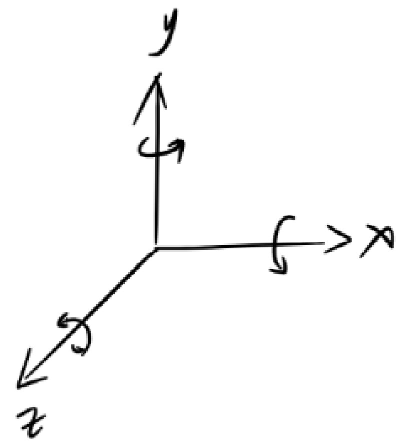
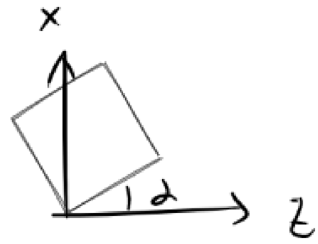
3D transformations

rotation



$$R_x(\alpha) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha & -s\alpha & 0 \\ 0 & s\alpha & c\alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$R_y(\alpha) = \begin{pmatrix} c\alpha & 0 & +s\alpha & 0 \\ 0 & 1 & 0 & 0 \\ -s\alpha & 0 & c\alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



$$R_z(\alpha) = \begin{pmatrix} c\alpha & -s\alpha & 0 & 0 \\ s\alpha & c\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\vec{y} \times \vec{z} = \vec{x}$$
$$\vec{z} \times \vec{x} = \vec{y}$$
$$\vec{x} \times \vec{y} = \vec{z}$$

(正偏转角⇒正轴)

rodrigues' rotation formula

$$\mathbf{R}(\mathbf{n}, \alpha) = \cos(\alpha)\,\mathbf{I} + (1 - \cos(\alpha))\,\mathbf{n}\mathbf{n}^T + \sin(\alpha) \underbrace{\begin{pmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{pmatrix}}_{\mathbf{N}}$$

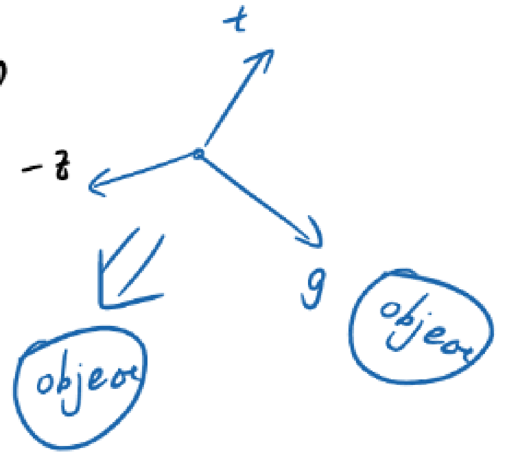
Transformation Cont

viewing(观测) transformation

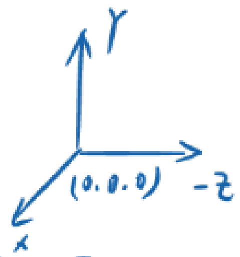
view(视图)/camera transformation

mvp: model → view → projection

camera origin, up y, look at -z



- Position $\vec{e}$
- look-at $\hat{g}$
- up direction $\hat{t}$



推导: $M_{view} = R_{view} T_{view}$

$$T_{view} = \begin{bmatrix} 1 & 0 & 0 & -x_e \\ 0 & 1 & 0 & -y_e \\ 0 & 0 & 1 & -z_e \end{bmatrix}$$

Rotate g to -z, t to Y, (g×t) to X

$$R^{-1}_{view} = \begin{bmatrix} x_{\hat{g}\times\hat{t}} & x_t & x_{-g} & 0 \\ y_{\hat{g}\times\hat{t}} & y_t & y_{-g} & 0 \\ z_{\hat{g}\times\hat{t}} & z_t & z_{-g} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

∵ 旋转正交

∴ $R^{-1}_{view} = R^{T}_{view}$

$$R_{view} = \begin{bmatrix} x_{\hat{g}\times\hat{t}} & y_{\hat{g}\times\hat{t}} & z_{\hat{g}\times\hat{t}} & 0 \\ x_t & y_t & z_t & 0 \\ x_{-g} & y_{-g} & z_{-g} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

