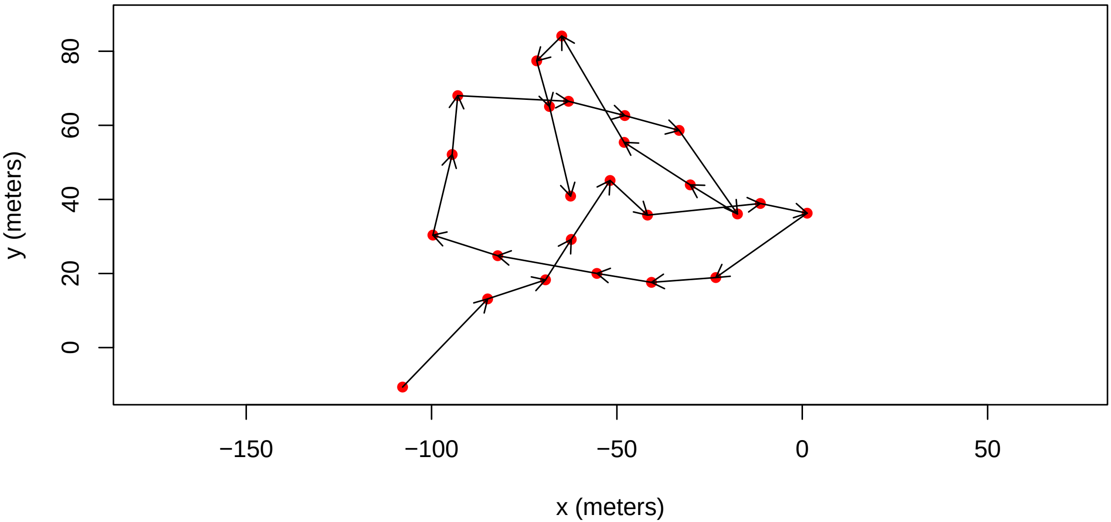

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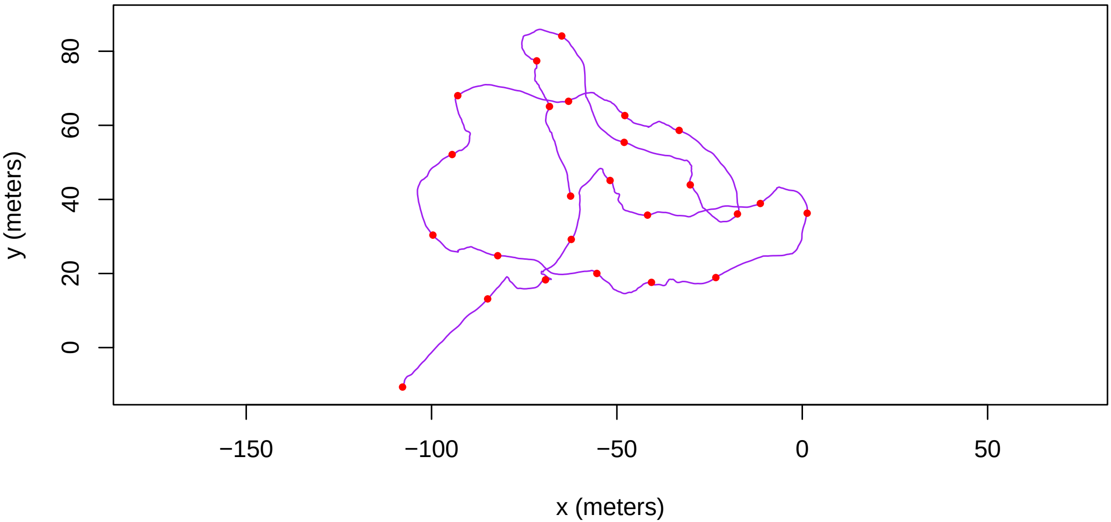

