Lec 14(2)-15(1) - 光的传播理论, Basic Radiometry 辐射度量学, 路径追踪与全局光照

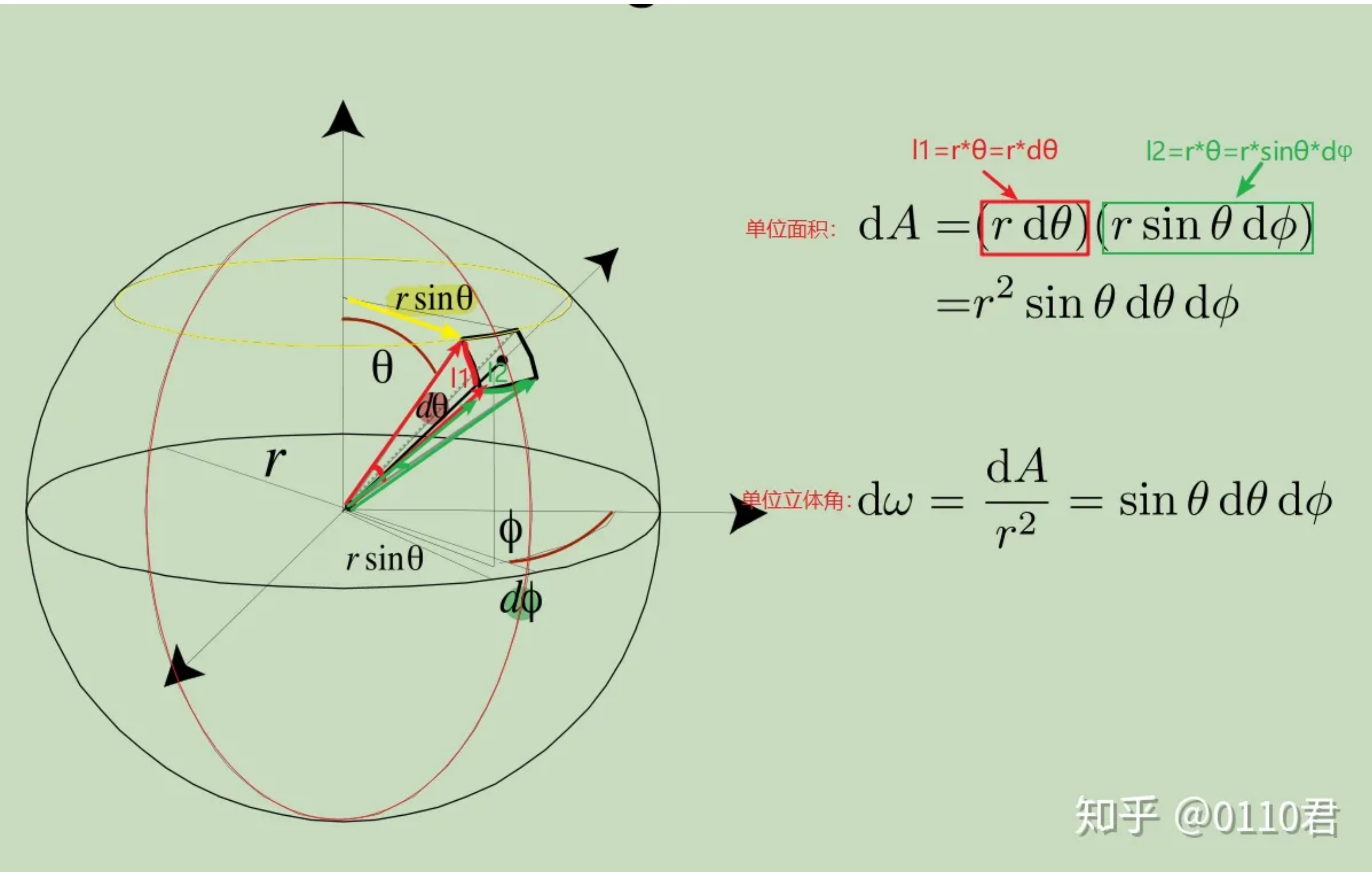
Whitted Style Ray Tracing 无法保证正确性

辐射度量学：精准度量光的一系列物理量 → Physically Based

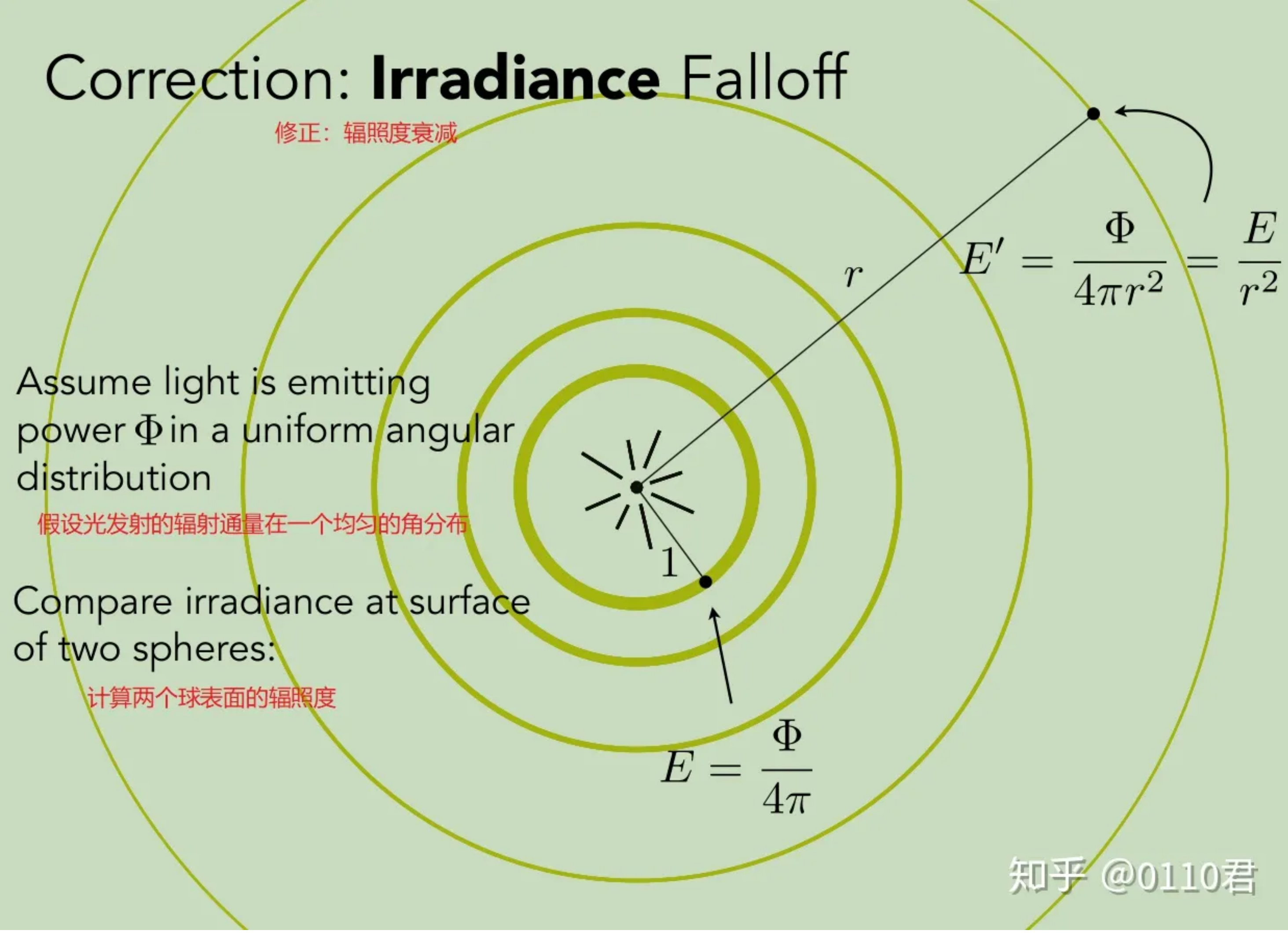
- the basics of Path Tracing
- Measurement system and units for illumination
- Accurately measure the spatial properties of light
    - New terms: Radiant flux, intensity, irradiance, radiance
- Perform lighting calculations in a physically correct manner

Radiant Energy and Flux (Power)

微分立体角



辐照度衰减



基础物理量

- Radiant energy: 能量，the energy of electromagnetic radiation
  - 符号：Q
  - 单位：J（Joule焦耳）
- Radiant flux (power): the energy emitted, reflected, transmitted or received, **per unit time**. （单位时间能量 → 功率）
  - 符号：ϕ （phi）
  - 单位：W（Watt），lm（lumen流明）
  - 单位时间 很重要
- Flux: number of photons flowing through a sensor in unit time

Important Light Measurements of Interest

- Radiant(luminous) **Intensity**: the power per unit solid angle (立体角) emitted by a point light source.
  - 符号定义：

$$I(\omega) = \frac{\mathrm{d}\Phi}{\mathrm{d}\omega}$$

  - 单位：W/sr，lm/sr=cd(candela坎德拉)
  - 立体角
    - 弧度：弧长/半径

- 弧度在三维的延伸→立体角：面积/半径^2，

$$\Omega = \frac{A}{r^2}$$

- 单位：sr, steradians
- 球面=4pi sr

