

A decorative graphic on the left side of the slide, consisting of a network of thin, gold-colored lines and small circles, resembling a circuit board or a stylized tree structure.

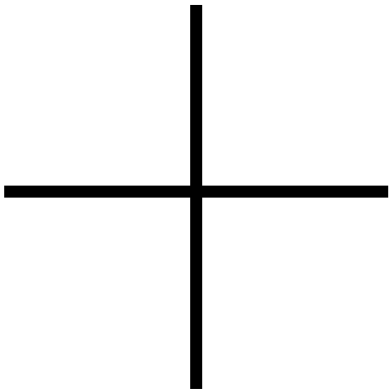
# Karnaugh Maps

**COMP311 Connor McMahon**

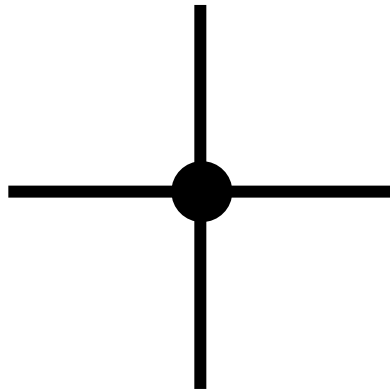
# Announcements

- Homework 2 due 2/8
- Lab 1 due 2/14
- Midterm 1 on 2/16

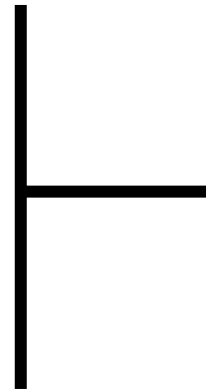
## Brief aside: wiring notation



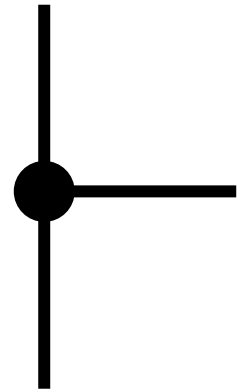
not connected



connected



connected  
(T-junction)

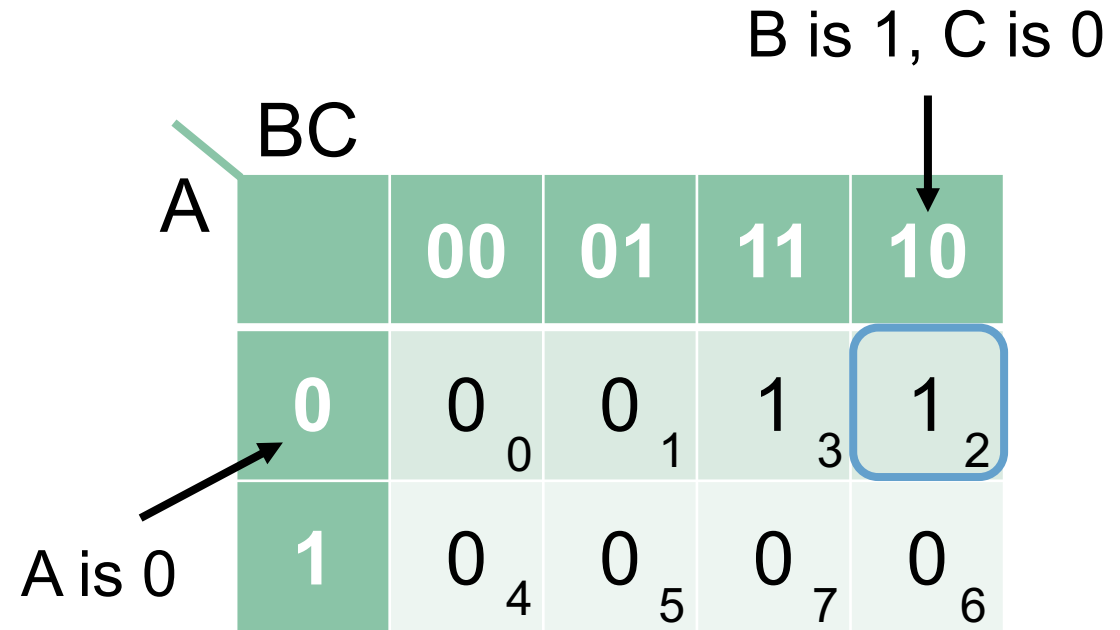


connected

# How to read K-map

| A | B | C | Y |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |

$\bar{A}B\bar{C}$



# How to read K-map

| A | B | C | Y |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |

$\bar{A}BC$

B and C are 1

BC

A

|   |                |                |                |                |
|---|----------------|----------------|----------------|----------------|
|   | 00             | 01             | 11             | 10             |
| 0 | 0 <sub>0</sub> | 0 <sub>1</sub> | 1 <sub>3</sub> | 1 <sub>2</sub> |
| 1 | 0 <sub>4</sub> | 0 <sub>5</sub> | 0 <sub>7</sub> | 0 <sub>6</sub> |

