Control Engineering (CEN4B)

CEN4B v20 planning

Week	*Inhoud*	*Studiemateriaal*	*Huiswerk uit het boek (end of chapter exercises)*
1	**Bode en Nyquistdiagrammen, 1e en 2e orde basissystemen:** Bode en Nyquist tekenregels. Omzetten van een naar de andere. Alle beeldverbanden van 1e 2e order systemen. 'Nyquist Stability' is buiten de scope!	**6.1** Frequency Response **6.3** Nyquist Stability Criterion **4.3.1 t/m 4.3.4** Basissystemen **3.3** Effect of Pole Locations	6.3 – 6.5 6.19 a-b-c
2	**Terugkoppeling en stabiliteit in het s-domein:** Principe van terugkoppeling; stabiliteitsonderzoek in s-domein. Stabiliteitsonderzoek in ω-domein; fasemarge, versterkingsmarge, ook in Nyquist; invloed terugkoppeling op pn-beeld;	**3.6.2** Stability of LTI Systems **6.4** Stability Margins **5.1** Root Locus of a Basic Feedback System	6.24 6.30 6.31
3	**Terugkoppeling en stabiliteit in het ω-domein; poolbanen:** constructieregels poolbanen	**5.2** Guidelines for Determining a Root Locus **5.3** Selected Illustrative Root Loci	5.3 t/m 5.7 5.12 5.13
4	**Ontwerpcriteria voor geregelde systemen:** Inleiding, ontwerpcriteria in het t- en s-domein, settlingtime, doorschot, offset	**3.3** Effect of Pole Locations **3.4** Time Domain Specifications	3.25 3.27 3.34
5	**Ontwerpcriteria voor geregelde systemen vervolg** **Ontwerpen van geregelde systemen:** Ontwerpcriteria in ω-domein, fase- en versterkingsmarge, bandbreedte; kwalitatieve beschouwing van de invloed van P-, I- en D-actie.	**6.6** Closed-Loop Frequency Response **5.4, 5.5** Design Using Dynamic Compensation (except 5.4.3)	6.43
6	**Ontwerpen van geregelde systemen vervolg:** Invloed van regelacties in t-, s- en ω-domein, praktische instelregels	**6.7** Compensation	6.46 6.54
7	**Oefenen:** Oefentoets		

Huiswerk

- Huiswerk via Socrative, niet verplicht.
- De uitwerkingen en de vragen worden niet gedeeld of online gezet, kom dus naar de les.
- Correlatie tussen “Readiness-%” en “Tentamenresultaat” = M4A: +30%
  M4B: +60%

READINESS-PERCENTAGE	tentamenresultaat
43,2	100
11,4	79
32,8	73
11,4	70
23,5	69
0,0	68
0,0	60
17,4	58
14,2	54
23,9	53
0,0	51
10,8	49
11,3	45
0,0	44
44,2	43
14,2	38
18,5	34
5,8	34
0,0	30
5,8	29
13,3	26
11,3	26
0,0	26
26,8	19
23,4	19
11,4	19
5,3	16
8,6	15
14,2	13
22,7	10
7,1	10
0,0	10

READINESS-PERCENTAGE	tentamenresultaat
30,0	89
76,4	81
50,0	81
45,6	81
39,6	75
27,8	75
22,0	69
12,6	69
12,6	69
2,8	69
0,0	65,0
16,6	64
24,0	61
64,4	60
27,6	59
25,2	59
0,0	55
18,6	50
6,6	50
5,8	44
0,0	41
26,8	38
0,0	38,0
0,0	31
0,0	26
13,4	10

VAKBOEK

Het officiële Boek: Feedback Control of Dynamic Systems, Franklin

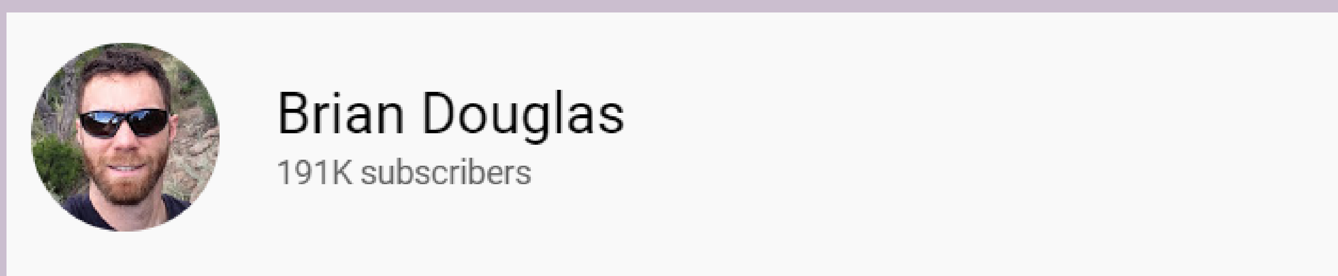
ICE3->**CEN4A->CEN4B-**>***ACE7->OBS7***

'->2^e differentiatie

Geadviseerd Boek: Regeltechniek voor het HBO, 5^e druk

Schrage, Van Daal, Stroeken, Van der Pol en Thomasse

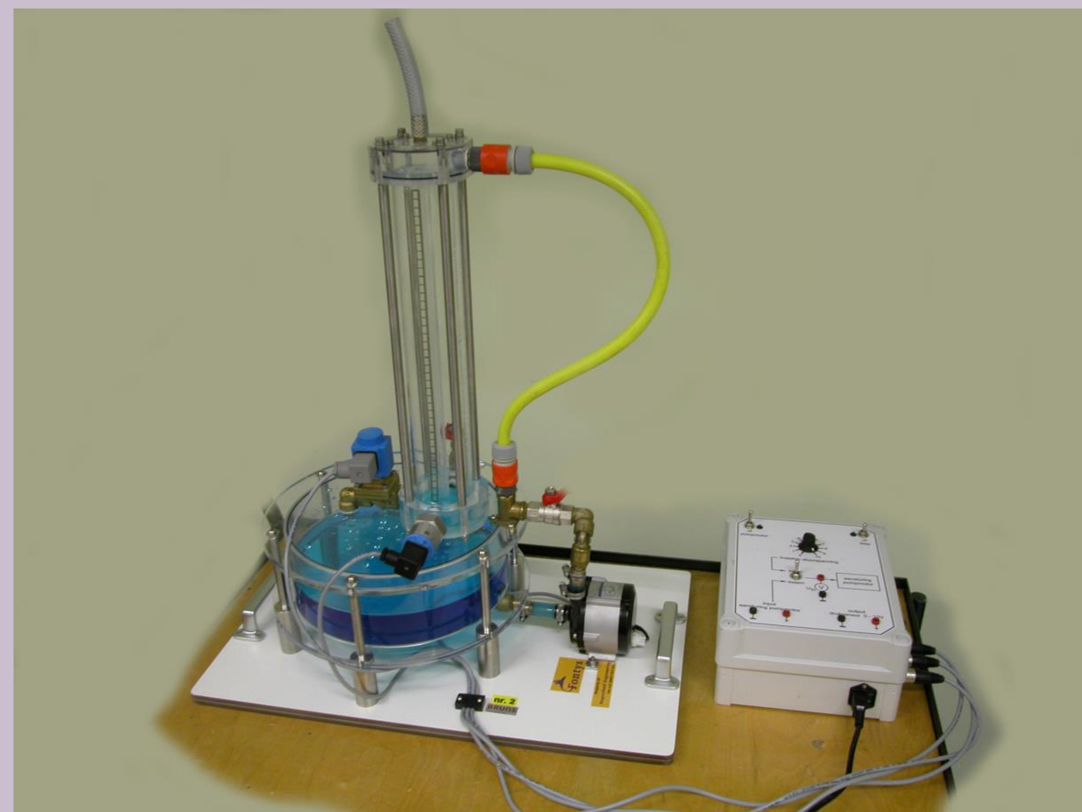
Youtube



https://www.youtube.com/user/ControlLectures/

Mededelingen: Practicumopstelling

Nog te bepalen aan de hand van RIVM besluit



Mededelingen: CEN4A Tentamen

CEN4A Tentamen:

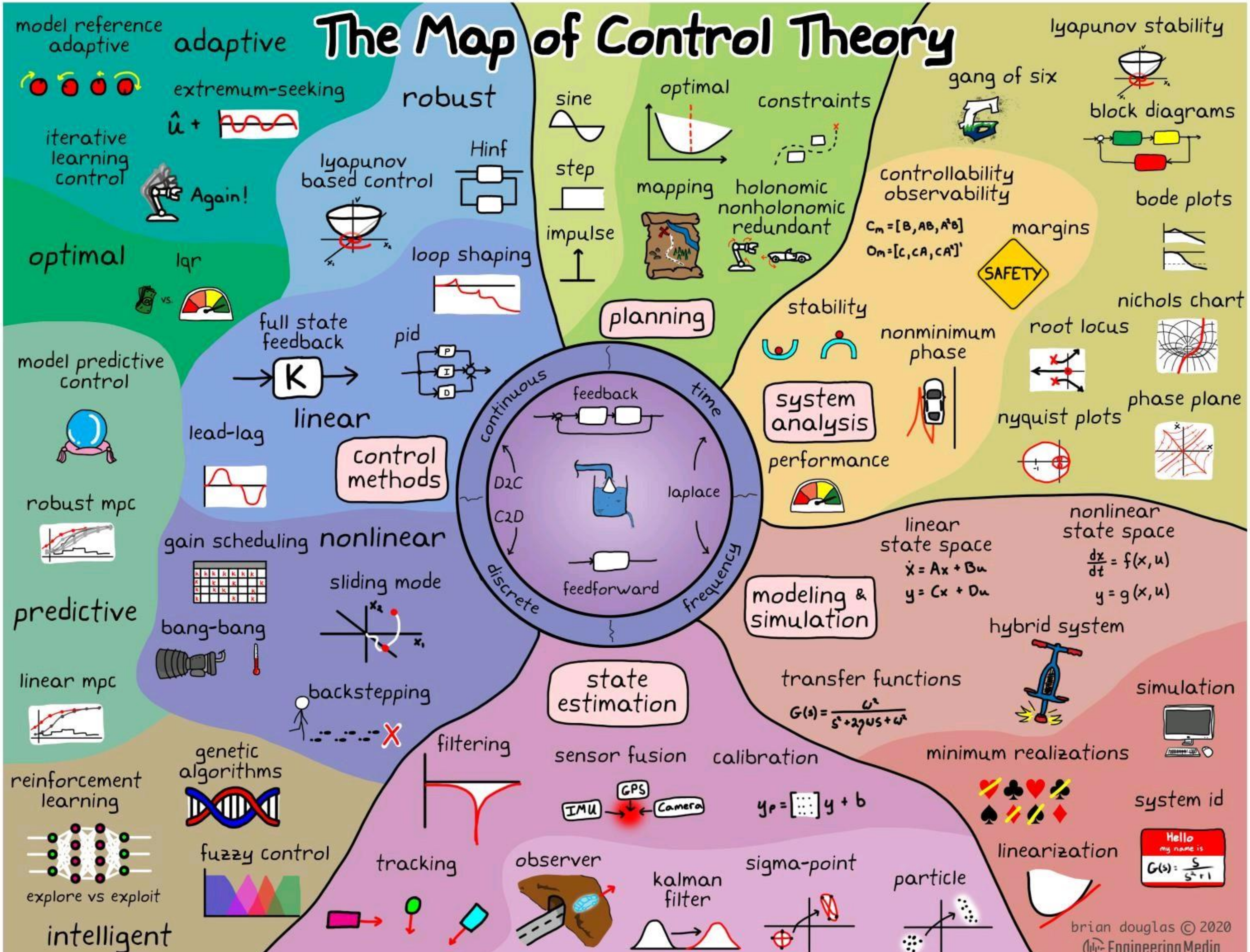
Nog te bepalen aan de hand van RIVM besluit

Regeltechniek in de industrie

- Robot Arms voor assamblage bedrijven
- Autonomous vehicles
- Path planning voor 3D printers
- Industriële automatisering
- Waffer stage van ASML
- Motor control


ALTEN

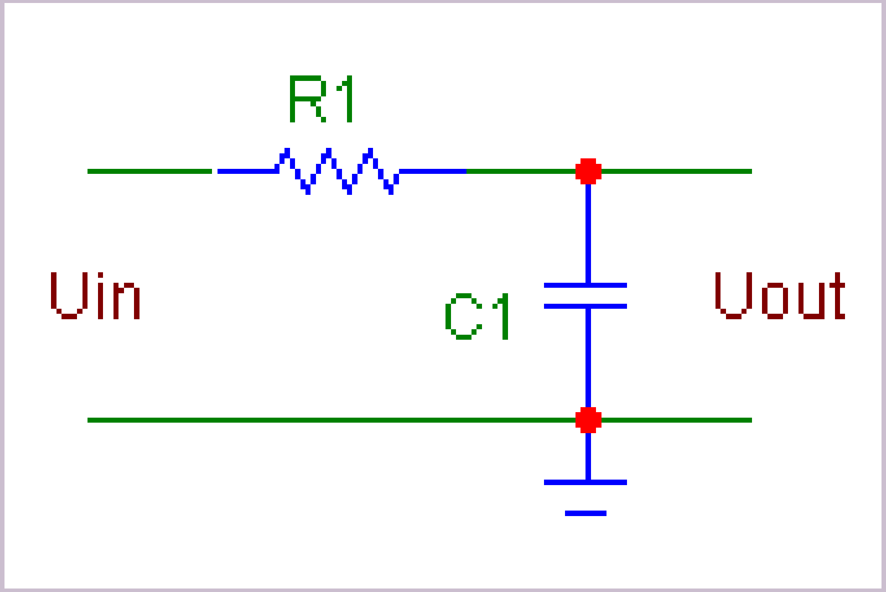
Andere voorbeelden?


