

Control Engineering (CEN4B)

CEN4B v20 planning

Week	Inhoud	Studiemateriaal	Huiswerk uit het boek (end of chapter exercises)
1	Bode en Nyquistdiagrammen, 1^e en 2^e orde basissystemen: Bode en Nyquist tekenregels. Omzetten van een naar de andere. Alle beeldverbanden van 1 ^e 2 ^e order systemen. ‘Nyquist Stability’ is buiten de scope!	6.1 Frequency Response 6.3 Nyquist Stability Criterion 4.3.1 t/m 4.3.4 Basissystemen 3.3 Effect of Pole Locations	6.3 – 6.5 6.19 a-b-c
2	Terugkoppeling en stabiliteit in het s-domein: Principe van terugkoppeling; stabiliteitsonderzoek in s-domein. Stabiliteitsonderzoek in ω -domein; fasemarge, versterkingsmarge, ook in Nyquist; invloed terugkoppeling op pn-beeld;	3.6.2 Stability of LTI Systems 6.4 Stability Margins 5.1 Root Locus of a Basic Feedback System	6.24 6.30 6.31
3	Terugkoppeling en stabiliteit in het ω-domein; poolbanen: constructieregels poolbanen	5.2 Guidelines for Determining a Root Locus 5.3 Selected Illustrative Root Loci	5.3 t/m 5.7 5.12 5.13
4	Ontwerpcriteria voor geregelde systemen: Inleiding, ontwerpcriteria in het t- en s-domein, settlingtime, doorschot, offset	3.3 Effect of Pole Locations 3.4 Time Domain Specifications	3.25 3.27 3.34
5	Ontwerpcriteria voor geregelde systemen vervolg Ontwerpen van geregelde systemen: Ontwerpcriteria in ω -domein, fase- en versterkingsmarge, bandbreedte; kwalitatieve beschouwing van de invloed van P-, I- en D-actie.	6.6 Closed-Loop Frequency Response 5.4, 5.5 Design Using Dynamic Compensation (except 5.4.3)	6.43
6	Ontwerpen van geregelde systemen vervolg: Invloed van regelacties in t-, s- en ω -domein, praktische instelregels	6.7 Compensation	6.46 6.54
7	Oefenen: Oefentoets		

Huiswerk

- Huiswerk via Socrative, niet verplicht.
- De uitwerkingen en de vragen worden niet gedeeld of online gezet, kom dus naar de les.
- Correlatie tussen “Readiness-%” en “Tentamenresultaat” = M4A: +30%
M4B: +60%

READYNESS-PERCENTAGE	tentamenr esultaat
43,2	100
11,4	79
32,8	73
11,4	70
23,5	69
0,0	68
0,0	60
17,4	58
14,2	54
23,9	53
0,0	51
10,8	49
11,3	45
0,0	44
44,2	43
14,2	38
18,5	34
5,8	34
0,0	30
5,8	29
13,3	26
11,3	26
0,0	26
26,8	19
23,4	19
11,4	19
5,3	16
8,6	15
14,2	13
22,7	10
7,1	10
0,0	10

READYNESS-PERCENTAGE	tentamenr esultaat
30,0	89
76,4	81
50,0	81
45,6	81
39,6	75
27,8	75
22,0	69
12,6	69
12,6	69
2,8	69
0,0	65,0
16,6	64
24,0	61
64,4	60
27,6	59
25,2	59
0,0	55
18,6	50
6,6	50
5,8	44
0,0	41
26,8	38
0,0	38,0
0,0	31
0,0	26
13,4	10

VAKBOEK

Het officiële Boek: Feedback Control of Dynamic Systems,
Franklin

ICE3->**CEN4A->CEN4B->ACE7->OBS7**
'->2^e differentiatie

Geadviseerd Boek: Regeltechniek voor het HBO, 5^e druk
Schrage, Van Daal, Stroeken, Van der Pol en
Thomasse

Youtube



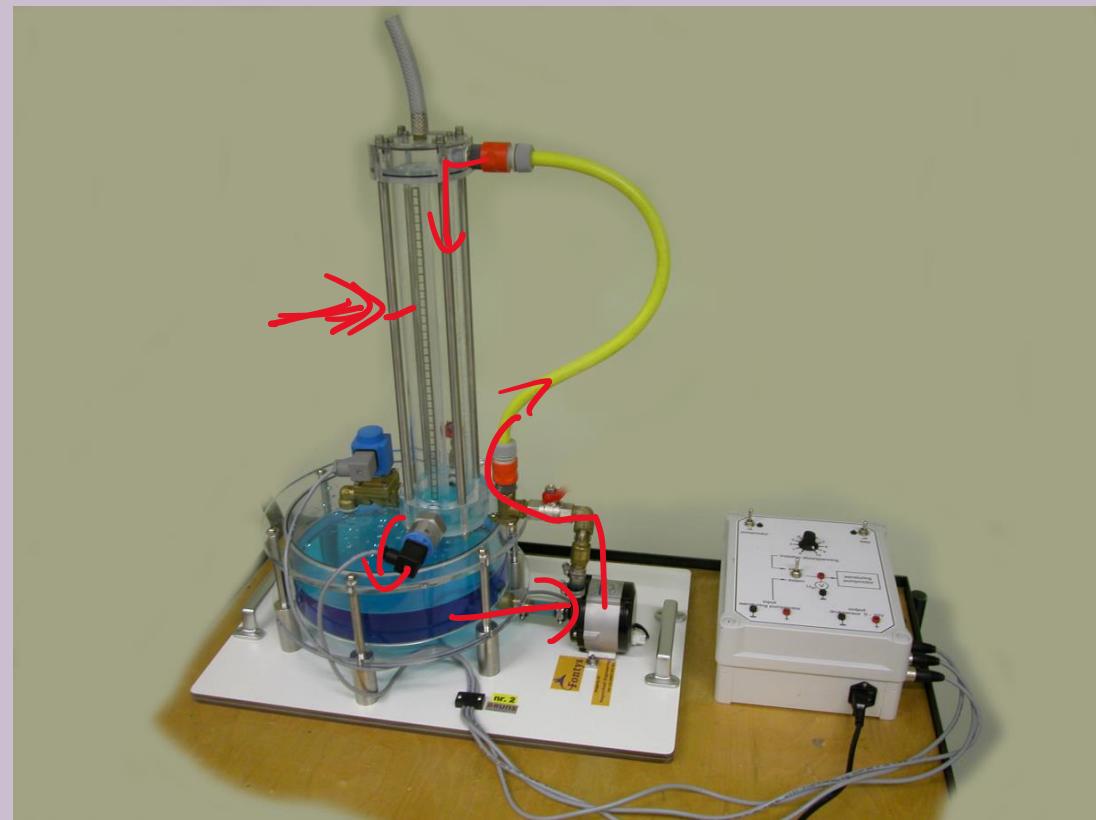
Brian Douglas

191K subscribers

<https://www.youtube.com/user/ControlLectures/>

Mededelingen: Practicumopstelling

Nog te bepalen aan de hand van RIVM besluit



Mededelingen: CEN4A Tentamen

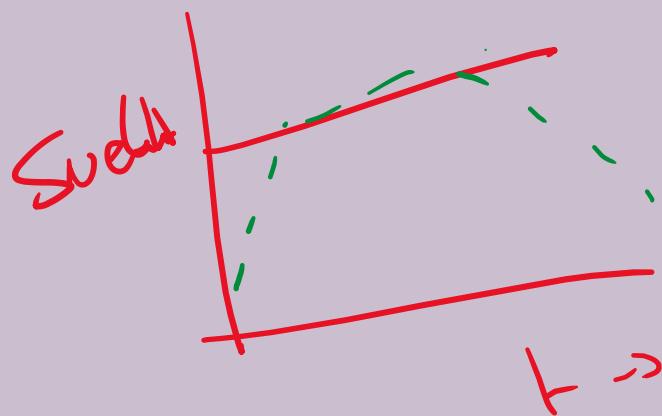
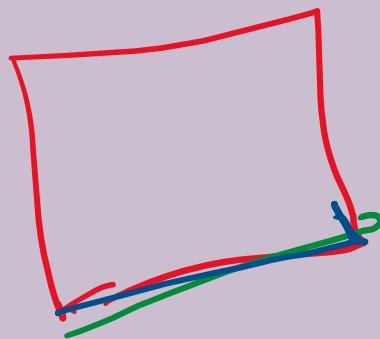
CEN4A Tentamen:

Nog te bepalen aan de hand van RIVM besluit

Regeltechniek in de industrie

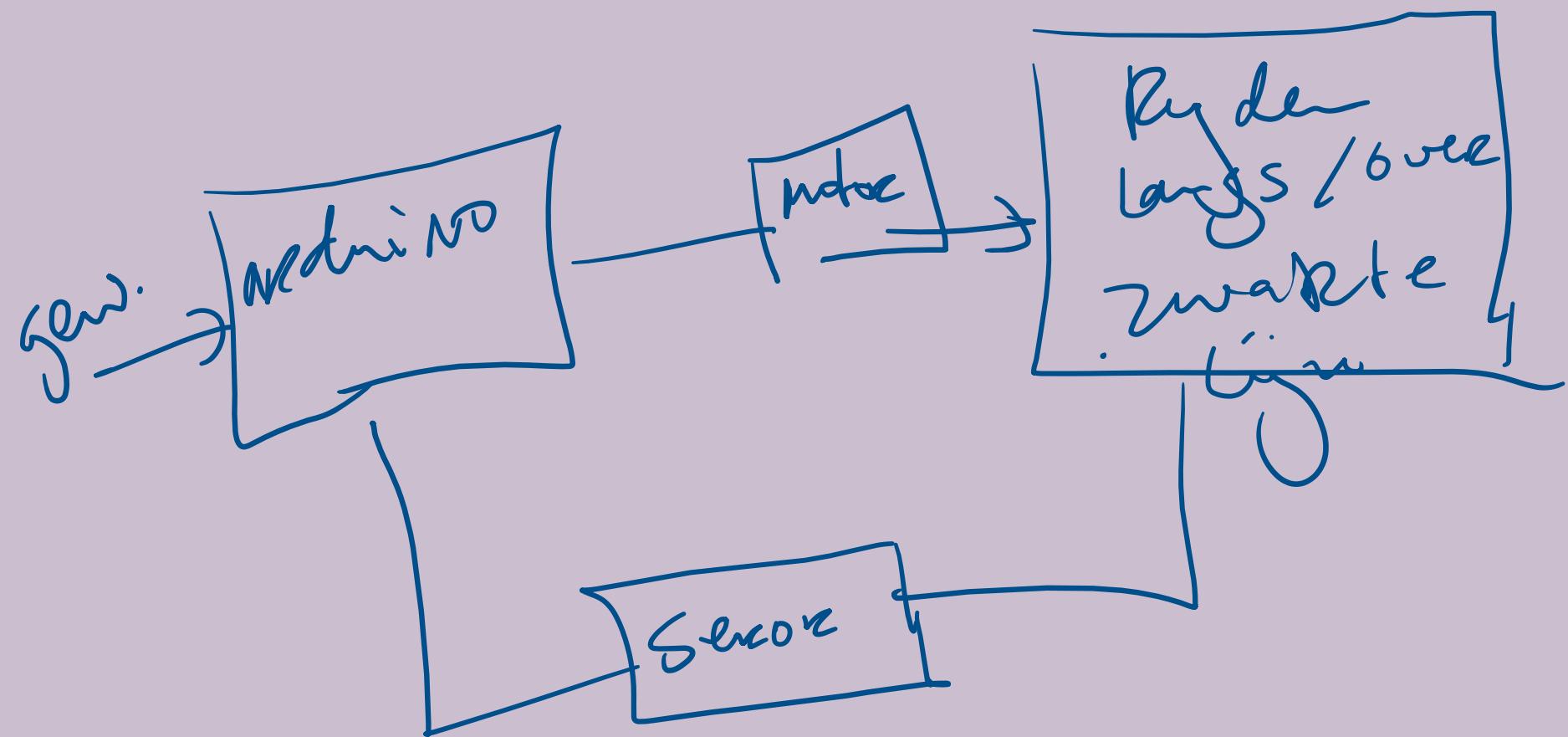
- Robot Arms voor assamblage bedrijven
- Autonomous vehicles
- Path planning voor 3D printers
- Industriële automatisering
- Waffer stage van ASML
- Motor control

path planning

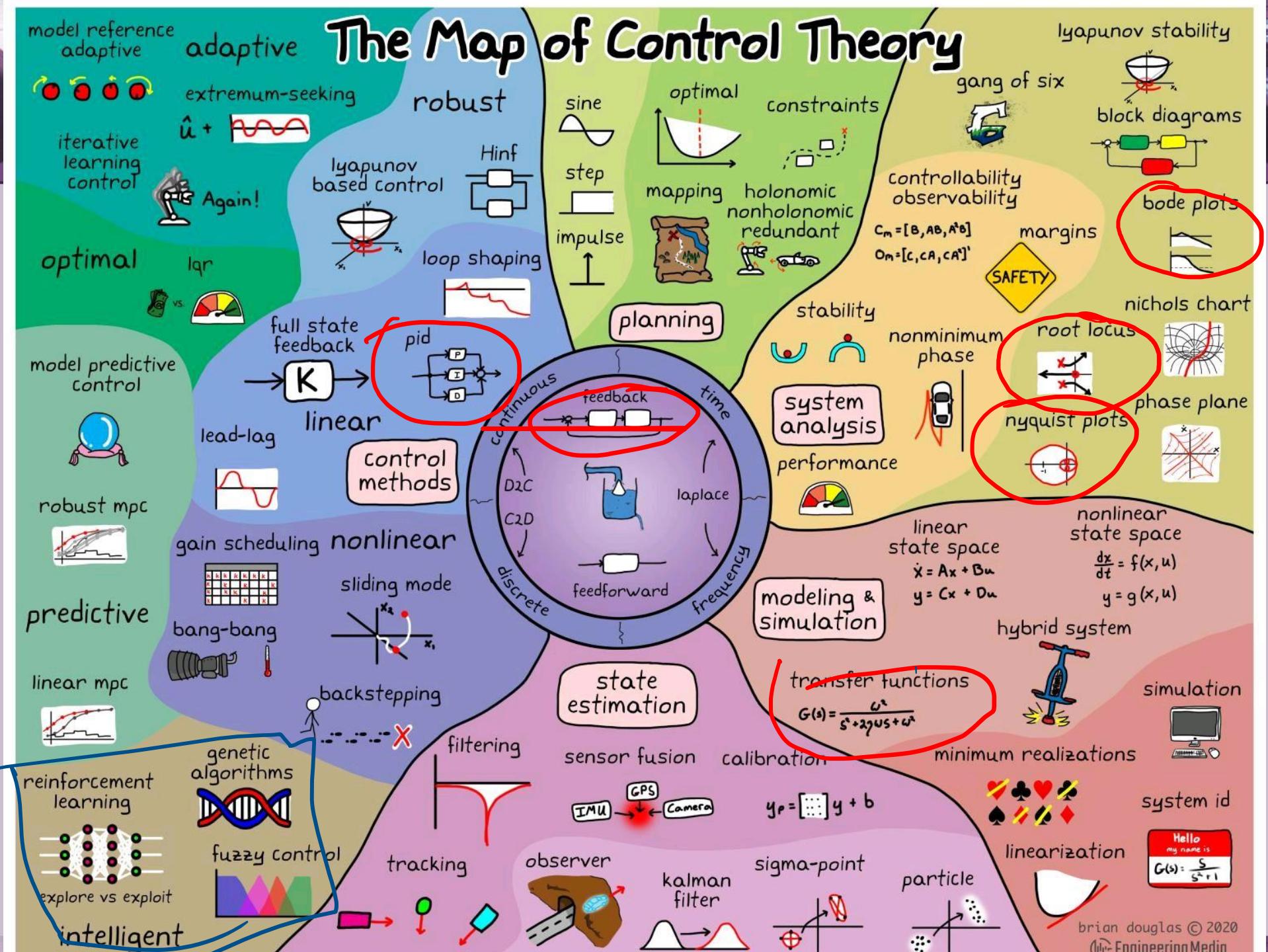




Andere voorbeelden?



The Map of Control Theory



system id
Hello my name is
 $G(s) = \frac{s}{s+1}$

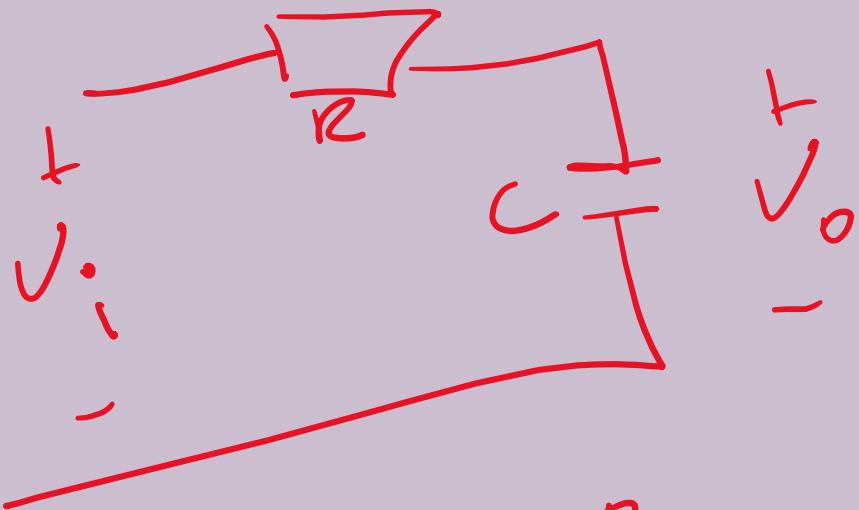
brian douglas © 2020
Engineering Media



Gereedschappen in Freq. Domein

(Chapter 6.1 en 6.3)

- Bodediagrammen
- Polaire Figuur (Nyquist Diagram)



$$V_o = \frac{X_C}{R + X_C} \cdot V_i$$

$$X_C = \frac{1}{j\omega C}$$

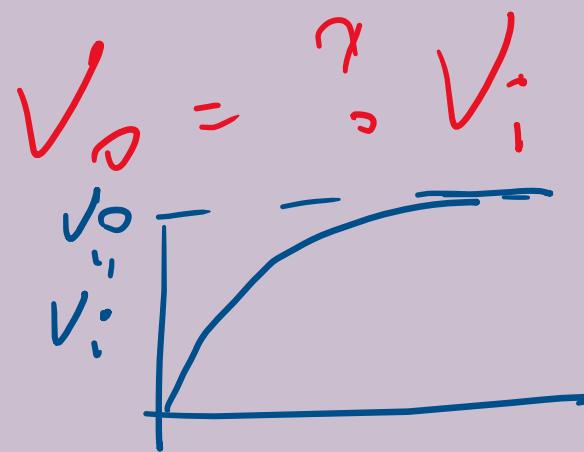
DC $\Rightarrow V_o = 0 \rightarrow V_o = V_i$

$$\omega = 10 \text{ Hz}$$

$$V_o = 20 \text{ Hz}$$

100° Hz

?



$$V_o = \frac{X_C}{R + X_C} \cdot V_i \rightarrow \frac{\cancel{Y} j\omega C}{R + \cancel{Y} j\omega C} \cdot V_i$$

$$V_o = \frac{1}{j\omega C + 1} \cdot V_i \quad \begin{array}{l} \omega = 0 \Rightarrow 0 = \frac{1}{2} \cdot V_i \\ \omega \neq 0 \end{array}$$

$$V_i \xrightarrow{H} V_o \quad H = \frac{1}{j\omega RC + 1}$$

willekeurige $\omega \Rightarrow$
 $|H| \leftarrow \text{Verstrekking / Vertraging}$
 $Q(H) \leftarrow \text{fase van de ring}$

$$H = \frac{1}{j\omega RC + 1} \quad |H| = \sqrt{\frac{1^2}{(\omega RC)^2 + 1^2}}$$

$$\varphi(H) = \arctan \left(\frac{\text{Im}}{\text{Re}} \right) = \arctan \left(\frac{1}{\sqrt{(\omega RC)^2 + 1}} \right)$$

$$\frac{1}{j\omega RC + 1} \cdot \frac{-j\omega RC + 1}{-j\omega RC + 1}$$

$$\frac{\text{Im}/\text{Re} \rightarrow \frac{-\omega RC}{(\omega RC)^2 + 1}}{1/(\omega RC)^2 + 1}$$