$$\mathrm{d}\omega = \frac{\mathrm{d}A}{r^2} = \sin\theta \mathrm{d}\theta \mathrm{d}\phi$$

- 方向性
  - 用 ω 来表示方向
- 点光源：

$$I = \frac{\phi}{4\pi}$$

- Irradiance: power per unit area (perpendicular/ projected)
  - Light Falling On A Surface

$$E(\mathbf{x}) \equiv \frac{\mathrm{d}\Phi(\mathbf{x})}{\mathrm{d}A}$$

  - 单位：W/m^2 | lm/m^2 = lux
  - 需要注意，是垂直方向
    - Lambert's Cosine Law
    - 地球的四季
    - Falloff: 点光源的能量散布
  - 无方向性
- **Radiance: power per unit solid angle, per projected unit area.**
  - describes the distribution of light in an environment
  - Light Traveling Along A Ray

$$L(\mathrm{p}, \omega) \equiv \frac{\mathrm{d}^2\Phi(\mathrm{p}, \omega)}{\mathrm{d}\omega \mathrm{d}A \cos\theta}$$

  - 某个单位面积往某个单位立体角辐射的能量密度
  - Radiance: **Irradiance** per solid angle
  - Radiance: **Intensity** per projected unit area
  - 有方向性
  - Incident radiance is the irradiance per unit solid angle arriving at the surface. 某个小面接受来自某个方向的光线

$$L(\mathrm{p}, \omega) = \frac{\mathrm{d}E(\mathrm{p})}{\mathrm{d}\omega \cos\theta}$$

- Exiting surface radiance is the intensity per unit projected area leaving the surface. 某个小面发出向某个方向的光线

$$L(\mathrm{p}, \omega) = \frac{\mathrm{d}I(\mathrm{p}, \omega)}{\mathrm{d}A \cos\theta}$$

