Counting: Multiplying Possibilities

E.g. Suppose 3 students running for class president

5 students running for vice president

6 students running for treasurer

2 students running for secretary

1 student for each office. Do not about overlaps, for instance, say no student runs for more than one office. (Or, if they are allowed to run for more than one office, they can simultaneously accept more than one position.)

$$(3 \text{ ways Pres})(5 \text{ ways VP})(6 \text{ ways treas})(2 \text{ ways sec.})$$

15 ways

90 ways

180 ways total

Much more generally, if there is a process with j stages and n_1 ways stage 1 can happen (e.g. chose Pres) and for each of these ways, n_2 ways stage 2 can happen (e.g. chose VP) and for each of those pairs of ways, n_3 ways stage 3 can happen (e.g. chose tres.) ......

and for the last stage, n_j ways that the last stage can happen.

Then altogether there are $(n_1)(n_2)(n_3)\cdots(n_j)$ ways to do the process.

In particular, if sampling with replacement from n items, and we chose r items total, there are:

n ways to pick item 1

n ways to pick item 2 $\bigg\} n^2 \text{ ways} \bigg\} n^3 \text{ ways}$

n ways to pick item 3

$\vdots$

n ways to pick item r

n^r ways altogether, to pick n items with replacement, where the order of selection matters.