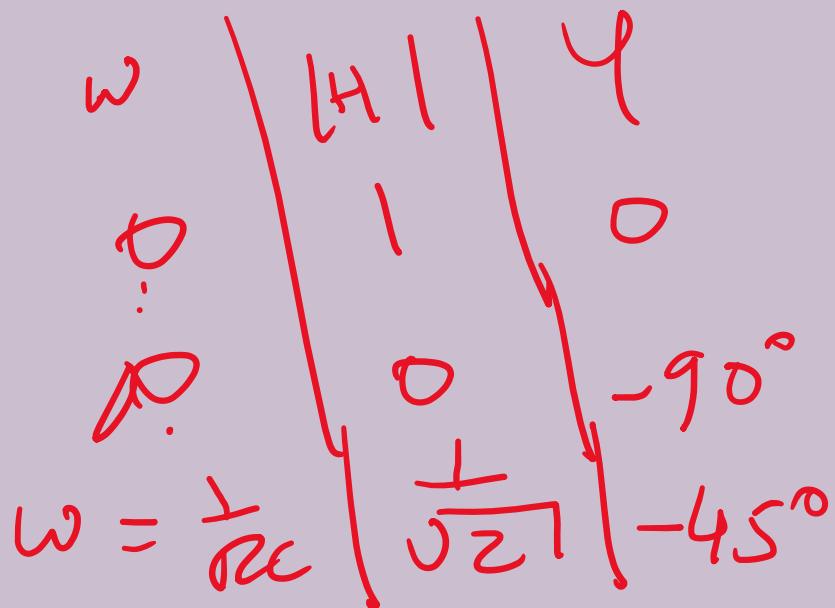
$$\frac{-j\omega RC + 1}{(\omega RC)^2 + 1}$$

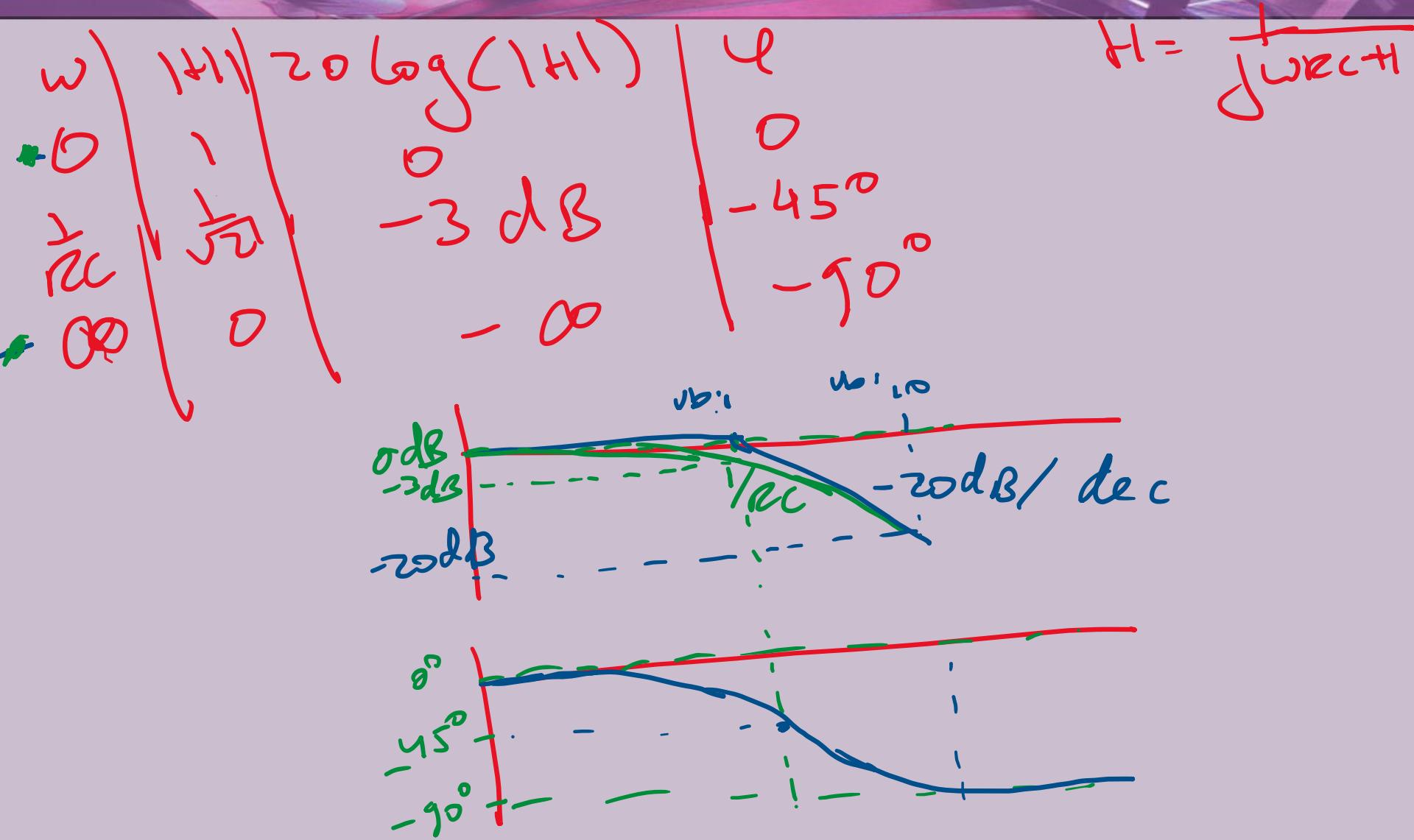
$$\varphi = \arctan(-\omega RC)$$

$$H = \frac{1}{j\omega R C + 1}$$

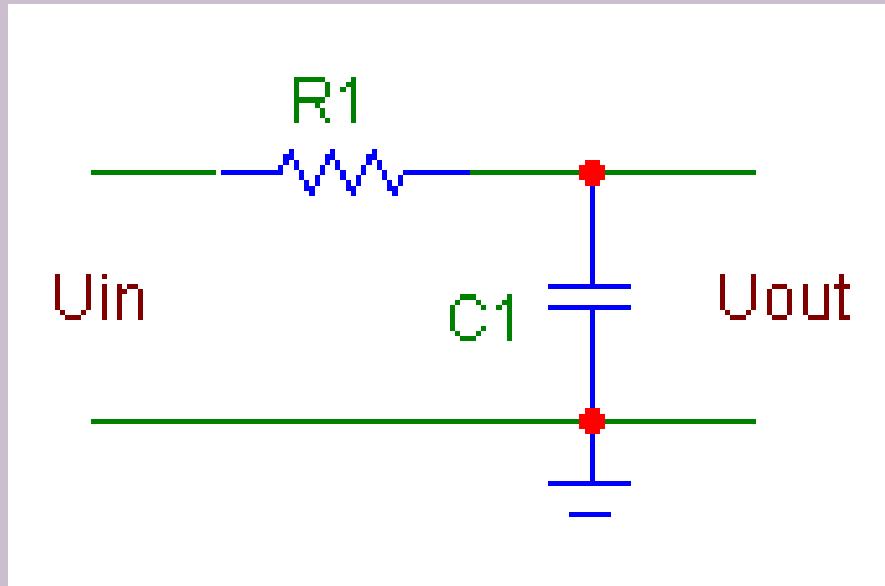
$$|H| = \sqrt{(\omega R C)^2 + 1}$$

$$\ell(\omega) = \arctan(-\omega R C)$$





RC-netwerk



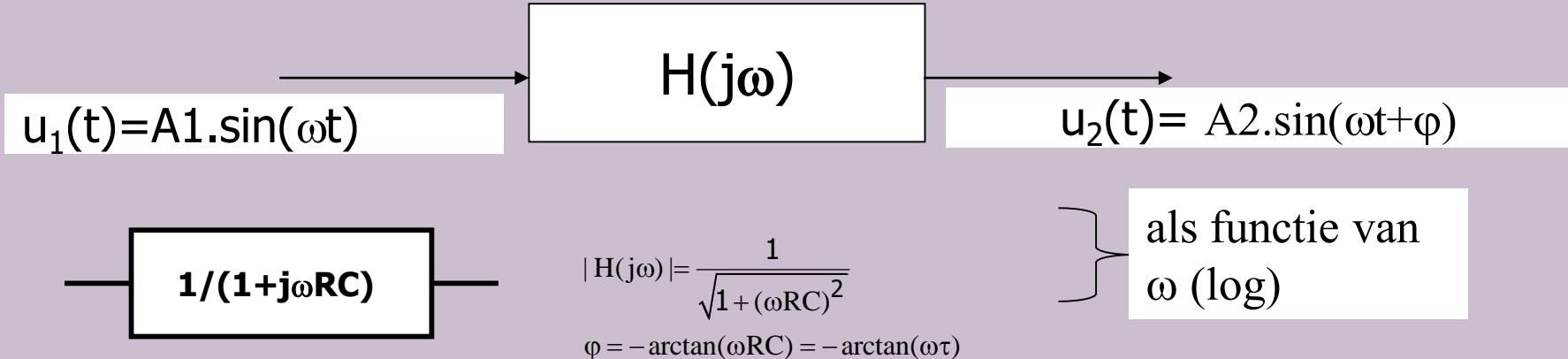
$$H(s) = \frac{U_o(s)}{U_i(s)} = \frac{1}{sR_1C_1 + 1}$$

$$s = \lambda + j\omega \Big|_{\lambda=0}$$

$$H(s) = \frac{U_o(s)}{U_i(s)} = \frac{\frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}} = \frac{1}{j\omega R_1 C_1 + 1}$$

$$H(s) = \frac{U_o(s)}{U_i(s)} = \frac{\frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}} = \frac{1}{j\omega R_1 C_1 + 1}$$

Bode amplitude- en fasediagram



amplitudediagram: $20\log|H(j\omega)| = 20\log(A_2/A_1)$ dB

fasediagram: $\varphi = \arg\{H(j\omega)\}$

ω	$ H(j\omega) $	$20\log H(j\omega) $	φ
0	1	0 dB	0°
$\frac{1}{\tau} = \frac{1}{RC}$	$\frac{1}{2}\sqrt{2}$	-3	-45°
∞	0	$-\infty$	-90°

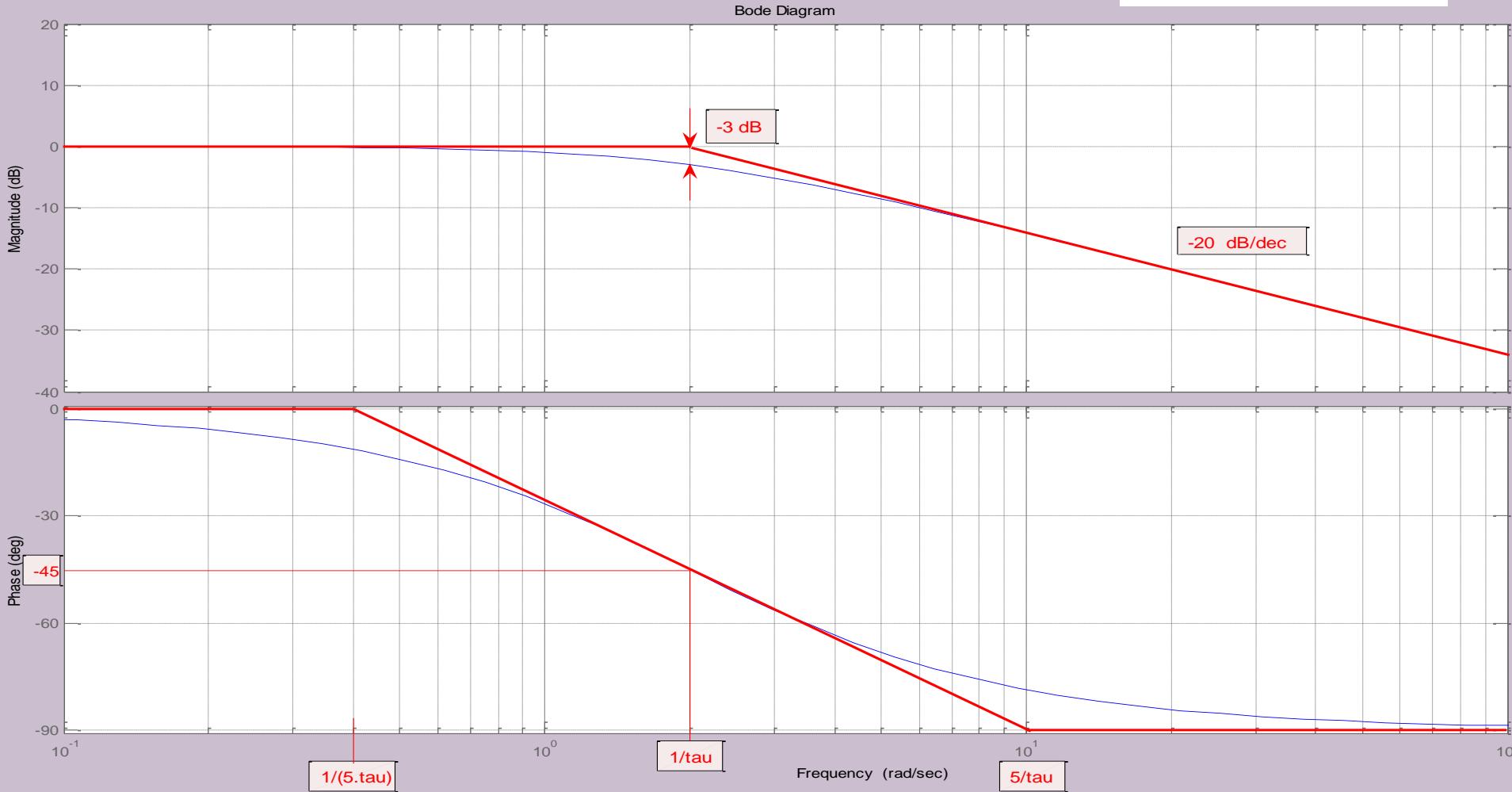
kantelfrequentie \rightarrow

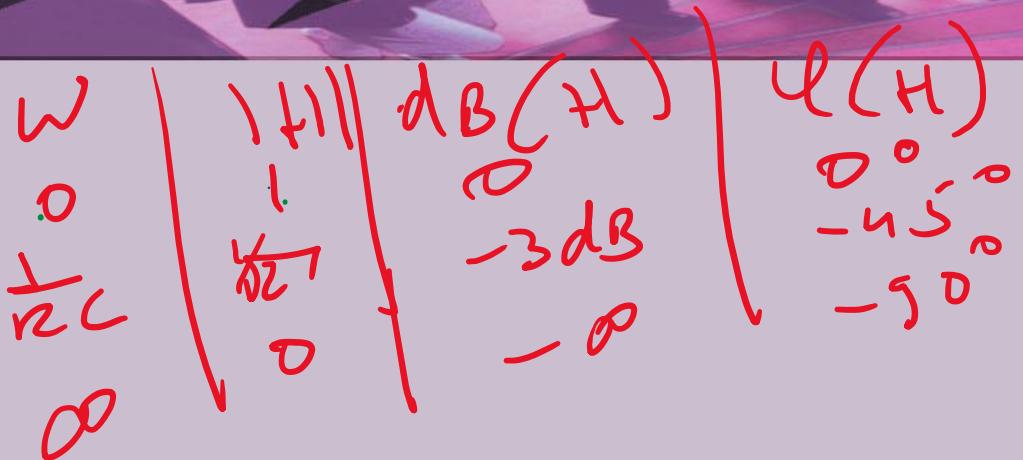
$$\frac{1}{\tau} = \frac{1}{RC}$$

Bode amplitude- en fasediagram

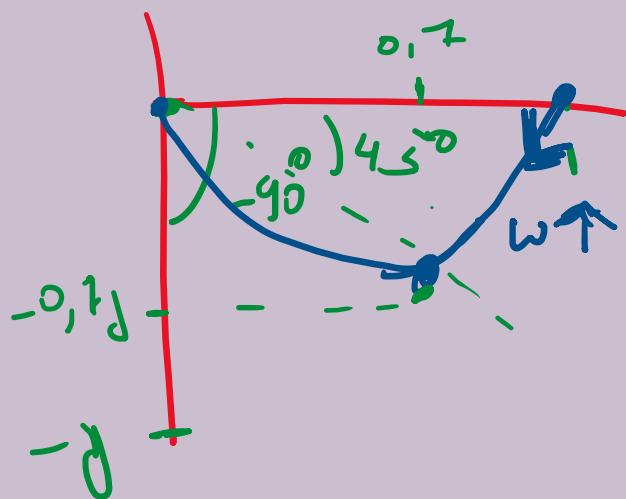
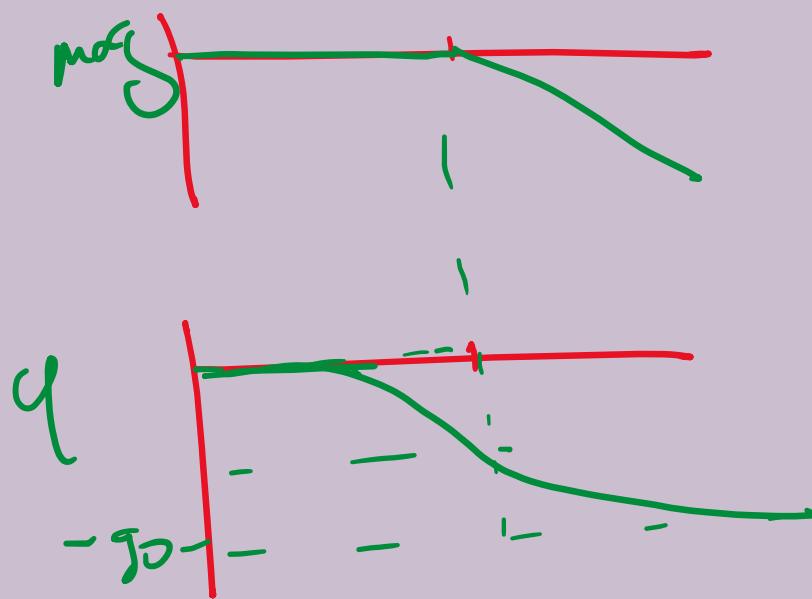
matlab: bode(1,[0.5 1])

$\tau = 0,5 \text{ sec}$





$$|H| < \varphi(H)$$



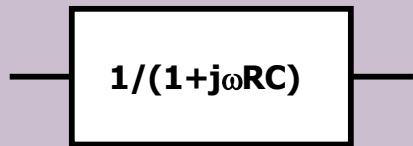
Polaire figuur (= Nyquist-diagram)

$$H(j\omega) = \operatorname{Re}\{H(j\omega)\} + j \cdot \operatorname{Im}\{H(j\omega)\}$$

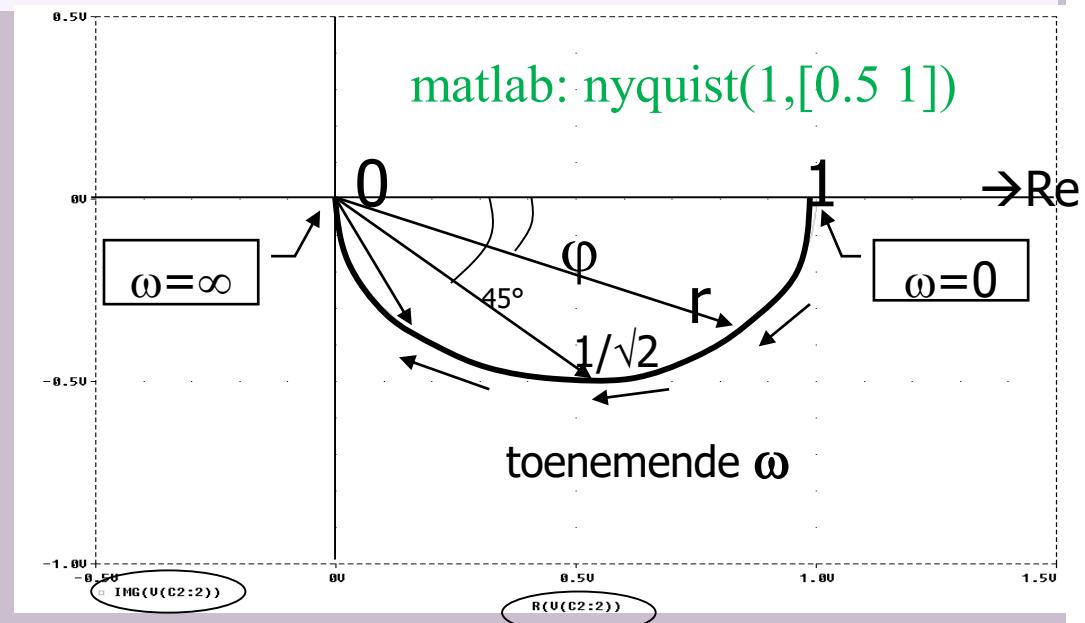
$$= x + j \cdot y = r \cdot e^{j\varphi}$$

met: $r = |H(j\omega)|$ en $\varphi = \arg\{H(j\omega)\}$

De polaire figuur is een afbeelding van $H(j\omega)$ in het complexe vlak als functie van ω !



