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OFFICE HOURS : $\underbrace{\text{FREE}}$

Whenever you want, provided you send an e-mail

BIBLIOGRAPHY : $\underbrace{\text{JOSÉ A. CUESTA - CALCULUS I}}$

AULA GLOBAL

PROGRAM : 4 BLOCKS

- NUMBERS / FUNCTIONS
- SEQUENCE / SERIES
- DIFFERENTIAL CALCULUS
- INTEGRAL "

EVALUATION : 2 MID-TERM TESTS) 40%

1 FINAL EXAM) 60%

$$M = \underbrace{0.2\, m_1 + 0.2\, m_2}_{40\%} + \underbrace{0.6\, M_f}_{60\%}$$

A few WARNINGS

- HARD WORK
  - A LOT OF TOPICS in JUST A few Months
  - STUDY STEP by STEP
  - do as many EXERCISES AS POSSIBLE
- COUNT ON ME
  - ASK QUESTIONS
  - TUTORSHIP

☒ WHAT IS MATHEMATICS?

→ LANGUAGE

- not so SUITABLE : Cinema
- SUITABLE : Physics
  Biomedic Sciences

☒ LETTER?

→ NUMBERS

Basic Set of Numbers

$\mathbb{N} = \{0, 1, 2, 3 \dots\}$

$10 \in \mathbb{N}$? Yes

$-3 \notin \mathbb{N}$? No

$\in \stackrel{def}{=}$ Belongs, In

Intuition $\longrightarrow$ Count by Hands

Formal Definition

1) $\exists!\ 0 \in \mathbb{N}$

   $\hookrightarrow$ First

2) $\forall n \in \mathbb{N}\ \exists\ \underbrace{n+1}_{\text{Successor}} \in \mathbb{N}$

$\exists \stackrel{def}{=}$ Exists

$\exists! \stackrel{def}{=}$ " one and only one

$\forall \stackrel{def}{=}$ For All

NUMBERS

↳ QUANTIFY

↳ COMPARE, ESTABLISH HIERARCHY

ORDER RELATIONSHIP $\leq$

- $m \leq m \quad \forall m \in \mathbb{N}$ (REFLEXIVITY)
- $n \leq m$ and $m \leq n \quad \forall m, n \in \mathbb{N}$

  $\Rightarrow m = n$ (ANTISYMMETRY)

  ↳ IMPLIES
- $n \leq m \leq k \Rightarrow n \leq k \quad \forall$ - - (TRANSITIVITY)

WE CAN INTRODUCE OPERATIONS TO COMBINE NUMBERS...

ADDITION +

SKIP

- $m + n = n + m \quad \forall \ldots$ (COMMUTATIVITY)
- $m + (n+k) = (m+n) + k$ (ASSOCIATIVITY)
- $m + 0 = m$ (NEUTRAL)

Compare with ORDER RELATIONSHIP...

$$n \leq m \Rightarrow n+k \leq m+k \quad \forall n, m, k \in \mathbb{N}$$

QUESTION for YOU

$$2 + n = 5 \Rightarrow n = 3 \in \mathbb{N} \quad \checkmark$$

$$2 + n = 0 \Rightarrow n = -2 \notin \mathbb{N} \quad \times$$

NO SOLUTION in $\mathbb{N}$

We introduce another set of numbers

$\mathbb{Z} = \{ \ldots -3, -2, -1, 0, 1, 2 \ldots \}$

INTEGERS

$+$ → Previous properties are valid

↳ new one

$\forall k \in \mathbb{Z} \ \exists -k \in \mathbb{Z} : k - k = 0$

↳ inverse

$\mathbb{Z}$ closed to $+$

NEW OPERATION

Multiplication $\times$, $\cdot$

COMMUTATIVE? YES $ab = ba \ \forall a, b \in \mathbb{Z}$

ASSOCIATIVE? " $a(bc) = (ab)c$ "

NEUTRAL? " $\exists 1 \in \mathbb{Z} : a \cdot 1 = a$

Compare with Order Rel. $m \leq n \Rightarrow mk \leq nk \ (k > 0)$

" $\geq$ " "<"

INVERSE? NO

<u>EXAMPLE</u> $3 + 2m = 0 \Rightarrow m = -\frac{3}{2} \notin \mathbb{Z}$

WE INTRODUCE ANOTHER OPERATION

$\mathbb{Q} = \{ \frac{m}{n} : m, n \in \mathbb{Z}, n \neq 0 \}$ RATIONALS

$\forall q \in \mathbb{Q} / \{0\} \ \exists p \in \mathbb{Q} : qp = 1$

$\longrightarrow$ $\mathbb{Q}$ CLOSED $+$, $\times$

PROPERTY JOINING TWO OPERATIONS

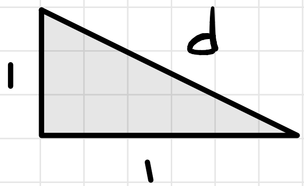
$\forall a, b, c \in \mathbb{Q} \quad a(b+c) = ab + ac$

(DISTRIBUTIVE)