The Map of Control Theory
model reference adaptive
adaptive
extremum-seeking
robust
iterative learning control
Again!
Hinf
lyapunov based control
optimal
lqr
vs.
loop shaping
full state feedback
K
pid
P
I
D
linear
model predictive control
lead-lag
control methods
robust mpc
gain scheduling
nonlinear
sliding mode
predictive
bang-bang
linear mpc
backstepping
genetic algorithms
reinforcement learning
fuzzy control
explore vs exploit
intelligent
sine
step
impulse
optimal
constraints
mapping
holonomic
nonholonomic
redundant
planning
lyapunov stability
gang of six
block diagrams
controllability
observability
$C_m = [B, AB, A^2B]$
$O_m = [C, CA, CA^2]'$
margins
SAFETY
bode plots
stability
nonminimum phase
nichols chart
root locus
system analysis
phase plane
nyquist plots
performance
continuous
time
feedback
D2C
C2D
laplace
feedforward
discrete
frequency
linear state space
$\dot{x} = Ax + Bu$
$y = Cx + Du$
nonlinear state space
$\frac{dx}{dt} = f(x, u)$
$y = g(x, u)$
modeling & simulation
hybrid system
state estimation
transfer functions
$G(s) = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2}$
simulation
filtering
sensor fusion
calibration
GPS
IMU
Camera
$y_p = [\,\vdots\,] y + b$
minimum realizations
system id
Hello
my name is
$G(s) = \frac{s}{s^2+1}$
tracking
observer
kalman filter
sigma-point
particle
linearization
brian douglas © 2020
Engineering Media

Gereedschappen in Freq. Domein
(Chapter 6.1 en 6.3)

- Bodediagrammen
- Polaire Figuur (Nyquist Diagram)

RC-netwerk



$$H(s) = \frac{U_o(s)}{U_i(s)} = \frac{1}{sR_1C_1 + 1}$$

$$s = \lambda + j\omega\Big|_{\lambda=0}$$

$$H(s) = \frac{U_o(s)}{U_i(s)} = \frac{\frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}} = \frac{1}{j\omega R_1C_1 + 1}$$

$$H(s) = \frac{U_o(s)}{U_i(s)} = \frac{\frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}} = \frac{1}{j\omega R_1C_1 + 1}$$

Bode amplitude- en fasediagram

$$H(j\omega)$$

$u_1(t)=A1.\sin(\omega t)$ → $H(j\omega)$ → $u_2(t)= A2.\sin(\omega t+\varphi)$

$1/(1+j\omega RC)$

$$|H(j\omega)| = \frac{1}{\sqrt{1+(\omega RC)^2}}$$

$$\varphi = -\arctan(\omega RC) = -\arctan(\omega\tau)$$

als functie van ω (log)

amplitudediagram: $20\log|H(j\omega)| = 20\log(A2/A1)$ dB

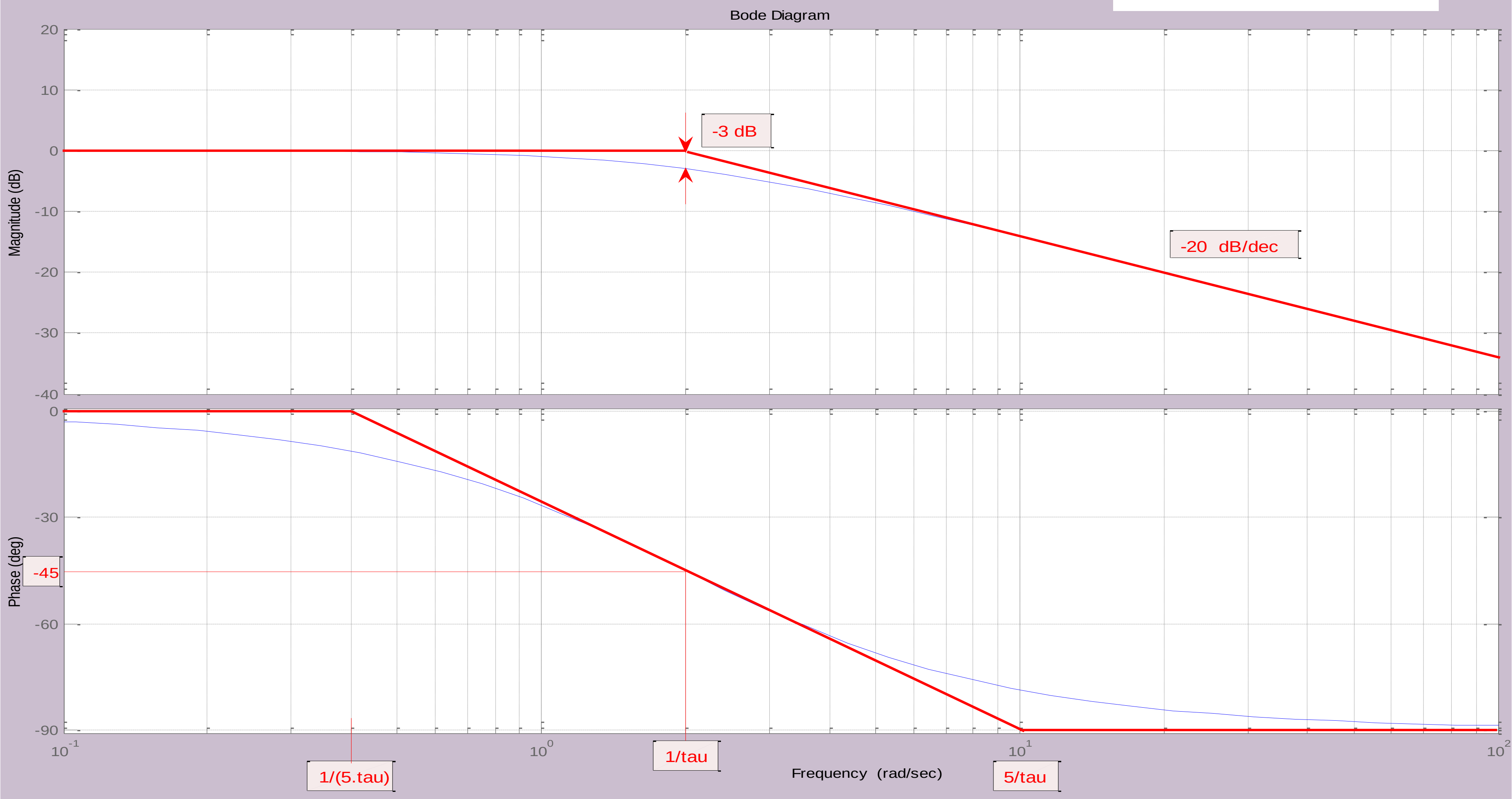
fasediagram: $\varphi = \arg\{H(j\omega)\}$

ω	$\lvert H(j\omega)\rvert$	$20\log\lvert H(j\omega)\rvert$	φ
0	1	0 dB	0°
kantelfrequentie → $\frac{1}{\tau} = \frac{1}{RC}$	$\frac{1}{2}\sqrt{2}$	-3	-45°
∞	0	$-\infty$	-90°

Bode amplitude- en fasediagram

matlab: bode(1,[0.5 1])

$\tau = 0{,}5$ sec



Polaire figuur (= Nyquist-diagram)

$H(j\omega)=\mathrm{Re}\{H(j\omega)\}+j.\mathrm{Im}\{H(j\omega)\}$

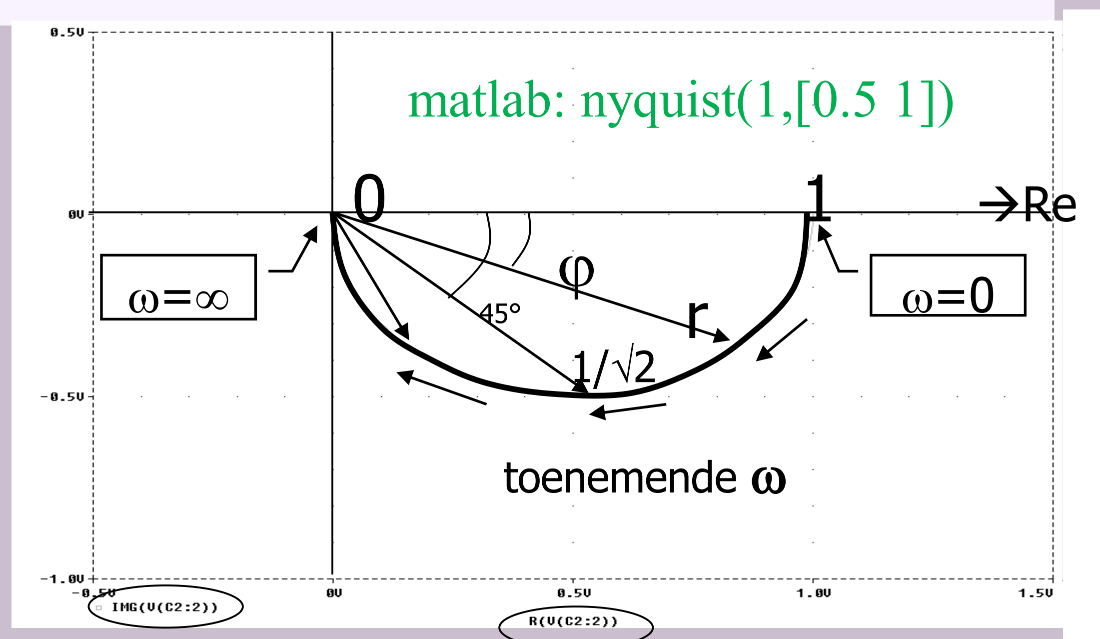
$\quad = x + j.y = r.e^{j\varphi}$

met: $r = |H(j\omega)|$ en $\varphi = \arg\{H(j\omega)\}$

De <u>polaire figuur</u> is een afbeelding van H(jω) in het complexe vlak als functie van **ω !**

1/(1+jωRC)

ω	$\lvert H(j\omega)\rvert$	$20\log\lvert H(j\omega)\rvert$	φ
0	1	0	0
$\frac{1}{RC}$	$\frac{1}{2}\sqrt{2}$	-3	-45°
∞	0	-∞	-90°



Basissystemen (Chapter 4.3.1 t/m 4.3.4)

Bij fysische processen treden slechts 6 essentieel verschillende kenmerken op. Deze karakteriseren de zogenaamde basissystemen.

Basissystemen

- Constante factor
- Integrator (zuiver – onzuiver)
- Differentiator (zuiver – onzuiver)
- Eerste-ordesysteem
- Tweede-ordesysteem
- Looptijd

De constante factor

$$\text{Mathematisch model}$$

$$y(t) = kx(t)$$

$$S-\text{domein}$$

$$Y(s) = kX(s)$$

$$j\omega \text{ domein}$$

$$Y(j\omega) = kX(j\omega)$$

$$H(j\omega) = k$$

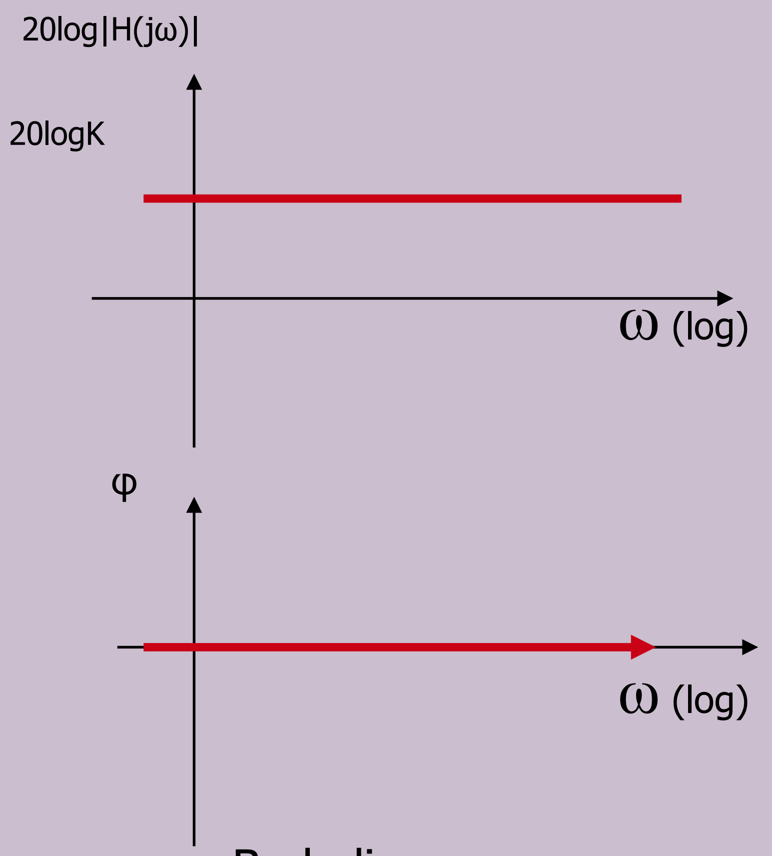
$$|H(j\omega)| = k \quad \text{en} \quad \varphi = 0$$

$$20\log|H(j\omega)| = 20\log K$$

tabel

ω	$\lvert H(j\omega)\rvert$	$20\log\lvert H(j\omega)\rvert$	φ
0	k	20log k	0
∞	k	20log k	0

De constante factor



Pn-beeld

Bodediagram

Zuivere integrator

Mathematisch model

$$y(t) = \int x(t)dt$$

S – domein

$$Y(s) = \frac{X(s)}{s}$$

$j\omega$ domein

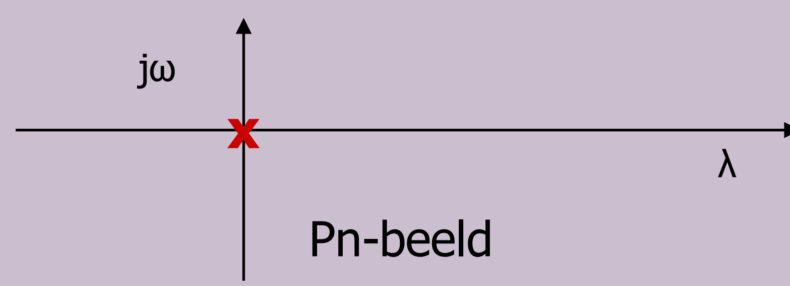
$$Y(j\omega) = \frac{1}{j\omega} X(j\omega)$$