ω	$ H(j\omega) $	$20\log H(j\omega) $	φ
0	1	0	0
$\frac{1}{RC}$	$\frac{1}{2}\sqrt{2}$	-3	-45°
∞	0	$-\infty$	-90°



Basissystemen (Chapter 4.3.1 t/m 4.3.4)

Bij fysische processen treden slechts 6 essentieel verschillende kenmerken op. Deze karakteriseren de zogenaamde basissystemen.

Basissystemen

- Constante factor
- Integrator (zuiver – onzuiver)
- Differentiator (zuiver – onzuiver)
- Eerste-orde systeem
- Tweede-orde systeem
- Looptijd

De constante factor

Mathematisch model

$$y(t) = kx(t)$$

S-domein

$$Y(s) = kX(s)$$

$j\omega$ domein

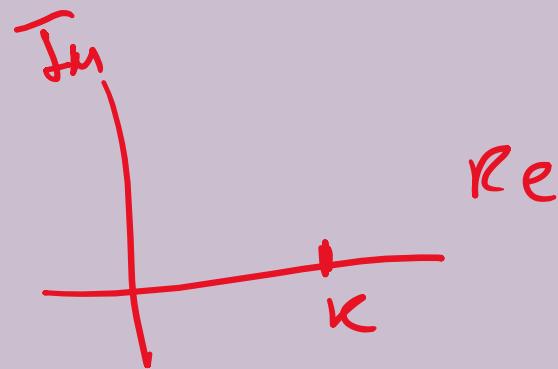
$$Y(j\omega) = kX(j\omega)$$

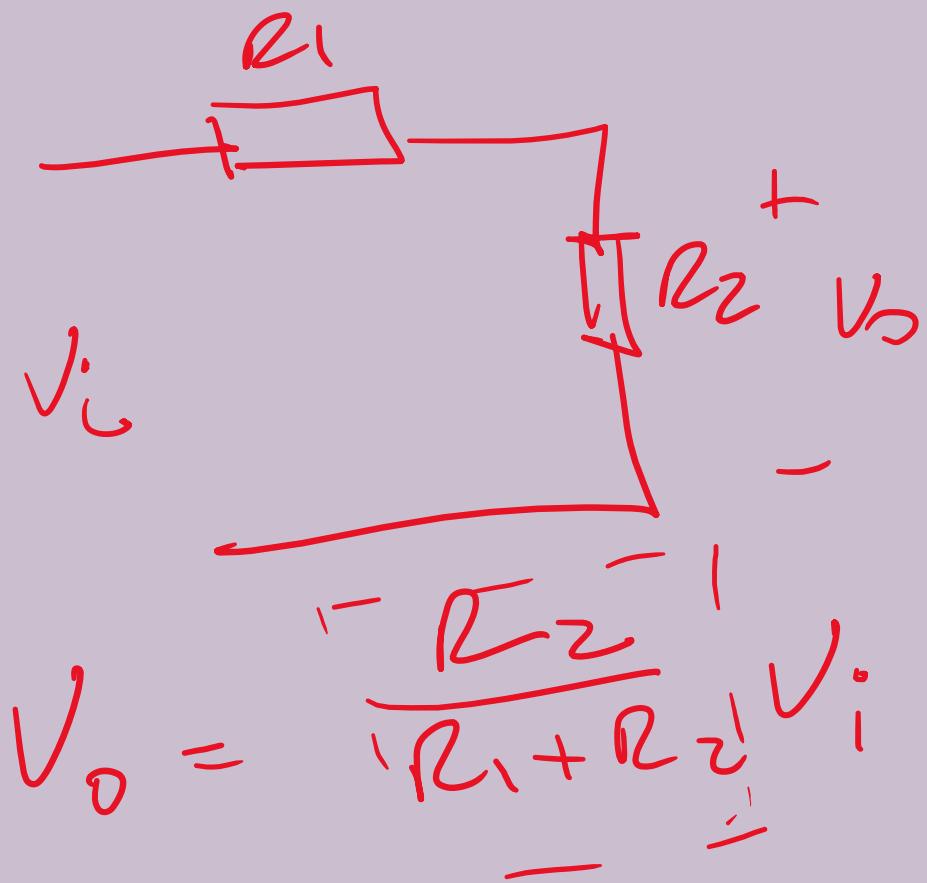
$$\boxed{H(j\omega) = k}$$

$$|H(j\omega)| = k \text{ en } \varphi = 0$$

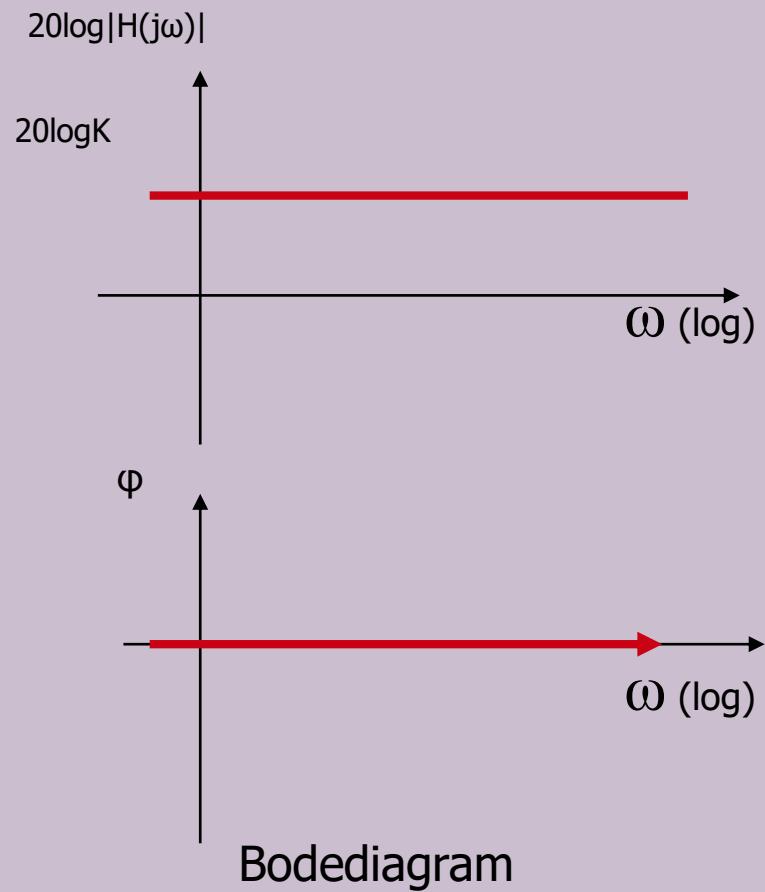
$$20 \log |H(j\omega)| = 20 \log K$$

tabel			
ω	$ H(j\omega) $	$20 \log H(j\omega) $	φ
0	k	$20 \log k$	0
∞	k	$20 \log k$	0





De constante factor



Zuivere integrator

$$\int \text{ (sigma)} \text{ (omega)}$$

Mathematisch model

$$y(t) = \int x(t)dt$$

S-domein

$$Y(s) = \frac{X(s)}{s}$$

$j\omega$ domein

$$Y(j\omega) = \frac{1}{j\omega} X(j\omega)$$

$$H(j\omega) = \frac{1}{j\omega}$$

$$|H(j\omega)| = \frac{1}{\omega} \quad \text{en} \quad \varphi = -90^\circ$$

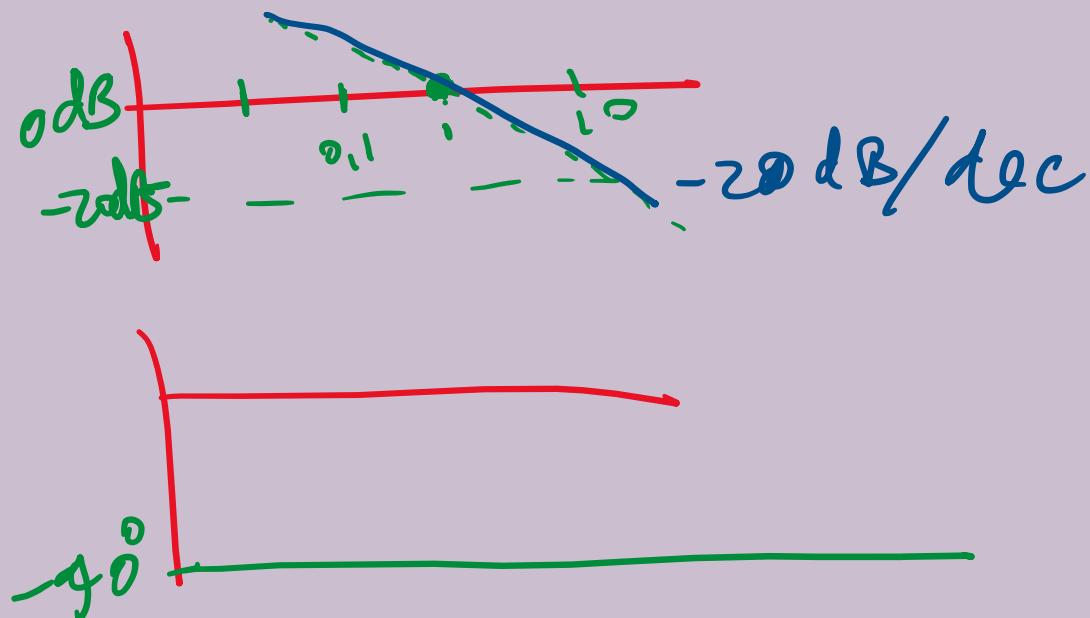
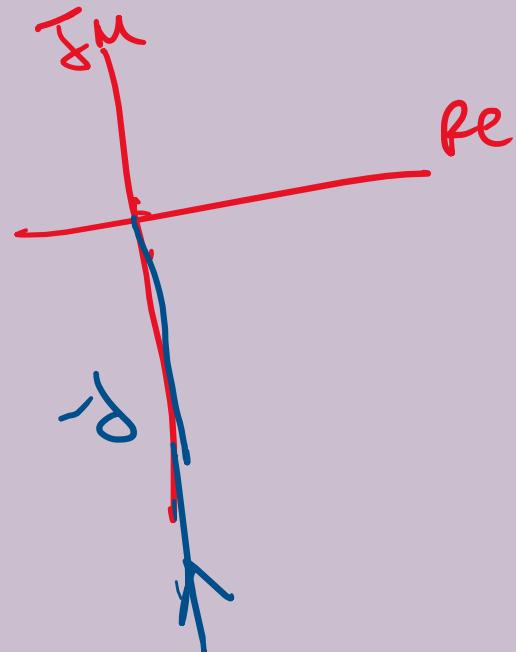
$$20 \log |H(j\omega)| = -20 \log \omega$$

ω	$ H(j\omega) $	$20 \log H(j\omega) $	φ
0	∞	$+\infty$	-90°
1	1	0	-90°
∞	0	$-\infty$	-90°

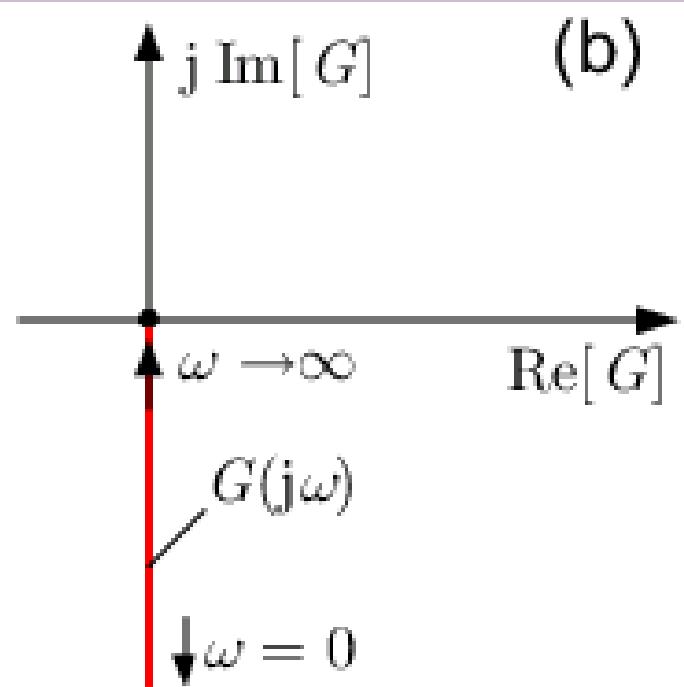
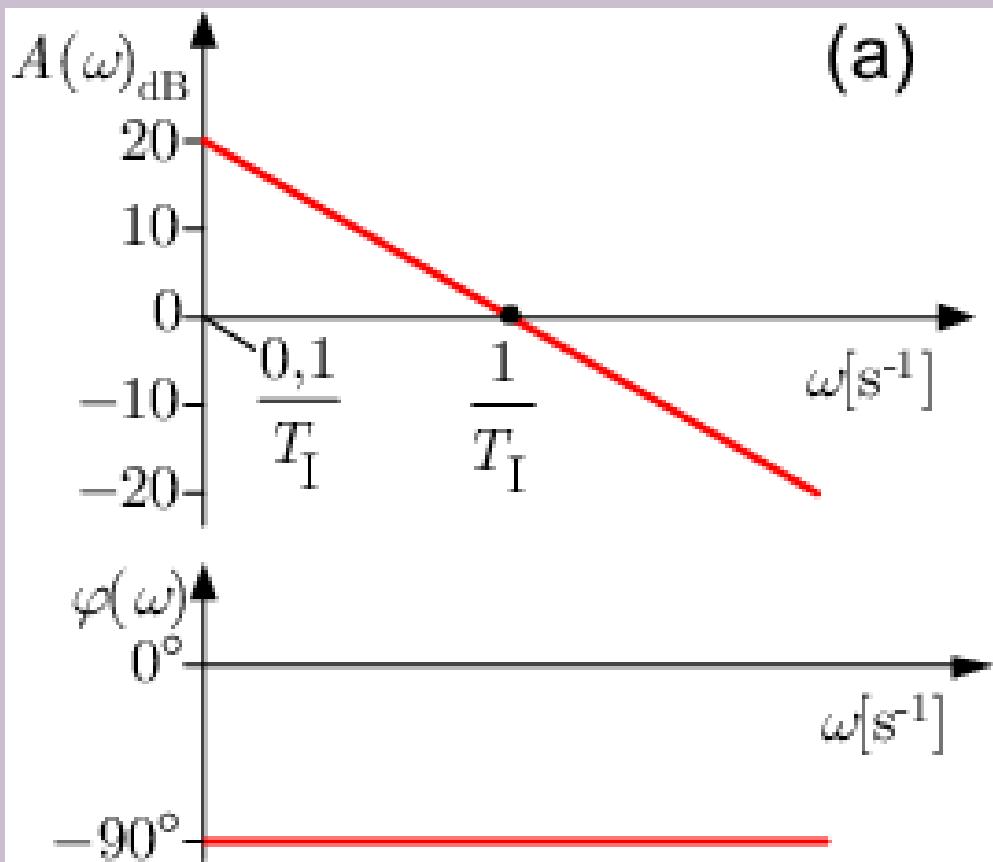


$$H = \frac{1}{j\omega} = \frac{-j}{\omega}$$

$$|H| = \frac{1}{\omega}$$



Zuivere integrator



Zuivere differentiator

$$s \frac{\sigma + j\omega}{\sigma - j\omega} \quad j\omega$$

Mathematisch model

$$y(t) = \frac{dx(t)}{dt}$$

S-domein

$$Y(s) = sX(s)$$

$j\omega$ domein

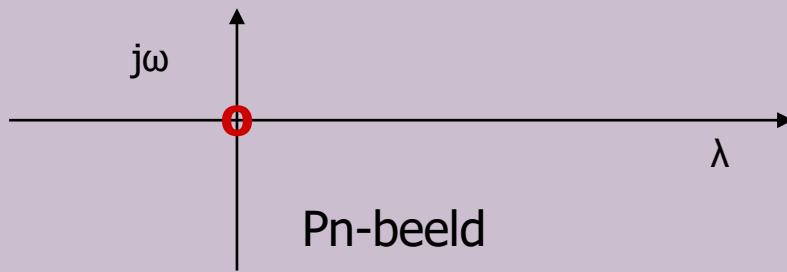
$$Y(j\omega) = j\omega X(j\omega)$$

$$\boxed{H(j\omega) = j\omega}$$

$$|H(j\omega)| = \omega \quad \text{en} \quad \varphi = 90^\circ$$

$$20 \log |H(j\omega)| = 20 \log \omega$$

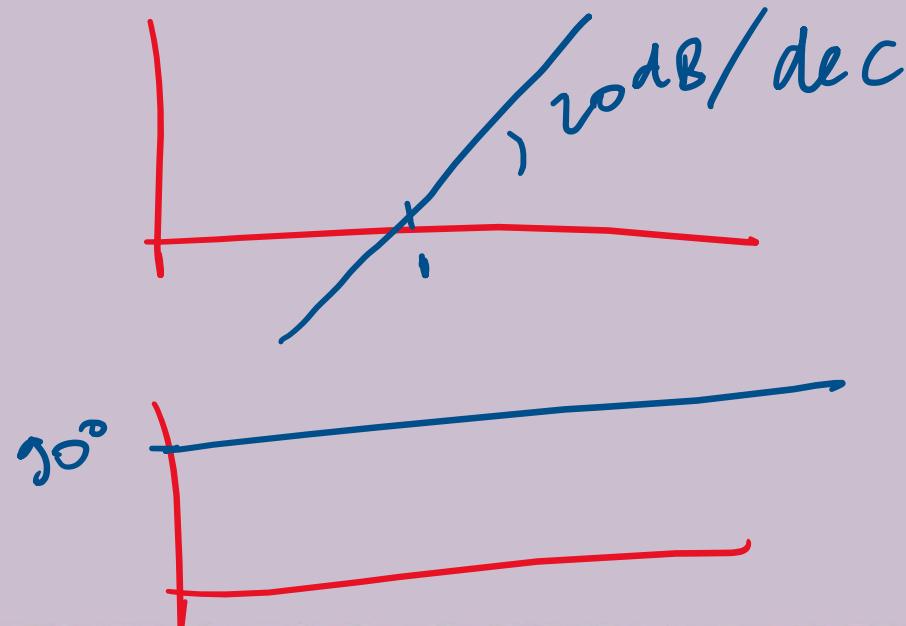
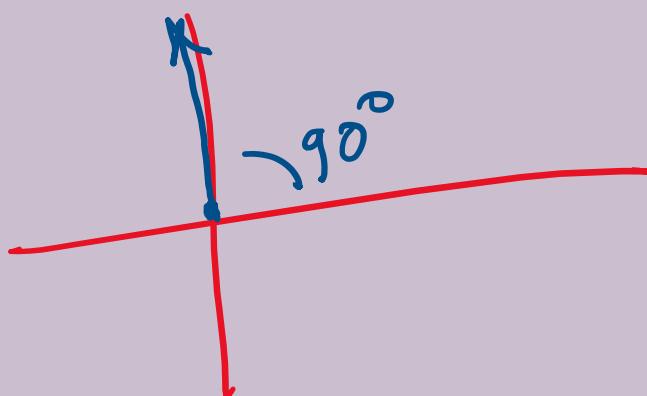
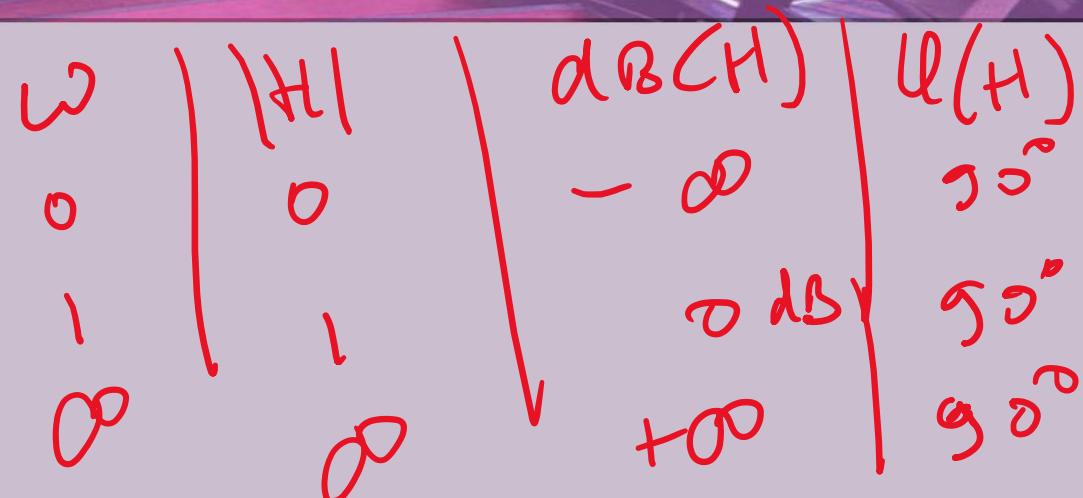
ω	$ H(j\omega) $	$20 \log H(j\omega) $	φ
0	0	$-\infty$	90°
1	1	0	90°
∞	$+\infty$	$+\infty$	90°



