PRACTICAL INFORMATION

1) ENERGY

Problem Lecture This Week

- Wednesday 13th 9-11 NO
- Friday 15th 13-15 Yes 2.3.C04

2) BIOMEDIC

Problem Lecture This Week

- Wednesday 13th 11-13 NO
- Tuesday 12th 13-15 Yes 7.1.J05

Thursday everything ok

LAST LESSON?

↳ SETS of NUMBERS

$\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \underbrace{\mathbb{R}}_{\text{SUBSETS}}$

THIS LESSON

↳ ESTABLISH RELATIONSHIPS BETWEEN SETS

FUNCTIONS

def A, B SETS

$$\left.\begin{aligned} f : A &\longrightarrow B \\ x &\longmapsto y = f(x) \end{aligned}\right\} \text{Any Association}$$

$$\forall x \in A \ \exists! \ y = f(x) \in B$$

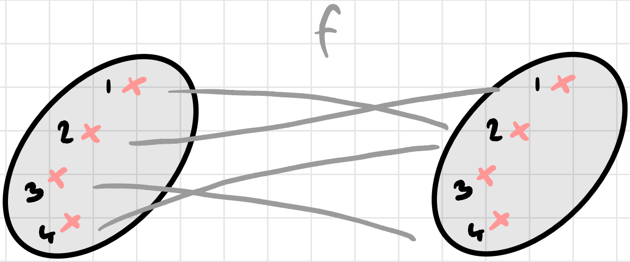
(also CALLED a MAP)

$A \stackrel{def}{=}$ DOMAIN

$B \stackrel{def}{=}$ CODOMAIN

IMAGE or RANGE $\quad \operatorname{Im} f \stackrel{def}{=} \{ y = f(x) \in B : x \in A \}$

EXAMPLE



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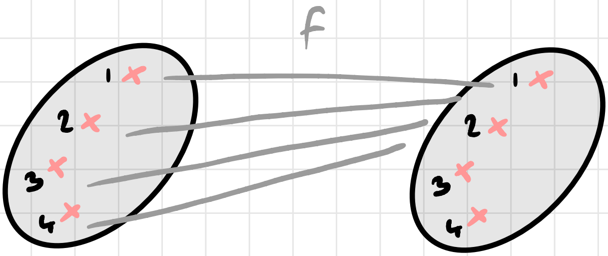
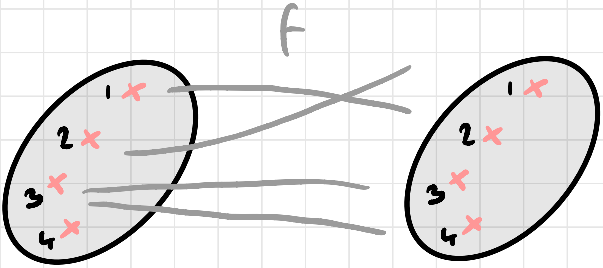


QUESTION: IS THIS a MAP?

↳ yes, CONSTANT



QUESTION IS THIS a MAP? NO (because of definition)

THIS IS a FIRST EXAMPLE of REPRESENTATION

OTHER REPRESENTATIONS ...

REAL FUN.