A is 0

# How to read K-map

| A | B | C | Y |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |

$\bar{A}B\bar{C}$

$\bar{A}BC$

|   |   |                |                |                |                |
|---|---|----------------|----------------|----------------|----------------|
|   |   | BC             |                |                |                |
| A |   | 00             | 01             | 11             | 10             |
|   | 0 | 0 <sub>0</sub> | 0 <sub>1</sub> | 1 <sub>3</sub> | 1 <sub>2</sub> |
|   | 1 | 0 <sub>4</sub> | 0 <sub>5</sub> | 0 <sub>7</sub> | 0 <sub>6</sub> |

$$Y = \bar{A}B\bar{C} + \bar{A}BC$$

# Simplification using k-map

| A | B | C | Y |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |

$\bar{A}B\bar{C}$

$\bar{A}BC$

| A | BC             |                |                |                |  |
|---|----------------|----------------|----------------|----------------|--|
|   | 00             | 01             | 11             | 10             |  |
| 0 | 0 <sub>0</sub> | 0 <sub>1</sub> | 1 <sub>3</sub> | 1 <sub>2</sub> |  |
| 1 | 0 <sub>4</sub> | 0 <sub>5</sub> | 0 <sub>7</sub> | 0 <sub>6</sub> |  |

The literals that the boxes have in common are  $A = 0$  and  $B = 1$

$$Y = \bar{A}B$$



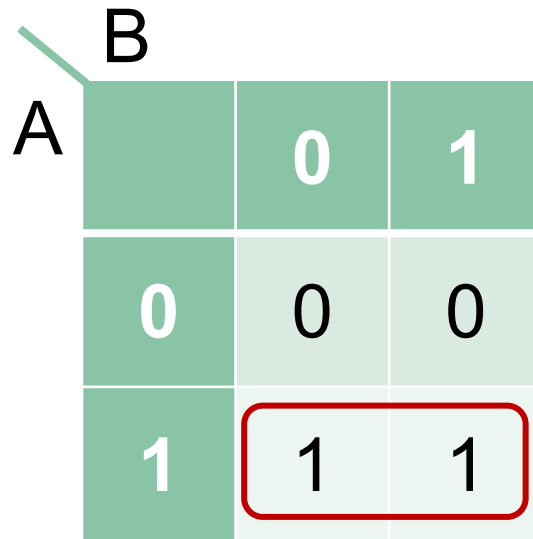
# K-Map Rules





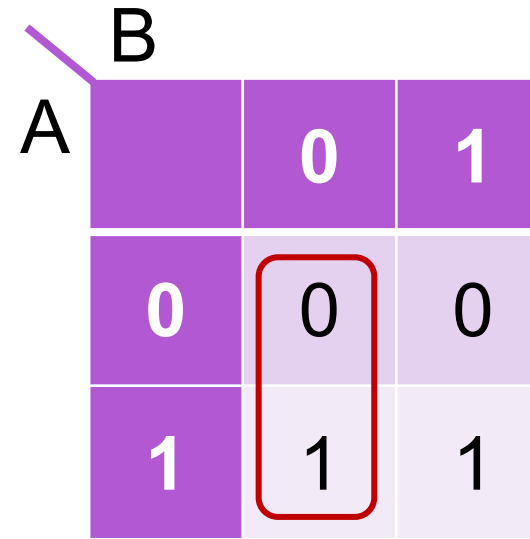
# K-Map Rules

All squares in each circle must contain 1's



|   |   |   |   |
|---|---|---|---|
|   | B |   |   |
| A |   | 0 | 1 |
|   | 0 | 0 | 0 |
|   | 1 | 1 | 1 |

Right

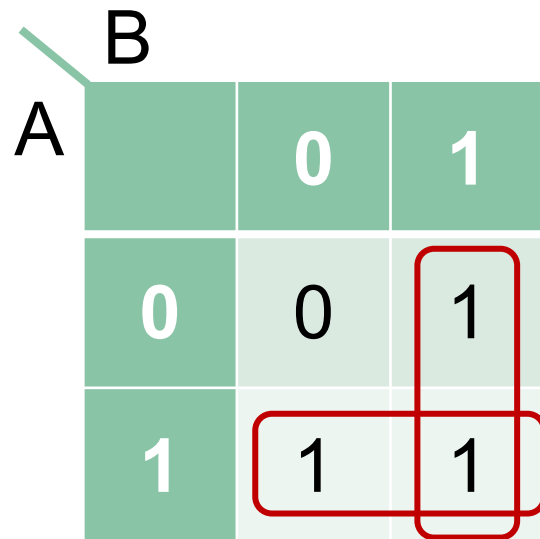


|   |   |   |   |
|---|---|---|---|
|   | B |   |   |
| A |   | 0 | 1 |
|   | 0 | 0 | 0 |
|   | 1 | 1 | 1 |

Wrong

# K-Map Rules

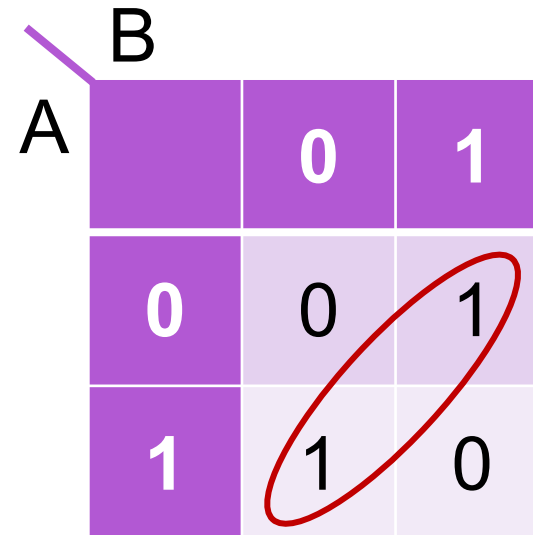
Groups may be horizontal or vertical, but not diagonal