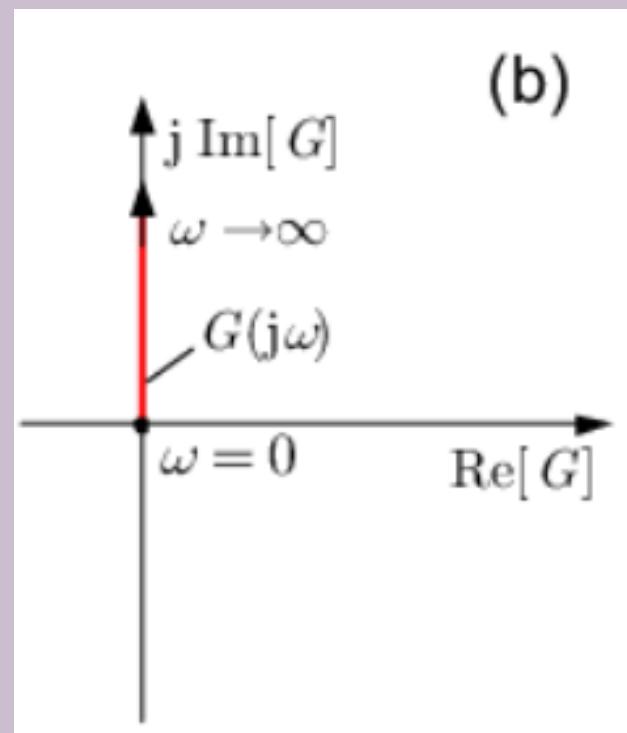
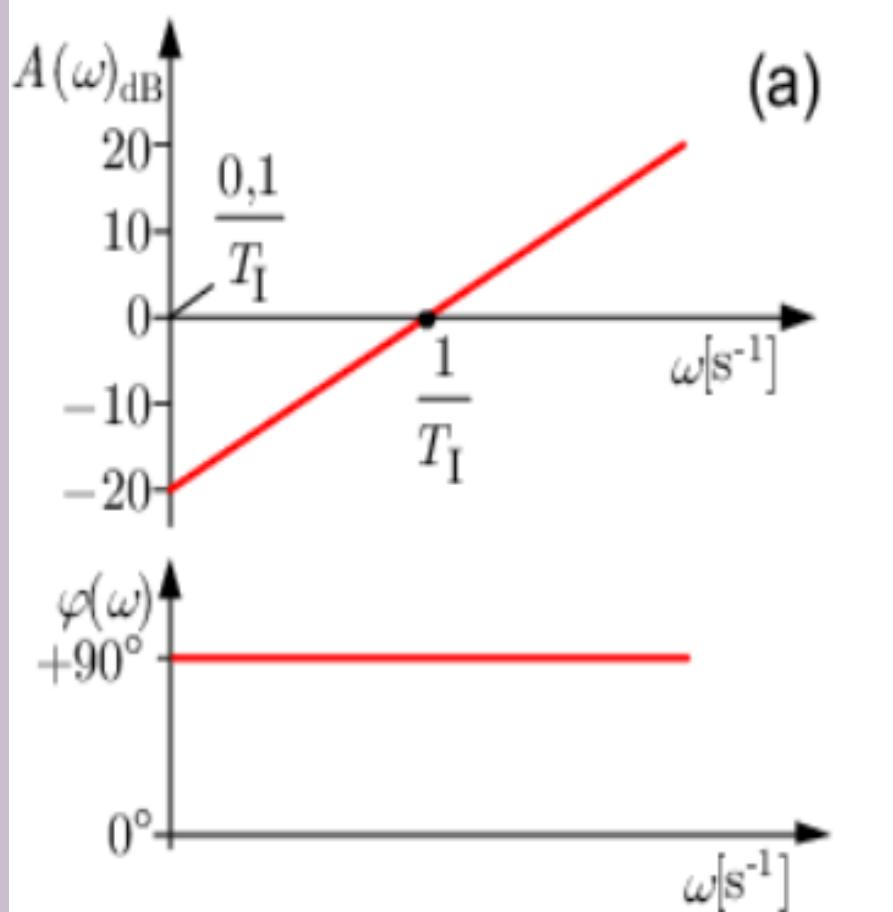
$$H = j\omega$$

$$|H| = \omega$$

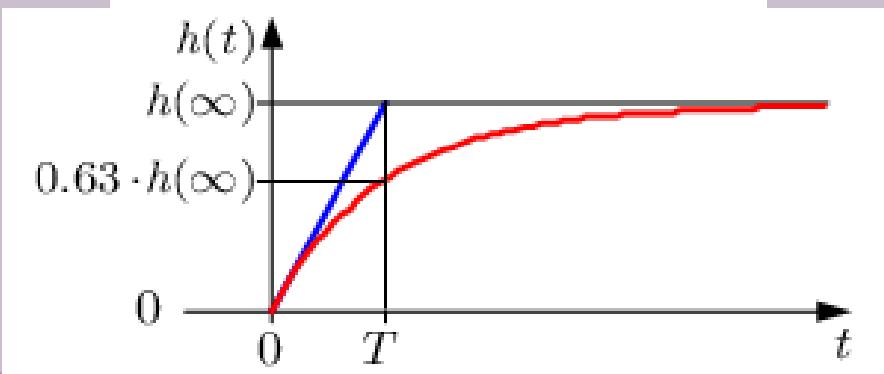
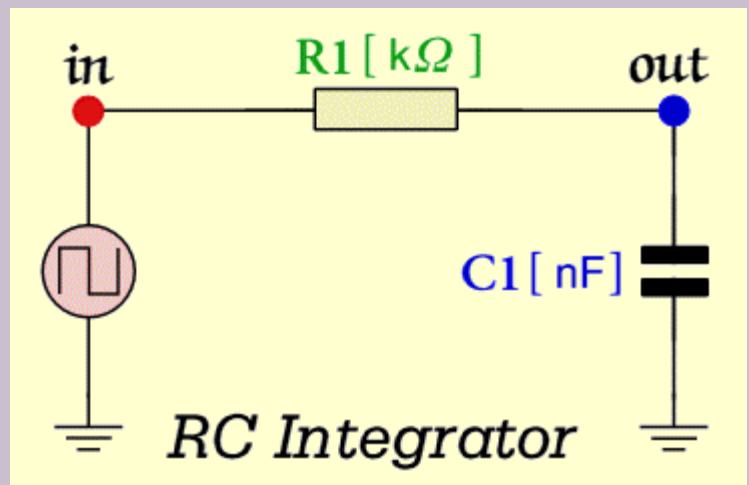
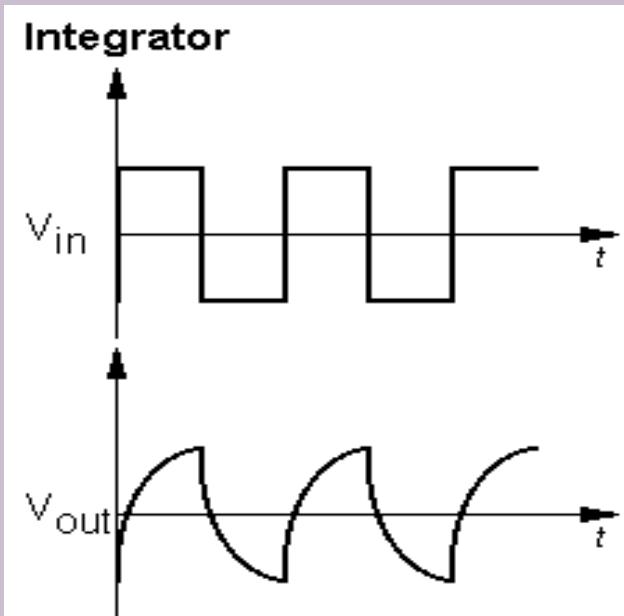
$$\ell(H) = +90^\circ$$



Zuivere differentiator



Onzuivere integrator = eerste-ordesysteem



Onzuivere integrator = eerste-orde systeem

Mathematisch model

$$\tau_i \frac{dy(t)}{dt} + y(t) = x(t)$$

S-domein

$$Y(s) = \frac{X(s)}{\tau_i s + 1}$$

$j\omega$ domein

$$Y(j\omega) = \frac{1}{1 + j\omega\tau_i} X(j\omega) \quad \text{geeft}$$

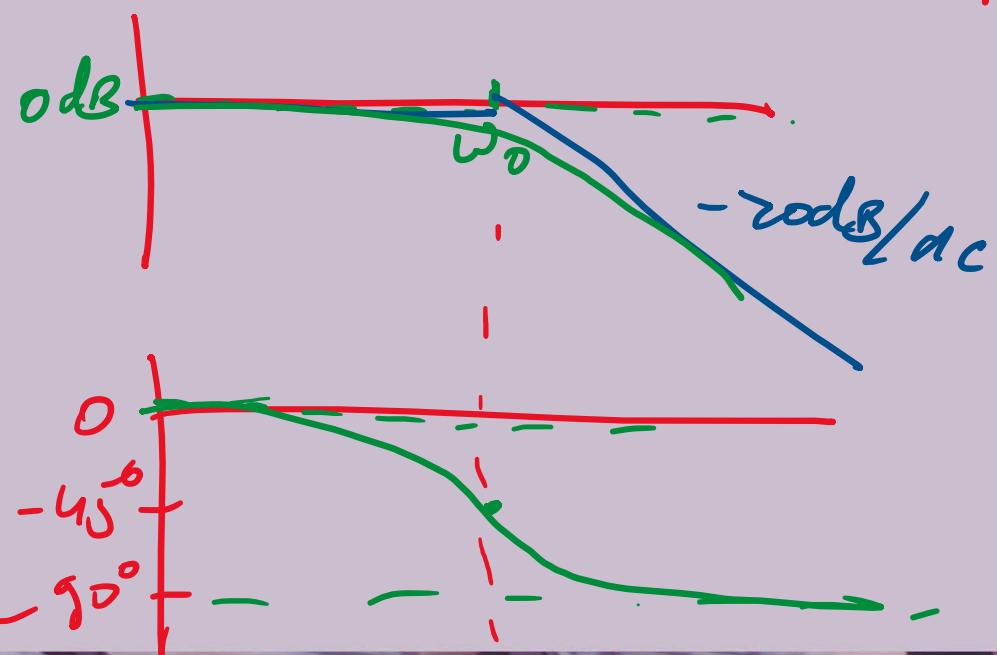
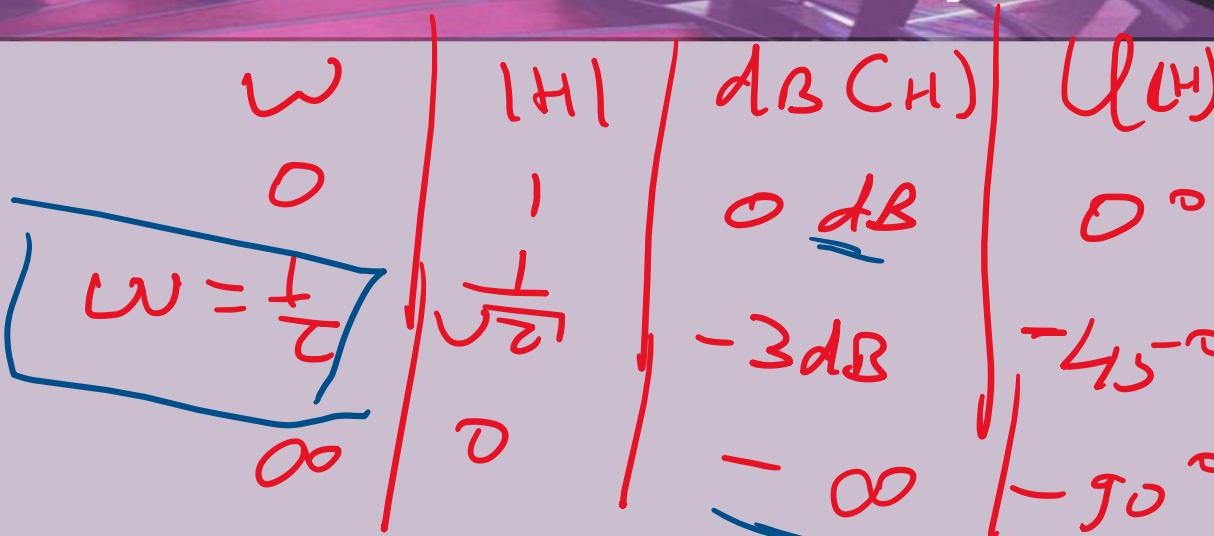
$$20 \log |H(j\omega)| = -\frac{1}{\sqrt{1+(\omega\tau_i)^2}} \quad \text{en} \quad \varphi = 0 - \arctan \omega\tau_i$$

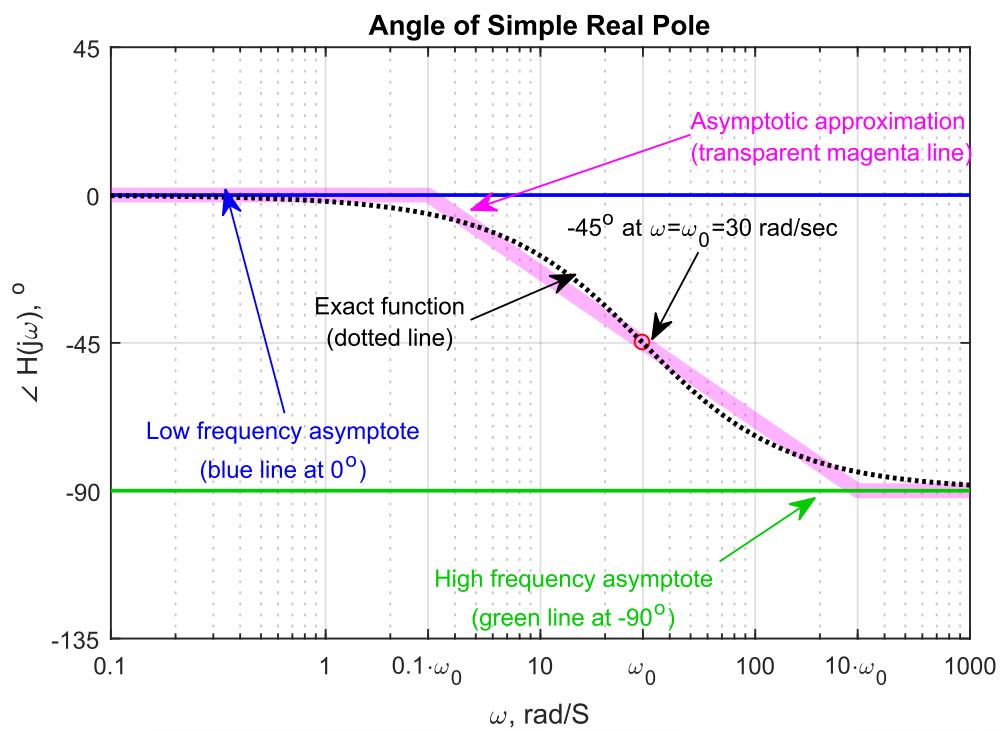
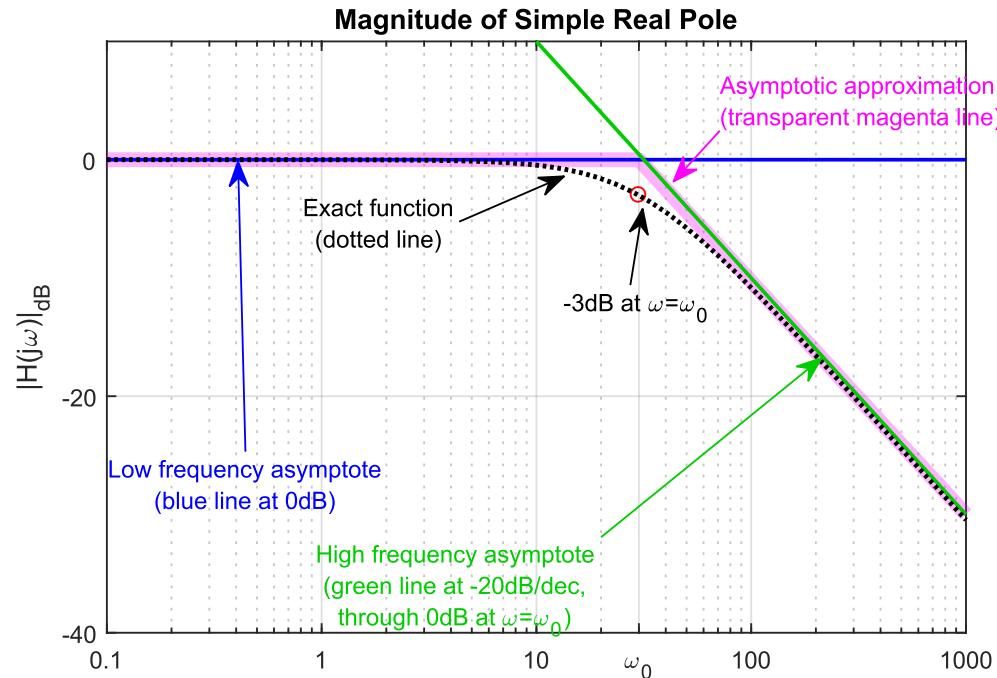
ω	$ H(j\omega) $	$20 \log H(j\omega) $	φ
0	1	0	0°
$1/\tau_i$	$\frac{1}{2}\sqrt{2}$	-3	-45°
∞	0	$-\infty$	-90°

$$H(j\omega) = \frac{1}{1 + j\omega\tau_i} \quad \Rightarrow \quad \tau \equiv RC$$

