Aviso: Próxima Semana

- Jueves, ISO
- Martes, SI 4/2

13:00

2.3.C04

Conjuntos en $\mathbb{R}^2$

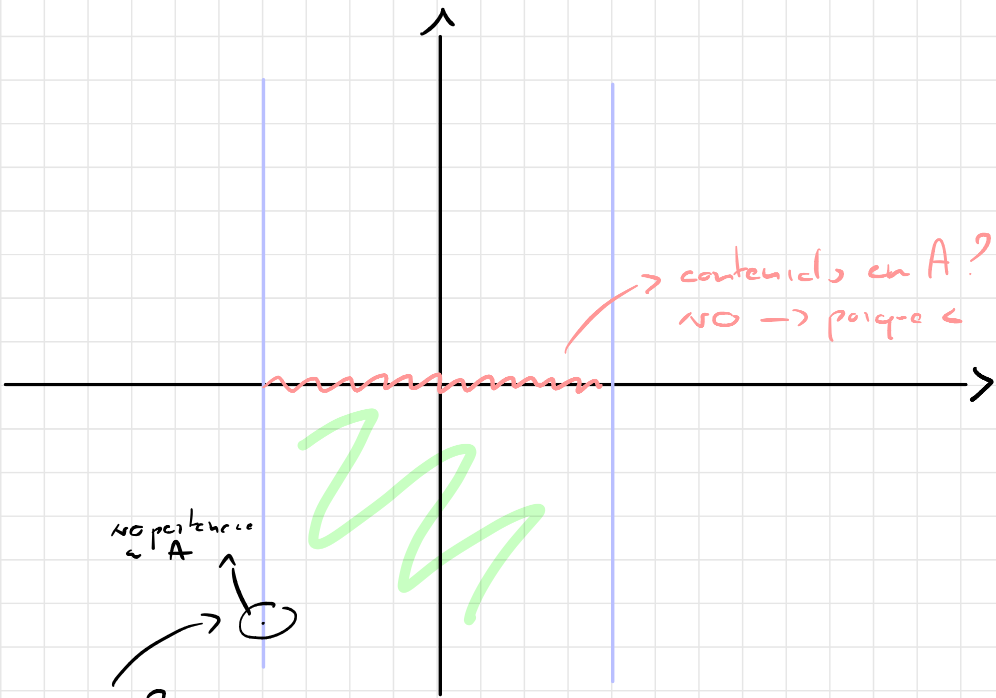
↳ dibujo

↳ Topología

↳ abierto, cerrado, frontera etc.

Ej 1.1 (i)

$$A = \{(x,y) \in \mathbb{R}^2 : \boxed{-1 \le x \le 1}, \boxed{y < 0}\}$$



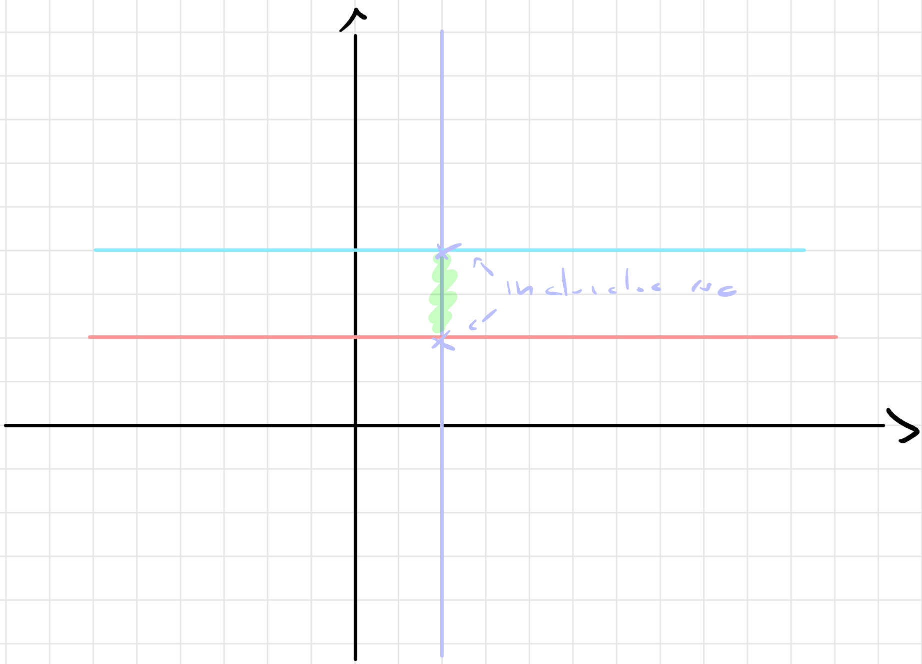
ABIERTO? NO

CERRADO? NO → $y < 0$

FRONTERA? $\partial_A = \{(x,y) \in \mathbb{R}^2 : \|x\| = 1, y = 0\}$

Ej 1.1.1 (ii)

