Example continued: $f_{X,Y}(x,y) = \frac{x}{16}(2-y)$ $\quad 0 \leq x \leq 4$
$\quad 0 \leq y \leq 2.$

Find $f_Y(y) = \int_0^4 \frac{x}{16}(2-y)\,dx = \frac{x^2}{32}(2-y)\Big|_{x=0}^{4} = \frac{16}{32}(2-y) = \frac{1}{2}(2-y)$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ for $0 \leq y \leq 2.$

Then $\quad P(Y \leq \tfrac{3}{4}) = \int_0^{3/4} \frac{1}{2}(2-y)\,dy = \frac{1}{2}(2y - y^2/2)\Big|_{y=0}^{3/4}$

$\qquad\qquad\qquad\qquad\qquad\qquad = \frac{1}{2}\left(\frac{3}{2} - \frac{9/16}{2}\right)$

$\qquad\qquad\qquad\qquad\qquad\qquad = \frac{1}{2}\left(\frac{48}{32} - \frac{9}{32}\right) = \frac{1}{2}\cdot\frac{39}{32} = \frac{39}{64}$

Since X,Y independent this is the same as

$\qquad P(Y \leq \tfrac{3}{4} \mid X = 3) = 39/64$

$\qquad P(Y \leq \tfrac{3}{4} \mid X \leq 3) = 39/64.$