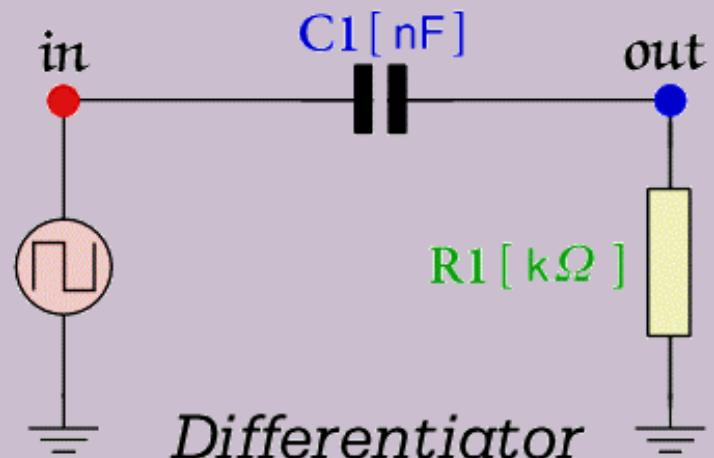
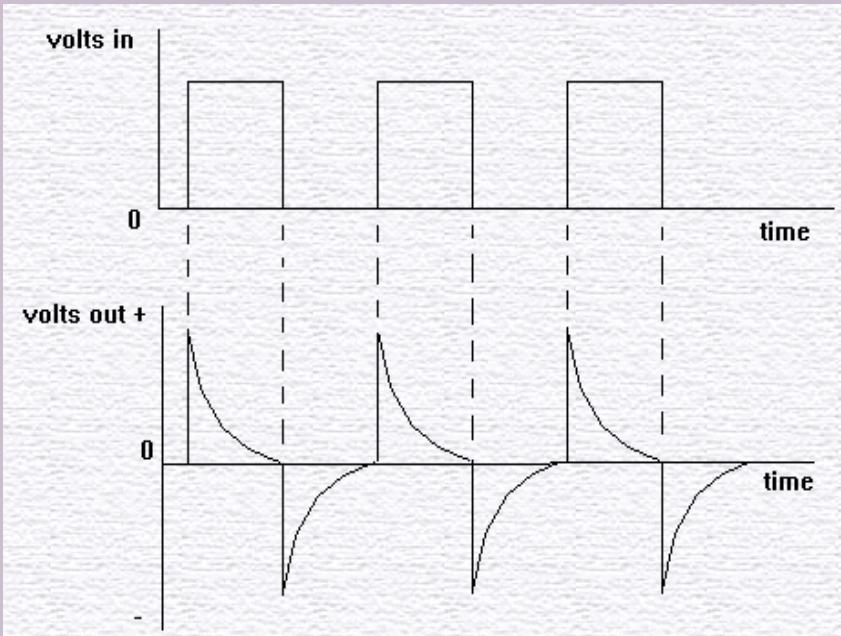
$$H = \frac{1}{j\omega\tau + 1}$$

$$\omega = \omega_0 = \frac{1}{\tau}$$

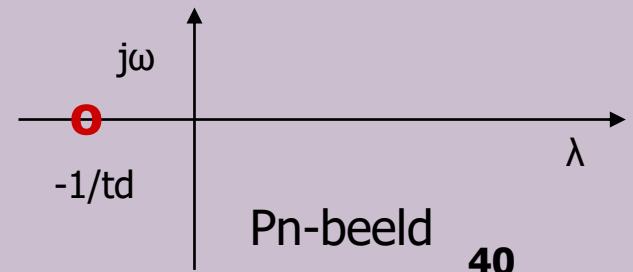
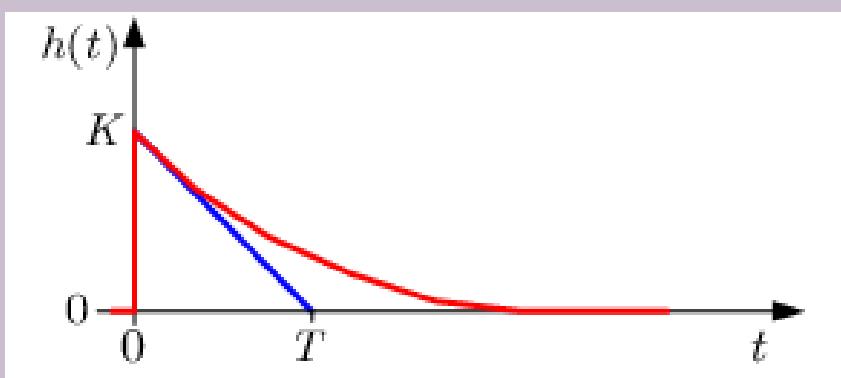




Onzuivere differentiator



Differentiator



Pn-beeld

40

Onzuijvere differentiator

Mathematisch model

$$y(t) = \tau_d \frac{dx(t)}{dt} + x(t)$$

S - domein

$$Y(s) = (s\tau_d + 1)X(s)$$

$j\omega$ domein

$$Y(j\omega) = (j\omega\tau_d + 1)X(j\omega)$$

$$\boxed{H(j\omega) = (j\omega\tau_d + 1)}$$

$$20\log|H(j\omega)| = 20\log\sqrt{(\omega\tau_d)^2 + 1}$$

$$\varphi = \arctan \omega\tau_d$$

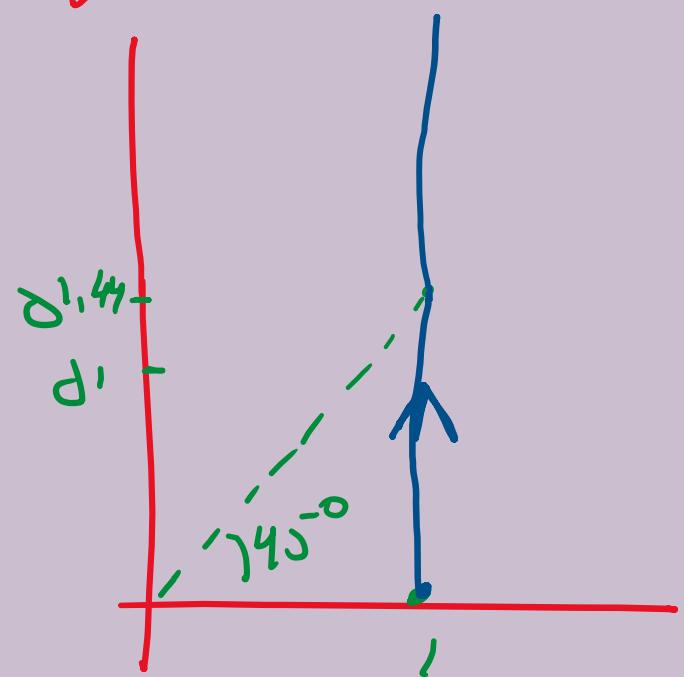
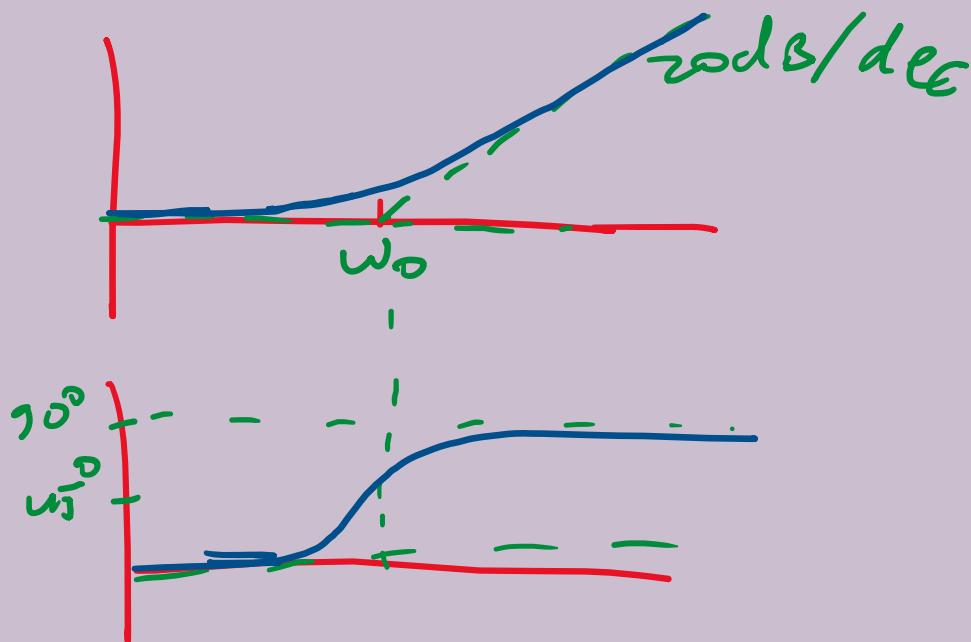
ω	$ H(j\omega) $	$20\log H(j\omega) $	φ
0	1	0	0°
$\frac{1}{\tau_d}$	$\sqrt{2}$	+3	45°
∞	$+\infty$	$+\infty$	90°

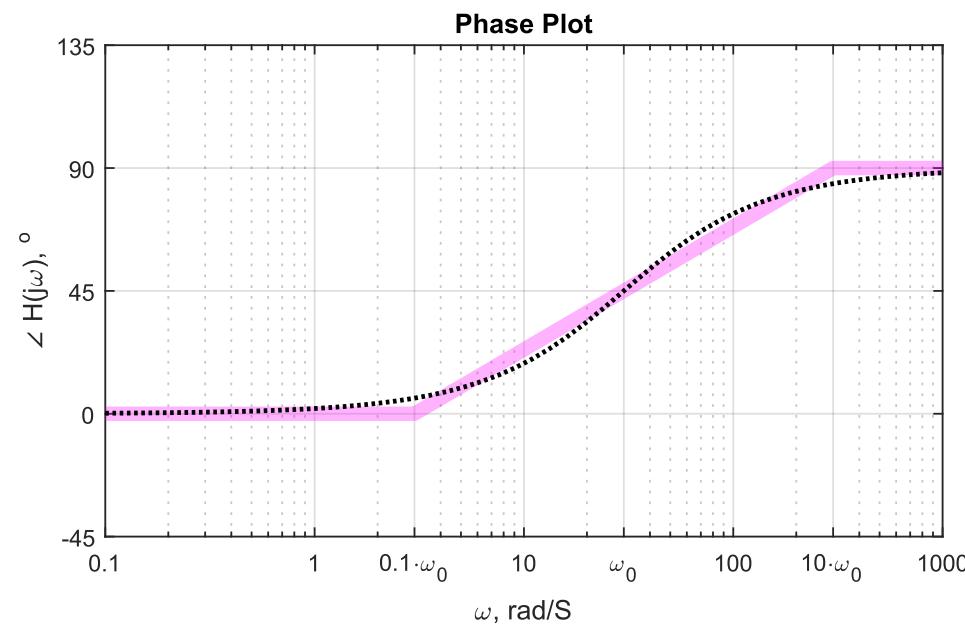
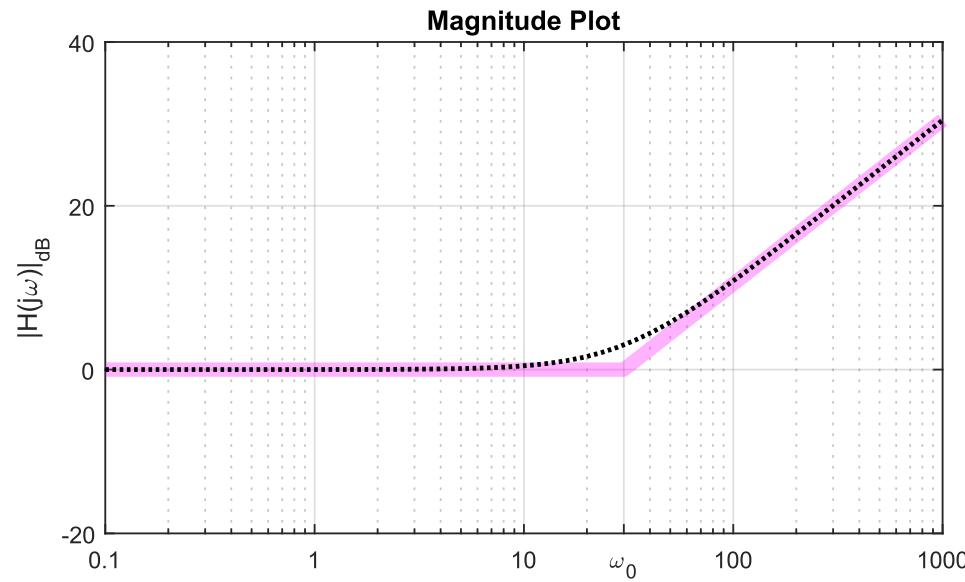
$$H = j\omega \tau + 1$$

$$\omega_0 = \frac{1}{\tau}$$

$$|H| = \sqrt{(\omega \tau)^2 + 1}$$

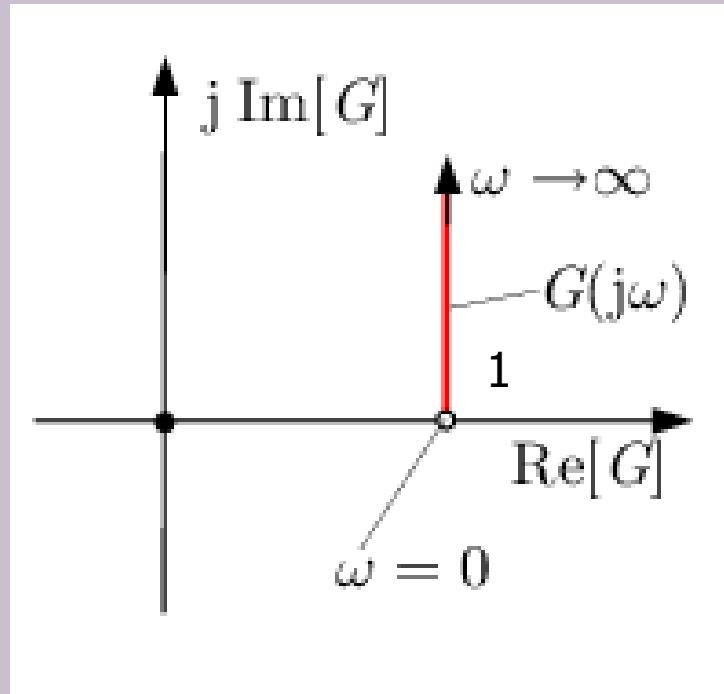
ω	$ H $	$dB(H)$	$\angle(H)$
0	1	0 dB	0°
ω_0	$\sqrt{2}$	+3 dB	45°
∞	∞	∞	90°





Onzuivere differentiator Bode en Nyquist diagram

ω	$ H(j\omega) $	$20 \log H(j\omega) $	φ
0	1	0	0°
$\frac{1}{\tau_d}$	$\sqrt{2}$	+3	45°
∞	$+\infty$	$+\infty$	90°



$$H(j\omega) = (j\omega\tau_d + 1)$$

Bodediagram cascadeschakeling



$$H(j\omega) = H_1(j\omega) \cdot H_2(j\omega) = r_1 e^{j\varphi_1} \cdot r_2 e^{j\varphi_2}$$

$$H(j\omega) = r \cdot e^{j\varphi} = r_1 \cdot r_2 \cdot e^{j(\varphi_1 + \varphi_2)}$$

$$20\log|H(j\omega)| = 20\log|H_1(j\omega)| \cdot |H_2(j\omega)| =$$

$$20\log|H_1(j\omega)| + 20\log|H_2(j\omega)| =$$

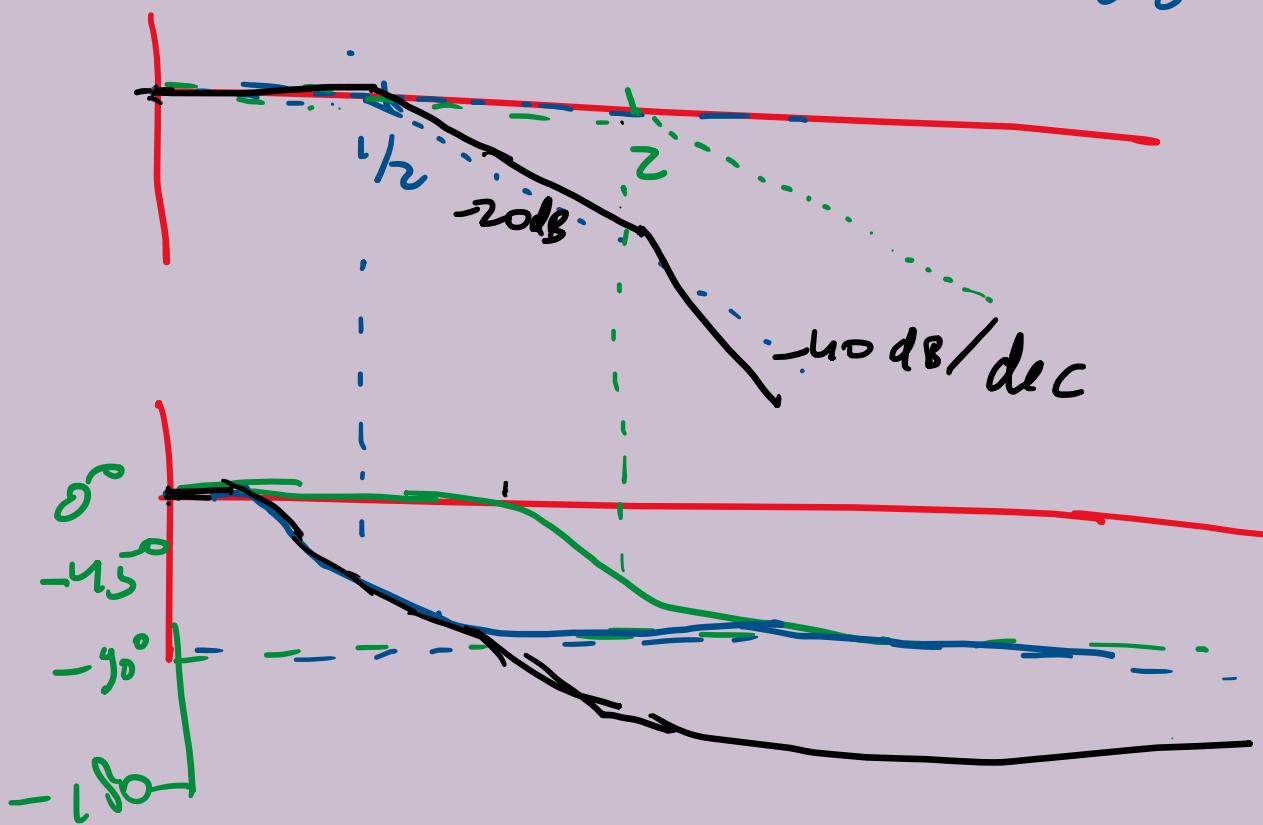
$$\boxed{|H_1(j\omega)|_{dB} + |H_2(j\omega)|_{dB}} \quad (\text{OPTELLEN})$$

en:

