E) 4.1.1 (i)

$$f(x,y) = 2xy^2$$

INTEGRALE CAMINO

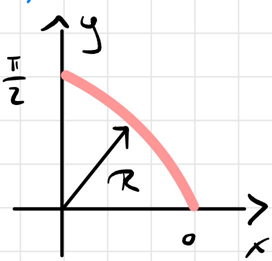
¿dónde? → Primo cuadrante Círculo radio R

Se TRATA de usar una fórmula

→ Vamos a Repetirlo

$$\int_a^b f(c(t))\,\|c'(t)\|\,dt$$

f → La Tenemos, y c?



$$c : [0, \tfrac{\pi}{2}] \to \mathbb{R}^2$$

$$t \longmapsto (R\cos t, R\sin t)$$

Podemos calcular

$$f(c(t)) = 2R^3\cos(t)\sin^2(t)$$

$$c'(t) = (-R\sin t, R\cos(t))$$

$$\|c'(t)\| = \sqrt{R^2(\sin^2 t + \cos^2 t)} = R$$

Tenemos que solucionar

$$\int_0^{\frac{\pi}{2}} 2R^4 \cos(t)\sin^2(t)\,dt$$

¿cómo se soluciona?

→ II Formas (equivalentes)

A) $z = \sin(t)$, $dz = \cos(t)\,dt$

$0 \longmapsto 0$, $\frac{\pi}{2} \longmapsto 1$

$$2R^4\int_0^1 z^2\,dz = \frac{2}{3}R^4$$

B) $$2R^4\int_0^{\frac{\pi}{2}} \frac{\partial}{\partial t}\left(\frac{\sin^3(t)}{3}\right)dt$$

$$= \frac{2R^4}{3}\sin^3(t)\Big|_0^{\frac{\pi}{2}} = \frac{2}{3}R^4$$

Ej 4.1.1 (ii) vamos a $\mathbb{R}^3$

$$f(x,y,z) = (x^2+y^2+z^2)^2$$

Integrale camino

¿cuál?

$$r(t) = (\cos(t), \sin(t), 3t)$$

entre $(1,0,0)$ y $(1,0,6\pi)$

Vamos a escribirlo bien:

$$c : [a,b] \longrightarrow \mathbb{R}^3$$

$$t \longmapsto r(t)$$

$a = ?$, $b = ?$

$a = 0$, $b = 2\pi$

nos sirven para los extremos de las integrales

Lo primero que calculamos

$$f(x,y,z) = (\cos^2 t + \sin^2 t + 9t^2)^2$$

$$= (1 + 9t^2)^2$$

Vamos a CALCULAR AHORA

$$c'(t) = (-\sin(t), \cos(t), 3)$$

$$\|c'(t)\| = \sqrt{(\sin t)^2 + (\cos t)^2 + 9}$$

$$= \sqrt{1+9} = \sqrt{10}$$

Tenemos

Polinomio

$$\sqrt{10} \int_0^{2\pi} (1+9t^2)^2 \, dt$$

$$= \sqrt{10} \int_0^{2\pi} 1 + 81t^4 + 18t^2 \, dt$$

$$= \sqrt{10} \left(t + \frac{81}{5} t^5 + 6t^3 \right) \Big|_0^{2\pi}$$

$$= 2\pi \sqrt{10} \left(1 + \frac{81}{5} (2\pi)^4 + 6(2\pi)^2 \right)$$

Vamos a ver ALGUNA APLICACIÓN

Ej 4.1.2

cuerda con forma

LONGITUD $\underbrace{\text{PARABOLA}}$ de $(0,0)$ a $(2,4)$

→ ¿ cómo se calcula ?

$$\longrightarrow \int_a^b f(c) \, \|c'\| \, dt$$

Lo primero

$$f(x,y) = 1$$ (Medida conjunto → se integra 1)

$$c : [a,b] \to \mathbb{R}^2$$

$$t \longmapsto (t, t^2)$$

$$a = 0 \,, \quad b = 2$$

entonces

$$c'(t) = (1, 2t)$$

$$\|c'(t)\| = \sqrt{1 + 4t^2}$$

entonces

$$L = \underbrace{\int_0^2 \sqrt{1+4t^2}\,dt}$$

¿cómo se calcula?

$$t = \frac{1}{2}\tan\theta, \qquad dt = \frac{1}{2}\frac{d\theta}{\cos^2\theta}$$

$$\sqrt{1+\tan^2\theta} = \frac{1}{\cos\theta} = \sec\theta$$

es un poco liosa

podemos no trasformar y volver después a x

$$0 \longmapsto 0, \qquad 2 \mapsto \arctan(4)$$

OBTENEMOS (CALCULAMOS LA PRIMITIVA)

$$I = \underbrace{\frac{1}{3}\int \sec^3\theta\, d\theta}_{\text{ejercicio}}$$

Vamos a CALCULAR

MASA CUERDA $\rightarrow$ densidad MASA $\rho(x,y) = x$

$$M = \int_0^2 x\sqrt{1+4x^2}\,dx$$

$z = 1 + 4x^2$, $dz = 8x\,dx$

$0 \longmapsto 1$, $2 \longmapsto 17$

$$M = \int_1^{17} \frac{1}{8}\frac{dz}{x} \times \sqrt{z} = \frac{1}{8}\int_1^{17} dz\,\sqrt{z}$$

$$= \frac{1}{8}\,\frac{2}{3}\,z^{\frac{3}{2}}\Big|_1^{17} = \frac{1}{12}\left(17^{\frac{3}{2}} - 1\right) ✓$$

Vamos ahora a los ...