Irradiance和Radiance之间的区别就在于是否有方向性

- Irradiance: total power received by area dA 四面八方的光线积分起来
- Radiance: power received by area dA from “direction” dω

$$dE(\mathrm{p}, \omega) = L_i(\mathrm{p}, \omega)\cos\theta \mathrm{d}\omega$$
$$E(\mathrm{p}) = \int_{H^2} L_i(\mathrm{p}, \omega)\cos\theta \mathrm{d}\omega$$

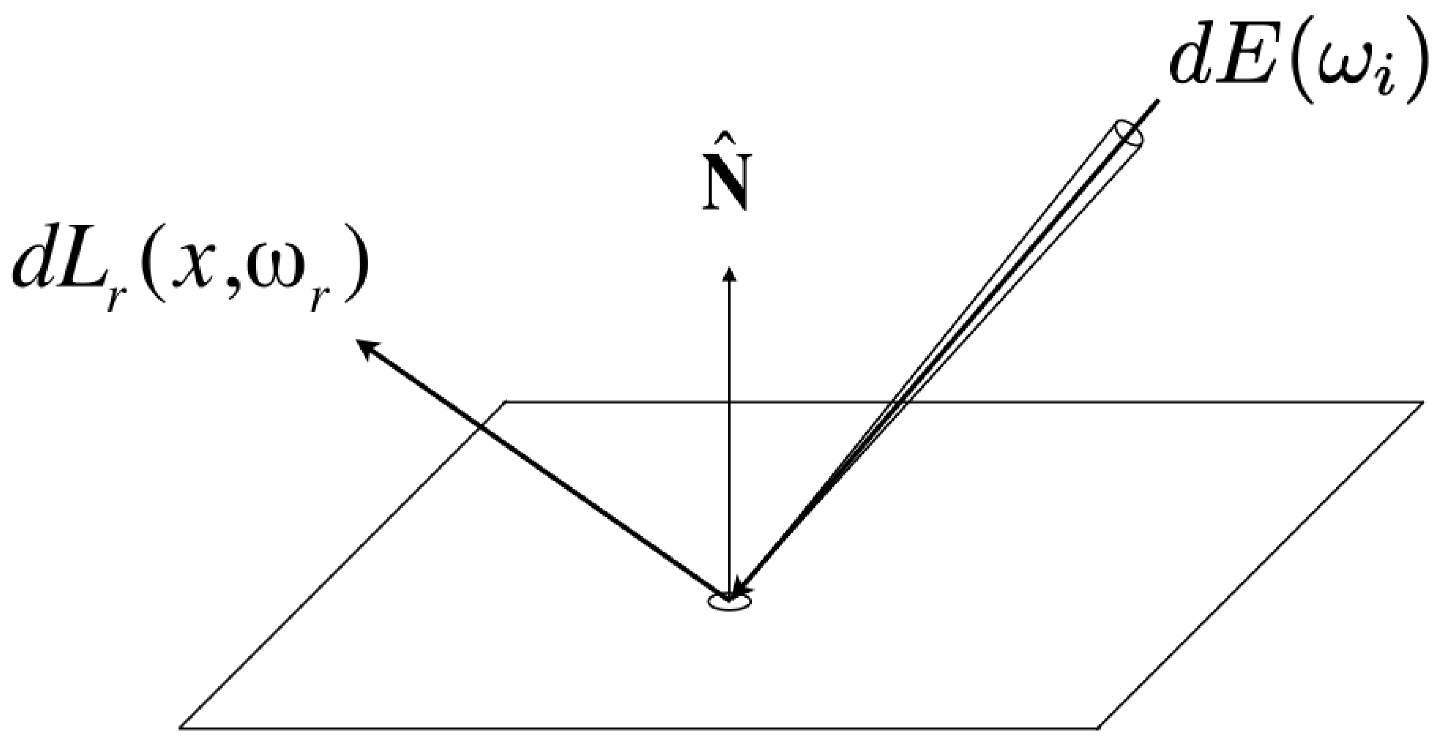
- H^2：半球面

Bidirectional Reflectance Distribution Function (BRDF) 双向反射分布函数

BRDF描述了从某个方向入射到一个点上的光线的能量会怎么反射，在不同的反射方向上会各分布多少能量

反射的理解：光线打到某个点，（被吸收了）然后反弹（发出）到其他地方

Radiance from direction ωi turns into the power E that dA（某个点） receives, Then power E will become the radiance to any other direction ωr



能量一般指功率，即单位时间内的能量。

某个点接受/发射光线总能量：Irradiance

某个点从某个方向接受/向某个方向发射光线能量：radiance

- Differential irradiance incoming:

$$dE(\omega_i) = L(\omega_i)\cos\theta_i d\omega_i$$

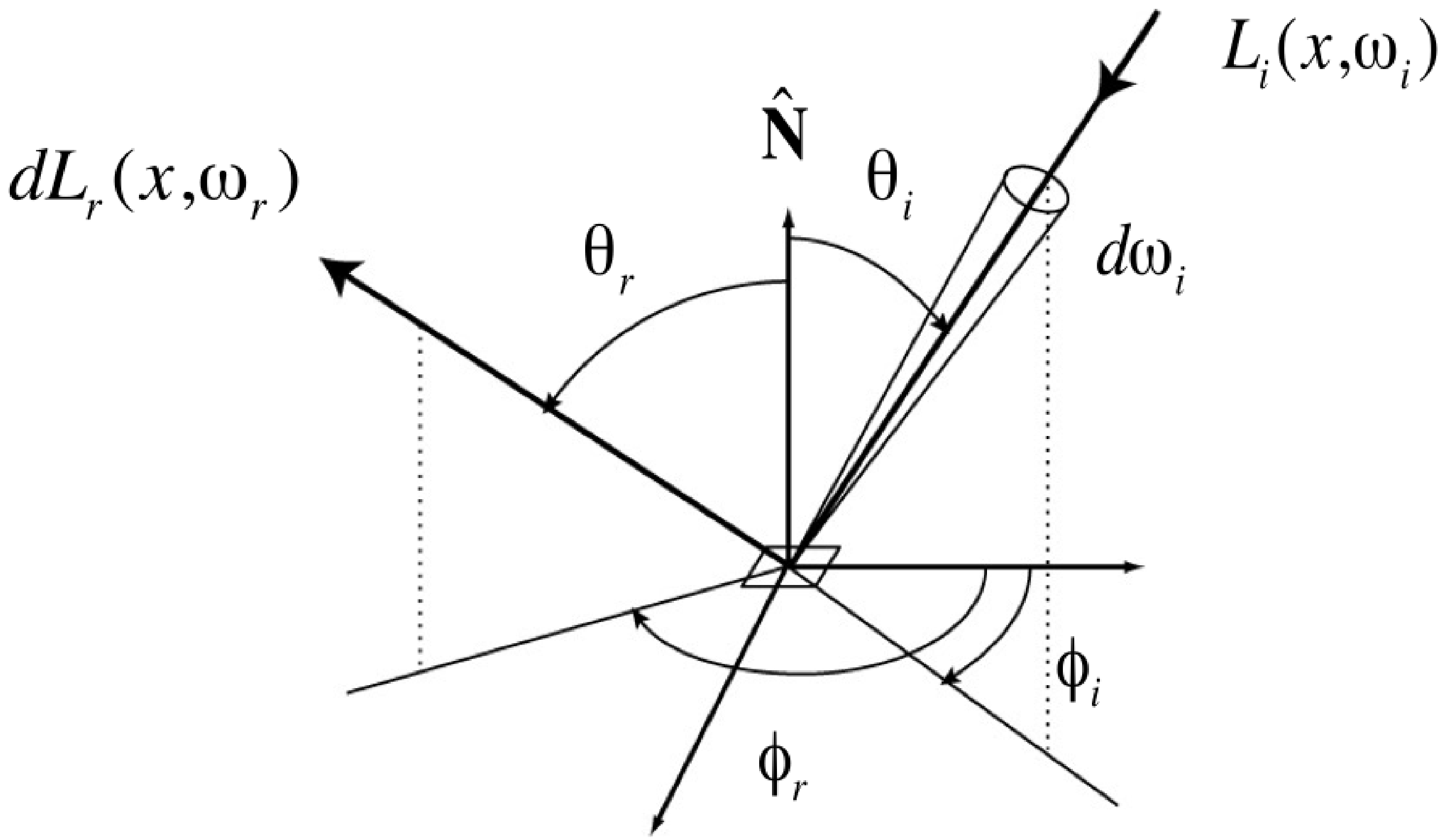
- Differential radiance exiting:

$$dL_r(\omega_r)$$

BRDF的几个细节：

- 针对一个输入源
- 描述不同方向的输出
- 定义了不同材质

BRDF：represents how much light is reflected into each outgoing direction dL r (! r) from each incoming direction



$$f_r(\omega_i \to \omega_r) = \frac{\mathrm{d}L_r(\omega_r)}{\mathrm{d}E_i(\omega_i)} = \frac{\mathrm{d}L_r(\omega_r)}{L_i(\omega_i)\cos\theta_i \mathrm{d}\omega_i}\left[\frac{1}{\mathrm{sr}}\right]$$

The Reflection Equation 反射方程

- 针对一个输出源（着色点），积分所有方向输入源（光照）的BRDF，获得最后的输出

$$L_r(\mathrm{p},\omega_r) = \int_{H^2} f_r(\mathrm{p},\omega_i \to \omega_r) L_i(\mathrm{p},\omega_i)\cos\theta_i \mathrm{d}\omega_i$$