$$H(j\omega) = \frac{1}{j\omega}$$

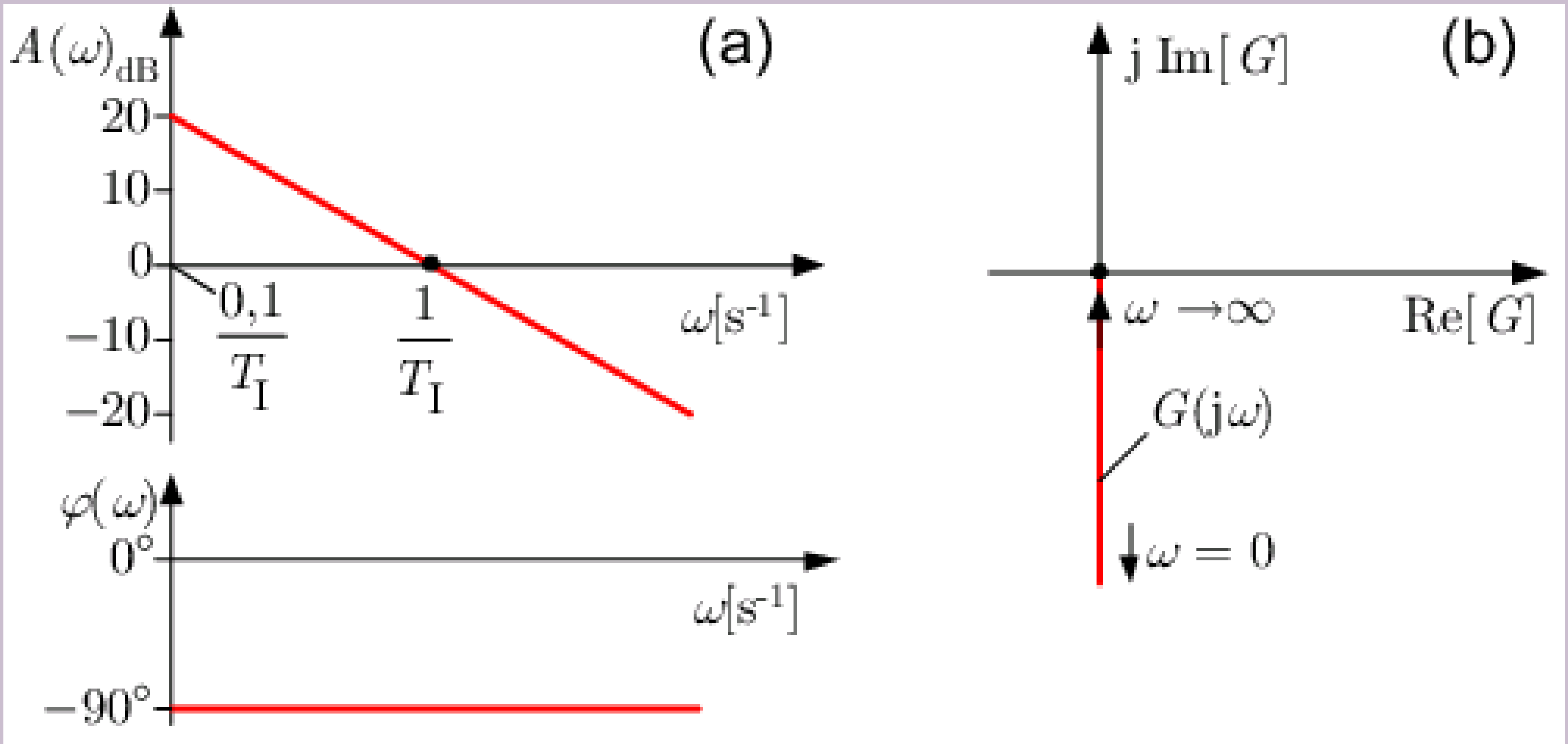
$$|H(j\omega)| = \frac{1}{\omega} \quad \text{en} \quad \varphi = -90^o$$

$$20\log|H(j\omega)| = -20\log\omega$$

ω	$\lvert H(j\omega)\rvert$	$20\log\lvert H(j\omega)\rvert$	φ
0	∞	$+\infty$	-90^o
1	1	0	-90^o
∞	0	$-\infty$	-90^o



Zuivere integrator



Zuivere differentiator

Fontys Hogescholen

Mathematisch model

$$y(t) = \frac{dx(t)}{dt}$$

$$S - \text{domein}$$

$$Y(s) = sX(s)$$

$$j\omega \text{ domein}$$

$$Y(j\omega) = j\omega X(j\omega)$$

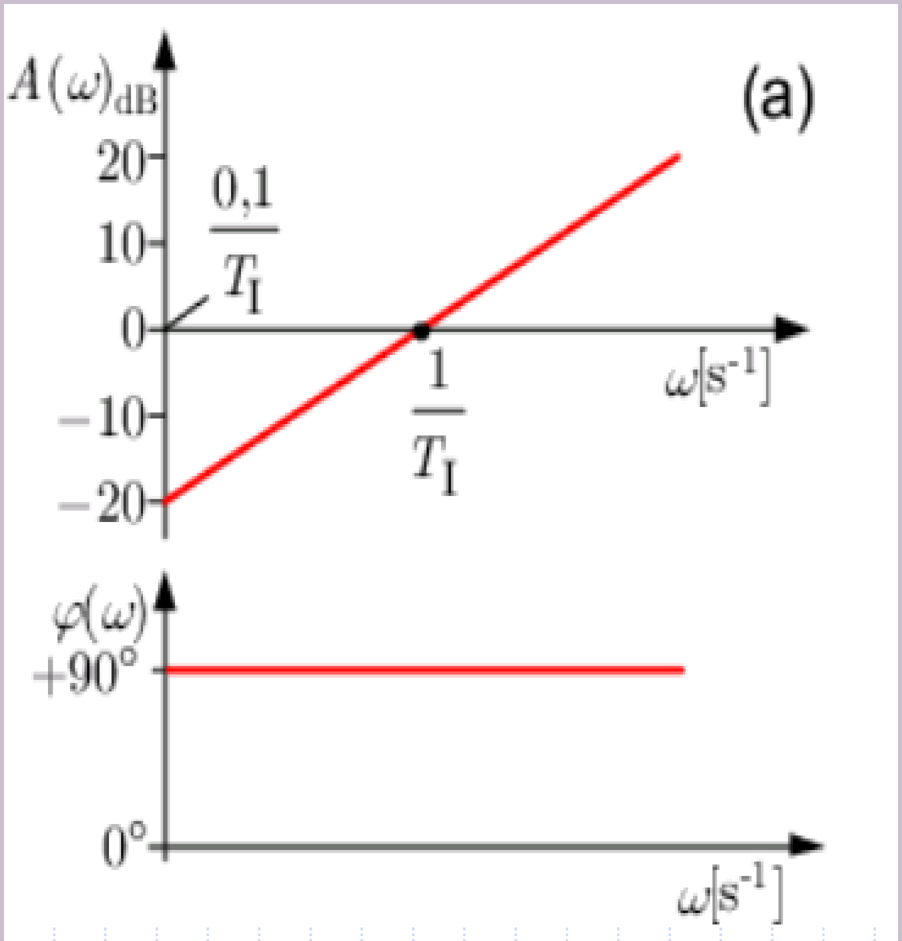
$$H(j\omega) = j\omega$$

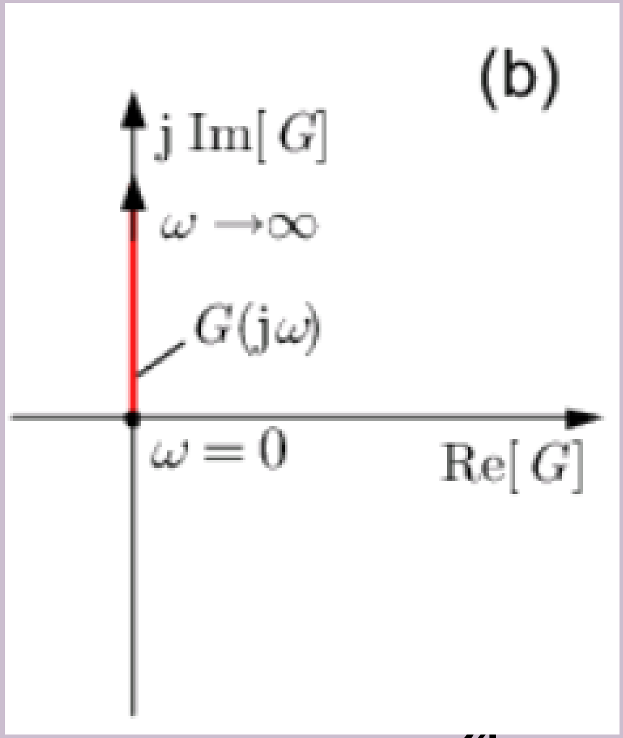
$$|H(j\omega)| = \omega \quad \text{en} \quad \varphi = 90^o$$

$$20\log|H(j\omega)| = 20\log\omega$$

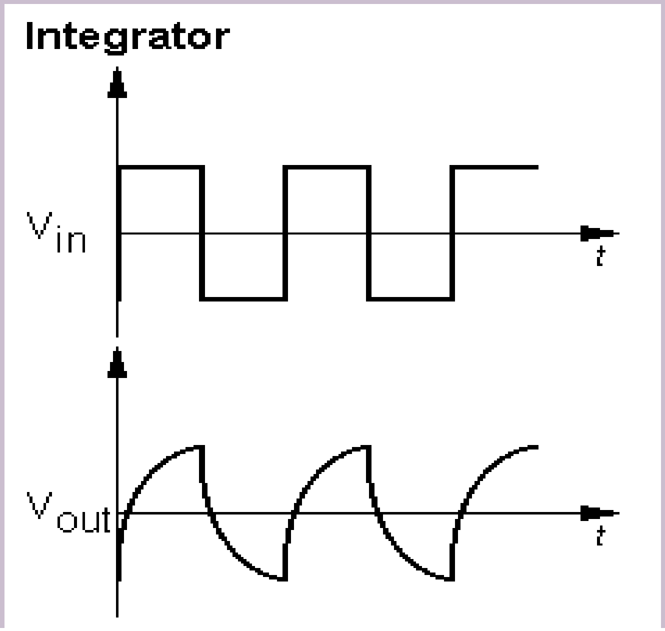
ω	$\lvert H(j\omega)\rvert$	$20\log\lvert H(j\omega)\rvert$	φ
0	0	-∞	90^o
1	1	0	90^o
∞	+∞	+∞	90^o

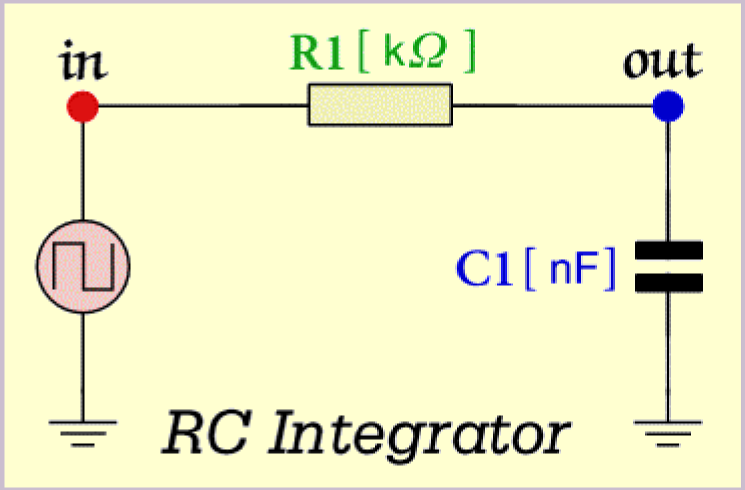
Zuivere differentiator

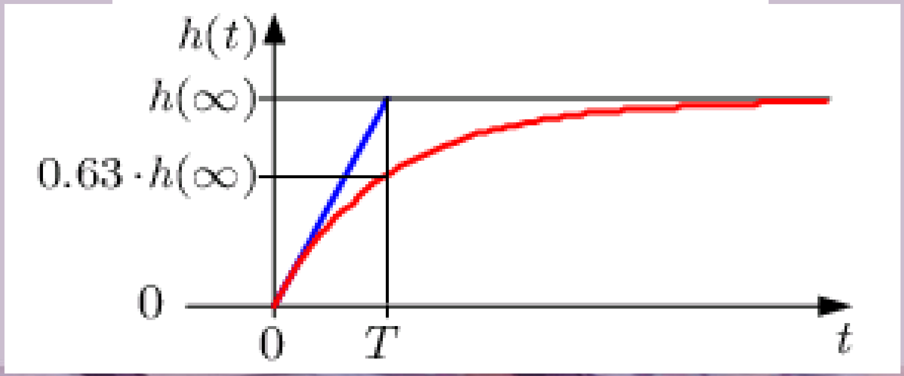




Onzuivere integrator = eerste-ordesysteem







Onzuivere integrator = eerste-ordesysteem

Mathematisch model

$$\tau_i \frac{dy(t)}{dt} + y(t) = x(t)$$

S – domein

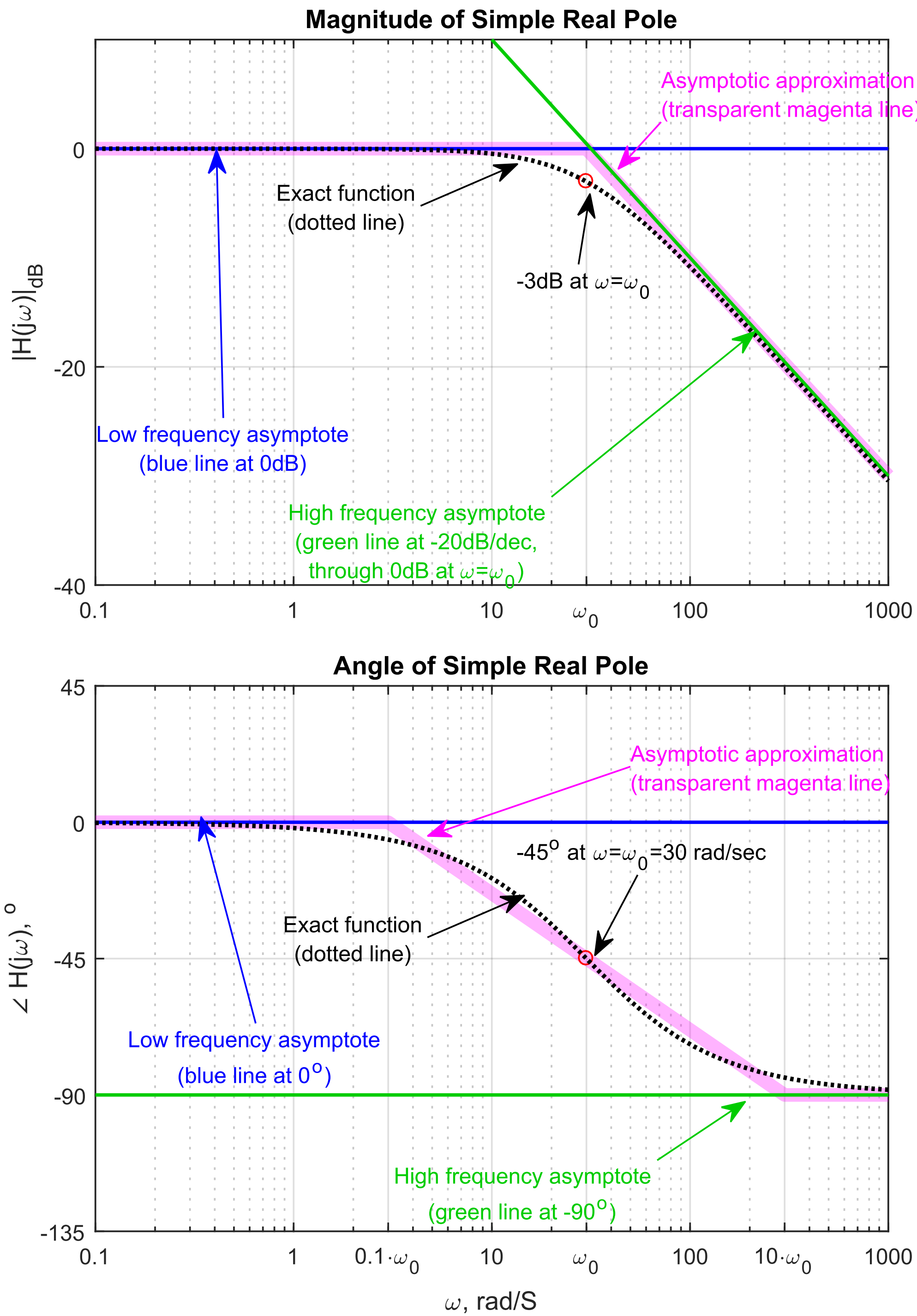
$$Y(s) = \frac{X(s)}{\tau_i s + 1}$$

$j\omega$ domein

$$Y(j\omega) = \frac{1}{1 + j\omega\tau_i} X(j\omega) \quad \text{geeft} \quad H(j\omega) = \frac{1}{1 + j\omega\tau_i}$$