A 3x3 Karnaugh map for variables A and B. The top row is labeled B and the left column is labeled A. The map contains 1s at (A=0, B=1), (A=1, B=0), (A=1, B=1), and (A=1, B=2). Two red rectangles highlight valid groups: a vertical group of two 1s in the third column (A=1) and a horizontal group of two 1s in the third row (B=0).

|     | B=0 | B=1 | B=2 |
|-----|-----|-----|-----|
| A=0 |     | 0   | 1   |
| A=1 | 0   | 0   | 1   |
| A=2 | 1   | 1   | 1   |

Right



A 3x3 Karnaugh map for variables A and B. The top row is labeled B and the left column is labeled A. The map contains 1s at (A=0, B=1), (A=1, B=0), (A=1, B=1), and (A=2, B=0). A red oval highlights an invalid diagonal group of two 1s at (A=1, B=1) and (A=2, B=0).

|     | B=0 | B=1 | B=2 |
|-----|-----|-----|-----|
| A=0 |     | 0   | 1   |
| A=1 | 0   | 0   | 1   |
| A=2 | 1   | 1   | 0   |

Wrong

# K-Map Rules

Groups must contain  $2^n$  cells where  $n = 0, 1, 2$ , etc.

|   |   |   |   |
|---|---|---|---|
| A | B |   |   |
|   |   | 0 | 1 |
|   |   | 0 | 0 |
|   |   | 1 | 0 |

Right

|   |    |    |    |    |    |
|---|----|----|----|----|----|
| A | BC |    |    |    |    |
|   |    | 00 | 01 | 11 | 10 |
|   |    | 0  | 0  | 0  | 0  |
|   |    | 1  | 1  | 1  | 0  |

Wrong

|   |    |    |    |    |    |
|---|----|----|----|----|----|
| A | BC |    |    |    |    |
|   |    | 00 | 01 | 11 | 10 |
|   |    | 0  | 0  | 0  | 0  |
|   |    | 1  | 1  | 1  | 0  |

Right

|   |    |    |    |    |    |
|---|----|----|----|----|----|
| A | BC |    |    |    |    |
|   |    | 00 | 01 | 11 | 10 |
|   |    | 0  | 1  | 1  | 0  |
|   |    | 1  | 1  | 1  | 0  |

Right

# K-Map Rules

Each group must be as large as possible

BC

A

|   |    |    |    |    |
|---|----|----|----|----|
|   | 00 | 01 | 11 | 10 |
| 0 | 1  | 1  | 1  | 1  |
| 1 | 0  | 0  | 1  | 1  |

Right

BC

A

|   |    |    |    |    |
|---|----|----|----|----|
|   | 00 | 01 | 11 | 10 |
| 0 | 1  | 1  | 1  | 1  |
| 1 | 0  | 0  | 1  | 1  |

Wrong

# K-Map Rules

Each cell containing a 1 must be in at least one group

BC

A

|   | 00 | 01 | 11 | 10 |
|---|----|----|----|----|
| 0 | 1  | 0  | 1  | 1  |
| 1 | 0  | 0  | 1  | 1  |

Right

BC

A

|   | 00 | 01 | 11 | 10 |
|---|----|----|----|----|
| 0 | 1  | 0  | 1  | 1  |
| 1 | 0  | 0  | 1  | 1  |

Wrong

# K-Map Rules

Groups may wrap around the table

A Karnaugh Map (K-Map) for a 2-variable function with variables A and BC. The map is a 2x4 grid. The columns are labeled with BC values: 00, 01, 11, 10. The rows are labeled with A values: 0, 1. The cells contain the function values (0 or 1). Two groups of 1s are highlighted with red curved lines, demonstrating that groups can wrap around the edges of the table.

| BC |   | 00 | 01 | 11 | 10 |
|----|---|----|----|----|----|
| A  |   |    |    |    |    |
| 0  | 1 | 1  | 0  | 0  | 1  |
| 1  | 1 | 1  | 0  | 0  | 1  |

# K-Map Rules

Every circle should contain at least one unique 1

BC

A

|   | 00 | 01 | 11 | 10 |
|---|----|----|----|----|
| 0 | 1  | 1  | 1  | 1  |
| 1 | 0  | 0  | 1  | 1  |

Right

BC

A

|   | 00 | 01 | 11 | 10 |
|---|----|----|----|----|
| 0 | 1  | 1  | 1  | 1  |
| 1 | 0  | 0  | 1  | 1  |

