DERIVADA DIRECCIONAL

$$f : A \subset \mathbb{R}^n \to \mathbb{R}$$

$$\bar{x} \longmapsto f_v(\bar{x}) = \lim_{h \to 0} \frac{f(\bar{x} + h\bar{v}) - f(\bar{x})}{h}$$

OBS $v = \hat{e}_i$ (BASE CANONICA) $\to$ D. DIR = D. PAR

$\to$ ¿CUÁNDO EXISTE?

f dif $x_0 \Rightarrow \exists\, f_v(x_0)$

$$f_v(x) = \nabla f\big|_x \cdot v$$

Ej 1.4.1

(iv) $f(x,y) = \dfrac{3x}{x-y}$ en $(1,0)$

dirección $(1,-\sqrt{3})$

$$f_x = \frac{3x-3y-3x}{(x-y)^2} = -\frac{3y}{(x-y)^2}$$

$$f_y = +\frac{3x}{(x-y)^2}$$

$$\nabla f(1,0) = (0,3)$$

$$f_{(1,-\sqrt{3})}(1,0) = (0,3)\cdot(1,-\sqrt{3}) = -3\sqrt{3}$$

(vii) $f(x,y,z) = \log \sqrt{x^2+y^2+z^2}$

en $(2,0,1)$ dirección $(1,2,0) \equiv v$

$$\frac{\partial}{\partial x} f = \frac{1}{\sqrt{x^2+y^2+z^2}} \; \frac{1}{\sqrt{x^2+y^2+z^2}} \cdot x$$

$$\text{Por simetría} \left\{ \begin{aligned} &\frac{\partial}{\partial y} \; '' = \quad '' \qquad '' \qquad y \\ &\frac{\partial}{\partial z} \; '' = \quad '' \qquad '' \qquad z \end{aligned} \right.$$

$$\nabla f(2,0,1) = \left(\frac{2}{5}, 0, \frac{1}{5}\right)$$

$$f_v(2,0,1) = \frac{2}{5}$$

Ej 1.4.3

$$f(x,y) = 1 + \sin(3x+y)$$

v TAL que $f_v(0,0) = 1$?

$$\nabla f(x,x) = (3\cos(3x+y), \cos(3x+y))$$

$$\nabla f(0,0) = (3, 1)$$

$$(3, 1)\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 1$$

$$\underbrace{3v_1 + v_2 = 1}$$

2 incognita, 1 eq!

elijo v unitario (quiero dirección)

$$v_1^2 + v_2^2 = 1$$

$$\begin{cases} 3v_1 + v_2 = 1 \\ v_1^2 + v_2^2 = 1 \end{cases}$$

$$v_2 = 1 - 3v_1$$

$$v_1^2 + (1 - 3v_1)^2 = 1$$

$$v_1^2 + \cancel{1} + 9v_1^2 - 6v_1 = \cancel{1}$$

$$10v_1^2 - 6v_1 = 0$$

$$v_1^2 - \frac{3}{5}v_1 = 0$$

$$v_1 \to 0 \Rightarrow v_2 = 1$$

$$v_1 \to \frac{3}{5} \Rightarrow v_2 = 1 - \frac{9}{5} = -\frac{4}{5}$$

Posibles direcciones $(0,1)$, $\left(\frac{3}{5}, -\frac{4}{5}\right)$

Ej 1.4.4

$$f(x,y) = \frac{x^2 - y^2}{x^2 + y^2}$$

1) v t.q. $f_v(1,1) = 0$

$$f_x(1,1) = \left.\frac{2x(x^2+y^2) - 2x(x^2-y^2)}{(x^2+y^2)^2}\right|_{(1,1)} = \frac{4xy^2}{(x^2+y^2)^2} = \frac{4}{4} = 1$$