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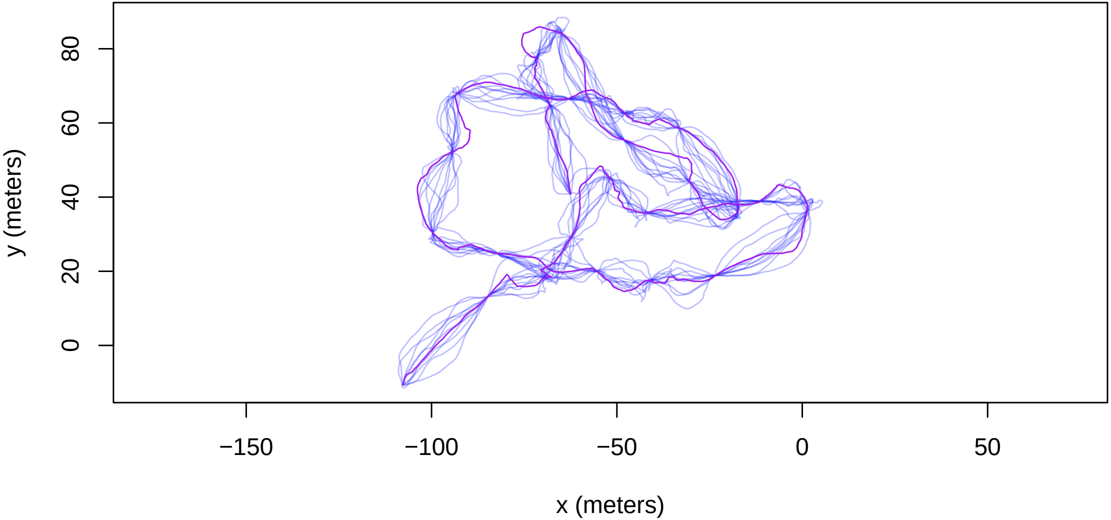

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  - Location error is easy to model (versus CRWs)

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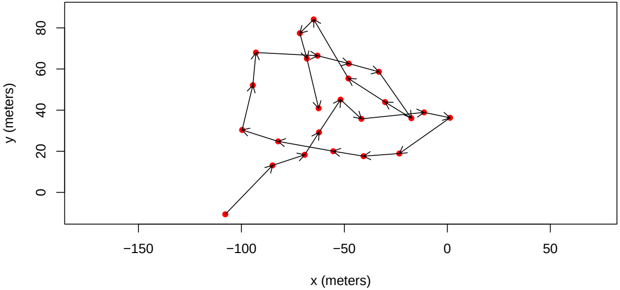

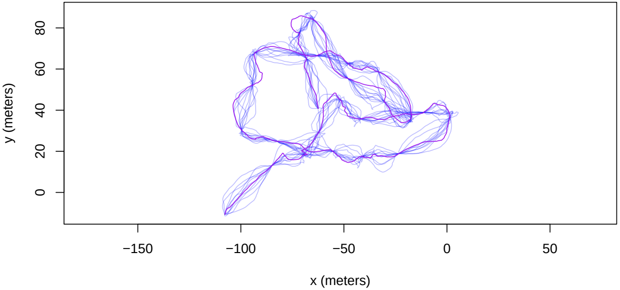

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- Brownian motion (Brownian bridge)

Accounting for non-independence: Stochastic process models

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- Integrated Ornstein-Uhlenbeck motion (`crawl`)
- OUF motion

Don't assume a model, select a model

Building-block continuous-time stochastic process models

- Independent locations (<1%)
- Brownian motion
- Ornstein-Uhlenbeck motion
- Integrated Ornstein-Uhlenbeck motion
- OUF motion

Don't assume a model, select a model

Accounting for non-independence: Stochastic process models

Building-block continuous-time stochastic process models

- Independent locations (<1%)
- Brownian motion
- Ornstein-Uhlenbeck motion (<30%)
- Integrated Ornstein-Uhlenbeck motion
- OUF motion

Don't assume a model, select a model

Building-block continuous-time stochastic process models

- Independent locations (<1%)
- Brownian motion
- Ornstein-Uhlenbeck motion (<30%)
- Integrated Ornstein-Uhlenbeck motion
- OUF motion (∼70%)

Don't assume a model, select a model

Accounting for non-independence: Stochastic process models

Building-block continuous-time stochastic process models

- Independent locations ($<$1%)
- Brownian motion
- Ornstein-Uhlenbeck motion ($<$30%)
- Integrated Ornstein-Uhlenbeck motion
- OUF motion ($\sim$70%)

Don't assume a model, select a model

Let's dive into `R` `ctmmlearn` "ctmm_intro.R"