Wrong

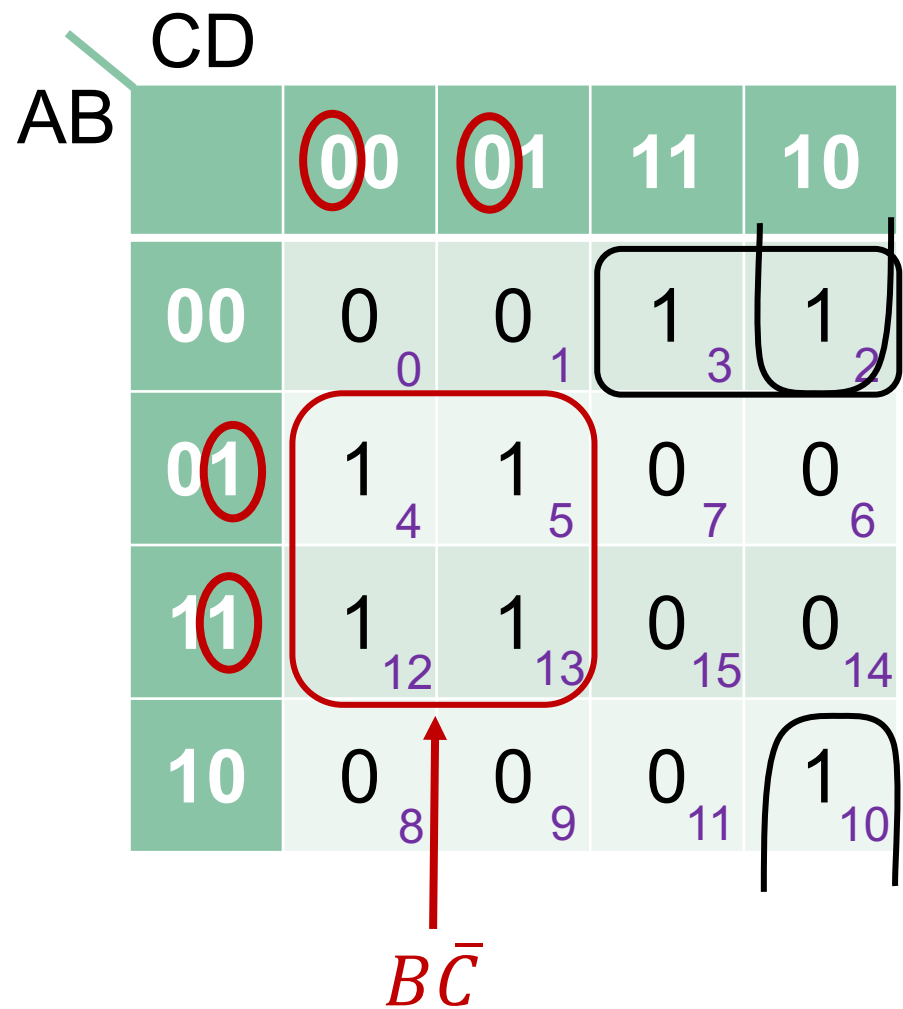
# 4-variable K-Map

| AB \ CD |                 |                 |                 |                 |  |
|---------|-----------------|-----------------|-----------------|-----------------|--|
|         | 00              | 01              | 11              | 10              |  |
| 00      | 0 <sub>0</sub>  | 0 <sub>1</sub>  | 1 <sub>3</sub>  | 1 <sub>2</sub>  |  |
| 01      | 1 <sub>4</sub>  | 1 <sub>5</sub>  | 0 <sub>7</sub>  | 0 <sub>6</sub>  |  |
| 11      | 1 <sub>12</sub> | 1 <sub>13</sub> | 0 <sub>15</sub> | 0 <sub>14</sub> |  |
| 10      | 0 <sub>8</sub>  | 0 <sub>9</sub>  | 0 <sub>11</sub> | 1 <sub>10</sub> |  |

|    | A | B | C | D | Y |
|----|---|---|---|---|---|
| 0  | 0 | 0 | 0 | 0 | 0 |
| 1  | 0 | 0 | 0 | 1 | 0 |
| 2  | 0 | 0 | 1 | 0 | 1 |
| 3  | 0 | 0 | 1 | 1 | 1 |
| 4  | 0 | 1 | 0 | 0 | 1 |
| 5  | 0 | 1 | 0 | 1 | 1 |
| 6  | 0 | 1 | 1 | 0 | 0 |
| 7  | 0 | 1 | 1 | 1 | 0 |
| 8  | 1 | 0 | 0 | 0 | 0 |
| 9  | 1 | 0 | 0 | 1 | 0 |
| 10 | 1 | 0 | 1 | 0 | 1 |
| 11 | 1 | 0 | 1 | 1 | 0 |
| 12 | 1 | 1 | 0 | 0 | 1 |
| 13 | 1 | 1 | 0 | 1 | 1 |
| 14 | 1 | 1 | 1 | 0 | 0 |
| 15 | 1 | 1 | 1 | 1 | 0 |