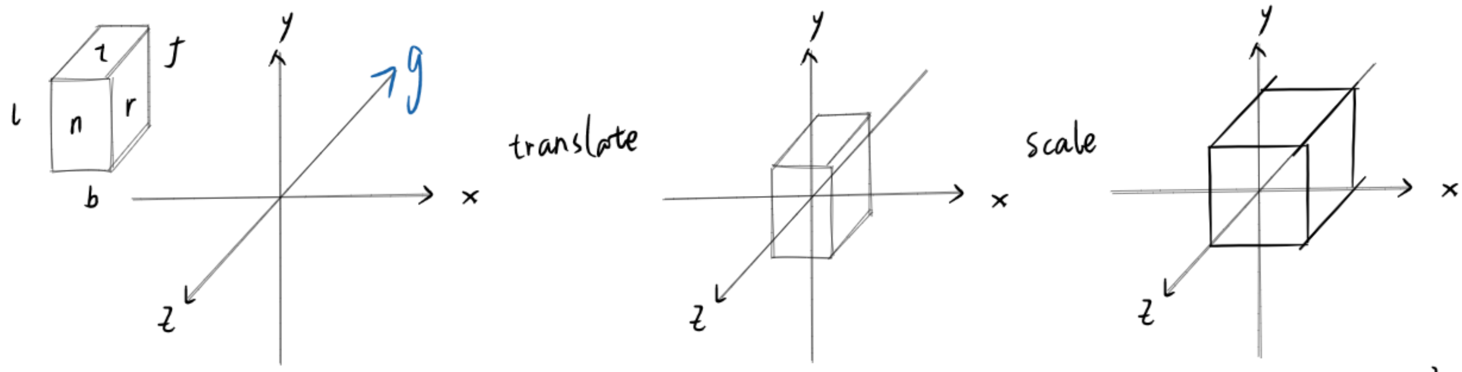
obj 同乘 R_v → 相对运动

位置关系不变

projection(投影) transformation

orthographic(正交) projection

$$[l,r] \times [b,t] \times [f,n] \quad (l<r,\ b<t,\ f<n)$$

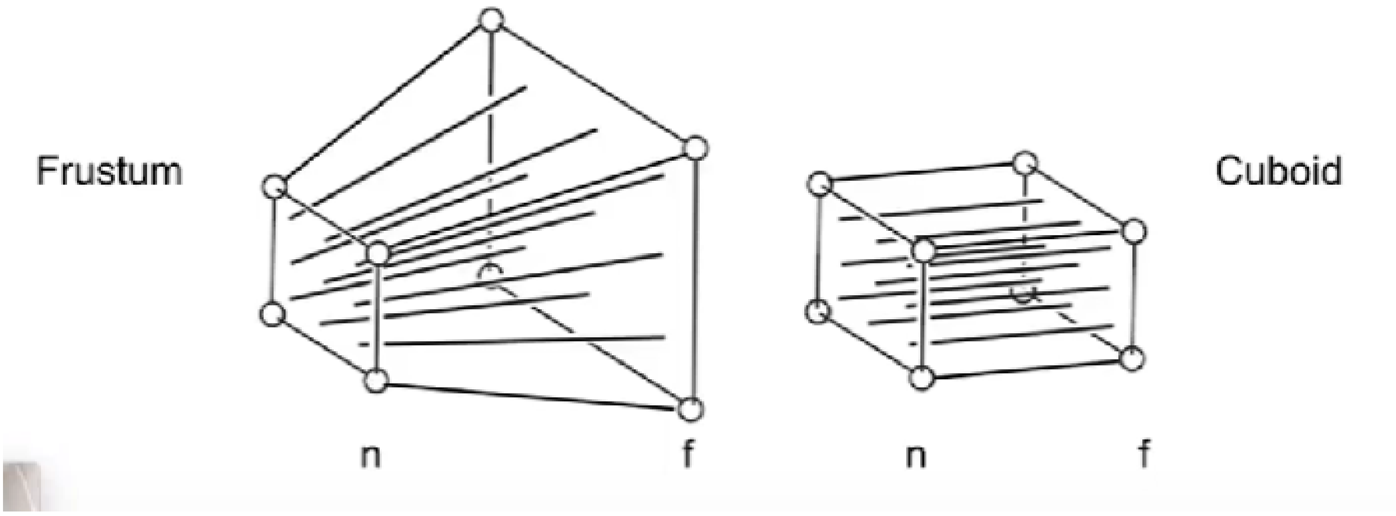


canonical cube $[-1,1]^3$

正则·标准

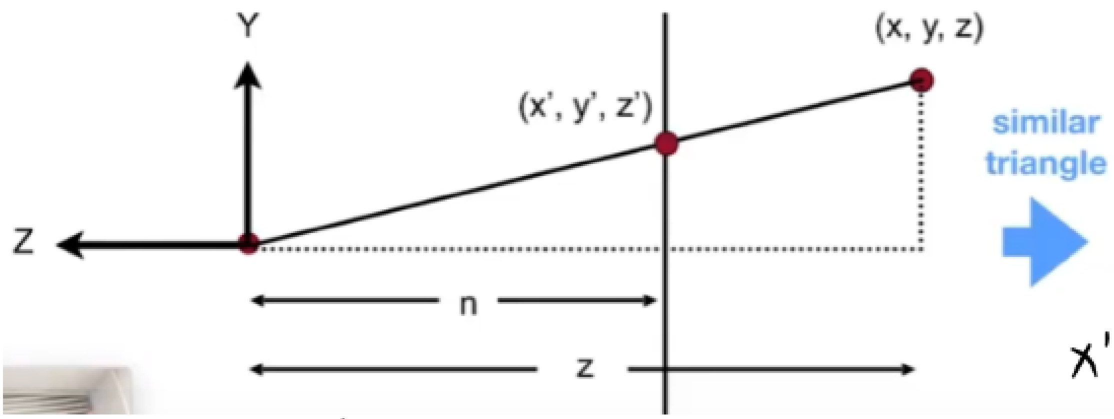
$$M_{ortho} = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & 0 \\ 0 & \frac{2}{t-b} & 0 & 0 \\ 0 & 0 & \frac{2}{n-f} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -\frac{r+l}{2} \\ 0 & 1 & 0 & -\frac{t+b}{2} \\ 0 & 0 & 1 & -\frac{n+f}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

perspective(透视) projection



规定 n面不变，f面z不变 中心不变

$$M_{persp} = M_{ortho} M_{persp \to ortho}$$



$$y' = \frac{n}{z} y$$

$$x' = \frac{n}{z} x$$

$$\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \xRightarrow{M_{o\to p}} \begin{pmatrix} nx/z \\ ny/z \\ \text{unknown} \\ 1 \end{pmatrix} \overset{\text{mult by } z}{==} \begin{pmatrix} nx \\ ny \\ \text{still uk} \\ z \end{pmatrix} \text{ 点不变}$$

$$M_{o\to p}^{(4\times 4)} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} nx \\ ny \\ uk \\ z \end{pmatrix}$$ 构造

