

Counting example: Suppose we have a 52 card deck, pick 5 cards, without replacement, and without keeping track of the order of selection.

Know if we kept of the order of selection, there are

$$(52)(51)(50)(49)(48) = \frac{52!}{47!} \text{ ways to pick the cards.}$$

Each 5-tuple appears $5!$ times in this list. Why?



$5!$ ways to arrange them, i.e. orders in which they could have appeared.

$$(5)(4)(3)(2)(1) = 5!$$

So we overcounted originally by a factor of $5!$. So if we went back and ignored the order of selection, there are

$$\frac{52!}{47!5!} = \binom{52}{5} \text{ to pick 5 out of 52 cards without replacement and without keeping track of the order of selection.}$$

In general, n items, choose r of them without replacement,

$$\text{there are } (n)(n-1)\cdots(n-r+1) = \frac{n!}{(n-r)!} \text{ ways to pick if we keep track of the order of selection. Know this!}$$

Now if we ignore the order of selection, each r -tuple appears $r!$ times in a list such as the one above, so we overcounted by a factor of $r!$. So

$$\frac{n!}{(n-r)!r!} = \binom{n}{r} \text{ ways to pick } r \text{ out of } n \text{ items without replacement and without regard to the order of selection.}$$