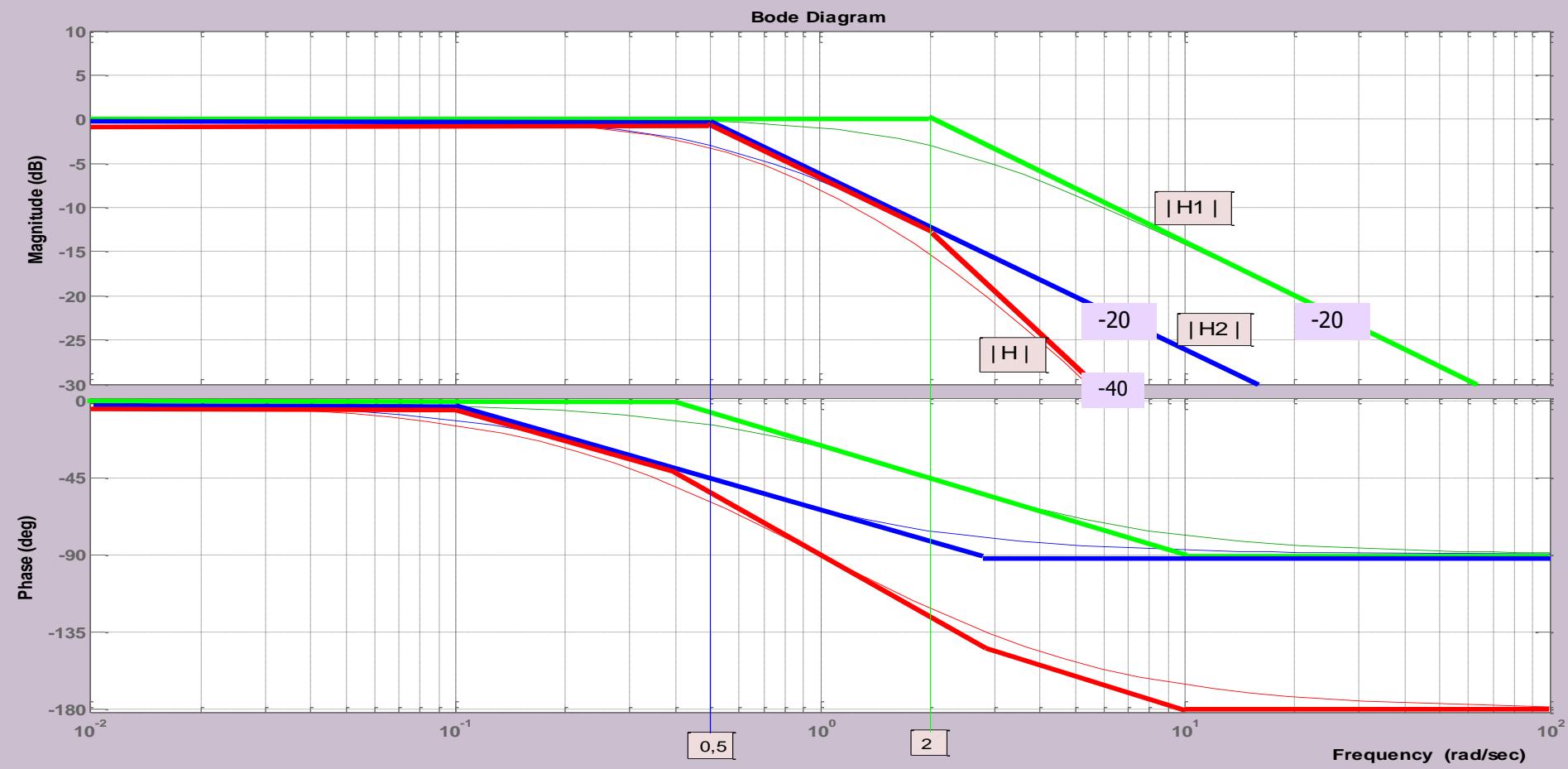
$$\varphi = \arg\{H(j\omega)\} = \arg\{H_1(j\omega) \cdot H_2(j\omega)\} =$$

$$\boxed{\arg\{H_1(j\omega)\} + \arg\{H_2(j\omega)\} = \varphi_1 + \varphi_2 \quad (\text{OPTELLEN})}$$

$$\begin{array}{c}
 \frac{1}{1 + j\omega_0} \\
 H_1 \omega_0 = 2
 \end{array}
 \quad
 \left\{
 \begin{array}{c}
 \frac{1}{1 + j\omega_2} \\
 H_2 \omega_0 = \frac{1}{2}
 \end{array}
 \right.
 \quad
 \begin{array}{l}
 H_1 + H_2 \\
 H_3 = \dots
 \end{array}$$



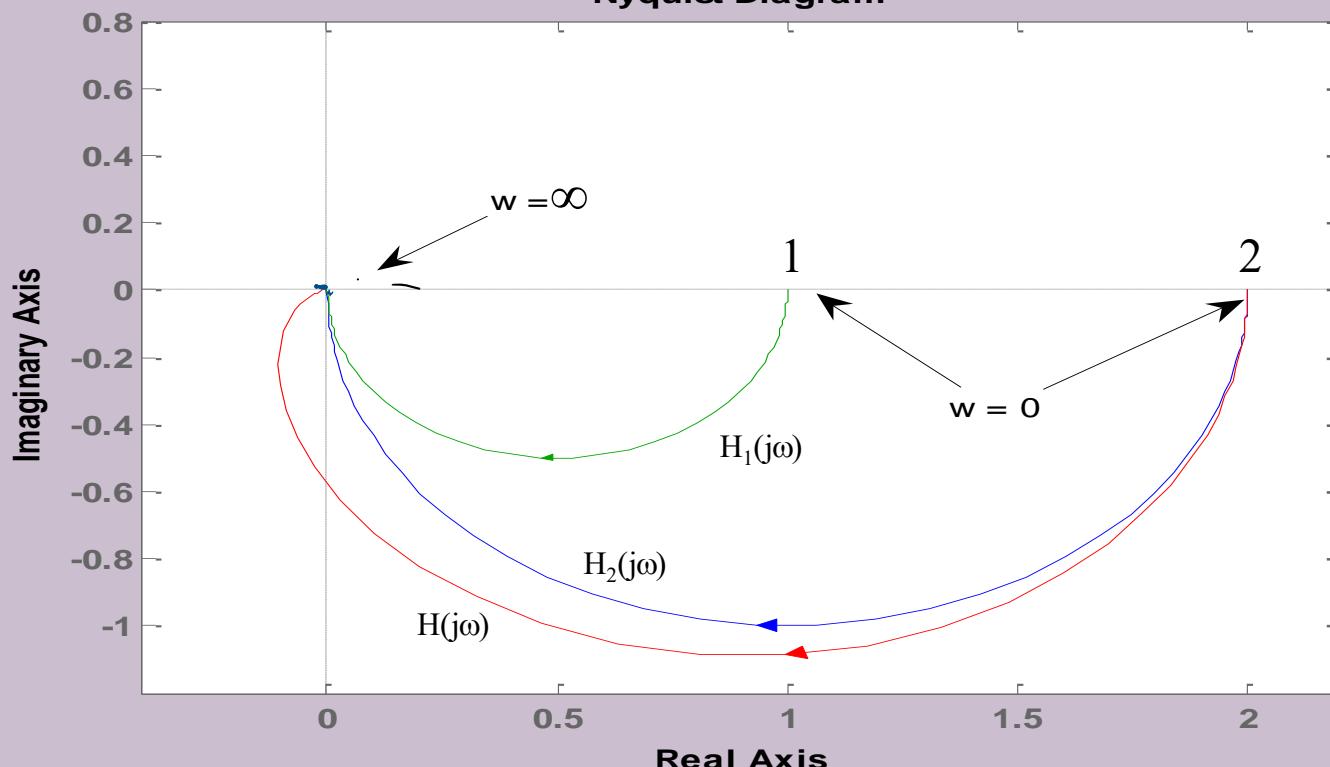
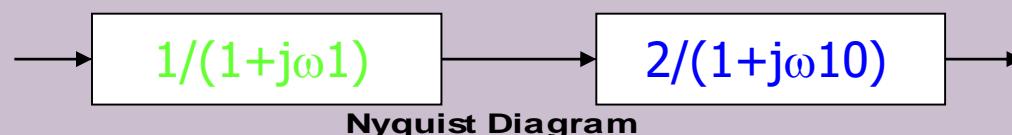
Bodediagram cascadeschakeling



Polaire figuur cascadeschakeling

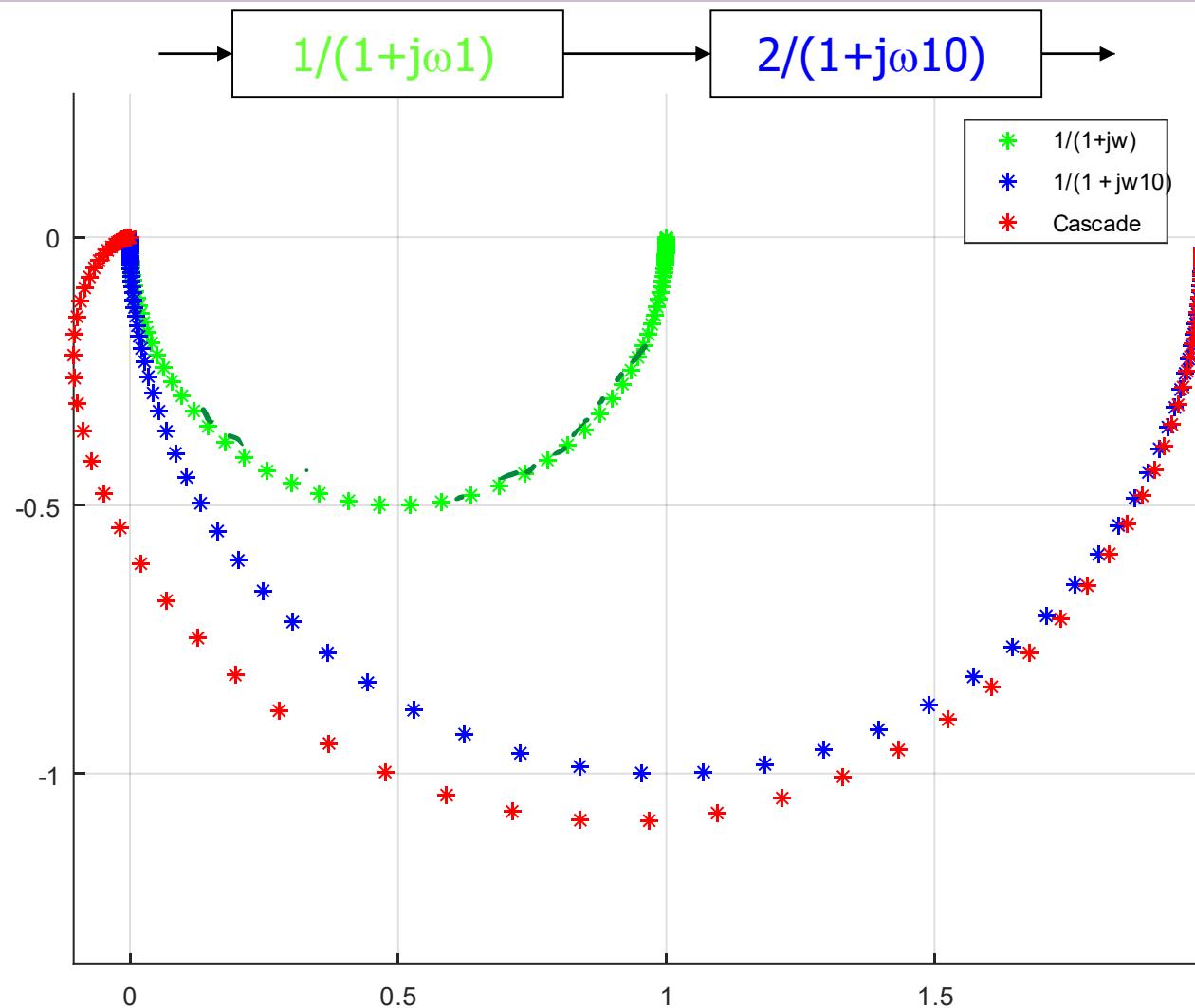
$$H_3 = H_1 \cdot H_2 \Rightarrow H_3 = |H_1| |H_2| \angle \varphi(H_1) + \varphi(H_2)$$

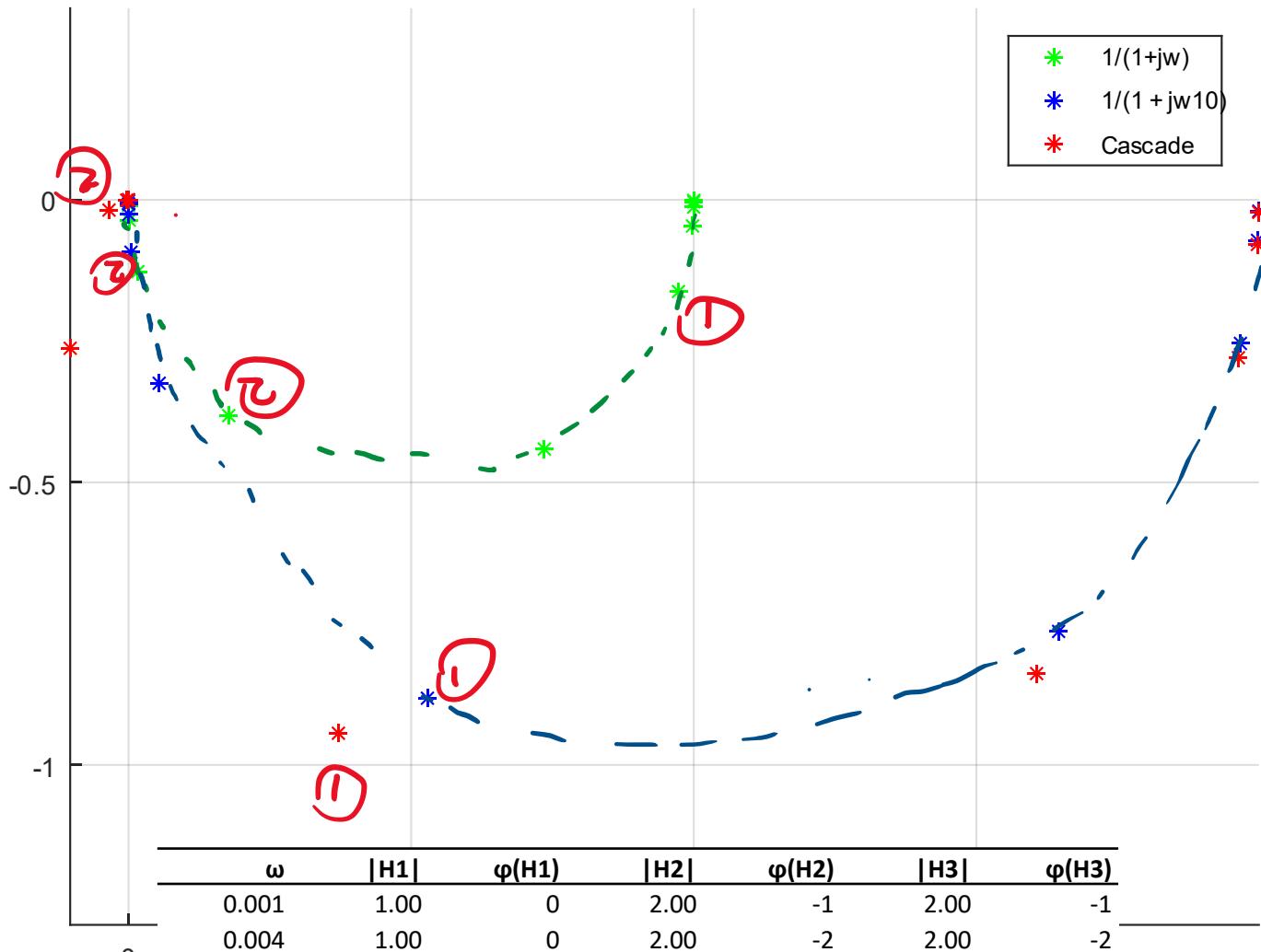
H_1 H_2



met matlab H_2 : nyquist(2,[10 1])

Polaire figuur cascadeschakeling





Oefenopgave 1

Teken de benaderde Bodediagrammen van
 $H_1(j\omega)$, $H_2(j\omega)$ en $H(j\omega) = H_1(j\omega) \cdot H_2(j\omega)$

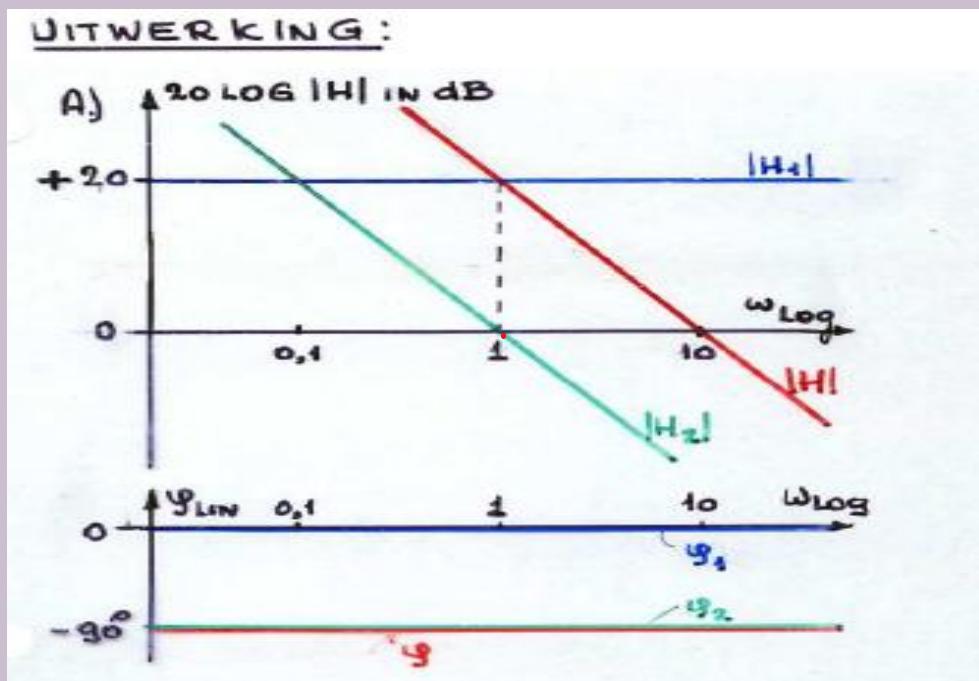
A) $H_1(j\omega) = 10$ en $H_2(j\omega) = 1/j\omega$

B) $H_1(j\omega) = 1 + j\omega$ en $H_2(j\omega) = -0,5$

A) $H_1(j\omega) = 10$ en $H_2(j\omega) = 1/j\omega$

$$\tilde{\ell} = 0 \quad \ell = -90^\circ$$

Bodediagram

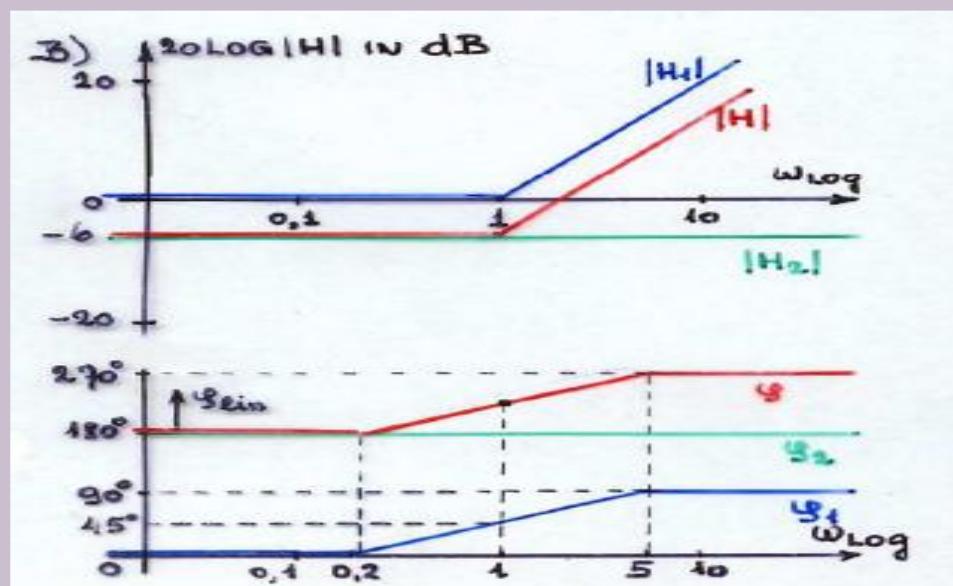


$$20 \log (-10) = -20 \text{ dB}$$

B) $H_1(j\omega) = 1 + j\omega$ en $H_2(j\omega) = -0,5$

-6 dB
 $\varphi(-0,5) = 180^\circ$

Bodediagram



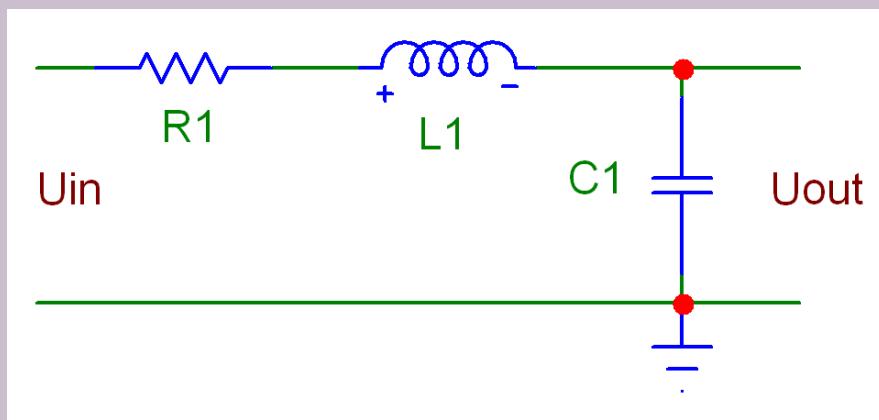
Tweede-orde normaalvorm

(Chapter 3.3)

