Ej 3.1.1 (iii) (c)

$$\iint_Q \sin^2 x \sin^2 y \, dx\, dy$$ FACTORIZA

$$Q = [0, \pi] \times [0, \pi]$$

$$\left(\int_0^\pi \sin^2 x \, dx\right)\left(\int_0^\pi \sin^2 y \, dy\right)$$

$\hookrightarrow$ variable muta

$$= \left(\int_0^\pi \sin^2 x \, dx\right)^2$$

$$\int_0^\pi \sin^2 x \, dx = \int_0^\pi \frac{1 - \cos(2x)}{2} \, dx$$

$$\frac{1}{2}\left(\pi - \overbrace{\int_0^\pi \cos(2x) \, dx}^{0}\right)$$

$$\iint_Q \sin^2 x \sin^2 y \, dx\, dy = \left(\frac{\pi}{2}\right)^2$$

E) 3.1.1 (iv) (d)

$$\iint_Q \sin(x+y)\,dx\,dy$$

$$Q = [0, \tfrac{\pi}{2}] \times [0, \tfrac{\pi}{2}]$$

$$\int_0^{\frac{\pi}{2}} \sin(x+y)\,dx = \int_y^{\frac{\pi}{2}+y} \sin(u)\,du$$

$$= \cos(y) - \cos(\tfrac{\pi}{2}+y)$$

$$\int_0^{\pi} dy \left(\cos y - \cos(\tfrac{\pi}{2}+y)\right)$$

$$= \underbrace{-\cos(y)\Big|_0^{\frac{\pi}{2}}}_{1} + \underbrace{\cos(u)\Big|_{\frac{\pi}{2}}^{\pi}}_{1} = 2$$

$\underbrace{\text{este no FACTORIZA}}$, el anterior sí

en realidad puede FACTORIZAR si APLICO FÓRMULA TRIGONOM.

E) 3.1.2 (ii)

$$\iint_Q xy - x^3 \, dx\, dy$$

$$Q = \{(x,y) \in \mathbb{R}^2,\ x \in [0,1],\ -1 \leq y \leq x\}$$

$$\int_0^1 dx \left(\int_{-1}^{x} dy \ \ xy - x^3 \right)$$

$$= \int_0^1 dx \left(x \frac{y^2}{2} \Big|_{-1}^{x} - x^3 y \Big|_{-1}^{x} \right)$$

$$= \int_0^1 dx \ -\frac{x^3}{2} - \frac{x}{2} - x^4 - \cancel{x^3}$$

$$= -\int_0^1 dx \left(x^4 + \frac{x^3}{2} + \frac{x}{2} \right) =$$

$$= -\left(\frac{x^5}{5} + \frac{x^4}{8} + \frac{x^2}{4} \right)\Big|_0^1 = -\left(\frac{1}{5} + \frac{1}{8} + \frac{1}{4} \right) = -\frac{8+5+10}{40}$$

$$= -\frac{23}{40}$$

Ej 3.1.3 (i) (a)

demonstrar (sin solucionar Explicitamente la integral)

$$4\pi \leq \int_D (x^2+y^2+1)\,dx\,dy \leq 20\pi$$

$$D = \underbrace{\{(x,y)\in\mathbb{R}^2 \; x^2+y^2=2^2\}}_{\text{DISCO RADIO 2}}$$

$$\int_D f\,dx\,dy \leq \underbrace{\max_D f}_{5} \underbrace{\iint dx\,dy}_{2\cdot 2\pi} = 20\pi$$

$2\cdot 2\pi \to$ CIRCUNFERENCIA

$\max_D (x^2+y^2+1)$ ← reemplazo max en D que es el radio al cuadrado

$= 4 + 1 = 5$

el otro lado lo obtengo con el

Ej 3.1.6

$$\iint_R \frac{1}{\sqrt{1-x^2}}\,dx\,dy$$

$$R = \{(x,y)\in \mathbb{R}^2 : x^2+(y-1)^2 \le 1 \, , \, x \ge 0\}$$

$$(y-1)^2 \le 1-x^2$$

$$y-1 \le \pm\sqrt{1-x^2}$$

$$-\sqrt{1-x^2} \le y-1 \le +\sqrt{1-x^2}$$

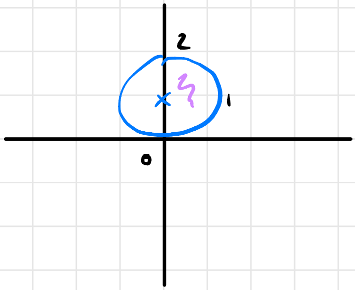
estoy en un círculo $\to$ max es 1

$$1-\sqrt{1-x^2} \le y \le 1+\sqrt{1-x^2}$$

$$\int_0^1 dx \int_{1-\sqrt{1-x^2}}^{1+\sqrt{1-x^2}} \frac{1}{\sqrt{1-x^2}}\,dy$$

$$= \int_0^1 dx \, \frac{1}{\cancel{\sqrt{1-x^2}}}\, 2\left(\cancel{1}+\cancel{\sqrt{1-x^2}}-\cancel{1}+\cancel{\sqrt{1-x^2}}\right) = 2$$

Lo podría hacer antes en el OTRO ORDEN?