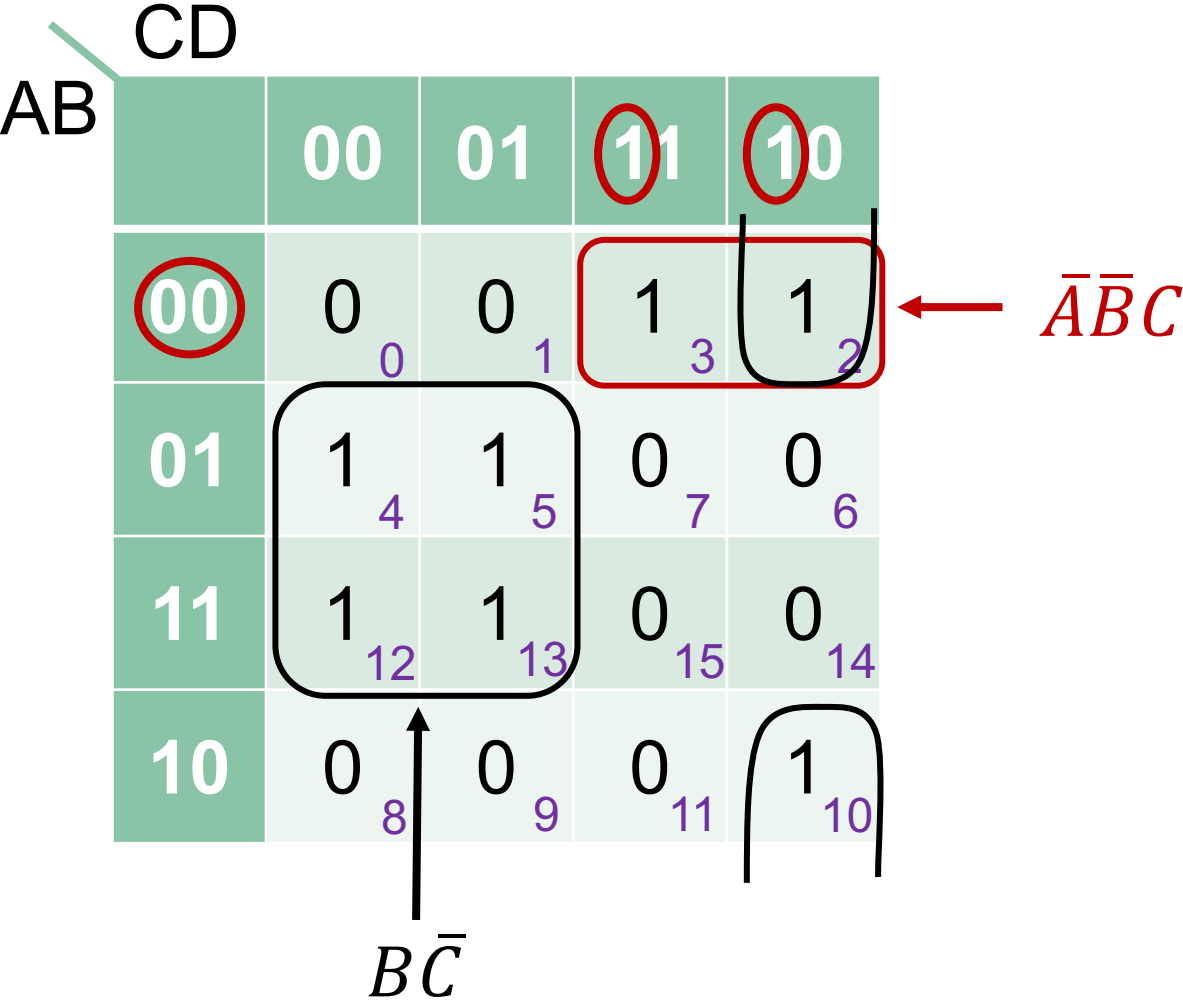


# 4-variable K-Map



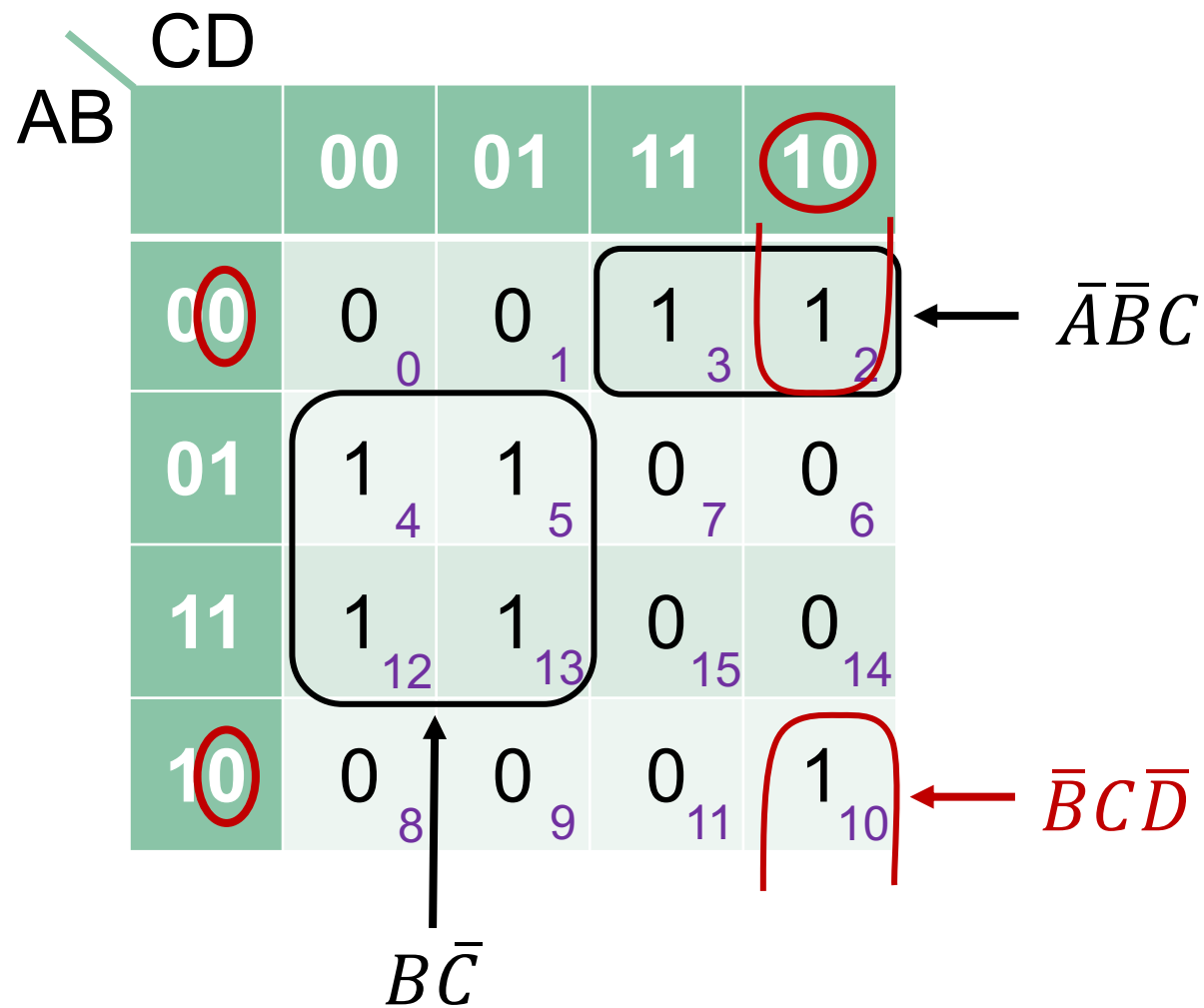
|    | A | B | C | D | Y |
|----|---|---|---|---|---|
| 0  | 0 | 0 | 0 | 0 | 0 |
| 1  | 0 | 0 | 0 | 1 | 0 |
| 2  | 0 | 0 | 1 | 0 | 1 |
| 3  | 0 | 0 | 1 | 1 | 1 |
| 4  | 0 | 1 | 0 | 0 | 1 |
| 5  | 0 | 1 | 0 | 1 | 1 |
| 6  | 0 | 1 | 1 | 0 | 0 |
| 7  | 0 | 1 | 1 | 1 | 0 |
| 8  | 1 | 0 | 0 | 0 | 0 |
| 9  | 1 | 0 | 0 | 1 | 0 |
| 10 | 1 | 0 | 1 | 0 | 1 |
| 11 | 1 | 0 | 1 | 1 | 0 |
| 12 | 1 | 1 | 0 | 0 | 1 |
| 13 | 1 | 1 | 0 | 1 | 1 |
| 14 | 1 | 1 | 1 | 0 | 0 |
| 15 | 1 | 1 | 1 | 1 | 0 |

# 4-variable K-Map



|    | A | B | C | D | Y |
|----|---|---|---|---|---|
| 0  | 0 | 0 | 0 | 0 | 0 |
| 1  | 0 | 0 | 0 | 1 | 0 |
| 2  | 0 | 0 | 1 | 0 | 1 |
| 3  | 0 | 0 | 1 | 1 | 1 |
| 4  | 0 | 1 | 0 | 0 | 1 |
| 5  | 0 | 1 | 0 | 1 | 1 |
| 6  | 0 | 1 | 1 | 0 | 0 |
| 7  | 0 | 1 | 1 | 1 | 0 |
| 8  | 1 | 0 | 0 | 0 | 0 |
| 9  | 1 | 0 | 0 | 1 | 0 |
| 10 | 1 | 0 | 1 | 0 | 1 |
| 11 | 1 | 0 | 1 | 1 | 0 |
| 12 | 1 | 1 | 0 | 0 | 1 |
| 13 | 1 | 1 | 0 | 1 | 1 |
| 14 | 1 | 1 | 1 | 0 | 0 |
| 15 | 1 | 1 | 1 | 1 | 0 |