Challenge: Recursive Equation

- 不只有光源才是输入源，其他物体的反射光也有可能是输入源
- 递归的计算方式，计算量爆炸！

The Rendering Equation 渲染方程（绘制方程）

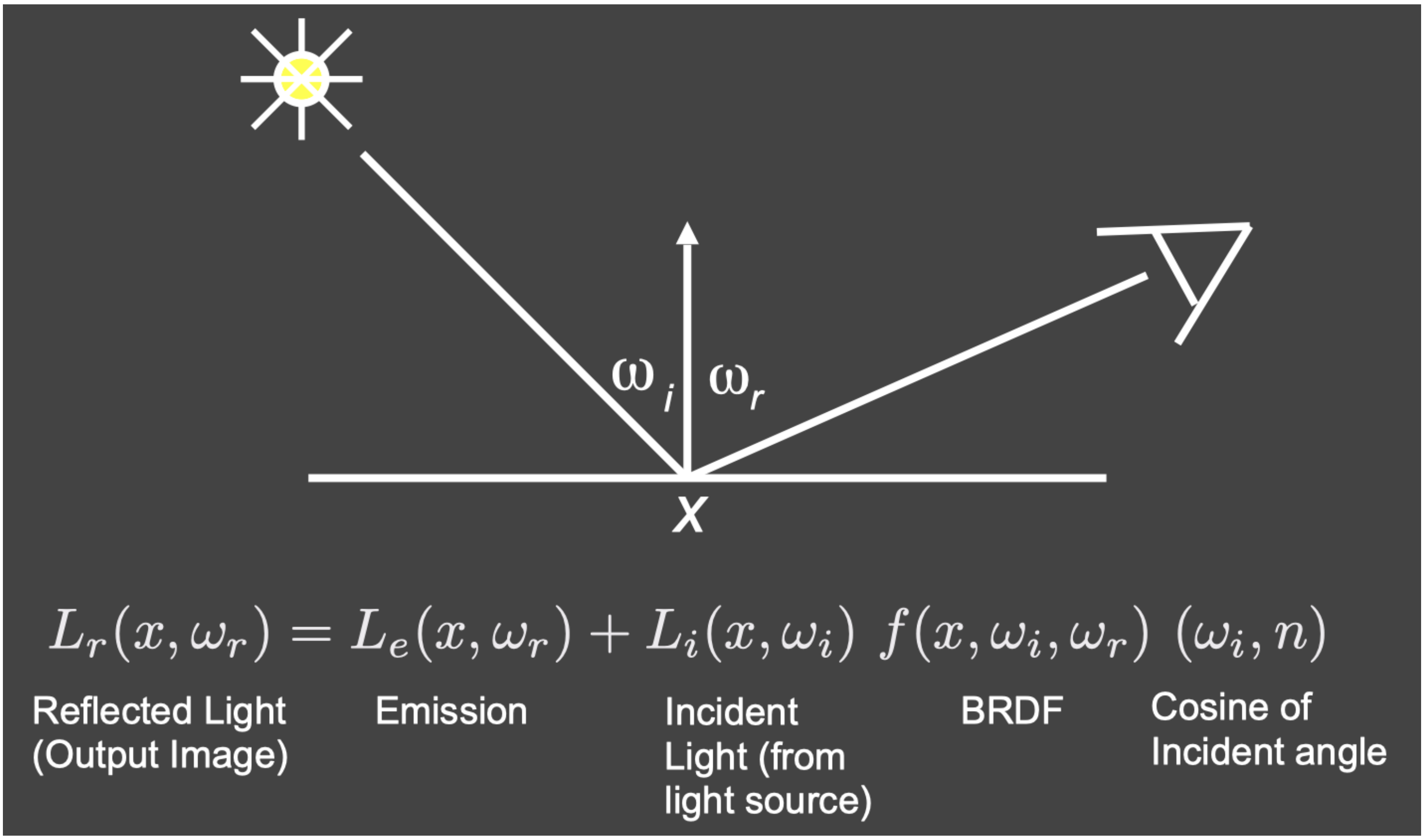
Rendering Equation (Kajiya 86) 跨时代

Adding an Emission term to the reflection equation to make it general!

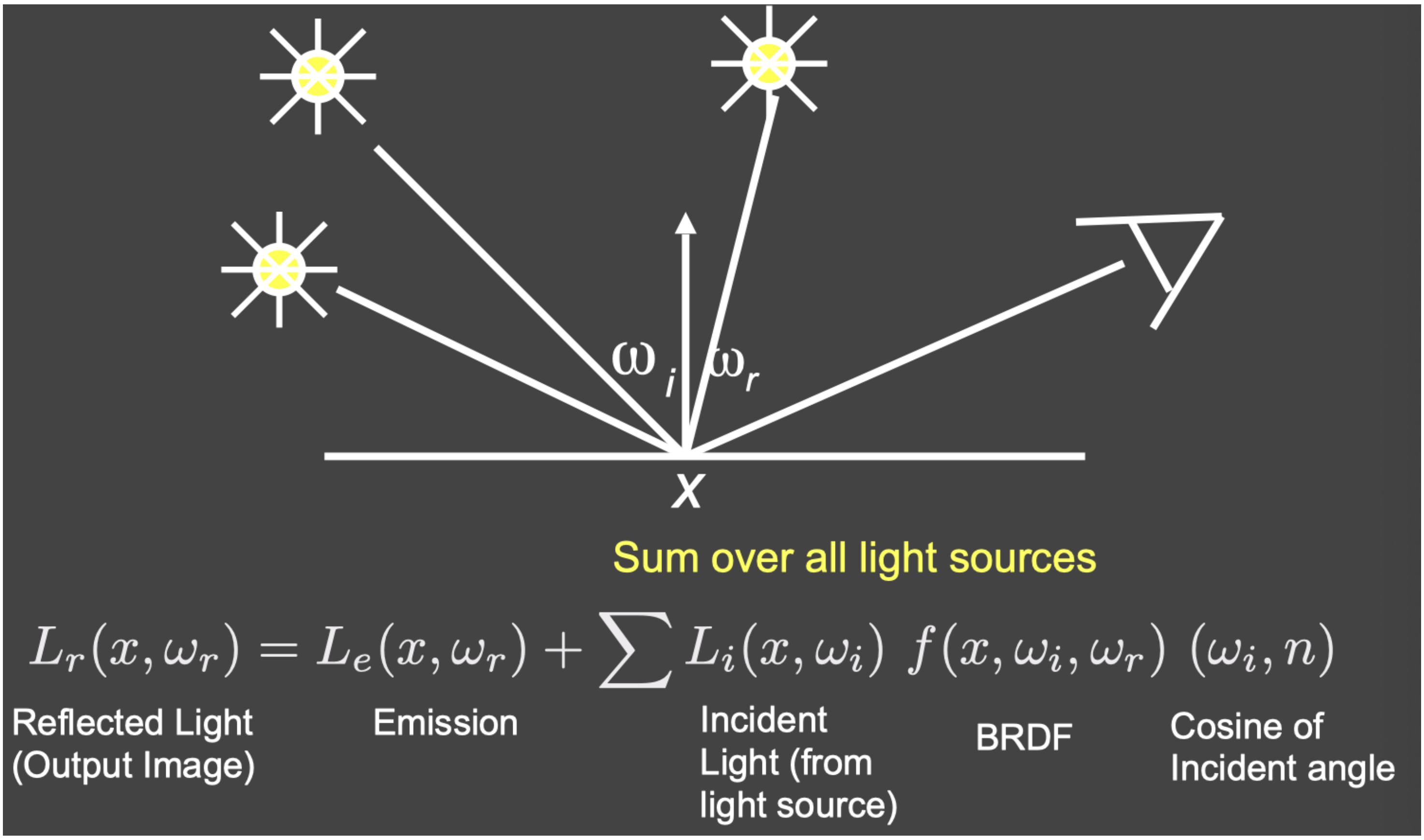
$$L_o(p,\omega_o) = L_e(p,\omega_o) + \int_{\Omega^+} L_i(p,\omega_i) f_r(p,\omega_i,\omega_o)(n\cdot\omega_i)\mathrm{d}\omega_i$$