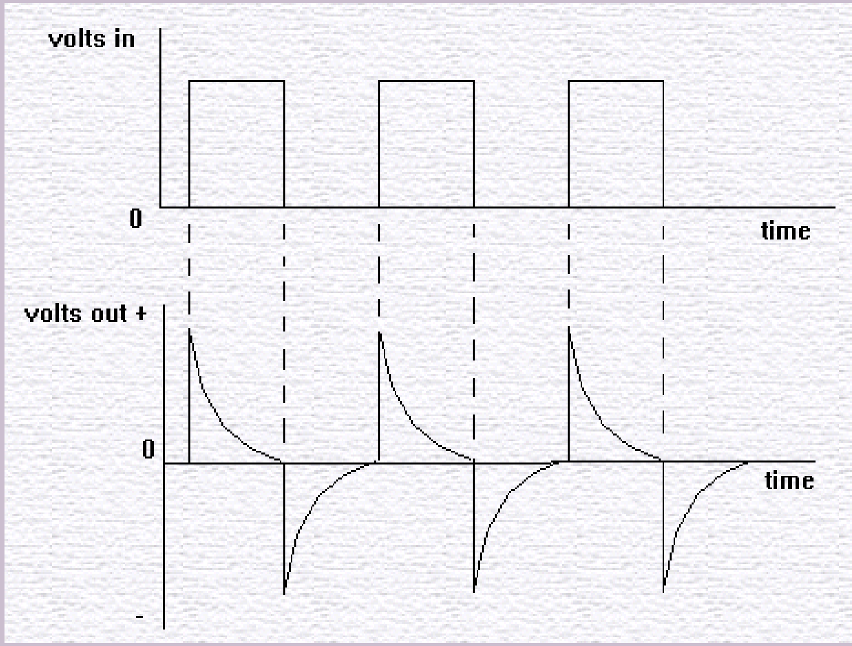
$$20\log|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega\tau_i)^2}} \quad \text{en} \quad \varphi = 0 - \arctan \omega\tau_i$$

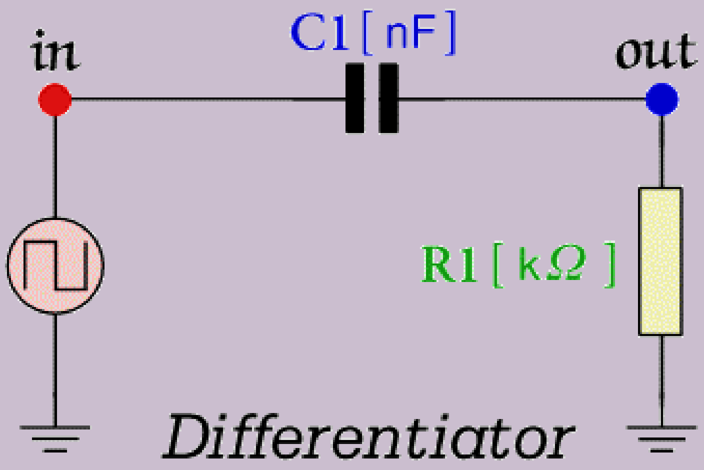
ω	$\lvert H(j\omega)\rvert$	$20\log\lvert H(j\omega)\rvert$	φ
0	1	0	0^o
$1/\tau_i$	$\frac{1}{2}\sqrt{2}$	-3	-45^o
∞	0	$-\infty$	-90^o

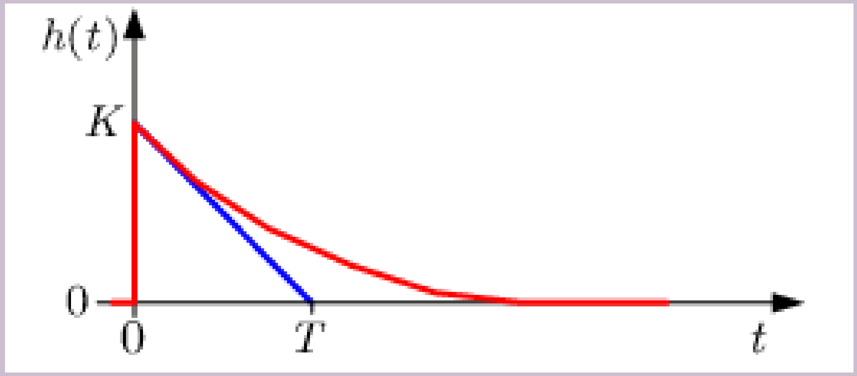


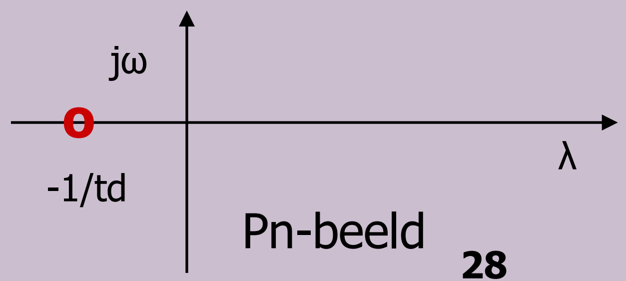
Magnitude of Simple Real Pole
Asymptotic approximation
(transparent magenta line)
Exact function
(dotted line)
-3dB at ω=ω0
Low frequency asymptote
(blue line at 0dB)
High frequency asymptote
(green line at -20dB/dec,
through 0dB at ω=ω0)
|H(jω)|dB
0
-20
-40
0.1
1
10
ω0
100
1000
Angle of Simple Real Pole
Asymptotic approximation
(transparent magenta line)
-45o at ω=ω0=30 rad/sec
Exact function
(dotted line)
Low frequency asymptote
(blue line at 0o)
High frequency asymptote
(green line at -90o)
∠ H(jω), o
45
0
-45
-90
-135
0.1
1
0.1·ω0
10
ω0
100
10·ω0
1000
ω, rad/S

Onzuivere differentiator









Onzuivere differentiator

Mathematisch model

$$y(t) = \tau_d \frac{dx(t)}{dt} + x(t)$$

S – domein

$$Y(s) = (s\tau_d + 1)X(s)$$

$j\omega$ domein

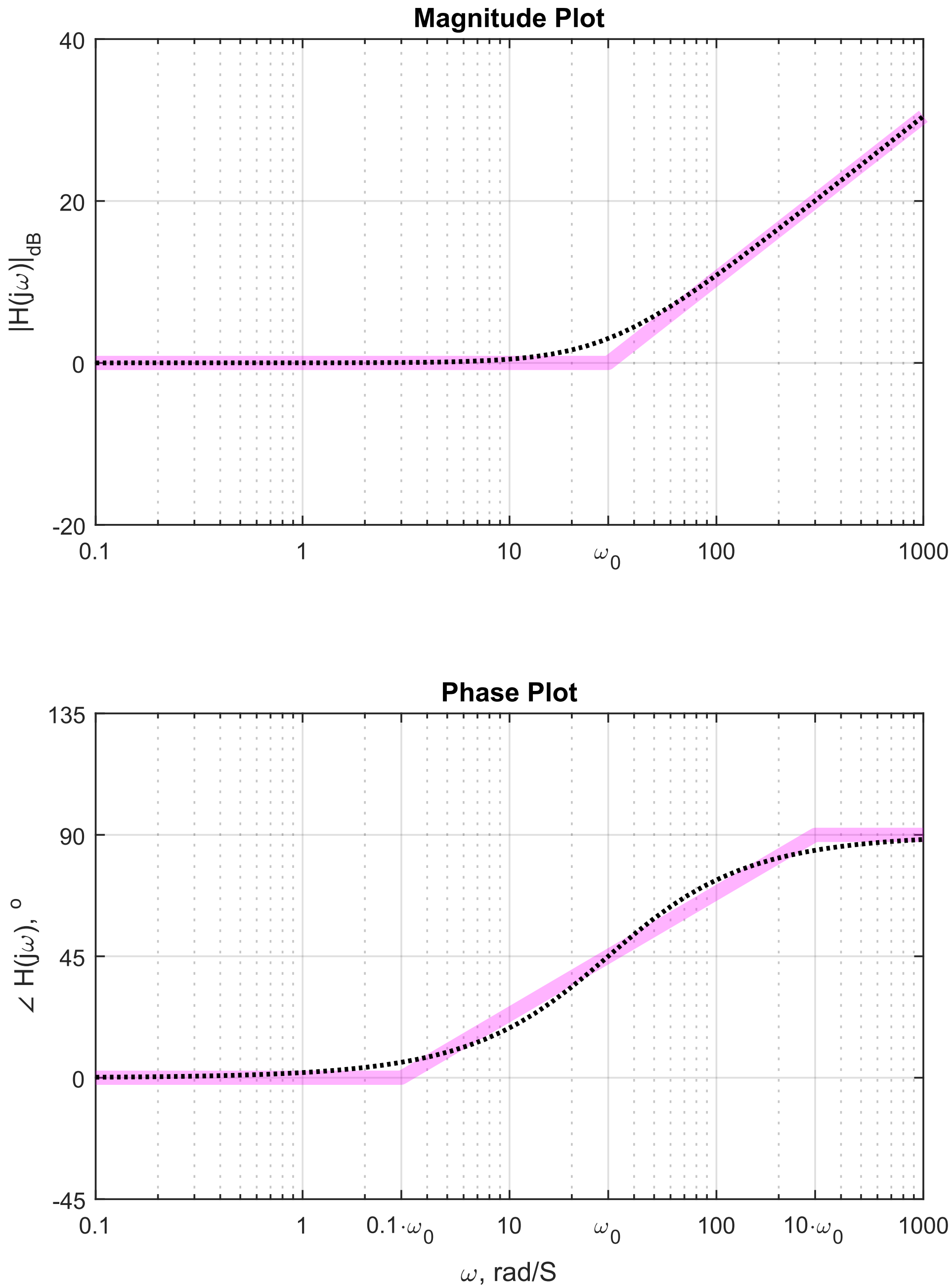
$$Y(j\omega) = (j\omega\tau_d + 1)X(j\omega)$$

$$H(j\omega) = (j\omega\tau_d + 1)$$

$$20\log\left|H(j\omega)\right| = 20\log\sqrt{\left(\omega\tau_d\right)^2 + 1}$$

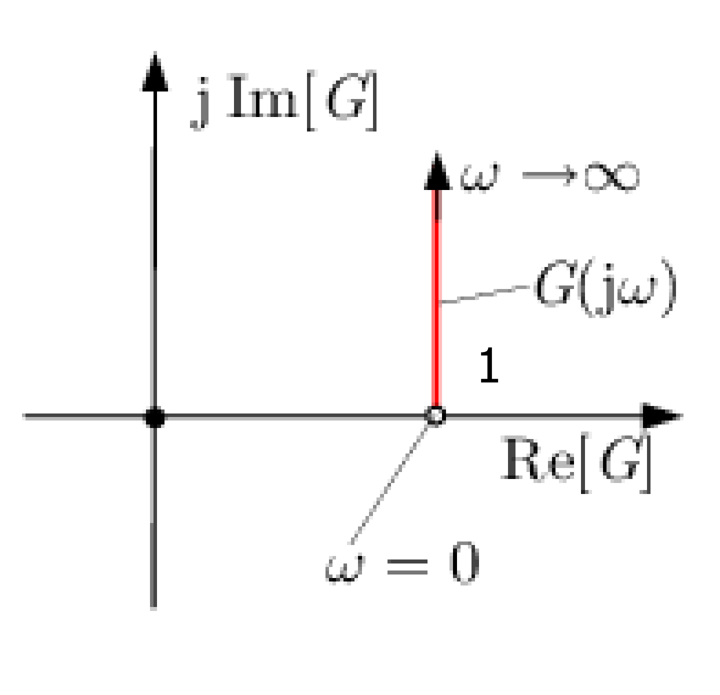
$$\varphi = \arctan \omega\tau_d$$

ω	$\left\|H(j\omega)\right\|$	$20\log\left\|H(j\omega)\right\|$	φ
0	1	0	0^o
$\frac{1}{\tau_d}$	$\sqrt{2}$	$+3$	45^o
∞	$+\infty$	$+\infty$	90^o


Magnitude Plot
40
20
0
-20
|H(jω)|dB
0.1
1
10
ω0
100
1000
Phase Plot
135
90
45
0
-45
∠ H(jω), °
0.1
1
0.1·ω0
10
ω0
100
10·ω0
1000
ω, rad/S

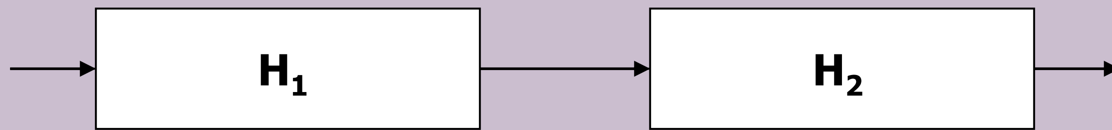
Onzuivere differentiator
Bode en Nyquist diagram