# 4-variable K-Map



|    | A | B | C | D | Y |
|----|---|---|---|---|---|
| 0  | 0 | 0 | 0 | 0 | 0 |
| 1  | 0 | 0 | 0 | 1 | 0 |
| 2  | 0 | 0 | 1 | 0 | 1 |
| 3  | 0 | 0 | 1 | 1 | 1 |
| 4  | 0 | 1 | 0 | 0 | 1 |
| 5  | 0 | 1 | 0 | 1 | 1 |
| 6  | 0 | 1 | 1 | 0 | 0 |
| 7  | 0 | 1 | 1 | 1 | 0 |
| 8  | 1 | 0 | 0 | 0 | 0 |
| 9  | 1 | 0 | 0 | 1 | 0 |
| 10 | 1 | 0 | 1 | 0 | 1 |
| 11 | 1 | 0 | 1 | 1 | 0 |
| 12 | 1 | 1 | 0 | 0 | 1 |
| 13 | 1 | 1 | 0 | 1 | 1 |
| 14 | 1 | 1 | 1 | 0 | 0 |
| 15 | 1 | 1 | 1 | 1 | 0 |

# 4-variable K-Map

|    |    |                 |                 |                 |                 |
|----|----|-----------------|-----------------|-----------------|-----------------|
|    |    | CD              |                 |                 |                 |
| AB |    | 00              | 01              | 11              | 10              |
|    | 00 | 0 <sub>0</sub>  | 0 <sub>1</sub>  | 1 <sub>3</sub>  | 1 <sub>2</sub>  |
|    | 01 | 1 <sub>4</sub>  | 1 <sub>5</sub>  | 0 <sub>7</sub>  | 0 <sub>6</sub>  |
|    | 11 | 1 <sub>12</sub> | 1 <sub>13</sub> | 0 <sub>15</sub> | 0 <sub>14</sub> |
|    | 10 | 0 <sub>8</sub>  | 0 <sub>9</sub>  | 0 <sub>11</sub> | 1 <sub>10</sub> |

$$Y = B\bar{C} + \bar{A}\bar{B}C + \bar{B}C\bar{D}$$

|    | A | B | C | D | Y |
|----|---|---|---|---|---|
| 0  | 0 | 0 | 0 | 0 | 0 |
| 1  | 0 | 0 | 0 | 1 | 0 |
| 2  | 0 | 0 | 1 | 0 | 1 |
| 3  | 0 | 0 | 1 | 1 | 1 |
| 4  | 0 | 1 | 0 | 0 | 1 |
| 5  | 0 | 1 | 0 | 1 | 1 |
| 6  | 0 | 1 | 1 | 0 | 0 |
| 7  | 0 | 1 | 1 | 1 | 0 |
| 8  | 1 | 0 | 0 | 0 | 0 |
| 9  | 1 | 0 | 0 | 1 | 0 |
| 10 | 1 | 0 | 1 | 0 | 1 |
| 11 | 1 | 0 | 1 | 1 | 0 |
| 12 | 1 | 1 | 0 | 0 | 1 |
| 13 | 1 | 1 | 0 | 1 | 1 |
| 14 | 1 | 1 | 1 | 0 | 0 |
| 15 | 1 | 1 | 1 | 1 | 0 |

# Q1: Reading a K-Map

| AB \ CD | CD |    |    |    |  |
|---------|----|----|----|----|--|
|         | 00 | 01 | 11 | 10 |  |
| 00      | 1  | 0  | 0  | 0  |  |
| 01      | 1  | 1  | 1  | 0  |  |
| 11      | 1  | 1  | 1  | 1  |  |
| 10      | 1  | 0  | 0  | 0  |  |

## Q1: Reading a K-Map



| AB \ CD |    |    |    |    |
|---------|----|----|----|----|
|         | 00 | 01 | 11 | 10 |
| 00      | 1  | 0  | 0  | 0  |
| 01      | 1  | 1  | 1  | 0  |
| 11      | 1  | 1  | 1  | 1  |
| 10      | 1  | 0  | 0  | 0  |

$$Y = \bar{C}\bar{D} + BD + AB$$

## Q2: Reading a K-Map

|    |    | CD |    |    |    |    |
|----|----|----|----|----|----|----|
|    |    |    | 00 | 01 | 11 | 10 |
| AB |    |    | 00 | 01 | 11 | 10 |
|    | 00 | 1  | 1  | 1  | 1  |    |
|    | 01 | 1  | 1  | 1  | 1  |    |
|    | 11 | 0  | 0  | 0  | 0  |    |
|    | 10 | 1  | 0  | 0  | 1  |    |

## Q2: Reading a K-Map



| AB \ CD | CD |    |    |    |
|---------|----|----|----|----|
|         | 00 | 01 | 11 | 10 |
| 00      | 1  | 1  | 1  | 1  |
| 01      | 1  | 1  | 1  | 1  |
| 11      | 0  | 0  | 0  | 0  |
| 10      | 1  | 0  | 0  | 1  |

$$Y = \bar{A} + \bar{B}\bar{D}$$



### Q3: Reading a K-Map

|    |    | CD |    |    |    |    |
|----|----|----|----|----|----|----|
|    |    |    | 00 | 01 | 11 | 10 |
| AB |    |    | 00 | 01 | 11 | 10 |
|    | 00 | 0  | 0  | 1  | 1  |    |
|    | 01 | 1  | 1  | 1  | 1  |    |
|    | 11 | 1  | 1  | 0  | 0  |    |
|    | 10 | 0  | 0  | 1  | 1  |    |