- 包括了物体自己会发光的情况，包括了所有光线的传播情况
- assume that all directions are pointing outwards!
- H^2和Ω^+ 都代表半球
- 怎么解这个方程呢？下节课

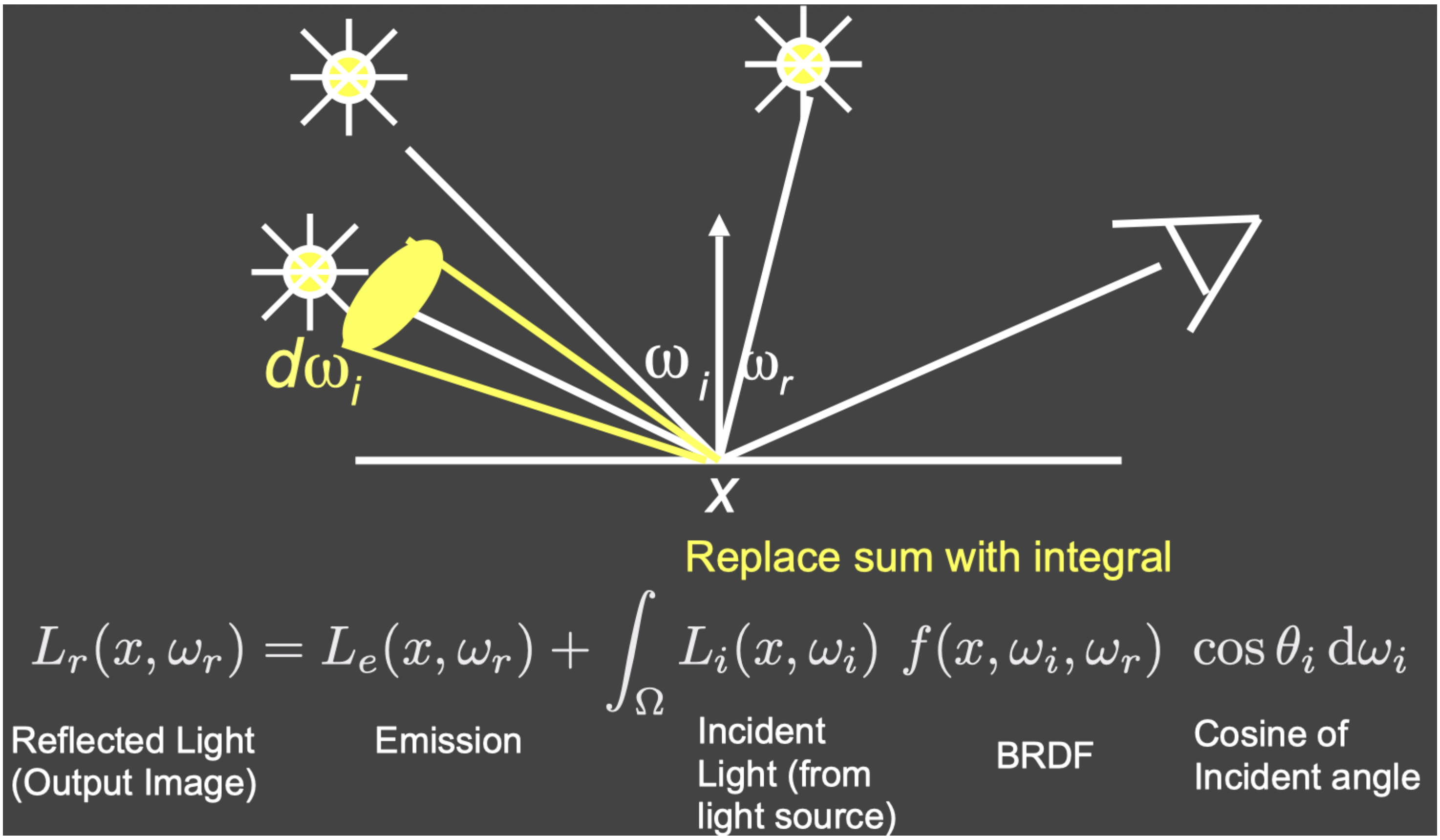
单个点光源：



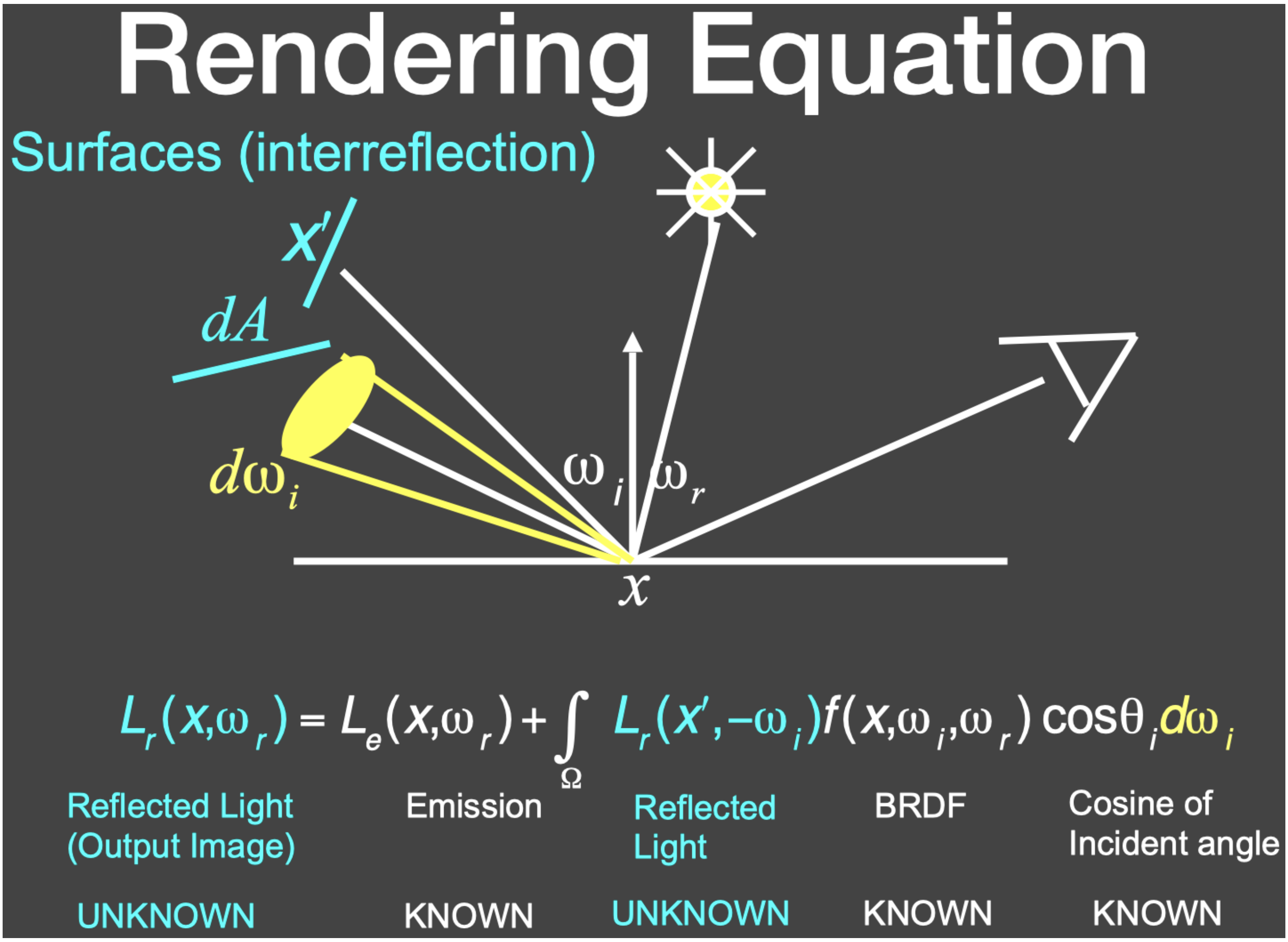
多个点光源：加起来



还有面光源：积分起来