$$x^2 + (y-1)^2 \leq 1$$

$$x^2 \leq 1 - (y-1)^2$$

$$x^2 \leq 1 - y^2 + 2y - 1 = 2y - y^2$$

$$\int_0^2 dy \underbrace{\int_0^{\sqrt{2y-y^2}} dx}_{x \geq 0} \frac{1}{\sqrt{1-x^2}}$$

$$= \underbrace{\int_0^2}_{\text{diametro círculo}} dy \ \arcsin \sqrt{2y - y^2}$$ difícil

misma pregunta para

$$g(x,y) = \sin(y-1)$$

$$\int_0^1 dx \int_{1-\sqrt{1-x^2}}^{1+\sqrt{1-x^2}} \sin(y-1)\, dy$$

$$u = y-1 \quad , \quad du = dy$$

$$1 \pm \sqrt{1-x^2} \longmapsto \pm\sqrt{1-x^2}$$

$$\int_0^1 dx \underbrace{\int_{-\sqrt{1-x^2}}^{+\sqrt{1-x^2}} \sin(u)\, du}_{=0} = 0$$

INTEGRAL FUNCIÓN ANTISIMÉTRICA
en intervalo simétrico

Si intercambio orden

$$\int_0^2 dy \int_0^{\sqrt{2y-y^2}} dx \; \sin(y-1) = \underbrace{\int_0^2 dy \sqrt{2y-y^2}\, \sin(y-1)}_{\text{difícil}}$$

Ex 3.1.9 (iii)

$$\int_W (2x+3y+z)\,dx\,dy\,dz$$

$$W = [1,2]\times[-1,1]\times[0,1]$$

$$\int_0^1 dz \int_{-1}^{1} dy \int_1^2 dx\,(2x+3y+z)$$

$$u = 2x+3y+z, \qquad du = 2\,dx$$

$$2 \mapsto 4+3y+z, \qquad 1 \mapsto 2+3y+z$$

$$\int_{2+3y+z}^{4+3y+z} \frac{du}{2}\,u = \frac{1}{2}\frac{1}{2}\left[(4+3y+z)^2 - (2+3y+z)^2\right]$$

$$= \frac{1}{2^2}\left[16 + \cancel{9y^2} + \cancel{z^2} + 24y + 8z + \cancel{6yz}\right.$$
$$\left. - 4 - \cancel{9y^2} - \cancel{z^2} - 12y - 4z - \cancel{6yz}\right]$$

$+\ 12 \qquad + \qquad +$

$$= 3 + 3y + z$$

$$\int_{-1}^{1} dy \ (3 + 3y + z)$$

$$u = 3 + 3y + z \quad , \quad du = 3dy$$

$$1 \longmapsto 6 + z \quad , \quad -1 \longmapsto z$$

$$\int_{z}^{z+6} \frac{du}{3} \ u = \frac{1}{6} u^2 \Big|_{z}^{z+6}$$

$$= \frac{1}{6} \left[z^2 + 36 + 12z - z^2 \right]$$

$$= 2z + 6$$

$$\int_{0}^{1} 2z + 6 \ dz = 2 \int_{0}^{1} (z + 3) \, dz = 7$$

Ex 3.1.9 (ii)

$$\int_W dx\,dy\,dz\; e^{-xy}\, y$$

$$W = [0,1] \times [0,1] \times [0,1]$$

$$\int_0^1 dx \int_0^1 dy \int_0^1 dz\; e^{-xy}\, y$$

$$= \int_0^1 dx \int_0^1 dy\; e^{-xy}\, y$$

$$= \int_0^1 dy\; y \int_0^1 e^{-xy}\, dx$$

$$= \int_0^1 dy\; y\, \frac{1}{y} \left(e^{-y \cdot 0} - e^{-y} \right)$$

$$= \int_0^1 dy\; e^{-y} = e^{-0} - e^{-1} = \frac{1}{e}$$

Ej 3.1.9 (iv)

$$\int z e^{x+y} \, dz \, dx \, dy$$

$W = [0,1] \times [0,1] \times [0,1]$

Puede FACTORIZAR

→ VARIABLE MUTA

$$\underbrace{\left(\int_0^1 dz \; z \right)}_{\frac{1}{2}} \underbrace{\left(\int_0^1 e^x \, dx \right) \left(\int_0^1 e^y \, dy \right)}_{\left(\int_0^1 e^x \, dx \right)^2}$$

$$= \frac{1}{2} (e - 1)^2$$

Comentar que se puede FACTORIZAR

E) 3.1.10

$$\int_W x^2 \cos(x)\, dW$$

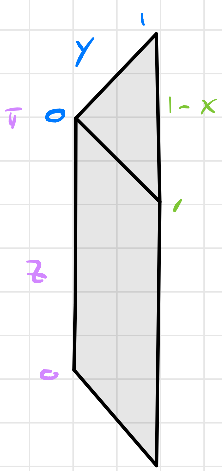
Región entre los planos

$z = 0 \quad , \quad z = \pi$

$y = 0 \quad , \quad y = 1$

$x = 0 \quad , \quad x + y = 1$

Vamos a dibujarlo



LA INTEGRAL se escribe como

$$\int_0^{\pi} dz \int_0^1 dx \int_0^{1-x} dy \; x^2 \cos(x) \, dx$$

$$= \int_0^{\pi} dz \int_0^1 dx \; x^2 (1-x) \cos(x) \, dx$$

$$= \int_0^1 \pi \, x^2 (1-X) \cos(x) \, dx \; dx$$

$$= \pi \int_0^1 x^2 \cos(x) \, dx \Big) \, a$$

$$- \pi \int_0^1 x^3 \cos(x) \, dx \Big) \, b$$

$\rightarrow$ x Partes

$$\int_0^1 x^2\cos(x)\,dx = x^2\sin(x)\Big|_0^1 - \int_0^1 2x\sin(x)$$

$$= \sin(1) + 2\cos(1) - 2\cos(1) + 2$$

Lo que quiero subraya es que podía TB calcula.

$$\int_0^{\pi} dz \int_0^1 dy \underbrace{\int_0^{1-y} dx\, x^2\cos(x)}_{\text{Tenéis una integral MÁS}}$$