Probabilities and random variables.

Remember: random variables themselves do not have probabilities.
E.g. makes no sense to say $P(X) = \_\_$.

Only events have probabilities. (If I say an outcome has a probability, I really mean an event with just that outcome has a probability)

What does $P(X=5)$ mean? It really means $P(A)$ where A is an event contains all outcomes that give $X=5$.

Roll two dice
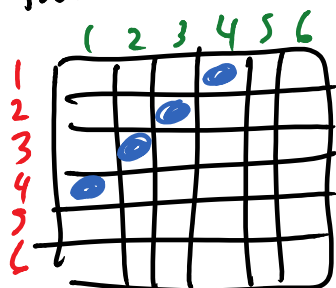
$P(X=5)$ means
$$P(\{(1,4),(2,3),(3,2),(4,1)\}) = \frac{4}{36} = \frac{1}{9}$$
Say for simplicity $P(X=5) = \frac{1}{9}$
but the above meaning is understood.