$$f_y(1,1) = -\frac{4x^2y}{(x^2+y^2)^2} = -1$$

$$\nabla f(1,1) = (1,-1)$$

$$\nabla f(1,1) \cdot (v_1, v_2) = 0$$

$$\Rightarrow v_1 - v_2 = 0$$

Considerando $v_1^2 + v_2^2 = 1$

$$2v^2 = 1, \quad v = \frac{1}{\sqrt{2}}(1,1)$$

(ii) MISMA pregunta con punto arbitrario

$$f_x = \frac{4xy^2}{(x^2+y^2)^2}$$

$$f_y = -\frac{4x^2y}{(x^2+y^2)^2}$$

$$\nabla f(x_0, y_0) \cdot (v_1, v_2) = 0$$

$$\frac{4x_0y_0}{(x_0^2+y_0^2)^2}(y_0, x_0)(v_1, v_2) = 0$$

$$v_1 y_0 - x_0 v_2 = 0$$

$$v_2 = \frac{y_0}{x_0} v_1$$

$$v_1^2 + v_2^2 = 1 \quad , \quad \left[\left(\frac{y_0}{x_0}\right)^2 + 1\right] v_1^2 = 1$$

$$v_1 = \sqrt{\frac{x_0^2}{x_0^2+y_0^2}} = \frac{|x_0|}{x_0^2+y_0^2} = \pm \sim$$

$$v_2 = \sqrt{\frac{y_0^2}{x_0^2+y_0^2}} \quad \frac{|y_0|}{x_0^2+y_0^2}$$

E) 1.4.5

(i) $x^2 + y^2 + z^2 = 3$

ECUACIÓN PLANO TANGENTE en $(1,1,1)$

↳ 2 maneras

1° MÉTODO Aplicar definición $z = f(x,y)$

→ NECESITO z en función de las otras

$$z = \sqrt{3 - x^2 - y^2}$$

$$z(1,1) = \sqrt{3-1-1} = 1$$ (ya lo tenía)

$$\frac{\partial}{\partial x} z \Big|_{(1,1)} = -\frac{\cancel{2}x}{\cancel{2}\sqrt{3-x^2-y^2}} = -1$$

$$\frac{\partial}{\partial y} z \Big|_{(1,1)} = -\frac{2y}{2\sqrt{3-x^2-y^2}} = -1$$

$$z(x,y) = 1 - (x-1) - (y-1)$$

O SI no _ _

PLAN TANGENTE → SUPERFICIE NIVEL
FUNCIÓN 3D

↳ es lo que tenemos

SUPERFICIE NIVEL $\perp$ ∇f

$$\nabla f \Big|_{(1,1,1)} = (2x, 2y, 2z)\Big|_{(1,1,1)} = (2,2,2)$$

$$(2,2,2)(x-1, y-1, z-1) = 0$$

$$x-1+y-1+z-1 = 0$$

$$z = 1-(x-1)-(y-1)$$

MISMO RESULTADO

Ej 1.4.12

POSICIÓN PARTÍCULA

$c : \mathbb{R} \to \mathbb{R}^3$

TRAYECTORIA → $t \mapsto c(t) = (e^{t^2}, e^{-t^2}, \log(1+t^2))$

Potencial

$V : \mathbb{R}^3 \to \mathbb{R}$

$(x,y,z) \mapsto (xy)^2 = e^{2\log xy}$

RESTRICCIÓN POTENCIAL TRAYECTORIA

→ DERIVADA (Fuerza a lo largo trayectoria)

$$h'(t) = \frac{d}{dt} V \circ c = \nabla V(c(t))\, c'(t)$$

$$c'(t) = \left(2te^{t^2}, -2te^{-t^2}, \frac{2t}{1+t^2}\right)$$

$$\nabla V = (2xy^2, 2x^2y, 0)$$

$$\nabla V(c(t)) = ?$$

$$2xy^2\Big|_{c(t)} = 2\cancel{e^{t^2}}\underbrace{\cancel{e^{-t^2}}\,e^{-t^2}}_{\left(e^{-t^2}\right)^2} = 2e^{-t^2}$$

(because of y^2)

$$2x^2y\Big|_{c(t)} = 2\underbrace{e^{t^2}\,\cancel{e^{t^2}}}_{\left(e^{t^2}\right)^2}\,\cancel{e^{-t^2}} = 2e^{t^2}$$

$$h'(t) = \nabla V(c(t)) \cdot c'(t)$$

$$= \left(2e^{-t^2}, 2e^{t^2}, 0\right) \cdot \left(2te^{t^2}, -2te^{-t^2}, \frac{2t}{1+t^2}\right)$$

$$= 4t - 4t = 0$$