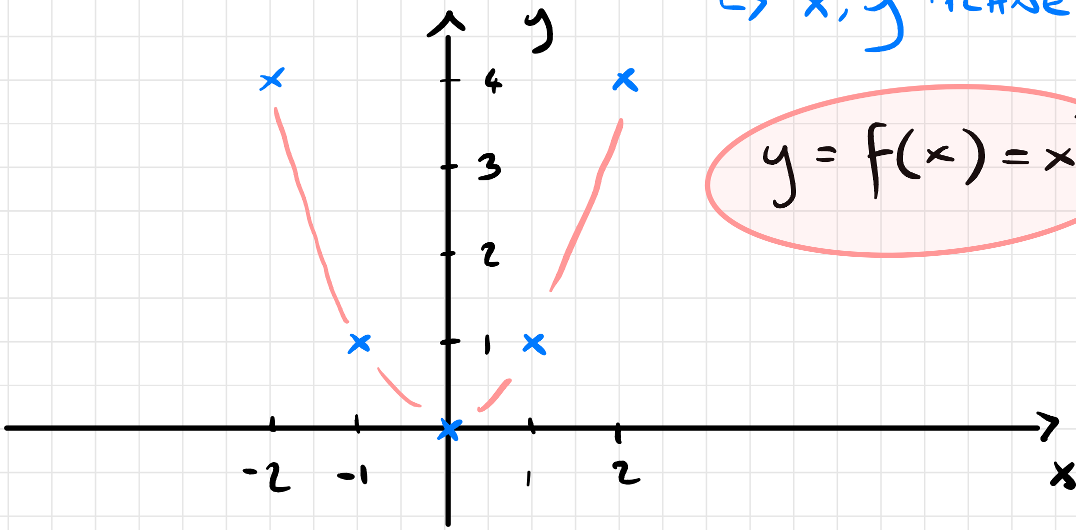
$f : A \subset \mathbb{R} \to \mathbb{R}$

OTHER REPRESENTATIONS

def

GRAPH f $\quad G_f = \{ (x,y) \in \mathbb{R}^2 : x \in A, \, y \in f(x) \}$

↳ x, y PLANE



$y = f(x) = x^2$

$x = 1 \longmapsto y = 1$

$x = -1 \longmapsto y = 1$

$x = 2 \longmapsto y = 4$

$x = -2 \longmapsto y = 4$

$x = 0 \longmapsto y = 0$

THIS IS A PARTICULAR CASE OF

<u>Ex</u> POLYNOMIAL $P_n(x) = \sum_{k=0}^{n} a_k x^k \quad (\underset{\text{degree}}{n \in \mathbb{N}},\ a_k \in \mathbb{R})$

DOMAIN ?

- THE SET ON WHICH THE FUNCTION IS DEFINED
- ROUGHLY SPEAKING : THE NUMBER THE FUNCTION IS ABLE TO EAT

<u>Ex</u> RATIONAL FUN. $f(x) = \dfrac{P_n(x)}{Q_n(x)}$

P, Q POLYNOMIALS

$$D_f = \{ x \in \mathbb{R} : Q_n(x) \neq 0 \}$$

- THIS IS A QUESTION THAT COULD APPEAR in the EXAM
- OTHER PROPERTIES

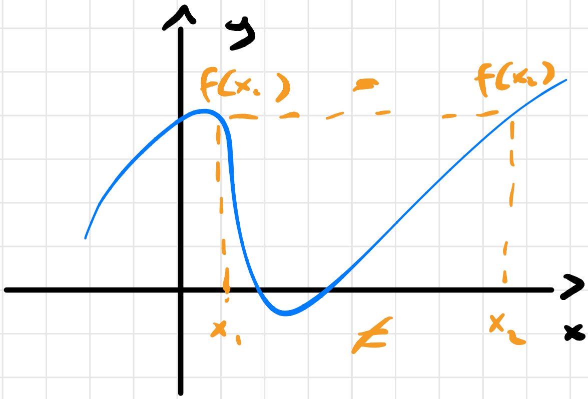
def f INJECTIVE $\iff \forall x_1, x_2 \in A \quad x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