ω	$\lvert H(j\omega)\rvert$	$20\log\lvert H(j\omega)\rvert$	φ
0	1	0	0^o
$\frac{1}{\tau_d}$	$\sqrt{2}$	$+3$	45^o
∞	$+\infty$	$+\infty$	90^o



$$H(j\omega) = (j\omega\tau_d + 1)$$

Bodediagram cascadeschakeling



$$H(j\omega) = H_1(j\omega).H_2(j\omega) = r_1 e^{\,j\varphi_1}.\ r_2 e^{\,j\varphi_2}$$

$$H(j\omega) = r.e^{\,j\varphi} = r_1.r_2\,.e^{\,j(\varphi_1 + \varphi_2)}$$

$$20\log|H(j\omega)| = 20\log|H_1(j\omega)|.|H_2(j\omega)| =$$

$$20\log|H_1(j\omega)| + 20\log|H_2(j\omega)| =$$

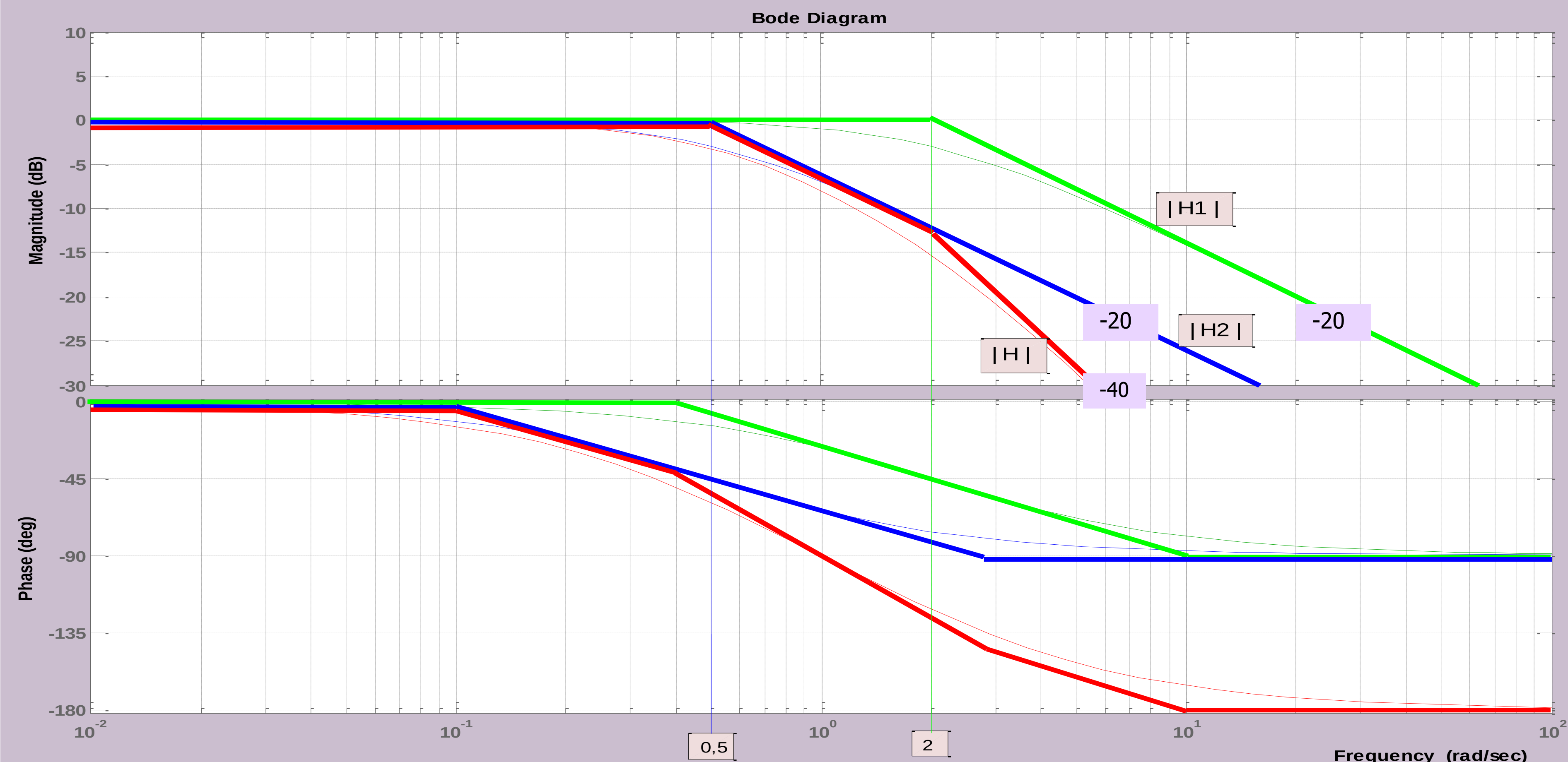
$$|H_1(j\omega)|_{dB} + |H_2(j\omega)|_{dB} \qquad \text{(OPTELLEN)}$$

en:

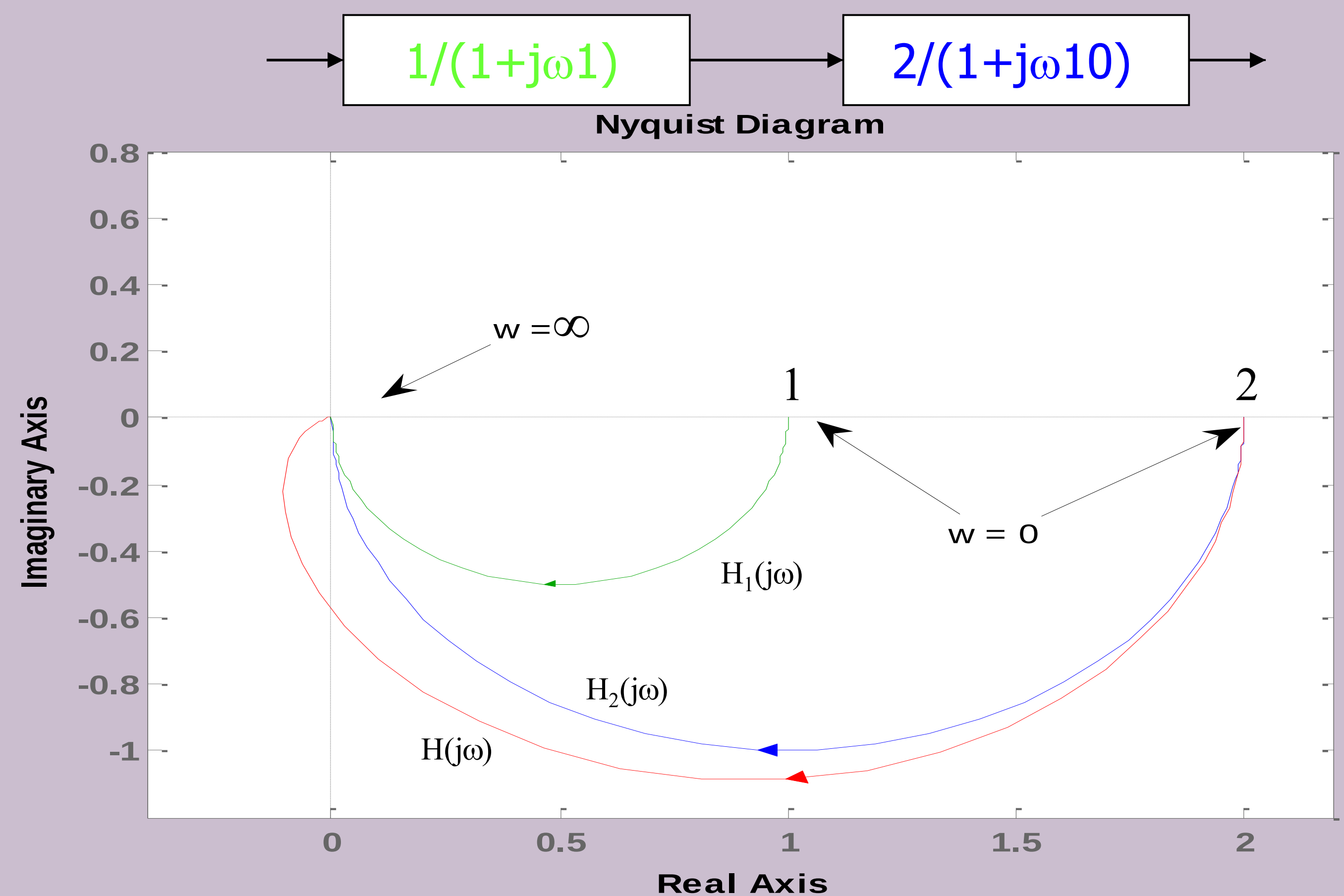
$$\varphi = \arg\{H(j\omega)\} = \arg\{H_1(j\omega).H_2(j\omega)\} =$$

$$\arg\{H_1(j\omega)\} + \arg\{H_2(j\omega)\} = \varphi_1 + \varphi_2 \qquad \text{(OPTELLEN)}$$

Bodediagram cascadeschakeling



Polaire figuur cascadeschakeling



met matlab H_2: nyquist(2,[10 1])

Oefenopgave 1

Teken de benaderde Bodediagrammen van

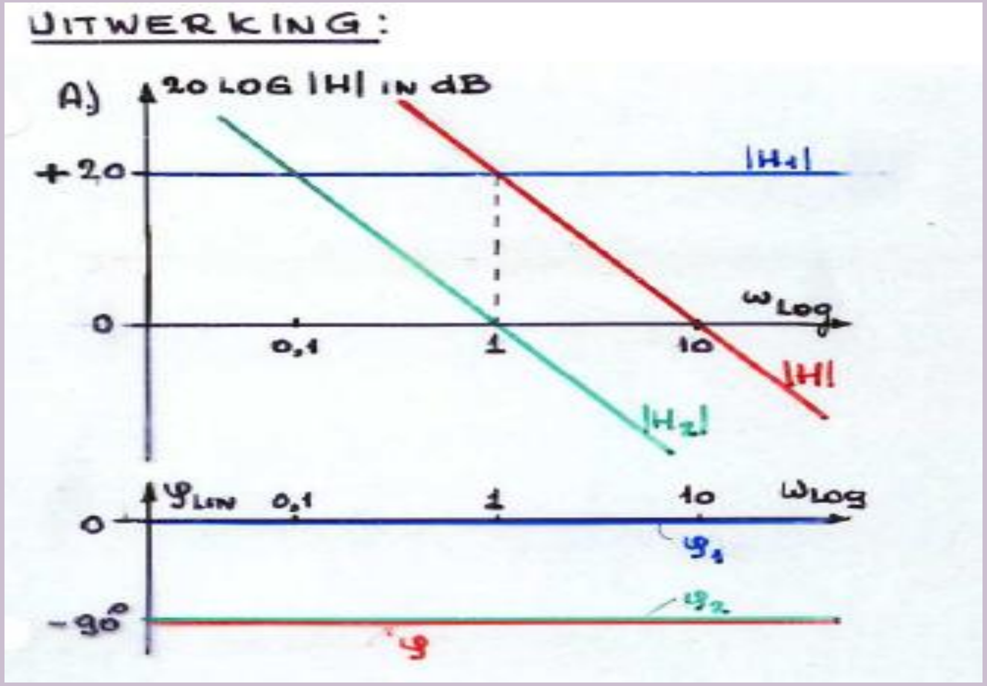
$H1(j\omega)$, $H2(j\omega)$ en $H(j\omega)= H1(j\omega)\cdot H2(j\omega)$

A) $H1(j\omega)=10$ en $H2(j\omega)=1/j\omega$

B) $H1(j\omega)=1+j\omega$ en $H2(j\omega)=-0,5$

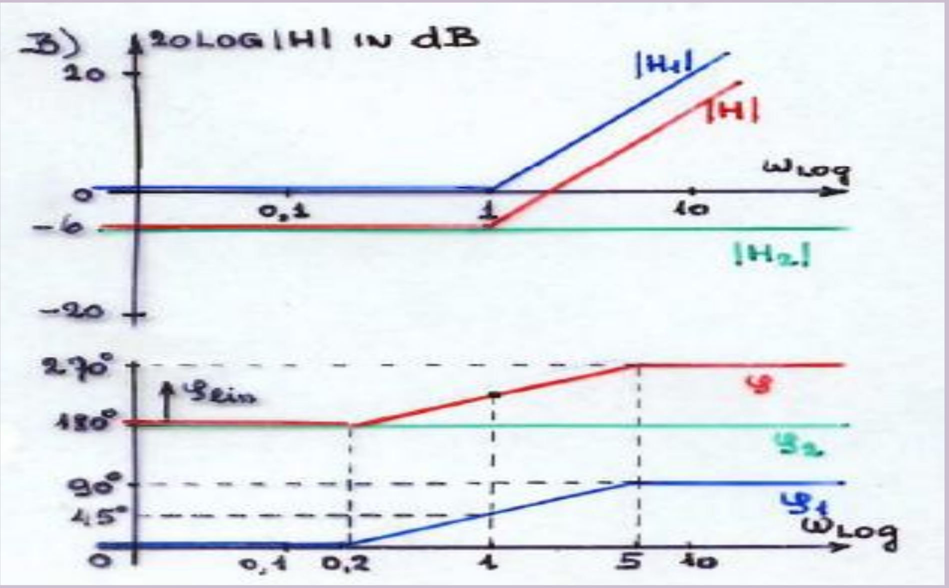
A) H1(jω)=10 en H2(jω)=1/jω

Bodediagram

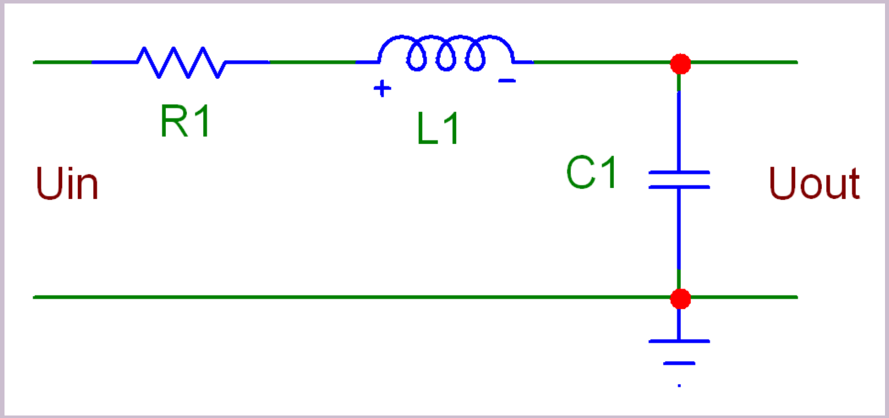


B) H1(jω)=1+jω en H2(jω)=-0,5

Bodediagram



Tweede-orde normaalvorm
(Chapter 3.3)



$$H(s) = \frac{1}{LCs^2 + RCs + 1}$$

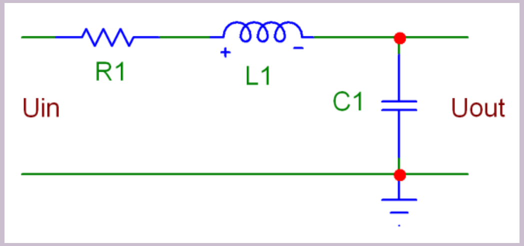
normaalvorm

$$H(s) = \frac{\omega_0^{\;2}}{s^2 + 2\beta\omega_0 s + \omega_0^{\;2}}$$

β = relatieve dempingsfactor

3 situaties:
- β >1 overkritische demping
- β =1 kritische demping
- β <1 onder kritische demping