INTEGRALES LINEA

→ ¿qué diferencia?

→ CAMPO VECTORIAL

Ej 4.1.2 (i)

$f(x,y) = (x^2 - 2xy, y^2 - 2xy)$

Linea? $y = x^2$; de $(-1,1)$ a $(1,1)$

$$\int_a^b f(c(t)) \cdot c'(t)\, dt$$

$$c : [-1, 1] \longrightarrow \mathbb{R}^3$$

$$x \longmapsto (x, x^2)$$

$$c'(x) = (1, 2x)$$

$$f(c(x)) = (x^2 - 2x^3, x^4 - 2x^3)$$

$$f(c(x)) \cdot c'(x) = x^2 - 2x^3 + 2x\,(x^4 - 2x^3)$$

Resulta

$$\int_{-1}^{1} dx \left(x^2 - \overset{\to 0}{2x^3} + \overset{\to 0}{2x^5} - 4x^4 \right)$$

$$= \left.\frac{x^3}{3}\right|_{-1}^{1} - 4 \left.\frac{x^5}{5}\right|_{-1}^{1}$$

$$= 2\left(\frac{1}{3} - \frac{4}{5}\right) = 2\,\frac{5-12}{15} = -\frac{14}{15}$$

E) 4.1.3 (iii)

$$F(x,y,z) = (y^2 - z^2, 2yz, -x^2)$$

$$r(t) = (t, t^2, t^3), \quad t \in [0,1]$$

$$\int_0^1 dt\, F(r(t)) \cdot r'(t)\, dt$$

$$r'(t) = (1, 2t, 3t^2)$$

$$F(r(t)) = ((t^2)^2 - (t^3)^2, 2t^2t^3, -(t)^2)$$

$$= (t^4 - t^6, 2t^5, -t^2)$$

$$\int_0^1 (t^4 - t^6, 2t^5, -t^2)(1, 2t, 3t^2)\, dt$$

$$= \int_0^1 t^4 - t^6 + 4t^6 - 3t^4\, dt$$

$$= \int_0^1 3t^6 - 2t^4\, dt = \frac{3}{7} - \frac{2}{5} = \frac{15-14}{35} = \frac{1}{35}$$

Ej 4.1.5 (ii)

$$\int_C x^3 dy - y^3 dx \quad , \quad C \text{ círculo unitario}$$

$$C : [0, 2\pi] \to \mathbb{R}^2$$
$$t \mapsto (\cos(t), \sin(t))$$

Lo volvemos a escribir como

$$\int_C \bar{F} \cdot d\bar{s} \quad , \quad \bar{F}(x,y) = (-y^3, x^3)$$
$$d\bar{s} = (dx, dy)$$

$$= \int_C F(c(t)) \cdot c'(t) \, dt$$

$$F(c(t)) = (-\sin^3 t, \cos^3 t)$$
$$c'(t) = (-\sin t, \cos t)$$

$$\int_0^{2\pi} \sin^4 t + \cos^4 t \, dt$$

Vamos a solucionar (el otro se hace igual)

$$\int_0^{2\pi} \sin^4 t \, dt = \frac{1}{4}\int_0^{2\pi} \left(1 - \cos(2t)\right)^2 dt$$

$\sin^2 t = \frac{1}{2}(1-\cos(2t))$ $$= \frac{1}{4}\int_0^{2\pi} 1 + \cos^2(2t) - 2\cos(2t)\, dt$$

$\cos^2 2t = \frac{1+\cos 4t}{2}$ $$= \frac{1}{4}\int_0^{2\pi} 1 + \frac{1}{2} + \cos(4t) - 2\cos(2t)\, dt$$

$$= \frac{1}{4}\int_0^{2\pi} \frac{3}{2} + \overbrace{\cos(4t) - 2\cos(2t)}^{\text{ESTOY integrando en todo el p}}\, dt$$

$$= \frac{1}{4}\left(3 \cdot 2\pi - \frac{1}{4}\sin(4t)\Big|_0^{2\pi} - \sin(2t)\Big|_0^{2\pi}\right)$$

$$= \frac{3}{2}\pi$$

E) Partícula masa $m=1$

$$r(t) = (t^2, \sin t, \cos t), \quad t \in [0,1]$$

TRABAJO W

$$W = \int_0^1 \bar{F} \cdot d\bar{s} = \int_0^1 F(t) \cdot r'(t)\,dt$$

$$F = m\, r''(t) \overset{\to m=1}{=} r''(t)$$

$$= \underbrace{(2, -\sin t, -\cos t)}_{\text{ya lo tengo en } t}$$

→ no hace FALTA Rep.

$$\int_0^1 (2, -\sin t, -\cos t) \cdot (2t, \cos t, -\sin t)\,dt$$

$$= \int_0^1 4t - \sin t \cos t + \sin t \cos t = \int_0^1 4t = 2$$