Hogescholen



Fontys



$$H(s) = \frac{1}{LCs^2 + RCs + 1}$$

normaalvorm

$$H(s) = \frac{\omega_0^2}{s^2 + 2\beta\omega_0 s + \omega_0^2}$$

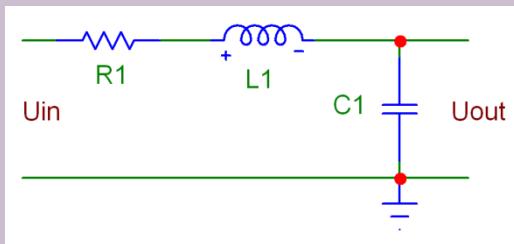
β = relatieve dempingsfactor

3 situaties: $\beta > 1$ overkritische demping

$\beta = 1$ kritische demping

$\beta < 1$ onder kritische demping

Tweede-orde normaalvorm



$$H(s) = \frac{\omega_0^2}{s^2 + 2\beta\omega_0 s + \omega_0^2}$$

β Relatieve dempingsfactor

ω_0 Ongedempte eigenfrequentie (ongedempte natuurlijke frequentie).

$$\omega_g = \omega_0 \sqrt{1 - \beta^2}$$

Gedempte eigenfrequentie;
zichtbaar in stapresponsie

$$\omega_r = \omega_0 \sqrt{1 - 2\beta^2}$$

Resonantie frequentie = frequentie waarbij de opslingering optreedt;
zichtbaar in het bodediagram

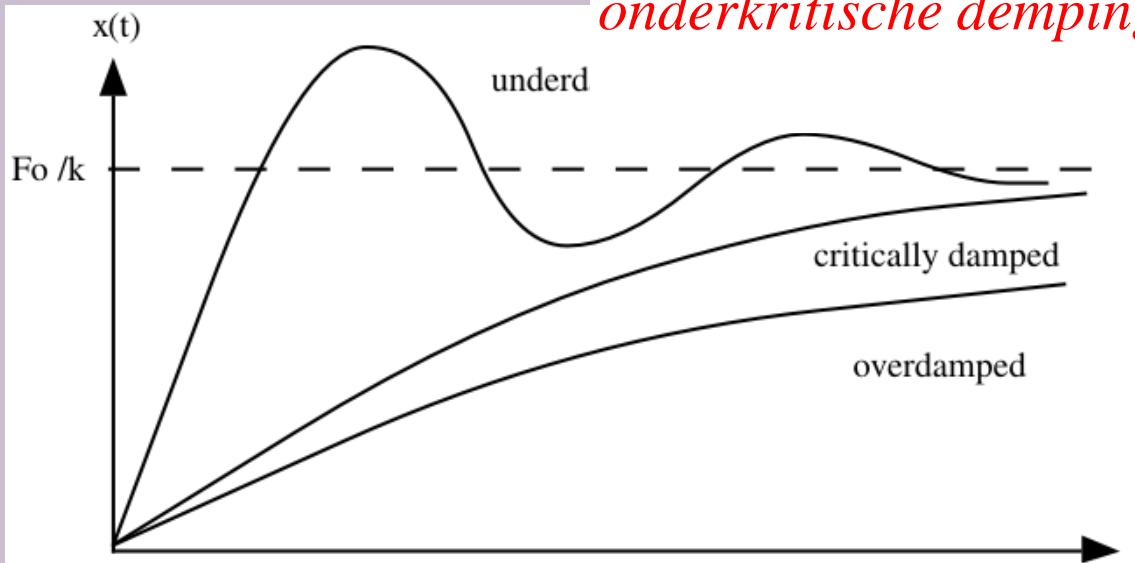
Tweede orde en demping

karakteristieke vergelijking $\rightarrow s^2 + 2\beta\omega_0 s + \omega_0^2 = 0$

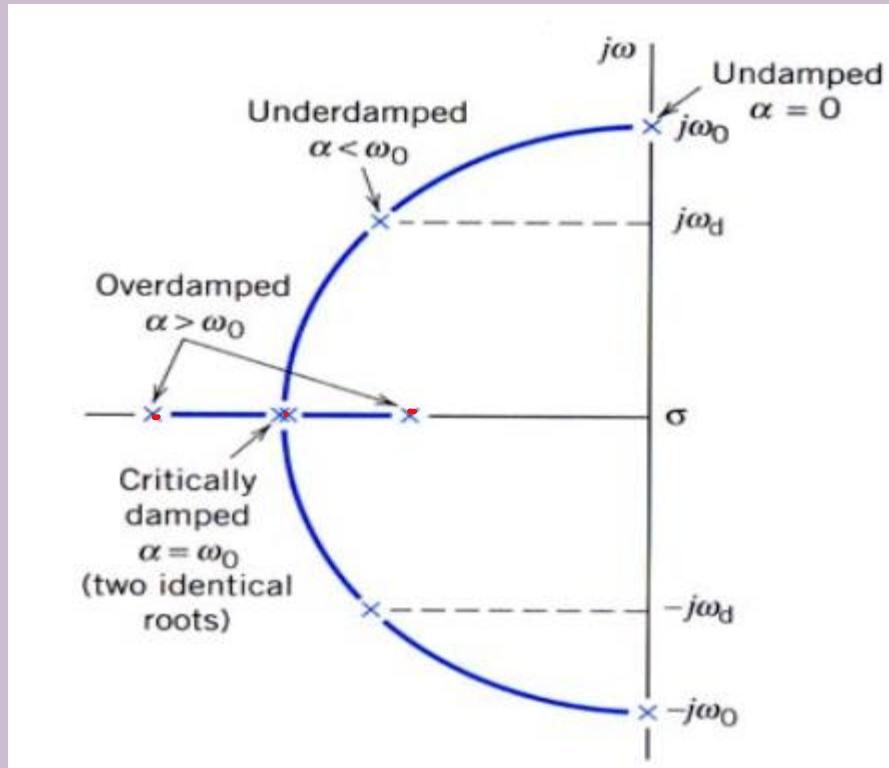
$D > 0 \rightarrow$ 2 separate 1^e orde systemen;
overkritische demping

$D = 0 \rightarrow$ 2 samenvallende 1^e orde systemen;
kritische demping

$D < 0 \rightarrow$ 2 complexe oplossingen dus opslingerig;
onderkritische demping



Tweede orde en demping



$$H(s) = \frac{\omega_0^2}{s^2 + 2\beta\omega_0 s + \omega_0^2}$$

$\beta > 1$; overkritische demping

=> 2 reele ongelijke polen

$\beta = 1$; kritische demping

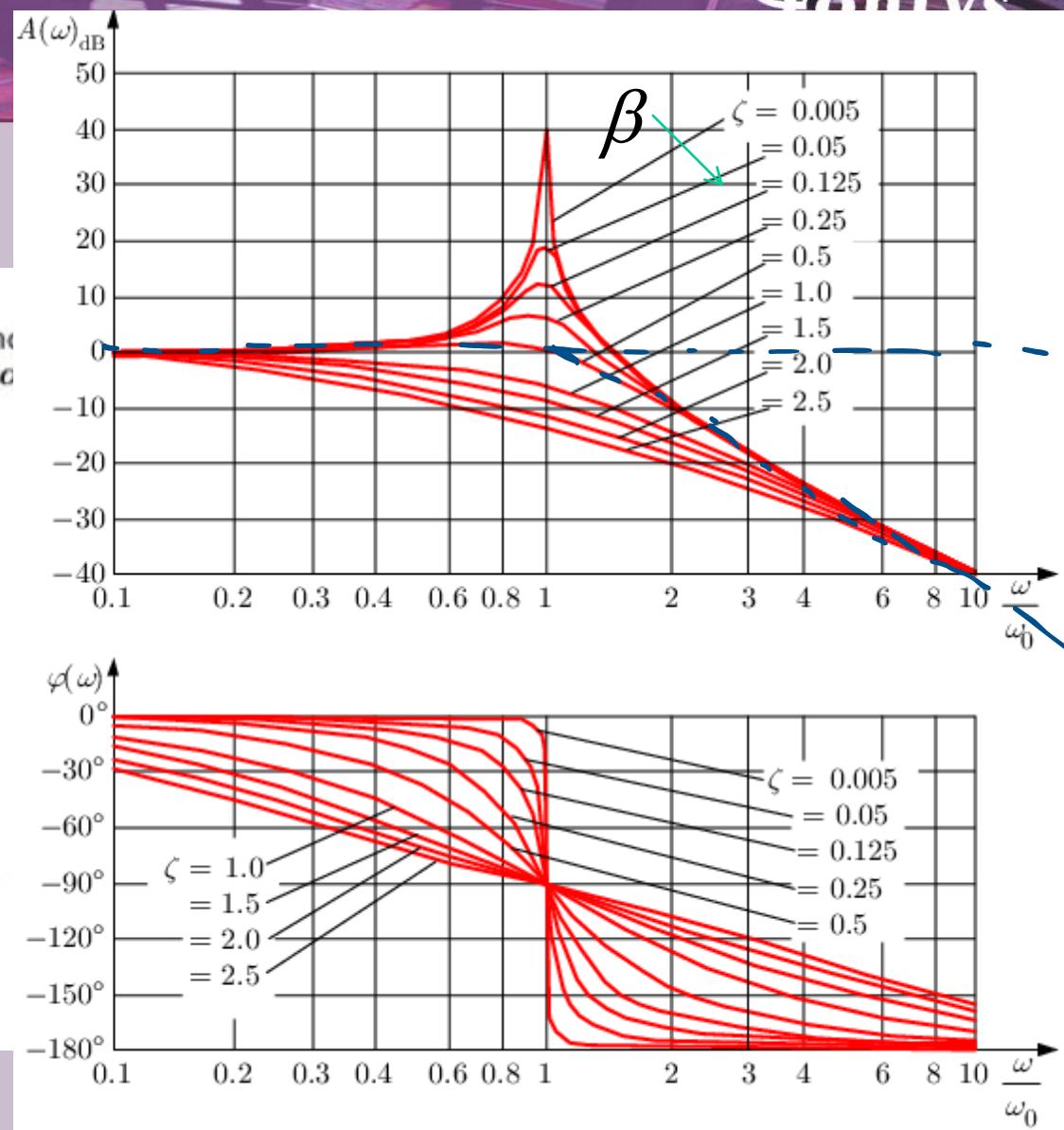
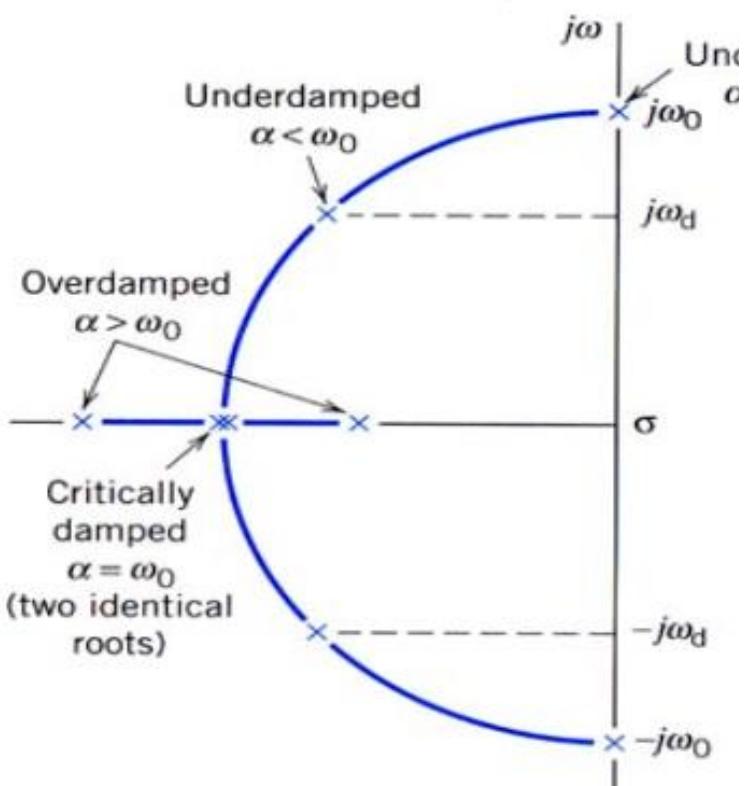
=> 2 reele gelijke polen

$\beta = 0$; 2 zuiver complexe polen;
=> ongedempte hoekfrequentie
ofwel oscillatie

$0 < \beta < 1$; onder kritische demping

=> 2 toegevoegd complexe polen

Tweede orde pn-beeld en Bode



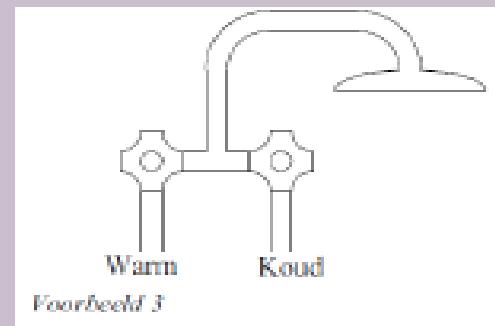
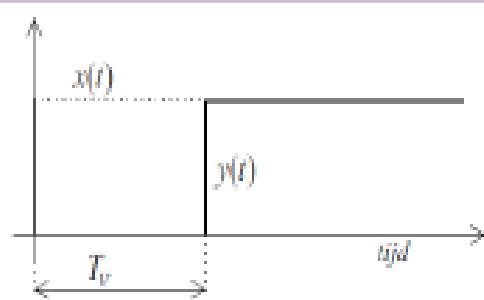
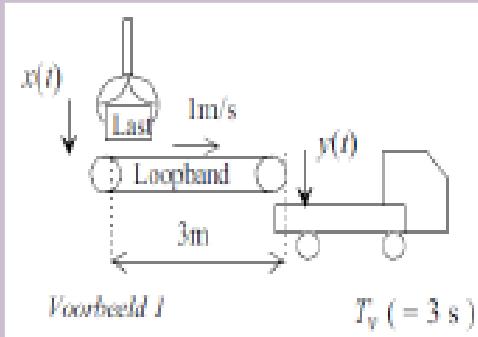
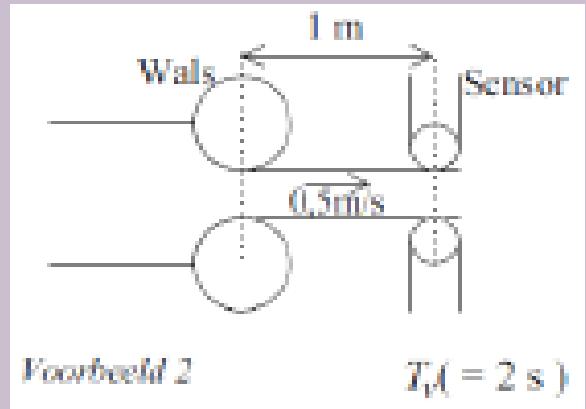
Looptijd

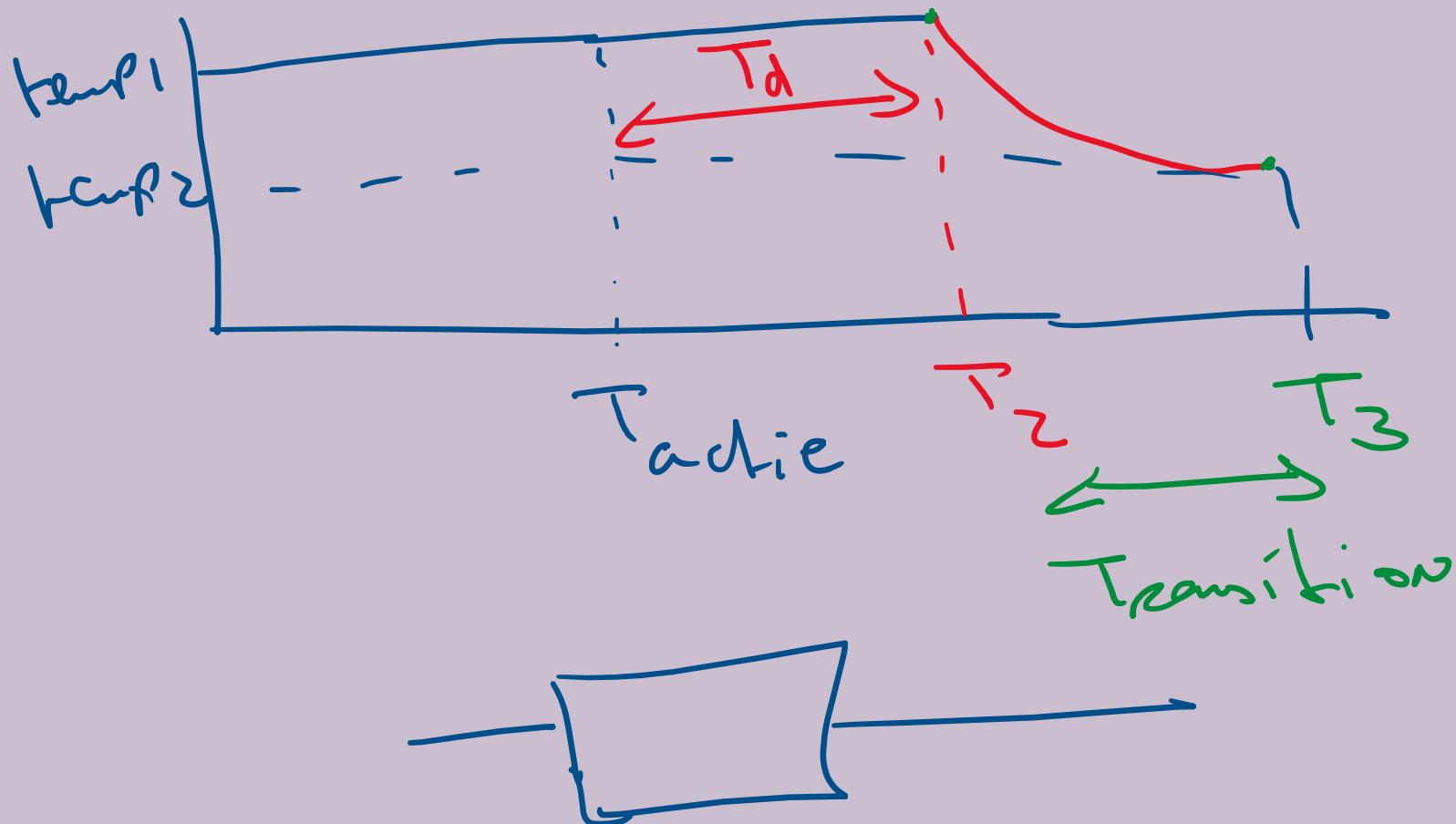
Mathematisch model: $y(t) = x(t - T_v)$

s-domein: $Y(s) = e^{-sT_v} \cdot X(s)$

$j\omega$ -domein: $Y(j\omega) = e^{-j\omega T_v} \cdot X(j\omega)$

$$H(j\omega) = e^{-j\omega T_v} \quad |H(j\omega)| = 1 \quad (0 \text{ dB}) \quad \varphi = -\omega T_v$$