Tweede-orde normaalvorm



$$H(s) = \frac{\omega_0^{\,2}}{s^2 + 2\beta\omega_0 s + \omega_0^{\,2}}$$

β Relatieve dempingsfactor

ω_0 Ongedempte eigenfrequentie (ongedempte natuurlijke frequentie).

$$\omega_g = \omega_0\sqrt{1-\beta^2}$$

Gedempte eigenfrequentie; zichtbaar in stapresponsie

$$\omega_r = \omega_0\sqrt{1-2\beta^2}$$

Resonantie frequentie = frequentie waarbij de opslingering optreedt; zichtbaar in het bodediagram

Tweede orde en demping

$karakteristieke\ vergelijking \rightarrow s^2 + 2\beta\omega_0 s + \omega_0{}^2 = 0$

$D > 0 \rightarrow$ 2 separate 1^e orde systemen;

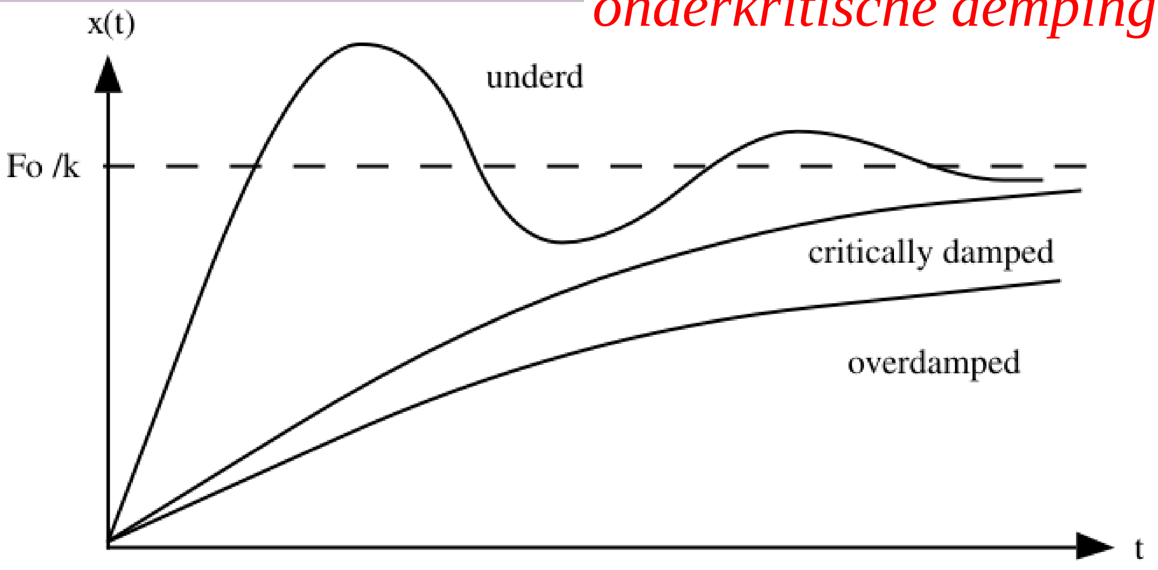
overkritische demping

$D = 0 \rightarrow$ 2 samenvallende 1^e orde systemen;

kritische demping

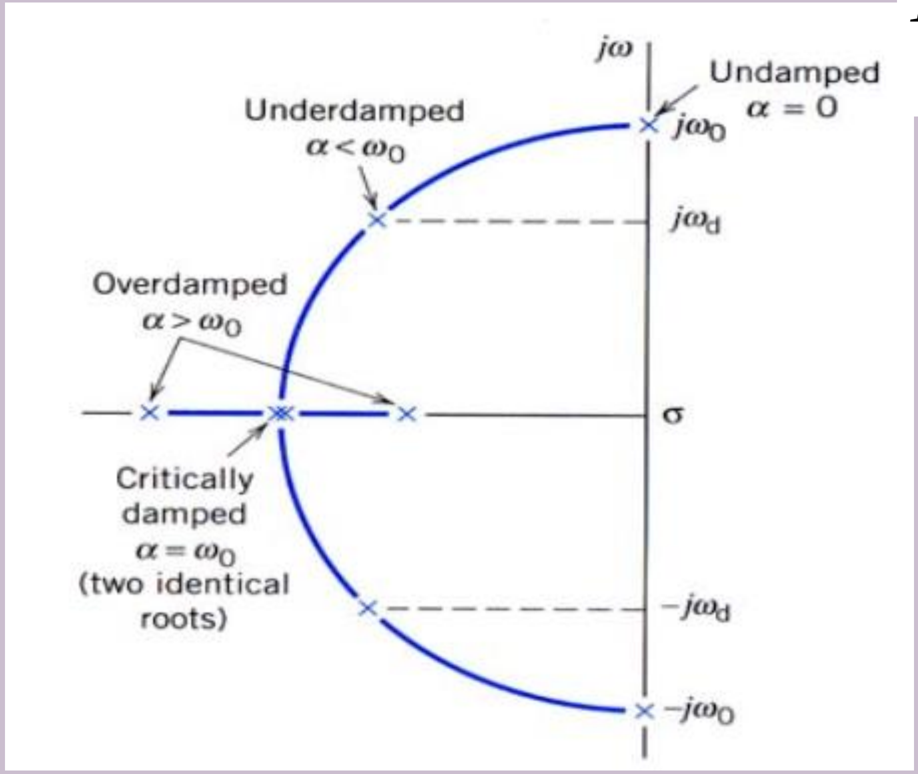
$D < 0 \rightarrow$ 2 complexe oplossingen dus opslingering;

onderkritische demping



Tweede orde en demping

$$H(s) = \frac{\omega_0^{\;2}}{s^2 + 2\beta\omega_0 s + \omega_0^{\;2}}$$



β>1; overkritische demping

=> 2 reele ongelijke polen

β = 1; kritische demping

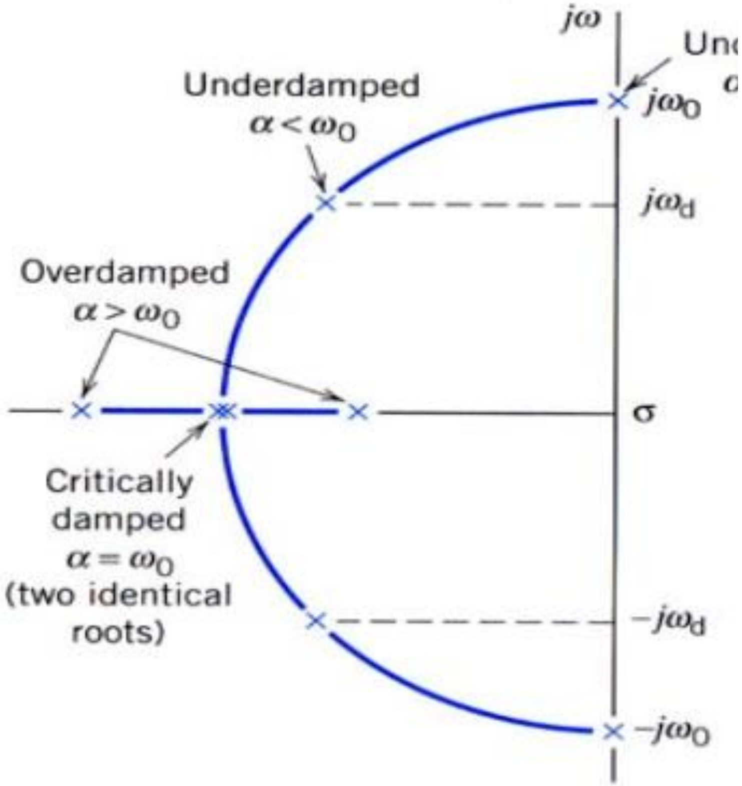
=> 2 reele gelijke polen

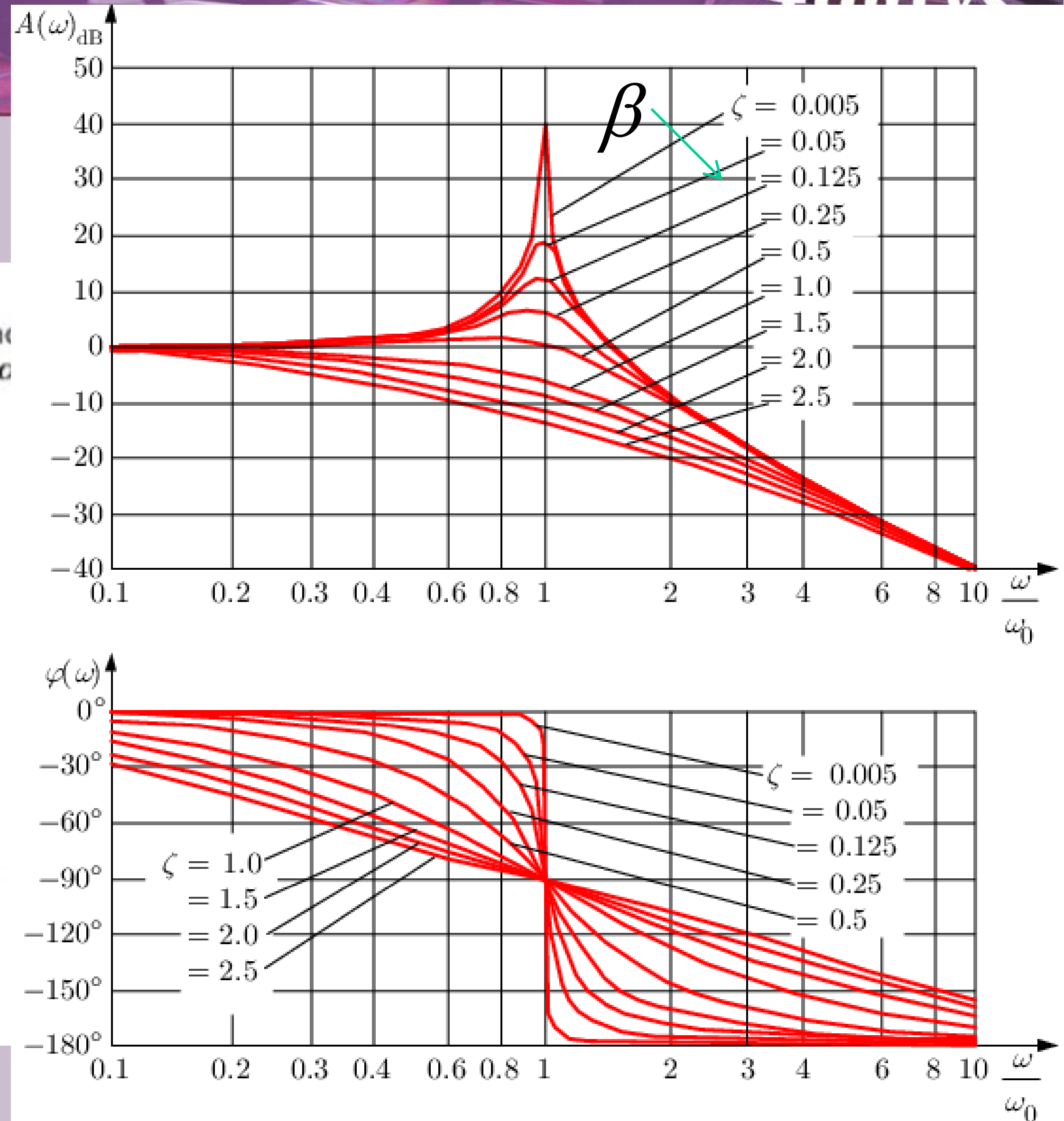
β =0; 2 zuiver complexe polen;
=> ongedempte hoekfrequentie
ofwel oscillatie