Ex $f(x) = x^2$ INJECTIVE ? NO

$\exists\, x_1 = 1,\ x_2 = -1,\ x_1 \neq x_2,$

But $f(x_1) = f(x_2)$

Ex $f(x) = x^3$ INJECTIVE ? YES



NO INJECTIVE

def f SURJECTIVE $\iff \mathrm{Im}_f = \mathbb{R}$

Ex $f(x) = x^2$ SURJECTIVE ? NO

$\mathrm{Im} f = \{ x \in \mathbb{R} : x > 0 \} \equiv \mathbb{R}^+$

Ex $f(x) = x^3$ SURJECTIVE ? YES

def f BIJECTIVE $\iff f$ INJECTIVE & SURJECTIVE

Ex $f(x) = x^3$ BIJECTIVE

def f EVEN $\iff \forall x \in A \quad f(x) = f(-x)$

def " ODD " " $f(-x) = -f(x)$

Ex $f(x) = x^2$ EVEN

" x^3 ODD

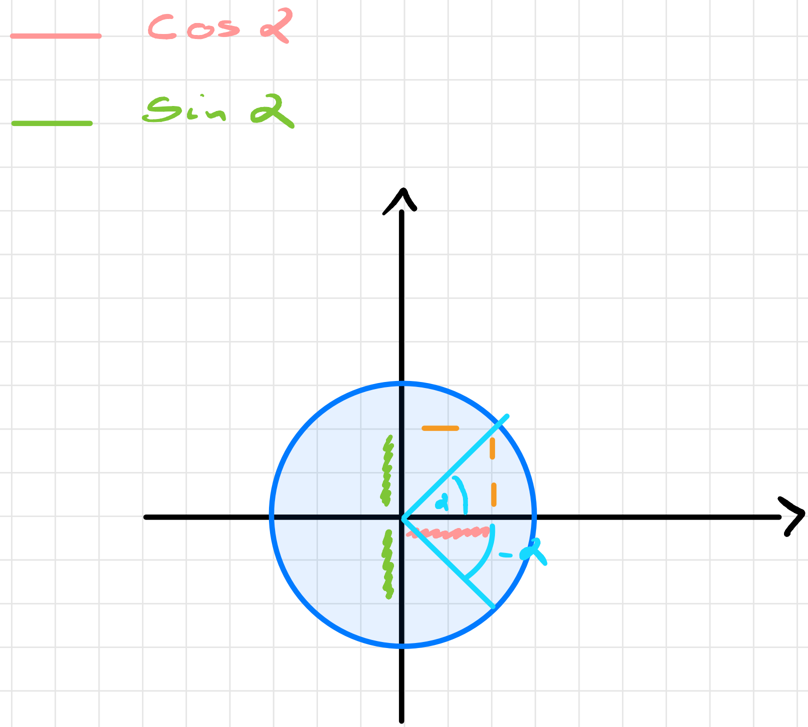
" $|x|$ EVEN

" x ODD (IDENTITY)

" c EVEN (CONSTANT)

WHAT ELSE ?

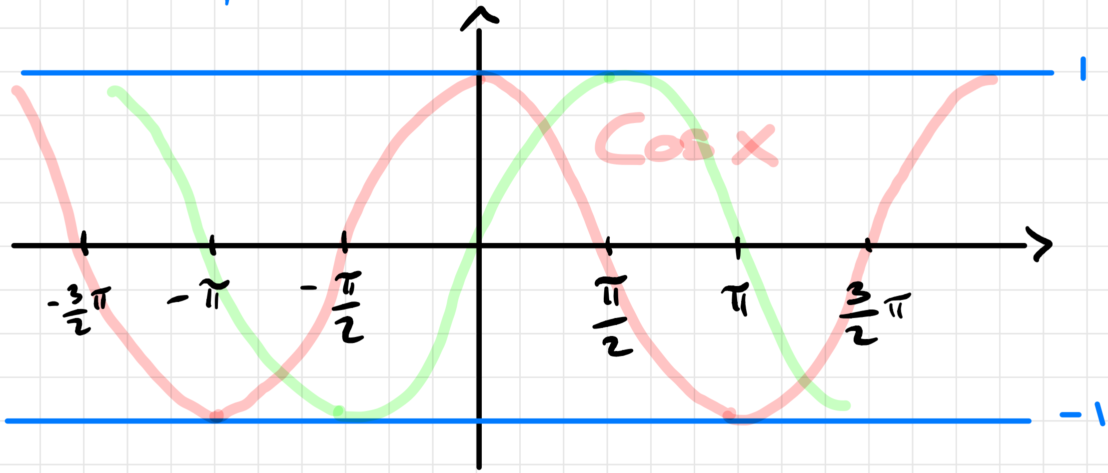
Trigonometric Functions



$\cos(-\alpha) = \cos(\alpha)$ EVEN

$\sin(-\alpha) = -\sin(\alpha)$ ODD

GRAPH of TRIGONOMETRIC FUNCTIONS



GRAPH of
- EVEN f Symm.
- ODD " Antisymm.