$B = \{(x,y) \in \mathbb{R}^2,\ x = 1,\ 1 < y < 2\}$

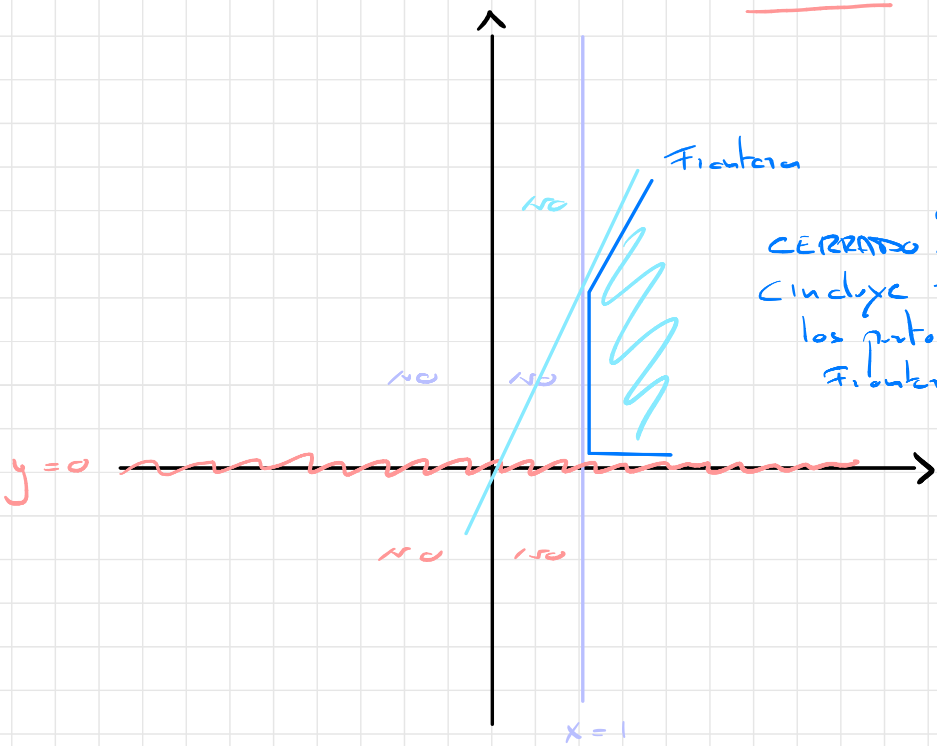


Cerrado? no

Abierto? no

Ej 1.1.1 (iii)

$$C = \{ (x,y) \in \mathbb{R}^2 : x \geq 1 , \ 0 \leq y \leq 2x \}$$



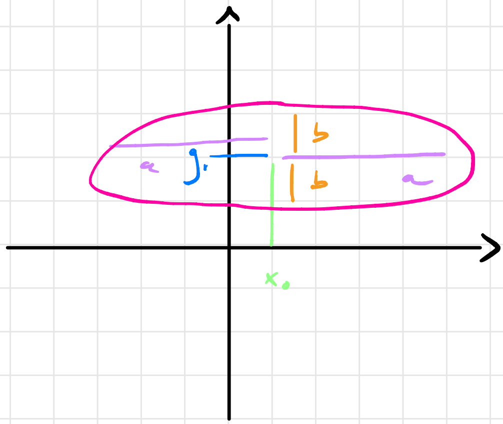
CERRADO? SI (incluye todo los puntos y su frontera)

Ej. 1.1.1. (v)

$$E = \{(x,y) : \underbrace{x^2 + 3y^2 \le 2}\}$$

ELIPSE

$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$$



$a = b \Rightarrow$ CÍRCULO

$x_0 = y_0 = 0 \Rightarrow$ CENTRADO en O

$$\left(\frac{x}{\sqrt{2}}\right)^2 + \left(\frac{y}{\sqrt{\frac{2}{3}}}\right)^2 \le 1$$

Parte interior elipse

- centrada en O
- $a = \sqrt{2}$, $b = \sqrt{\frac{2}{3}}$

$\Big\}$ cerrado? SI

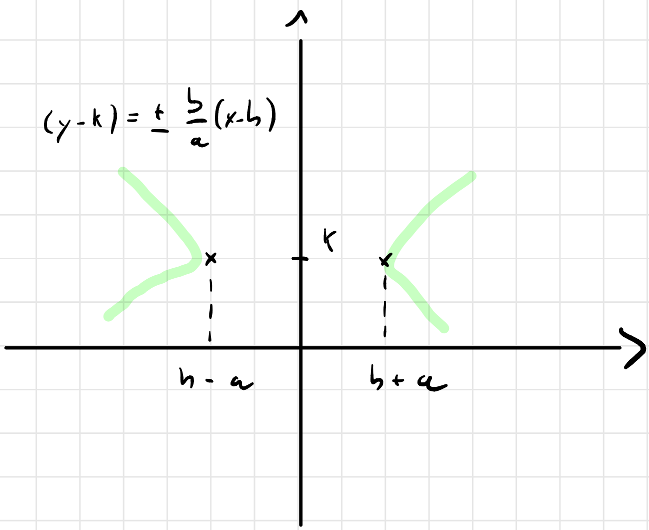
E) 1.1.1 (vii)

$$G = \{ (x,y) \in \mathbb{R}^2 : x^2 - 3y^2 = 2 \}$$

ojo?