0 < β <1; onder kritische demping

=> 2 toegevoegd complexe polen

Tweede orde pn-beeld en Bode



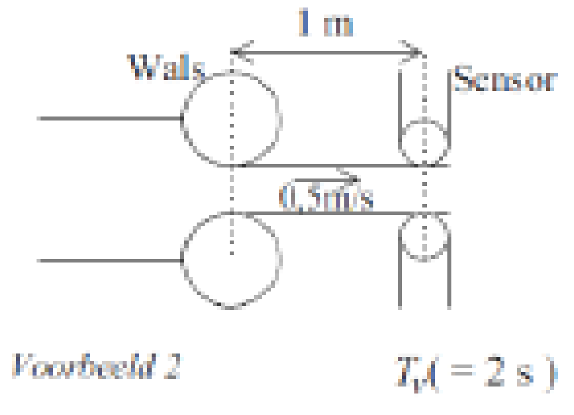


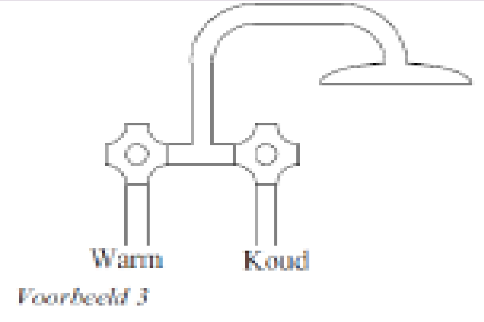
Looptijd

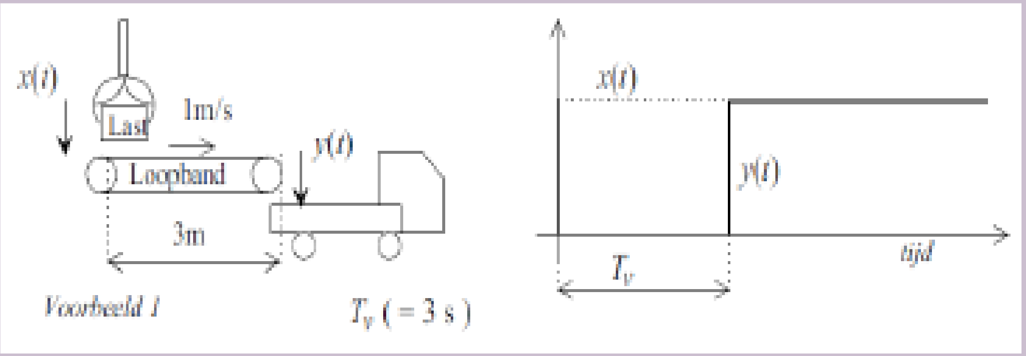
Mathematisch model: $y(t)=x(t-Tv)$

s-domein: $Y(s)=e^{-sTv}\cdot X(s)$

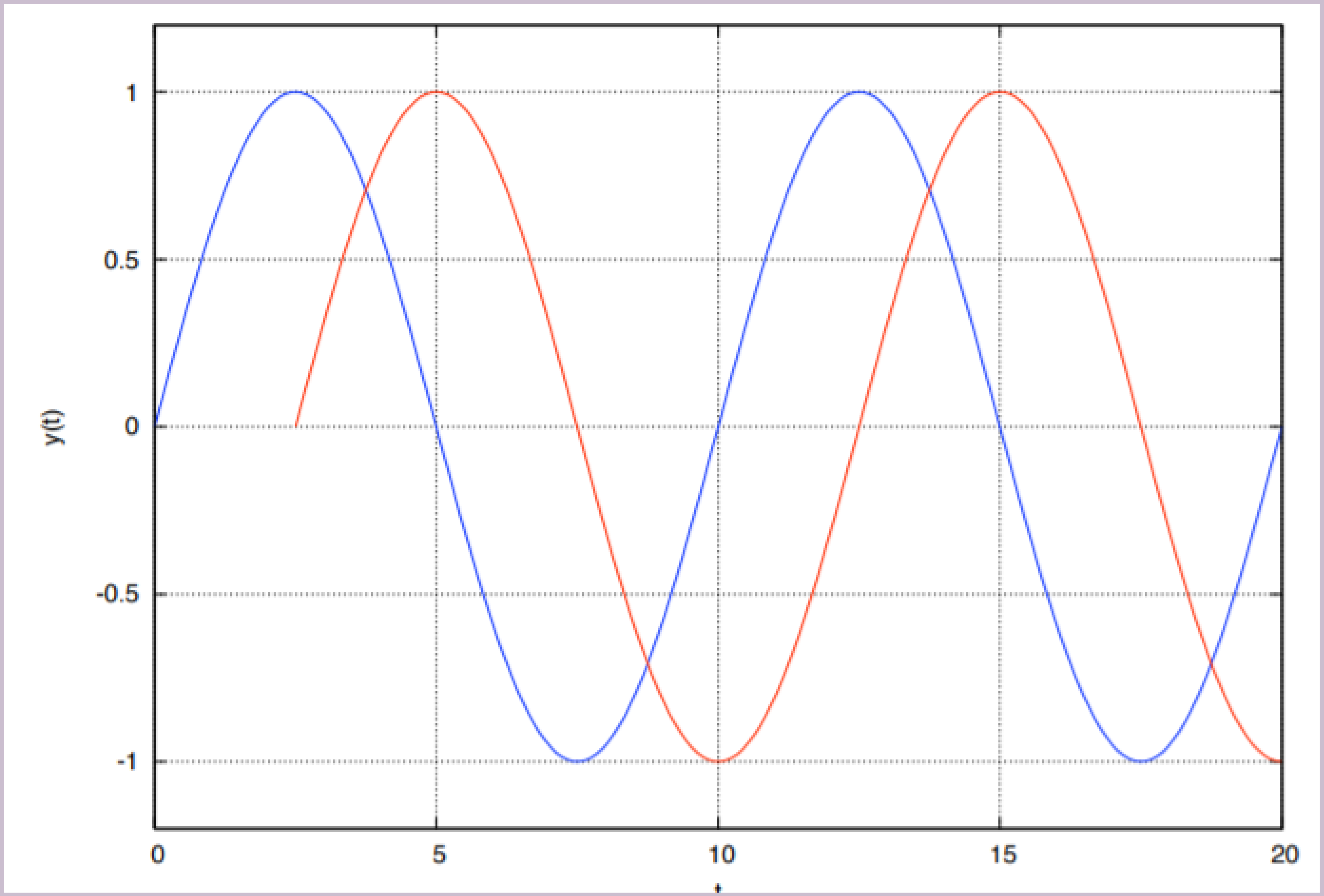
jω-domein: $Y(j\omega)=e^{-j\omega Tv}\cdot X(j\omega)$
$H(j\omega)=e^{-j\omega Tv}$ $|H(j\omega)|=1$ (0 dB) $\varphi=-\omega Tv$







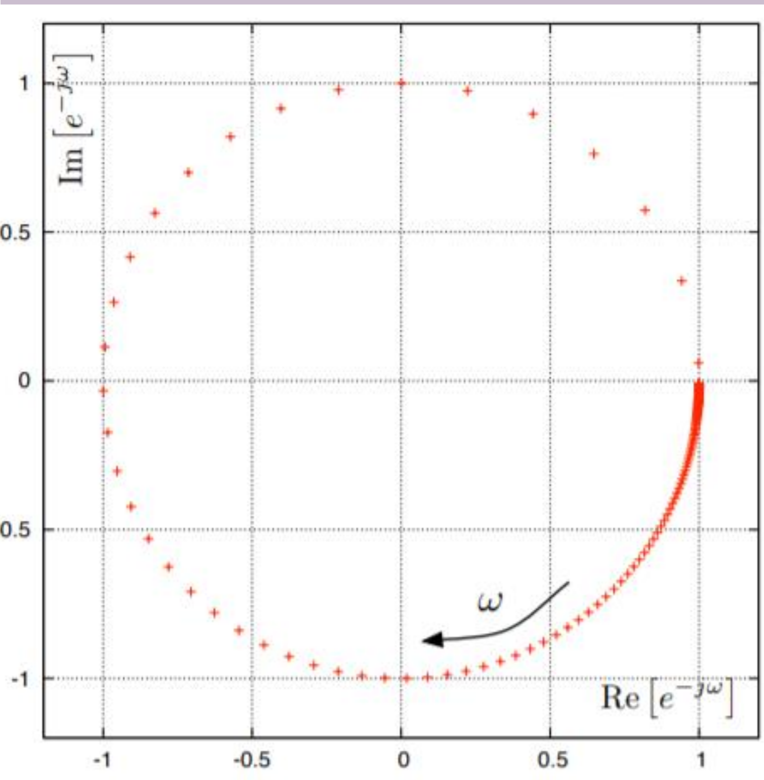
Looptijd

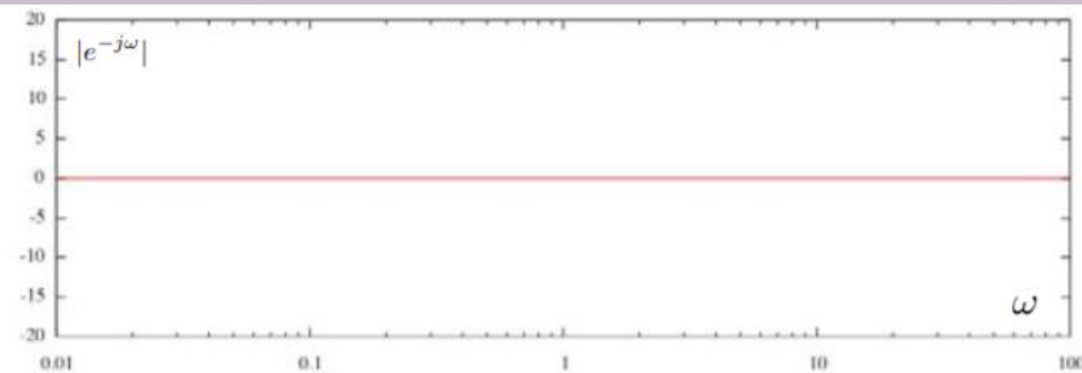


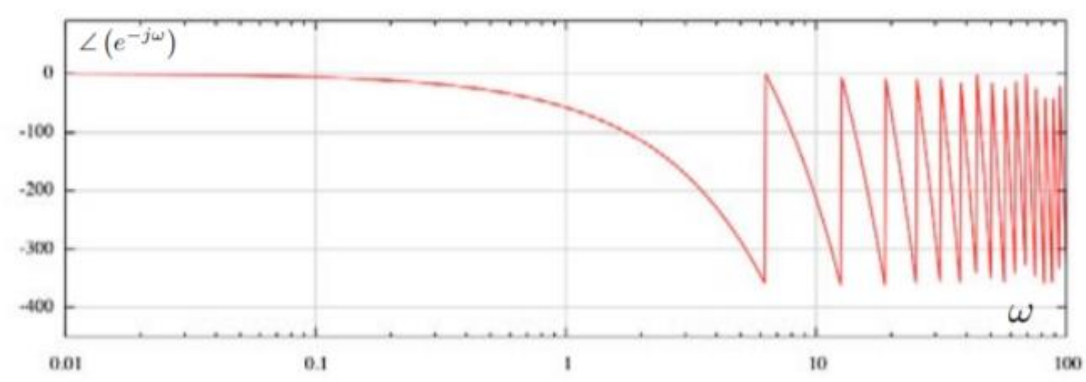
Looptijd

$$\mathrm{Delay}_{\mathrm{T}}(s) = e^{-sT}$$

$$|e^{j\omega T}| = 1, \qquad \angle\left(e^{-j\omega T}\right) = -\omega T.$$







Looptijd

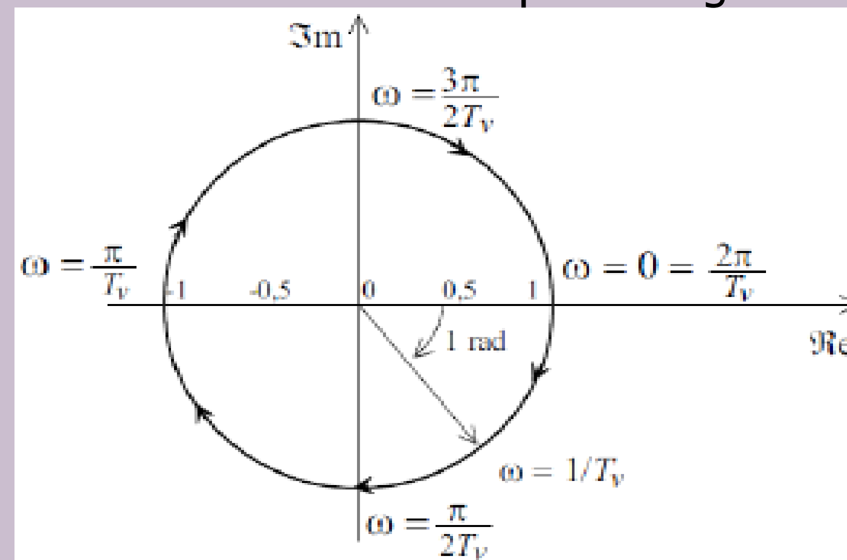
jω-domein:

$Y(j\omega)=e^{-j\omega Tv}\cdot X(j\omega)$

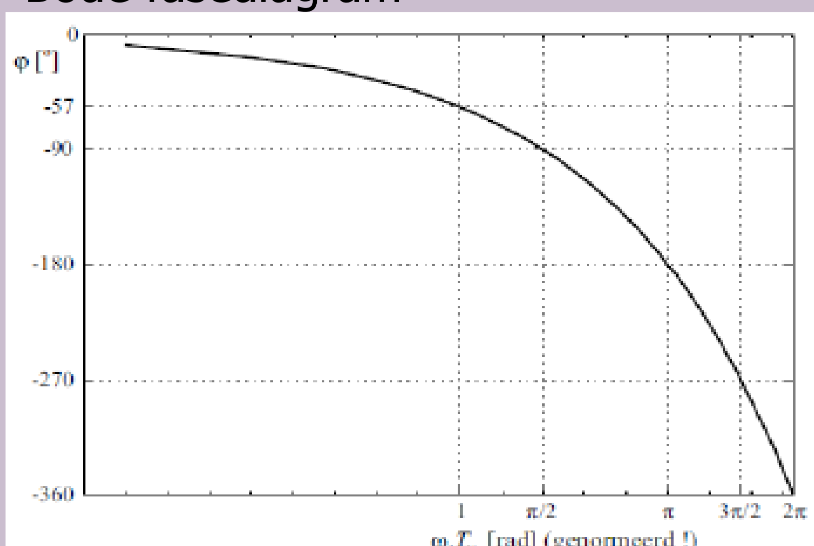
$H(j\omega)=e^{-j\omega Tv}$

$|H(j\omega)|=1 \quad \varphi=-\omega Tv$

polaire figuur



Bode fasediagram



ω	$\varphi=-\omega Tv$
0	0
$1/T_v$	1
$0{,}5\pi T_v$	$\pi/2$
π/T_v	π
$1{,}5\pi/T_v$	$3\pi/2$
$2\pi/T_v$	2π

Zelfstudiemateriaal/ Achtergrondinformatie

- Vanaf deze slide.