<u>QUESTION</u>
- DOMAIN? $\mathbb{R}$
- INJECTIVE? NO

$\exists\, x_1 \neq x_2 \,,\; x_1 = x_2 + 2\pi n \quad (n \in \mathbb{Z})$

$\sin(x_1) = \sin(x_2)$

TRIGONOMETRIC FUNCTIONS HAS THE FOLLOWING PROPERTY

def f PERIODIC $\Leftrightarrow \exists c \in \mathbb{R} : \forall x \in D_f \; f(x+c) = f(x)$

↳ PERIOD

Ex $\sin(x+2\pi) = \sin(x) \quad \forall x \in \mathbb{R}$

QUESTION : sin / cos SURJECTIVE ? NO

$\longrightarrow \mathrm{Im}_f = [-1, 1] \neq \mathbb{R}$

THEY ARE ...

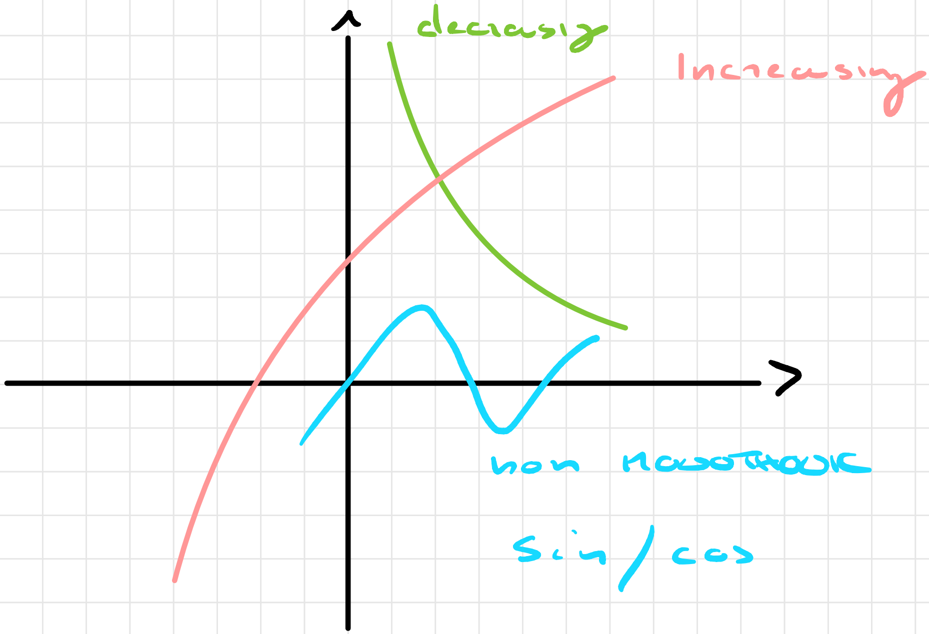
def f BOUNDED $\Leftrightarrow \exists M \in \mathbb{R} \;\; |f(x)| \leq M$

def f MONOTHONIC INCREASING $\Leftrightarrow$
(decreasing)

$$\forall x_1, x_2 \in D_f \quad x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2)$$

$$>\qquad\qquad\qquad >$$

Ex



A PARIDGMATIC EXAMPLE of MONOTHONIC FUNCTION

$$f(x) = e^{x} = (\underbrace{2.71\ldots}_{\text{IRRATIONAL}})^{x} \hookrightarrow \mathbb{R}$$

WE WILL BE BACK on a FORMAL Definition OF THIS FUNCTION

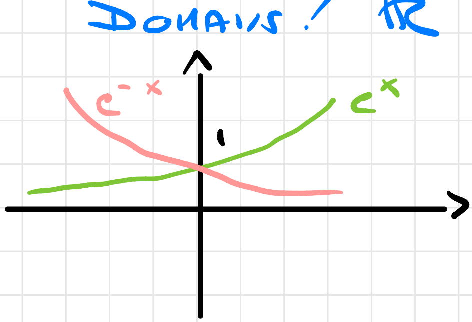
$e^{x} > 0$

$e^{x+y} = e^{x} e^{y}$

$(e^{x})^{y} = e^{xy}$

$e^{-x} = 1/e^{x}$

DOMAIN? $\mathbb{R}$



These are examples of Basic Functions

We can assemble them

OPERATIONS

$$f : A \subset \mathbb{R} \to \mathbb{R} \qquad g : B \subset \mathbb{R} \to \mathbb{R}$$
$$x \longmapsto f(x) \qquad x \longmapsto g(x)$$