考虑其他物体反射的光线（把光源也包括在内了）：→递归



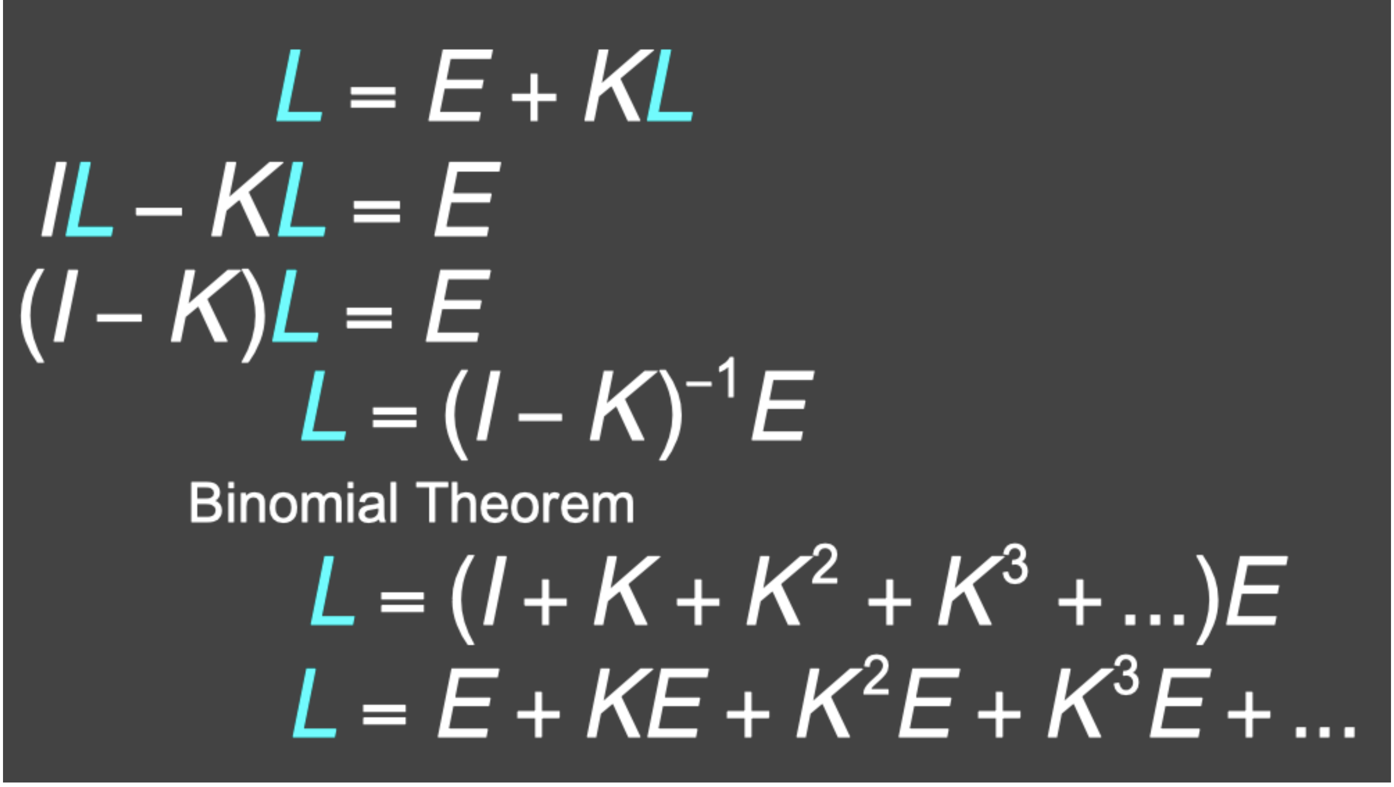
$$L_r\left(x, \omega_r\right)=L_e\left(x, \omega_r\right)+\int_{\Omega} L_r\left(x^{\prime},-\omega_i\right) f\left(x, \omega_i, \omega_r\right) \cos \theta_i d \omega_i$$

