Ej 2.3.1 (i)

$f(x,y) = x^2 + y^2$

Extremos en $A = \{\underbrace{xy}_{g} = 1\}$ → vínculo

Primer Paso → ∇ → $\nabla f = (2x, 2y)$; $\nabla g = (y, x)$

Segundo Paso → sistema $\begin{cases} \nabla f = \lambda \nabla g \\ g(x,y) = 1 \end{cases}$

Se obtiene

$$\begin{cases} 2x = \lambda y & a \\ 2y = \lambda x & b \\ xy = 1 & c \end{cases}$$

$a \rightarrow \lambda = \frac{2x}{y}$, $b \rightarrow \lambda = \frac{2y}{x}$

$\rightarrow \frac{2x}{y} = \frac{2y}{x} \rightarrow x^2 = y^2 \rightarrow x = \pm y$

$x = y \rightarrow x^2 = 1 \rightarrow x = \pm 1$

$x = -y \rightarrow -x^2 = 1$ (IMPOSIBLE)

Posible estremos $(1, 1)$, $(-1, -1)$

Ej 2.3.2

$$f(x,y) = x^2 + y^2 + 6x - 8y + 25$$

Extremos en $D = \{x^2 + y^2 \leq 16\}$

Primero → Puntos críticos

→ ∇f

$$\nabla f = (2x + 6, 2y - 8)$$

$$\nabla f = 0 \rightarrow x = -3, \; y = 4$$

$$\rightarrow (-3)^2 + (4)^2 \geq 16$$

$$\rightarrow (-3, 4) \notin D$$

Vamos a ver que pasa en ∂D

$$\partial D = \{x^2 + y^2 = 16\}$$

$$\rightarrow g(x,y) = x^2 + y^2 - 16 = 0$$

$$\nabla g = (2x, 2y)$$

Tengo que solucionar el sistema

$$\begin{cases} 2x + 6 = 2\lambda x & a \\ 2y - 8 = 2\lambda y & b \\ x^2 + y^2 = 16 & c \end{cases}$$

$$a \rightarrow \quad 3 = (\lambda - 1)x$$

$$b \rightarrow \quad 4 = (1 - \lambda)y$$

$$\frac{a}{b} \rightarrow \quad \frac{x}{y} = -\frac{3}{4} \quad \rightarrow \quad x = -\frac{3}{4}y$$

$$c \rightarrow \quad y^2 + \left(\frac{3}{4}\right)^2 y^2 = 16$$

$$y^2\left(1 + \left(\frac{3}{4}\right)^2\right) = 16$$

$$y^2\left(\frac{4^2 + 3^2}{4^2}\right) = 4^2$$

$$y = \pm \frac{4^2}{\sqrt{4^2 + 3^2}} = \pm \frac{16}{\sqrt{16 + 9}} = \pm \frac{16}{5}$$

$$y = \begin{cases} +\frac{16}{5} \longrightarrow x = -\frac{3}{4}\frac{16}{5} = -\frac{12}{5} \\ -\frac{16}{5} \longrightarrow x = \left(-\frac{3}{4}\right)\left(-\frac{16}{5}\right) = \frac{12}{5} \end{cases}$$

Extremos en la Frontera

$$\left(-\frac{12}{5}, \frac{16}{5}\right), \quad \left(\frac{12}{5}, -\frac{16}{5}\right)$$

¿Cuál es el MAX o MIN?

$$f\left(-\frac{12}{5}, \frac{16}{5}\right) = \left(-\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2 - \frac{12}{5} \cdot 6$$

$$- 8\left(\frac{16}{5}\right) + 25$$

$$= \frac{1}{5}\left(\frac{12^2}{5} + \frac{16^2}{5} - 12 \cdot 6 - 8 \cdot 16 + 5 \cdot 5\right)$$

$$= \left(\frac{144}{25} + \frac{256}{25} - \frac{72}{5} - \frac{128}{5} + 25\right)$$

$$\approx \left(6 + 10 - 15 - 25 + 25\right)$$

$$= 1$$

Vamos a Calcular el otro

$$f\left(+\frac{12}{5},-\frac{16}{5}\right)=\frac{144}{25}+\frac{256}{25}+\frac{72}{5}+\frac{128}{5}+25$$

$$\approx 6+10+15+25+25\approx 81$$

es el MAX es el MIN.

$\underline{E_j}$ 2.3.4 (ii)

$f(x,y,z) = x+y+z$

Extremos en $S = \{\underbrace{2x^2+3y^2+6z^2}_{g(x,y,z)} = 1\}$

$\nabla f = (1,1,1)$

$\nabla g = (4x, 6y, 12z)$

$$\begin{cases} 1 = 4\lambda x & a \\ 1 = 6\lambda y & b \\ 1 = 12\lambda z & c \\ 2x^2+3y^2+6z^2 = 1 & d \end{cases}$$

$\frac{a}{b} \rightarrow \frac{x}{y} = \frac{6}{4} = \frac{3}{2} \rightarrow y = \frac{2}{3}x$

$\frac{a}{c} \rightarrow \frac{x}{z} = \frac{12}{4} = 3 \rightarrow z = \frac{x}{3}$

$d \rightarrow 2x^2 + \frac{4}{3}x^2 + \frac{2}{3}x^2 = 1$

$\rightarrow \left(2 + \frac{4}{3} + \frac{2}{3}\right)x^2 = 1 \rightarrow x = \pm\sqrt{\frac{3}{12}} = \pm\frac{1}{2}$

$$x \quad \begin{array}{l} \frac{1}{2} \longrightarrow y = \frac{2}{3}\,\frac{1}{2} = \frac{1}{3}, \quad z = \frac{1}{6} \\[2ex] -\frac{1}{2} \longrightarrow y = -\frac{1}{3}, \quad z = -\frac{1}{6} \end{array}$$

Extremos $\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{6}\right)$

$$\left(-\frac{1}{2}, -\frac{1}{3}, -\frac{1}{6}\right)$$

Se ve claramente que uno es el mínimo y el otro el máximo

E) 2.3.7

$$f(x,y,z) = x + 2y + 3z$$

EXTREMOS en $\left.\begin{array}{l} x^2+y^2 = 2 \\ x + z = 1 \end{array}\right\}$ 2 vínculos

$$g_1(x,y) = x^2 + y^2 - 2$$
$$g_2(x,y) = x + z - 1$$

$$\nabla f = (1, 2, 3)$$
$$\nabla g_1 = (2x, 2y, 0)$$
$$\nabla g_2 = (1, 0, \ )$$

¿cómo se trata?

$$\nabla f = \lambda \nabla g_1 + \mu \nabla g_2$$

$$\begin{cases} 1 = 2\lambda x + \mu & a \\ 2 = 2\lambda y & b \quad \longrightarrow \quad \lambda \neq 0, \ y \neq 0 \\ 3 = \mu & c \\ x^2 + y^2 = 2 & d \\ x + z = 1 & e \end{cases}$$

$c \to \underline{\mu = 3}$

$$a \to \left.\begin{aligned} 1 &= 2\lambda x + 3 \\ -\cancel{2} &= \cancel{2}\lambda x \end{aligned}\right\} \to x = -\frac{1}{\lambda}$$

$b \to y = \frac{1}{\lambda}$

$d \to \frac{2}{\lambda^2} = 2 \to \lambda = \pm 1$

$$\lambda \begin{cases} \to +1 \to x = -1, \ y = +1, \ z = 2 \\ \to -1 \to x = 1, \ y = -1, \ z = 0 \end{cases}$$

$f(-1, 1, 2) = 7$ MAX

$f(1, -1, 0) = -1$ MIN