Complexe getallen

- Modulus (amplitude)
- Argument (fase)

$$20.\log\left|H(j\omega)\right| =$$

$$= 20.\log\sqrt{\operatorname{Re}\left\{H(j\omega)\right\}^2 + \operatorname{Im}\left\{H(j\omega)\right\}^2}$$

$$\varphi = Arg\left\{H(j\omega)\right\} = \arctan\frac{\operatorname{Im}\left\{H(j\omega)\right\}}{\operatorname{Re}\left\{H(j\omega)\right\}}$$

Amplitude en Fasedraaiing

$$H(j\omega) = \frac{1}{j\omega R_1 C_1 + 1}$$

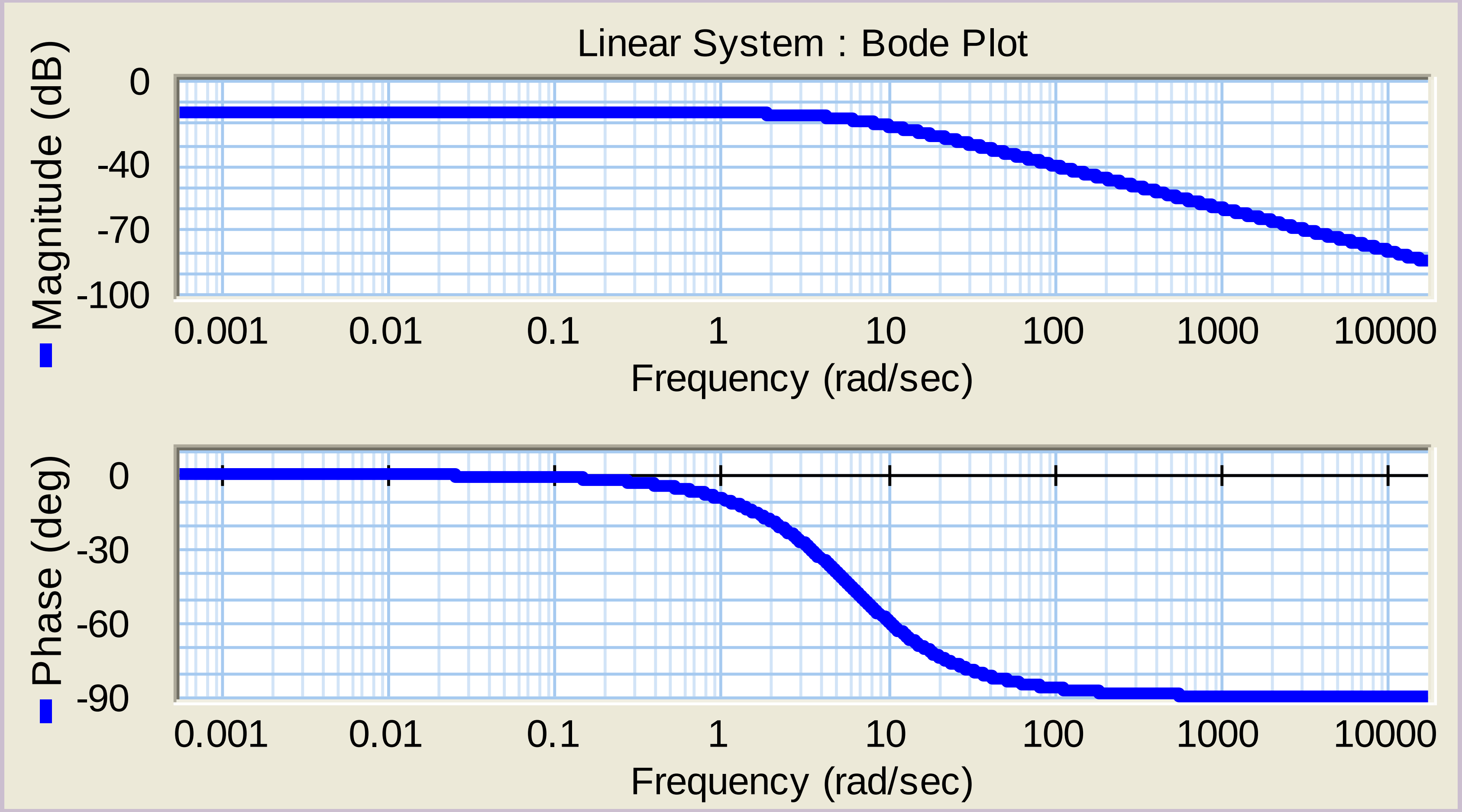
$$\left|H(j\omega)\right| = \frac{1}{\sqrt{\mathrm{Re}\left\{H(j\omega)\right\}^2 + \mathrm{Im}\left\{H(j\omega)\right\}^2}} = \frac{1}{\sqrt{\left(\omega R_1 C_1\right)^2 + 1^2}}$$

$$\varphi = Arg\left\{H(j\omega)\right\} = 1 - \arctan\frac{\mathrm{Im}\left\{H(j\omega)\right\}}{\mathrm{Re}\left\{H(j\omega)\right\}} = 0 - \arctan\omega R_1 C_1$$

Belangrijke punten

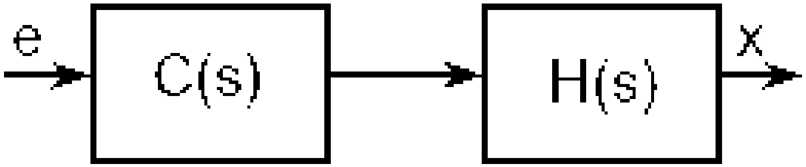
- $\omega \rightarrow 0$
- $\omega \rightarrow \infty$
- ω = kantelfrequenties

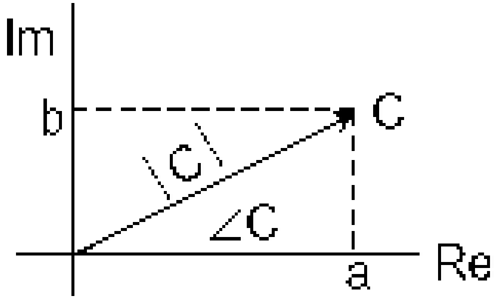
- $Fase = \left(\frac{90}{20}\right) * (helling\ van\ de\ amplitude)$


Linear System : Bode Plot
Magnitude (dB)
0
-40
-70
-100
0.001
0.01
0.1
1
10
100
1000
10000
Frequency (rad/sec)
Phase (deg)
0
-30
-60
-90
0.001
0.01
0.1
1
10
100
1000
10000
Frequency (rad/sec)

Term	Magnitude	Phase
Constant: K	$20\log_{10}(\|K\|)$	K>0: 0° K<0: ±180°
Pole at Origin (Integrator) $\frac{1}{s}$	-20 dB/decade passing through 0 dB at ω=1	-90°
Zero at Origin (Differentiator) s	+20 dB/decade passing through 0 dB at ω=1 *(Mirror image, around x axis,of Integrator)*	+90° *(Mirror image, around x axis, of Integrator about)*
Real Pole $$\frac{1}{\frac{s}{\omega_0}+1}$$	1. Draw low frequency asymptote at 0 dB 2. Draw high frequency asymptote at -20 dB/decade 3. Connect lines at ω_0.	1. Draw low frequency asymptote at 0° 2. Draw high frequency asymptote at -90° 3. Connect with a straight line from $0.1\cdot\omega_0$ to $10\cdot\omega_0$
Real Zero $$\frac{s}{\omega_0}+1$$	1. Draw low frequency asymptote at 0 dB 2. Draw high frequency asymptote at +20 dB/decade 3. Connect lines at ω_0. *(Mirror image, around x-axis, of Real Pole)*	1. Draw low frequency asymptote at 0° 2. Draw high frequency asymptote at +90° 3. Connect with a straight line from $0.1\cdot\omega_0$ to $10\cdot\omega_0$ *(Mirror image, around x-axis, of Real Pole about 0°)*
Underdamped Poles (Complex conjugate poles) $$\frac{1}{\left(\frac{s}{\omega_0}\right)^2+2\zeta\left(\frac{s}{\omega_0}\right)+1}$$ $$0<\zeta<1$$	1. Draw low frequency asymptote at 0 dB 2. Draw high frequency asymptote at -40 dB/decade 3. If ζ<0.5, then draw peak at ω_0 with amplitude $\|H(j\omega_0)\|=-20\cdot\log_{10}(2\zeta)$, else don't draw peak 4. Connect lines	1. Draw low frequency asymptote at 0° 2. Draw high frequency asymptote at -180° 3. Connect with straight line from $$\omega=\frac{\omega_0}{10^\zeta}\text{ to }\omega_0\cdot 10^\zeta$$ *You can also look in a textbook for examples*
Underdamped Zeros (Complex conjugate zeros) $$\left(\frac{s}{\omega_0}\right)^2+2\zeta\left(\frac{s}{\omega_0}\right)+1$$ $$0<\zeta<1$$	1. Draw low frequency asymptote at 0 dB 2. Draw high frequency asymptote at +40 dB/decade 3. If ζ<0.5, then draw peak at ω_0 with amplitude $\|H(j\omega_0)\|=+20\cdot\log_{10}(2\zeta)$, else don't draw peak 4. Connect lines *(Mirror image, around x-axis, of Underdamped Pole)*	1. Draw low frequency asymptote at 0° 2. Draw high frequency asymptote at +180° 3. Connect with straight line from $$\omega=\frac{\omega_0}{10^\zeta}\text{ to }\omega_0\cdot 10^\zeta$$ *You can also look in a textbook for examples.* *(Mirror image, around x-axis, of Underdamped Pole)*

The amplitude story





$$C = a + bj \qquad H = c + dj \qquad CH = ac + bcj + adj - bd$$

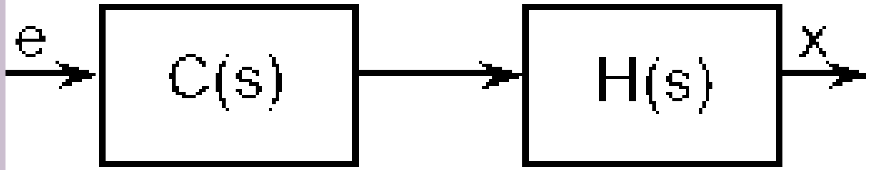
$$\mathrm{Re}(C) = a \qquad \mathrm{Re}(H) = c \qquad \mathrm{Re}(CH) = ac - bd$$

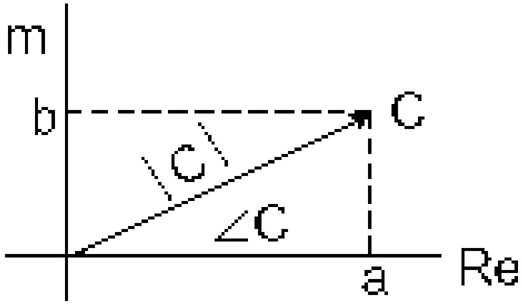
$$\mathrm{Im}(C) = b \qquad \mathrm{Im}(H) = d \qquad \mathrm{Im}(CH) = bc + ad$$

$$|C| = \sqrt{a^2 + b^2} \qquad |H| = \sqrt{c^2 + d^2} \qquad |CH| = \sqrt{(a^2 + b^2)\,(c^2 + d^2)}$$

$$\boxed{|C \cdot H| = |C| \cdot |H|}$$

The phase story

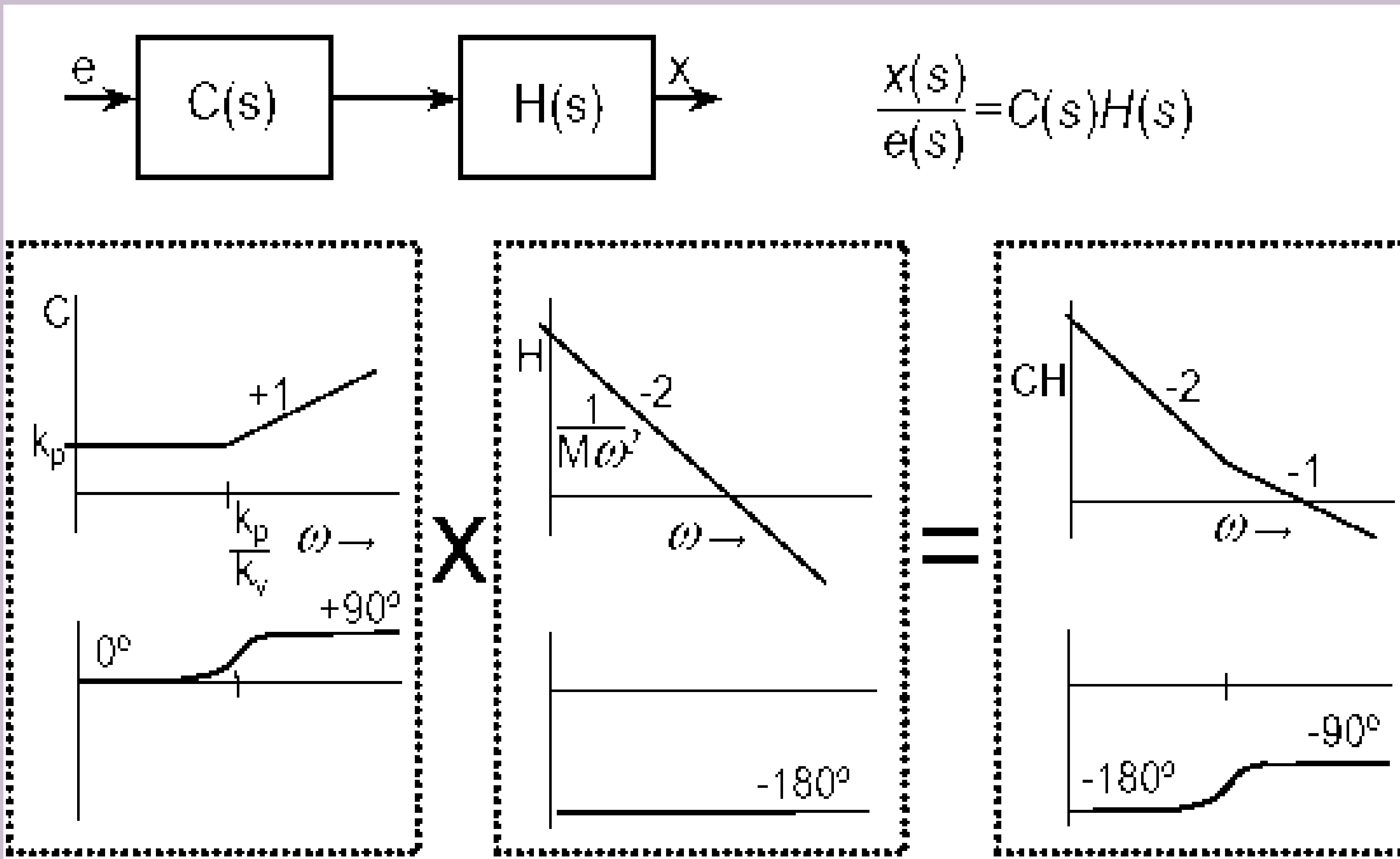




$$C = |C|\cdot[\cos(\varphi) + j\sin(\varphi)] \qquad H = |H|\cdot[\cos(\psi) + j\sin(\psi)]$$

$$\begin{aligned} C\cdot H &= |C|\cdot|H|\cdot[\cos(\varphi) + j\sin(\varphi)]\cdot[\cos(\psi) + j\sin(\psi)] \\ &= |C\cdot H|\cdot\big[\big(\cos(\varphi)\cos(\psi) - \sin(\varphi)\sin(\psi)\big) + \ldots \\ &\qquad \ldots j\,\big(\cos(\varphi)\sin(\psi) + \sin(\varphi)\cos(\psi)\big)\big] \\ &= |C\cdot H|\cdot[\cos(\varphi+\psi) + j\sin(\varphi+\psi)] \end{aligned}$$

$$\left.\begin{array}{l} \varphi = \angle C \\ \psi = \angle H \end{array}\right\} \longrightarrow \boxed{\angle(C\cdot H) = \angle C + \angle H}$$



e
C(s)
H(s)
x
$\frac{x(s)}{e(s)} = C(s)H(s)$
C
+1
k_p
$\frac{k_p}{k_v}$
$\omega \rightarrow$
0^o
$+90^o$
X
H
-2
$\frac{1}{M\omega^2}$
$\omega \rightarrow$
-180^o
=
CH
-2
-1
$\omega \rightarrow$
-180^o
-90^o