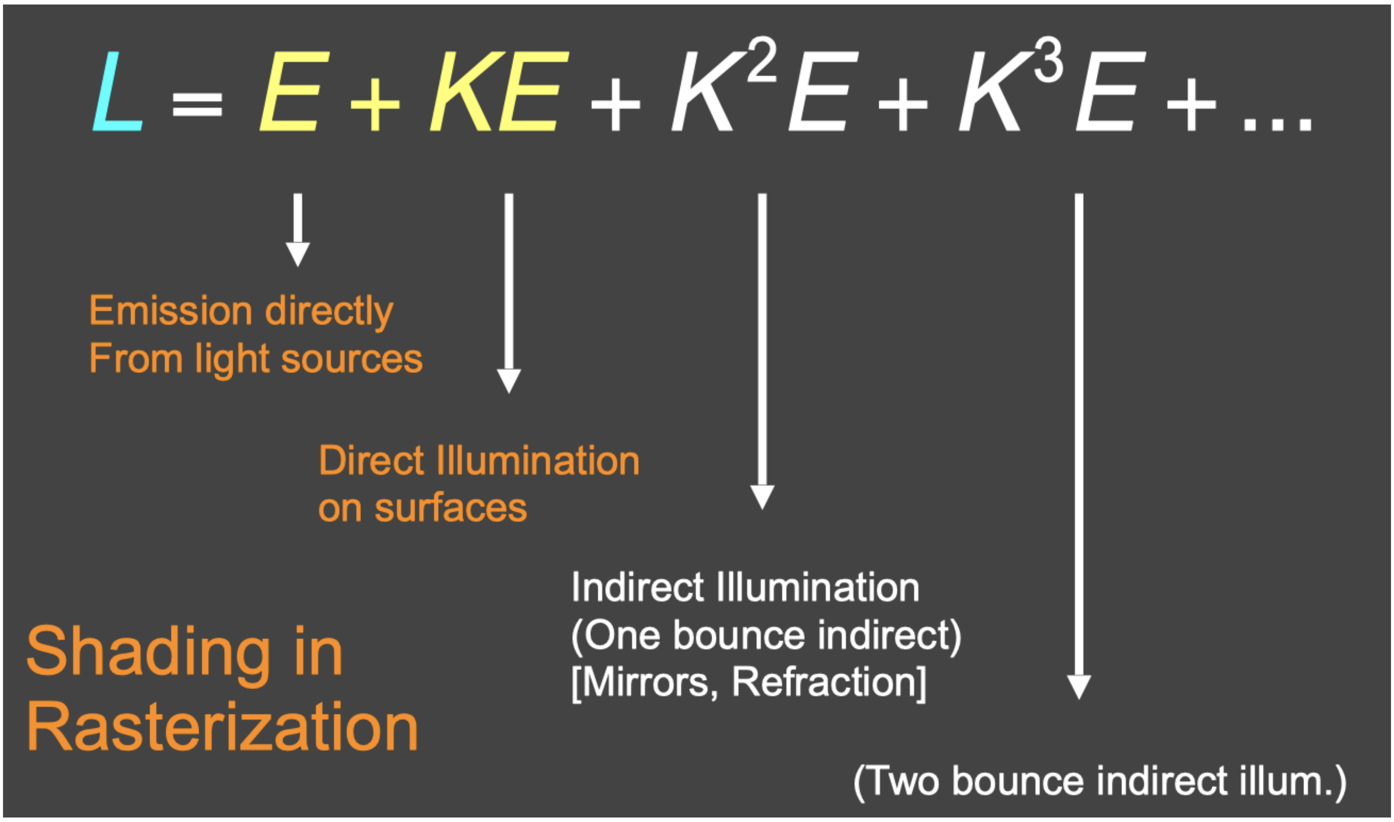
这个方程是 Fredholm Integral Equation of second kind [extensively studied numerically] with canonical form → 简写为如下形式

$$I(u)=\theta(u)+\int l(v) K(u, v) d v$$

通过算符的抽象还可极度简化成如下形式：L = E + KL （K：反射算符）其中L为未知数

- Can be discretized to a simple matrix equation [or system of simultaneous linear equations] (L, E are vectors, K is the light transport matrix)





- 光栅化的着色过程只有直接光照（间接光照需要间接计算）
- 全局光照：直接和间接光照的集合
  - 会收敛到一个亮度

如何解全局光照方程？Monte Carlo Path Tracing (Lec 16)