

General technique (not specific to hypergeometries)  
for finding  $E(X^2)$  when  $X$  is a sum of indicator random variables  
(or any kind of random variables, summed).

Idea: Suppose  $X = X_1 + X_2 + \dots + X_n$ .

$$\begin{aligned} \text{Find } E(X^2) &= E((X_1 + X_2 + \dots + X_n)(X_1 + X_2 + \dots + X_n)) \\ &= E(X_1X_1 + X_1X_2 + X_1X_3 + \dots + X_1X_n \\ &\quad + X_2X_1 + X_2X_2 + X_2X_3 + \dots + X_2X_n \\ &\quad + \dots + X_nX_1 + X_nX_2 + X_nX_3 + \dots + X_nX_n) \\ &= E(X_1X_1) + E(X_1X_2) + \dots \\ &\quad \dots + E(X_nX_n) \\ &= \sum_{i=1}^n \sum_{j=1}^n E(X_iX_j) \end{aligned}$$

Question:  
Can we do better, with regard to organizing. This is usually helpful!

Say  $n=4$

$$\begin{array}{cccc} E(X_1X_1) & + E(X_1X_2) & + E(X_1X_3) & + E(X_1X_4) \\ E(X_2X_1) & + E(X_2X_2) & + E(X_2X_3) & + E(X_2X_4) \\ E(X_3X_1) & + E(X_3X_2) & + E(X_3X_3) & + E(X_3X_4) \\ E(X_4X_1) & + E(X_4X_2) & + E(X_4X_3) & + E(X_4X_4) \end{array}$$

have two copies of all the off-diagonal terms.

$$\sum_{i=1}^4 E(X_iX_i) + 2 \sum_{1 \leq i < j \leq 4} E(X_iX_j)$$

Same thing works in general: Nothing special for  $n=4$ .

$$E((X_1 + \dots + X_n)(X_1 + \dots + X_n)) = \sum_{i=1}^n E(X_iX_i) + 2 \sum_{1 \leq i < j \leq n} E(X_iX_j)$$

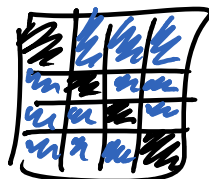
and all  $E(X_iX_j)$  terms are the same.

In the special case where all  $E(X_iX_i)$  terms are the same, this becomes:

$$nE(X_1X_1) + (n)(n-1)E(X_1X_2)$$

e.g. if  $n=4$

$$4E(X_1X_1) + (4)(3)E(X_1X_2)$$



$n^2 - n$   
on diagonal  
 $n$   
 $n(n-1)$   
off diagonal