OVERALL $(\mathbb{Q}, +, \times)$ FIELD

THIS WAS THE SITUATION until ANCIENT GREECE

IN THAT PERIOD, PEOPLE (ACTUALLY a GUY CALLED PITAGORA) STRUGGLED with THIS PROBLEM



$$d^2 = 1^2 + 1^2 = 2$$

→ NO SOLUTION in $\mathbb{Q}$

$$d = \sqrt{2} \notin \mathbb{Q}$$

$$= 1,41\underbrace{\text{-- -- --}}_{\infty \text{ non-periodic}}$$

RAUGHLY SPEAKING

RATIONALS → PERIODIC DIGITS

Ex $q = \frac{1}{3} = 0.33\ldots$

IRRATIONALS → NON PERIODIC

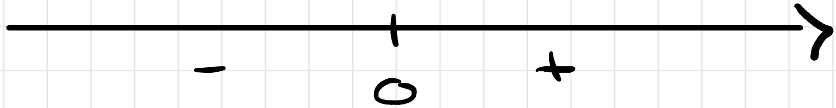
Ex $\pi = 3.14\ldots$

We introduce - - -

$\mathbb{R} = \mathbb{Q}$ COMPLETED by IRRATIONALS

I am going TO CLARIFY THIS WORD in a while

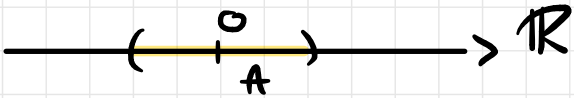
$\longrightarrow$ $\mathbb{R} \sim$ LINE



Let's try to clarify what "complete" means

Before a few definitions

$A \subset \mathbb{R}$, $A \neq \emptyset$, $x \in \mathbb{R}$

Ex 

def x upper bound $A \overset{def}{\iff} \forall a \in A \quad x \geq a$

x lower " " $\overset{def}{\iff}$ " " $\leq$ "

in the first case we say

- A upper bounded

in the second one

- A lower bounded

} Bounded

def $x = \sup A$ (supremum) $\overset{def}{\iff}$ x least upper bound

$x = \inf A$ (infimum) $\overset{def}{\iff}$ x max lower "

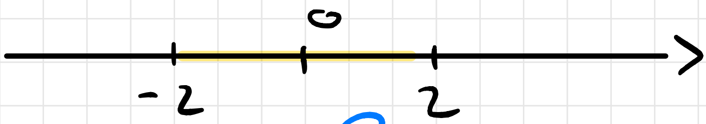
def $x = \sup A \in A \iff x \overset{def}{=} \max A$

" $\inf$ " " " " " $\min$ "

Ex $A = \{x \in \mathbb{R} : x^2 < 4\}$

$= \{x \in \mathbb{R} : -2 < x < 2\}$

$= (-2, 2)$ $\boxed{\text{OPEN}}$



$\sup A \overset{?}{=} 2 \overset{?}{\neq} \max A$ NO

$\inf A \overset{?}{=} 2 \overset{?}{\neq} \min A$ NO

Ex $A = \{x \in \mathbb{R} : x^2 \leq 4\}$

$= [-2, 2]$ $\boxed{\text{CLOSED}}$

$\sup A = 2 = \max A$

$\inf A = -2 = \min A$

Ex $A = (-2, 2]$ $\boxed{\text{SEMI OPEN}}$

$\sup A = 2 = \max A$

$\inf A = -2 \neq \min A$

WE CAN STATE

COMPLETENESS THEOREM

$A \subset \mathbb{R}$, $A \neq \emptyset$

i) A UPPER BOUNDED $\Rightarrow \exists \sup A$

ii) " LOWER " " " $\inf A$

Ex $A' = \{ x \in \mathbb{Q} : x^2 < 2 \}$

neither infimum, nor supremum

Induction Principle

- only valid for $\mathbb{N}$
- method to prove statements
  TRUE or FALSE

Ex $3 > 1$
$n^2 > 0$ } Statement / Proposition

Today is a Beautiful day

Hy 1 $\quad p(0)$ TRUE

Hy 2 $\quad \exists n \in \mathbb{N} \quad p(n)$ TRUE
$\Rightarrow p(n+1)$ TRUE

$\Rightarrow$ Th $\quad p(n)$ TRUE $\forall n \in \mathbb{N}$

Ex Prove that $\underbrace{\sum_{i=1}^{n} i = \frac{n(n+1)}{2}}_{p(n)}$ $\forall n \in \mathbb{N}$

$$\sum_{i=1}^{n} i = 1 + 2 + 3 + \dots + n$$

$$p(1) = \sum_{i=1}^{1} i = 1 = \frac{1\,\cancel{(1+1)}}{\cancel{2}} \quad \text{TRUE}$$

JUST REPLACE n by 1 in $p(n)$

Hy $\exists n \in \mathbb{N} : p(n)$ TRUE

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\underbrace{\Rightarrow p(n+1) \text{ TRUE}}_{\text{THIS IS WHAT you have to PROVE}} \; ?$$

$p(n+1)$?

JUST REPLACE $n+1$ into p

$$\sum_{i=1}^{n+1} i = \frac{(n+1)(n+2)}{2}$$

Are you ABLE TO GET $p(n+1)$ knowing previous Hypothesis?

$$\sum_{i=1}^{n+1} i = \underbrace{1+2+3+\_\,\_\,\_+n}_{\sum_{i=1}^{n} i}+n+1$$

$$= \underbrace{\sum_{i=1}^{n} i}_{\frac{n(n+1)}{2}} + (n+1) \quad \text{Hy}$$

$$= (n+1) + \frac{n(n+1)}{2} = (n+1)\left(1+\frac{n}{2}\right)$$

$$= \frac{1}{2}(n+1)(n+2)$$

DISTRIBUTIVE

$$\sum_{i=1}^{n+1} i = \frac{1}{2}(n+1)(n+2)$$

$p(n) \Rightarrow p(n+1)$

$p(n) \; \forall n \in \mathbb{N}$