$$\therefore M_{o\to p} = \begin{pmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ ? & ? & ? & ? \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

由规定n面不变，f面z不变 中心不变

① 对近端

面上 $\forall P = \begin{pmatrix} x \\ y \\ n \\ 1 \end{pmatrix} = \begin{pmatrix} nx \\ ny \\ n^2 \\ n \end{pmatrix}$

$$M_{o\to p} \begin{pmatrix} x \\ y \\ n \\ 1 \end{pmatrix} = \begin{pmatrix} nx \\ ny \\ n^2 \\ n \end{pmatrix}$$

$\therefore$ row3

$$(0\ 0\ A\ B) \begin{pmatrix} x \\ y \\ n \\ 1 \end{pmatrix} = n^2$$

② 对远端

取中心点 $P = \begin{pmatrix} 0 \\ 0 \\ f \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ f^2 \\ f \end{pmatrix}$

$$M_{o\to p} \begin{pmatrix} 0 \\ 0 \\ f \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ f^2 \\ f \end{pmatrix}$$

$\therefore$ row3

$$(0\ 0\ A\ B) \begin{pmatrix} x \\ y \\ f \\ 1 \end{pmatrix} = f^2$$

$$\Rightarrow \begin{cases} An + B = n^2 \\ Af + B = f^2 \end{cases}$$

解之 $\begin{cases} A = n+f \\ B = -nf \end{cases}$

$$M_{persp\to ortho} = \begin{pmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & (n+f) & (-nf) \\ 0 & 0 & 1 & 0 \end{pmatrix}$$