Hiperbola

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$



E) 1.1.2 (5)

$$\{ \bar{x} \in \mathbb{R}^n : \| \bar{x} \| > 3 \}$$

↳ notación vector

$$\underbrace{\| \bar{x} \| = \sum_i x_i^2}_{\text{MODULUS}}$$

$$\| \bar{x} \| = 3 \quad (\text{Hipersphere in } n \text{ dim})$$

el conjunto es el espacio fuera de esta ipersfera

Dominio funciones

→ Ej 1.1.5

(i) $f(x,y) = x^2 - y^2$

$D_f = \mathbb{R}^2$

(ii) $f(x,y) = \sqrt{x^2 - 4y^2}$

$D_f = \{ (x,y) \in \mathbb{R}^2 : x^2 - 4y^2 \geq 0 \}$

(iii) $f(x,y) = \dfrac{x^3 - y^2}{x - y}$

$D_f = \{ (x,y) \in \mathbb{R}^2 : x \neq y \}$

(vi) $f(x,y) = \arcsin(x+y)$

$D_f = \{ (x,y) \in \mathbb{R}^2 : -1 \leq x+y \leq 1$

(vii) $f(x,y) = e^{x/y}$

$D_f = \{ (x,y) \in \mathbb{R}^2 : y \neq 0 \}$

(viii) $f(x,y) = \log(xy)$

$$D_f = \{(x,y) \in \mathbb{R}^2 : xy > 0\}$$

(ix) $f(x,y) = \dfrac{1}{\cos(x-y)}$

$$D_f = \{(x,y) \in \mathbb{R}^2 : x-y \neq (2k+1)\frac{\pi}{2} \quad \underbrace{k \in \mathbb{Z}}_{?}\}$$

(x) $f(x,y) = \cos\left(\dfrac{1}{x-y}\right)$

$$D_f = \{(x,y) \in \mathbb{R}^2 : x \neq y\}$$

(xi) $f(x,y,z) = \dfrac{\sqrt{9-y^2}}{1+\sqrt{4-(x^2+y^2)}}$

$$D_f = \{(x,y,z) \in \mathbb{R}^3 : 9-y^2 \geq 0, \ x^2+y^2 \leq 4\}$$

(xii) $f(x,y) = \dfrac{\log(1+x) + \log(1+y)}{\log(1-x) + \log(1-y)}$

$$\left.\begin{array}{lll} \log(1+x) & \longrightarrow & x > -1 \\ \text{"} \quad 1+y & \longrightarrow & y > -1 \\ \text{"} \quad 1-x & \longrightarrow & x < 1 \\ \text{"} \quad 1-y & \longrightarrow & y < 1 \end{array}\right| \longrightarrow |x| < 1 \,,\ |y| < 1$$

$$\log(1-x) + \log(1-y) \neq 0$$

¿cómo se soluciona?

$$\underbrace{e^{\log(1-x) + \log(1-y)}}_{e^{\log(1-x)}\, e^{\log(1-y)}} \neq \underbrace{e^{0}}_{1}$$

$$(1-x)(1-y) \neq 1$$

$$1 - x - y - xy \neq 1$$

$$\underline{xy \neq x - y}$$

(xiii) $f(x,y) = \log \dfrac{(1+x)(1+y)}{(1-x)(1-y)}$

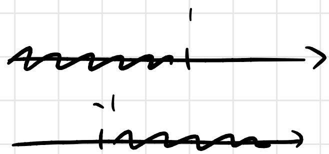
- $(1-x)(1-y) \neq 0$

  $x \neq 1$ and $y \neq 1$

- $\arg > 0$

  $1 - x > 0 \to x < 1$

  $1 + x > 0 \to x > -1$



$\Rightarrow |x| < 1$

lo mismo para y

Image

(i) $\to$ $\mathbb{R}$

(ii) $\to$ $[0, +\infty)$

(iii) $\to$ $\mathbb{R}$

(iv) $\to$ $[0, +\infty)$

(v) $\to$ $\mathbb{R}/\{0\}$

Level Curves

(i) $f(x,y) = xy$

$$xy = c \Rightarrow y = \frac{c}{x}$$

(ii) $f(x,y) = \log(x-y)$

$$\log(x-y) = c$$

$$y = x + e^c$$

(iii) $f(x,y) = \dfrac{x+y}{x-y}$

$$\frac{x+y}{x-y} = c \Rightarrow x+y = c(x-y)$$

$$y - cy = x + cx$$

$$y = \frac{1+c}{1-c}x$$