→ intersection

$\underline{\underline{\text{def}}}$ $f+g : A \cap B \to \mathbb{R}$

(sum) $x \longmapsto f(x) + g(x)$

$\underline{\underline{\text{Ex}}}$ $\sinh(x) = \frac{1}{2}\left(e^{x} - e^{-x}\right)$

$\cosh$ 〃 〃 $+$

Domain? $\mathbb{R}$, $D_{e^x} = \mathbb{R} \Rightarrow \mathbb{R} \cap \mathbb{R} = \mathbb{R}$

$\underline{\underline{\text{def}}}$ $fg : A \cap B \to \mathbb{R}$

(Product) $x \longmapsto f(x)\,g(x)$

$\underline{\underline{\text{def}}}$ $f/g \quad A \cap B' \to \mathbb{R}$

$x \longmapsto f(x)/g(x)$

$B' = \{x \in B : x \neq 0\}$

Ex $\tan x = \dfrac{\sin x}{\cos x}$

$$D_f = \{ x \in \mathbb{R} : \cos x \neq 0 \}$$
$$= \{ \quad // \quad : \frac{\pi}{2} + k\pi , k \in \mathbb{Z} \}$$

Ex $\cot x = \dfrac{\cos x}{\sin x}$

$$\sec = \frac{1}{\cos x}$$

$$\operatorname{cosec} = \frac{1}{\sin x}$$

HOMEWORK : DOMAIN

$$\underline{\underline{\text{def}}} \quad f \circ g : \left.\begin{aligned} C &\to \mathbb{R} \\ x &\to f(g(x)) \end{aligned}\right) \text{ COMPOSITION}$$

$$C = \{x \in B : g(x) \in A\}$$

$\underline{\underline{\text{Ex}}}$ $\quad f(x) = x^2, \quad g(x) = \sin x$

$$\left.\begin{aligned} f \circ g &= (\sin x)^2 \\ g \circ f &= \sin(x^2) \end{aligned}\right\} \text{ NON COMMUTATIVE}$$

COMPOSITION PAVES THE WAY TO _ _

$\underline{\underline{\text{def}}}$ $f : A \subset \mathbb{R} \to \mathbb{R}$

$f^{-1} : \operatorname{Im} f \to A$ INVERSE

$\Leftrightarrow f^{-1} \circ f = f \circ f^{-1} = \underbrace{id(x)}_{\text{identity}}$

(each number into itself)

Ex $f : \mathbb{R} \to [0, +\infty)$

$x \longmapsto y = e^x$

$f^{-1} : [0, +\infty) \longrightarrow \mathbb{R}$

$y \longmapsto x = \log y$

$$e^{\log x} = x$$

DOMAIN $[0, +\infty)$

Ex $f : [-\frac{\pi}{2}, \frac{\pi}{2}] \longmapsto [-1, 1]$

$x \longmapsto y = \sin x$

$f^{-1} : [-1, 1] \longmapsto [-\frac{\pi}{2}, \frac{\pi}{2}]$

$y \longmapsto x = \arcsin y$

Ex $f : [0, +\infty) \longrightarrow [0, +\infty)$

$$x \longmapsto y = x^2$$

$f^{-1} : [0, +\infty) \longrightarrow [0, +\infty)$

$$y \longmapsto x = \sqrt{y}$$

SQUARE ROOT

$D_{f^{-1}} = [0, +\infty)$

SQUARE ROOT ONLY TAKES POSITIVE X

$Im_{f^{-1}} = [0, +\infty)$

$\Rightarrow \forall x \in [0, +\infty) \quad \sqrt{x} \geq 0$

QUESTION

$\sqrt{x^2} = |x|$