Looptijd



Looptijd

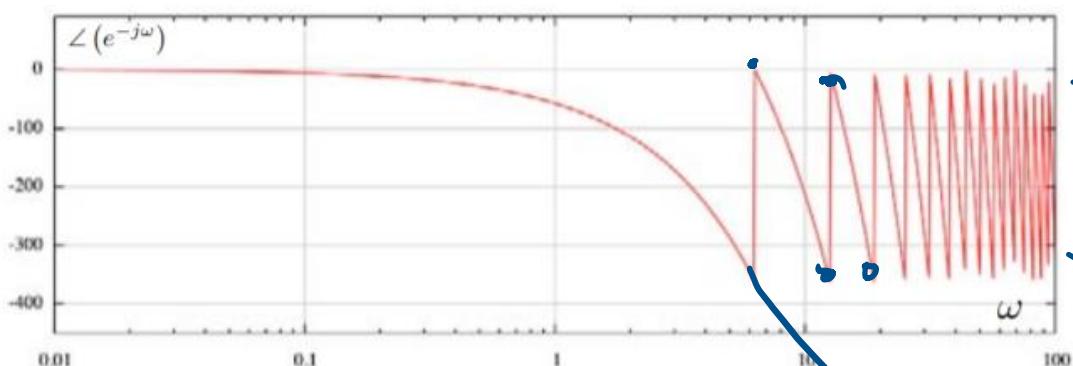
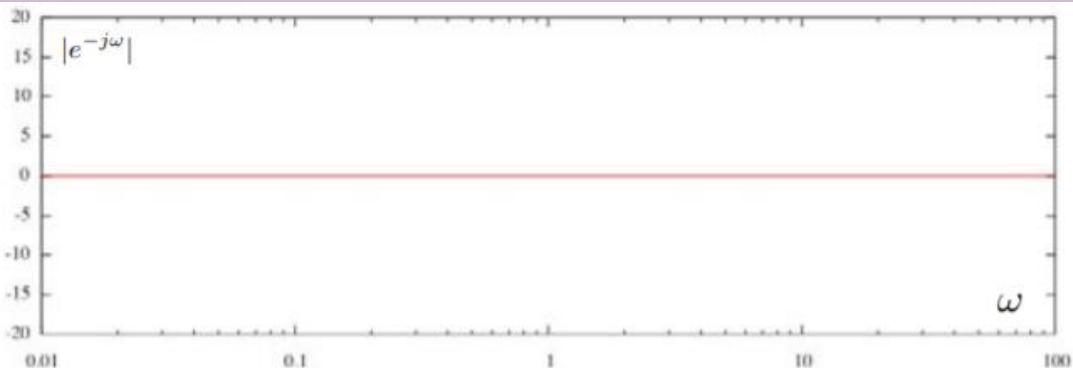
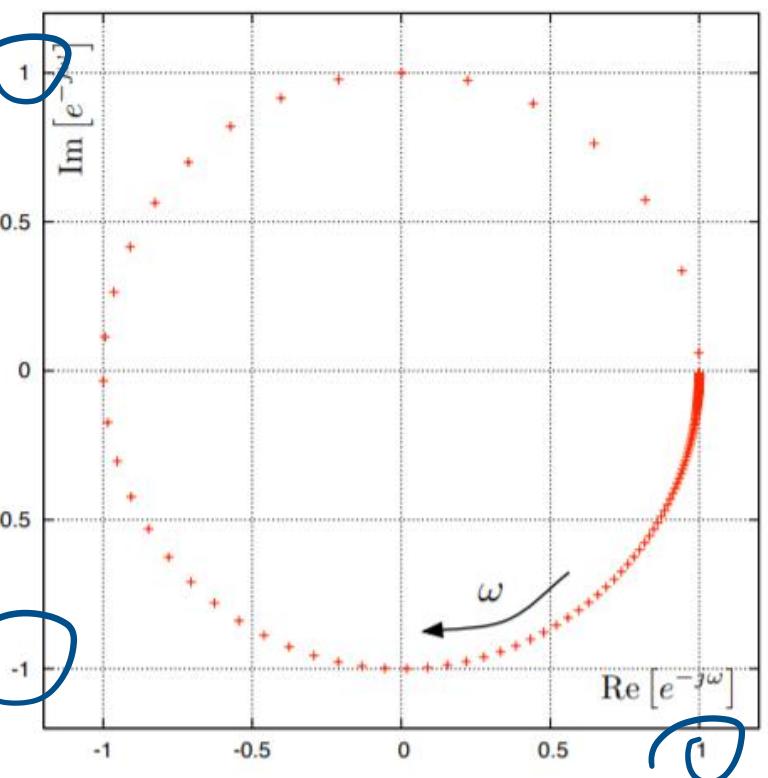
$$\text{Delay}_T(s) = e^{-sT}$$

$$\zeta = \sigma + j\omega$$

$$\sigma = 0$$

$$e^{-j\omega T}$$

$$|e^{j\omega T}| = 1, \quad \angle(e^{-j\omega T}) = -\omega T.$$



Looptijd

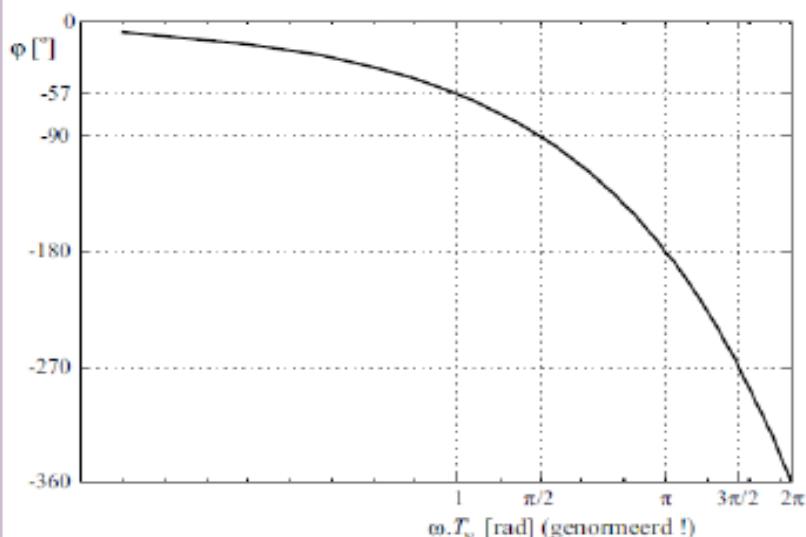
j ω -domein:

$$Y(j\omega) = e^{-j\omega T_v} \cdot X(j\omega)$$

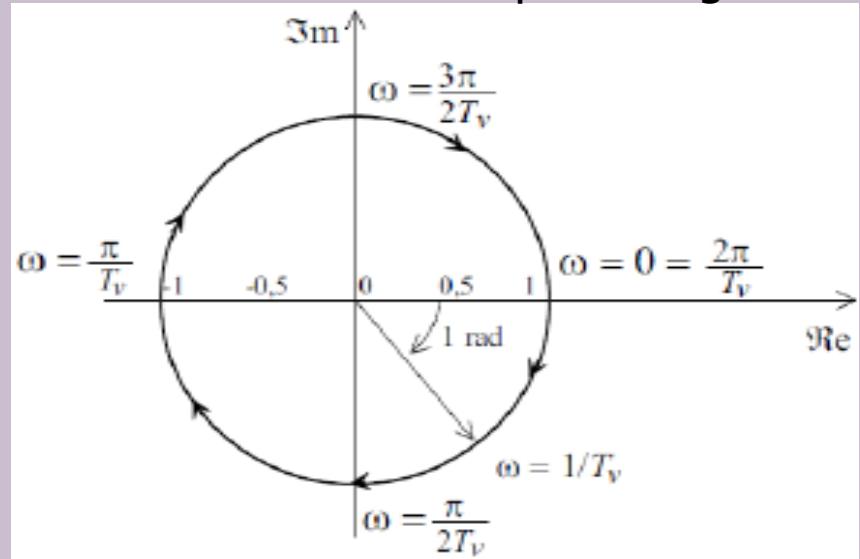
$$H(j\omega) = e^{-j\omega T_v}$$

$$|H(j\omega)| = 1 \quad \phi = -\omega T_v$$

Bode fasediagram



polaire figuur



ω	$\phi = -\omega T_v$
0	0
$1/T_v$	1
$0,5\pi T_v$	$\pi/2$
π/T_v	π
$1,5\pi T_v$	$3\pi/2$
$2\pi/T_v$	2π

Zelfstudiemateriaal/ Achtergrondinformatie

- Vanaf deze slide.

Complexe getallen

- Modulus (amplitude)
- Argument (fase)

$$20 \cdot \log |H(j\omega)| =$$

$$= 20 \cdot \log \sqrt{\operatorname{Re}\{H(j\omega)\}^2 + \operatorname{Im}\{H(j\omega)\}^2}$$

$$\varphi = \operatorname{Arg}\{H(j\omega)\} = \arctan \frac{\operatorname{Im}\{H(j\omega)\}}{\operatorname{Re}\{H(j\omega)\}}$$

Amplitude en Fasedraaiing

$$H(j\omega) = \frac{1}{j\omega R_1 C_1 + 1}$$

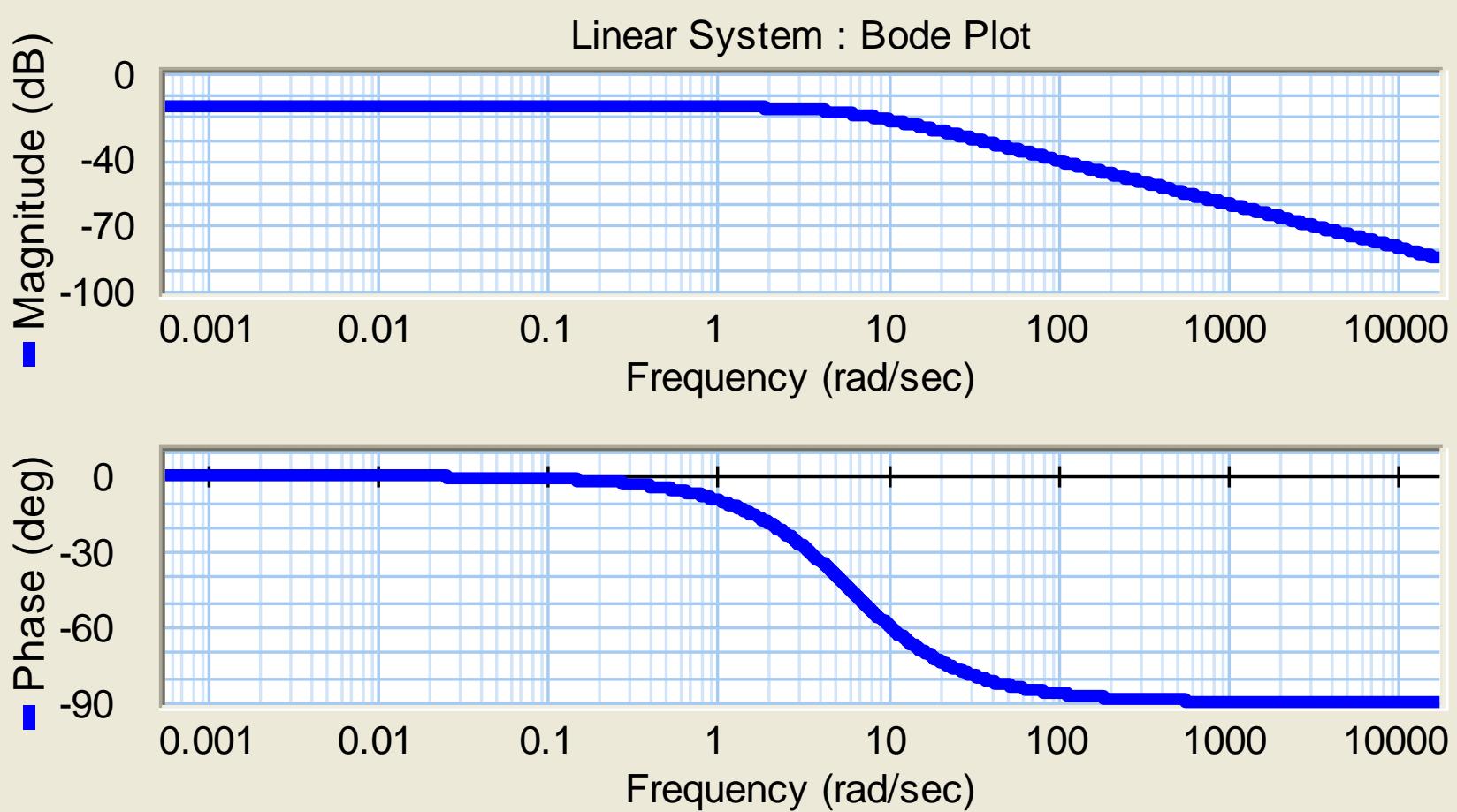
$$|H(j\omega)| = \frac{1}{\sqrt{\operatorname{Re}\{H(j\omega)\}^2 + \operatorname{Im}\{H(j\omega)\}^2}} = \frac{1}{\sqrt{(\omega R_1 C_1)^2 + 1^2}}$$

$$\varphi = \operatorname{Arg}\{H(j\omega)\} = 1 - \arctan \frac{\operatorname{Im}\{H(j\omega)\}}{\operatorname{Re}\{H(j\omega)\}} = 0 - \arctan \omega R_1 C_1$$

Belangrijke punten

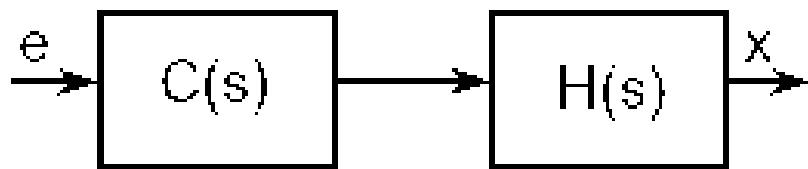
- $\omega \rightarrow 0$
- $\omega \rightarrow \infty$
- $\omega = \text{kantelfrequenties}$

- $Fase = \left(\frac{90}{20}\right) * (\text{helling van de amplitude})$



Term	Magnitude	Phase
Constant: K	$20\log_{10}(K)$	$K>0: 0^\circ$ $K<0: \pm 180^\circ$
Pole at Origin (Integrator) $\frac{1}{s}$	-20 dB/decade passing through 0 dB at $\omega=1$	-90°
Zero at Origin (Differentiator) s	+20 dB/decade passing through 0 dB at $\omega=1$ <i>(Mirror image, around x axis, of Integrator)</i>	+90° <i>(Mirror image, around x axis, of Integrator about)</i>
Real Pole $\frac{1}{\frac{s}{\omega_0} + 1}$	1. Draw low frequency asymptote at 0 dB 2. Draw high frequency asymptote at -20 dB/decade 3. Connect lines at ω_0 .	1. Draw low frequency asymptote at 0° 2. Draw high frequency asymptote at -90° 3. Connect with a straight line from $0.1 \cdot \omega_0$ to $10 \cdot \omega_0$
Real Zero $\frac{s}{\omega_0} + 1$	1. Draw low frequency asymptote at 0 dB 2. Draw high frequency asymptote at +20 dB/decade 3. Connect lines at ω_0 . <i>(Mirror image, around x-axis, of Real Pole)</i>	1. Draw low frequency asymptote at 0° 2. Draw high frequency asymptote at +90° 3. Connect with a straight line from $0.1 \cdot \omega_0$ to $10 \cdot \omega_0$ <i>(Mirror image, around x-axis, of Real Pole about 0°)</i>
Underdamped Poles (Complex conjugate poles) $\frac{1}{\left(\frac{s}{\omega_0}\right)^2 + 2\zeta\left(\frac{s}{\omega_0}\right) + 1}$ $0 < \zeta < 1$	1. Draw low frequency asymptote at 0 dB 2. Draw high frequency asymptote at -40 dB/decade 3. If $\zeta < 0.5$, then draw peak at ω_0 with amplitude $ H(j\omega_0) = -20 \cdot \log_{10}(2\zeta)$, else don't draw peak 4. Connect lines	1. Draw low frequency asymptote at 0° 2. Draw high frequency asymptote at -180° 3. Connect with straight line from $\omega = \frac{\omega_0}{10^\zeta} \text{ to } \omega_0 \cdot 10^\zeta$ <i>You can also look in a textbook for examples</i>
Underdamped Zeros (Complex conjugate zeros) $\left(\frac{s}{\omega_0}\right)^2 + 2\zeta\left(\frac{s}{\omega_0}\right) + 1$ $0 < \zeta < 1$	1. Draw low frequency asymptote at 0 dB 2. Draw high frequency asymptote at +40 dB/decade 3. If $\zeta < 0.5$, then draw peak at ω_0 with amplitude $ H(j\omega_0) = +20 \cdot \log_{10}(2\zeta)$, else don't draw peak 4. Connect lines <i>(Mirror image, around x-axis, of Underdamped Pole)</i>	1. Draw low frequency asymptote at 0° 2. Draw high frequency asymptote at +180° 3. Connect with straight line from $\omega = \frac{\omega_0}{10^\zeta} \text{ to } \omega_0 \cdot 10^\zeta$ <i>You can also look in a textbook for examples.</i> <i>(Mirror image, around x-axis, of Underdamped Pole)</i>

The amplitude story



$$C = a + bj$$

$$\text{Re}(C) = a$$

$$\text{Im}(C) = b$$

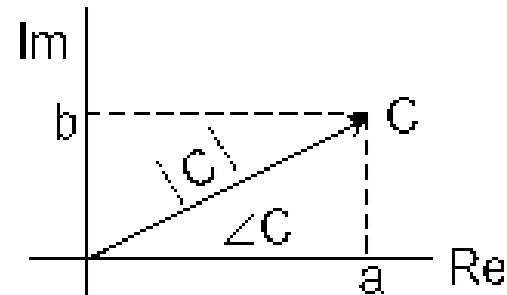
$$|C| = \sqrt{a^2 + b^2}$$

$$H = c + dj$$

$$\text{Re}(H) = c$$

$$\text{Im}(H) = d$$

$$|H| = \sqrt{c^2 + d^2}$$



$$CH = ac + bdj + adj - bd$$

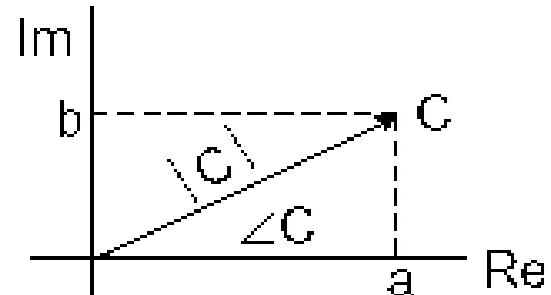
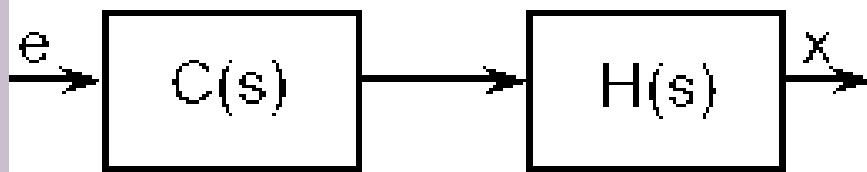
$$\text{Re}(CH) = ac - bd$$

$$\text{Im}(CH) = bc + ad$$

$$|CH| = \sqrt{(a^2 + b^2)(c^2 + d^2)}$$

$$|C \cdot H| = |C| \cdot |H|$$

The phase story



$$C = |C| \cdot [\cos(\varphi) + j \sin(\varphi)] \quad H = |H| \cdot [\cos(\psi) + j \sin(\psi)]$$

$$\begin{aligned} C \cdot H &= |C| \cdot |H| \cdot [\cos(\varphi) + j \sin(\varphi)] \cdot [\cos(\psi) + j \sin(\psi)] \\ &= |C \cdot H| \cdot [(\cos(\varphi)\cos(\psi) - \sin(\varphi)\sin(\psi)) + \dots] \end{aligned}$$

$$\dots j (\cos(\varphi) \sin(\psi) + \sin(\varphi) \cos(\psi))]$$

$$= |C \cdot H| \cdot [\cos(\varphi + \psi) + j \sin(\varphi + \psi)]$$

$$\left. \begin{array}{l} \varphi = \angle C \\ \psi = \angle H \end{array} \right\} \rightarrow \boxed{\angle(C \cdot H) = \angle C + \angle H}$$

