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Sheet 1P
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Scientific Computing II

Programming assignment 1: Smoothing Properties of (weighted) Jacobi Method

We want to numerically solve the Poisson problem

$$\begin{aligned} -\frac{d^2u}{dx^2} &= 0, \quad x \in (0,1), \\ u(0) = u(1) &= 0, \end{aligned}$$

with finite differences. Discretise the equation on a grid with $N+1$ points and mesh size $h := 1/N$.

(a) Implement an iteration of the Jacobi method in Matlab or in any other programming language.

(b) Carry out 10 relaxation steps with the initial condition $u_k(x) := \sin(\pi k x)$, $k = 1, 3, 7$, and mesh size $h := 1/8$. At which rate does the (discrete) error $e_k^{(n)} := \max_i |0 - u_{k,i}^{(n)}|$ decrease in each iteration step? ($u_{k,i}^{(n)}$ is the value of the i-th grid point at iteration n with initial condition $u_k(x)$.) Measure $r := e_k^{(n)} / e_k^{(n-1)}$ for this purpose. Compare your results with the analytical findings.

(c) What happens when decreasing the mesh size to $h := 1/16$ and keeping all other parameters unchanged?

(d) Carry out the same study for the weighted Jacobi method with $\omega = \frac{1}{3}, \frac{2}{3}, \frac{1}{2}$.

Note: Do not stick to the description given here. Try other values of h, N, and k. You can also try to introduce a right hand side $f(x)$.