United International University
Department of Computer Science and Engineering
CSI 227 Algorithms, Mid Exam, Spring 2016
Total Marks: 30, Time: 90 minutes

Answer any 5 questions ($5 \times 6 = 30$).

1. Perform runtime analysis of the following programs and express them in Big-Theta notation. [$3 \times 2 = 6$]

   (a)
```
void program1(int n)
{
    int sum=0;
    for(int i=1;i<n;i++)
        for(int j=n;j>0;j/=2)
            sum+=i;
}
```

   (b)
```
void program2(int n)
{
    if(n)
    {
        int prod=1;
        for(int j=1;j<=n;j++)
            prod*=j;
        program2(n/2);
    }
}
```

2. (a) What is the running time of **heapsort** algorithm on an array A of length n that is already in sorted order? [2]

   (b) Suppose you have a max-priority queue implemented using a max-heap stored in an array. Now, you want to insert a new key value. Write a **pseudo-code or code** to perform the task. A prototype is given below. [2]

```
void insertKey(int A[],int heapSize, int newValue);
```

   (c) You are given an array that is a max-priority-queue, $A = \langle 15, 13, 9, 5, 12, 8, 7, 4, 0, 6, 2 \rangle$. Now, suppose you insert a new element 10 into this priority queue. What will be the array after this insertion? [2]

3. (a) A mathematician has written the following code to find the value of **Catalan Number**.

```
int catalan(int n)
{
    if(n==0) return 1;
```

```
    else {
        int ans=1;
        for(int j=1;j<=n;j++)
            ans+=catalan(i-1)*catalan(n-i);
        return ans;
    }
}
```

How good is this code in terms of runtime complexity? Improve this function using **memoization or dynamic programming** to reduce the runtime complexity. Write down the improved code or pseudo-code. What is the runtime complexity of the improved code of yours? [1+2+1=4]

(b) What is the value of the 6th Catalan number (i.e. if this function is called with parameter $n = 5$)? [2]

4. (a) What are the necessary conditions to apply dynamic programming to solve a problem? [1]

   (b) Suppose, you have got an iterative and a recursive version of the same algorithm to solve a problem. Which one will you prefer and why? [1]

   (c) You are in-charge of a canteen's cash counter. You can give the changes of certain amount to customers using the combination of 50 cents, 25 cents, 10 cents and 1 cents coins. Now all the time customers want to get minimum number of coins. Propose an algorithm which can determine the optimal (in this case minimum) number of coins and which coins to give for a certain amount of change. [4]

5. Suppose, students of the algorithms course of Spring 2016 trimester are going to **Dream Holiday Park, Panchdona, Narsingdi** for an outing after their exam is over. The students have gathered in small groups. Each group consists of maximum 4 students and minimum 1 student. Now, they have have to call a number of taxis for transport. Each taxi can carry at most four passengers. What minimum number of cars will the children need if all members of each group should ride in the same taxi (but one taxi can take more than one group)? For example if there are five groups like {1,2,4,3,3}, minimum 4 taxis will be required. Another example, if there are 8 groups like {2,3,4,4,2,1,3,1}, minimum 5 taxis are required. This is called the **Taxi Problem**. Design and write down the pseudo-code or code of a **greedy algorithm** to solve this problem. [6]

6. (a) Solve the following recurrence. [2]

$$T(n) = 7T(2n/3) + n^2$$

   (b) What is the maximum and minimum number of items that can be stored in a heap with height h? [2]

   (c) Perform the worst case runtime analysis of **insertion sort** algorithm. [2]