### Q3: Reading a K-Map



| AB \ CD | CD |    |    |    |
|---------|----|----|----|----|
|         | 00 | 01 | 11 | 10 |
| 00      | 0  | 0  | 1  | 1  |
| 01      | 1  | 1  | 1  | 1  |
| 11      | 1  | 1  | 0  | 0  |
| 10      | 0  | 0  | 1  | 1  |

$$Y = B\bar{C} + \bar{B}C + \bar{A}C$$

| AB \ CD | CD |    |    |    |
|---------|----|----|----|----|
|         | 00 | 01 | 11 | 10 |
| 00      | 0  | 0  | 1  | 1  |
| 01      | 1  | 1  | 1  | 1  |
| 11      | 1  | 1  | 0  | 0  |
| 10      | 0  | 0  | 1  | 1  |

$$Y = B\bar{C} + \bar{B}C + \bar{A}B$$

# Definition: Implicant

Any product term that is a 1 for a given Boolean Equation (i.e. any group of 1s)

| A | BC |    |    |    |    |
|---|----|----|----|----|----|
|   |    | 00 | 01 | 11 | 10 |
| 0 | 0  | 0  | 0  | 1  | 0  |
| 1 | 1  | 0  | 1  | 1  | 0  |

## Implicants

$$\bar{A}BC$$

$$A\bar{B}C$$

$$ABC$$

$$BC$$

$$AC$$

# Definition: Prime Implicant

An implicant that is not a subset of any other implicant

| AB \ CD |    |    |    |    |  |
|---------|----|----|----|----|--|
|         | 00 | 01 | 11 | 10 |  |
| 00      | 1  | 1  | 0  | 0  |  |
| 01      | 0  | 1  | 1  | 0  |  |
| 11      | 1  | 1  | 1  | 0  |  |
| 10      | 1  | 1  | 0  | 0  |  |

Q: Is this a prime implicant?

# Definition: Prime Implicant



An implicant that is not a subset of any other implicant

| AB \ CD | CD |    |    |    |  |
|---------|----|----|----|----|--|
|         | 00 | 01 | 11 | 10 |  |
| 00      | 1  | 1  | 0  | 0  |  |
| 01      | 0  | 1  | 1  | 0  |  |
| 11      | 1  | 1  | 1  | 0  |  |
| 10      | 1  | 1  | 0  | 0  |  |

Q: Is this a prime implicant?

Yes!

# Definition: Prime Implicant

An implicant that is not a subset of any other implicant

| AB \ CD | 00 | 01 | 11 | 10 |
|---------|----|----|----|----|
| 00      | 1  | 1  | 0  | 0  |
| 01      | 0  | 1  | 1  | 0  |
| 11      | 1  | 1  | 1  | 0  |
| 10      | 1  | 1  | 0  | 0  |

Q: Is this a prime implicant?

# Definition: Prime Implicant



An implicant that is not a subset of any other implicant

| AB \ CD | CD |    |    |    |  |
|---------|----|----|----|----|--|
|         | 00 | 01 | 11 | 10 |  |
| 00      | 1  | 1  | 0  | 0  |  |
| 01      | 0  | 1  | 1  | 0  |  |
| 11      | 1  | 1  | 1  | 0  |  |
| 10      | 1  | 1  | 0  | 0  |  |

Q: Is this a prime implicant?

No, it can be covered by another group

# Definition: Prime Implicant

An implicant that is not a subset of any other implicant

| AB \ CD |    |    |    |    |
|---------|----|----|----|----|
|         | 00 | 01 | 11 | 10 |
| 00      | 1  | 1  | 0  | 0  |
| 01      | 0  | 1  | 1  | 0  |
| 11      | 1  | 1  | 1  | 0  |
| 10      | 1  | 1  | 0  | 0  |

Q: Is this a prime implicant?



# Definition: Prime Implicant



An implicant that is not a subset of any other implicant

| CD |    |    |    |    |    |
|----|----|----|----|----|----|
| AB |    | 00 | 01 | 11 | 10 |
|    | 00 | 1  | 1  | 0  | 0  |
|    | 01 | 0  | 1  | 1  | 0  |
|    | 11 | 1  | 1  | 1  | 0  |
|    | 10 | 1  | 1  | 0  | 0  |

Q: Is this a prime implicant?

No, it can be covered by another group

# Definition: Essential Prime Implicant

A prime implicant with at least one element that is not covered by one or more prime implicants (i.e. we must use this group in our final solution in order to cover all 1s)

| AB \ CD | CD |    |    |    |  |
|---------|----|----|----|----|--|
|         | 00 | 01 | 11 | 10 |  |
| 00      | 1  | 1  | 0  | 0  |  |
| 01      | 0  | 1  | 1  | 0  |  |
| 11      | 1  | 1  | 1  | 0  |  |
| 10      | 1  | 1  | 0  | 0  |  |

Q: Is this an essential prime implicant?

# Definition: Essential Prime Implicant



A prime implicant with at least one element that is not covered by one or more prime implicants

| AB \ CD |    |    |    |    |  |
|---------|----|----|----|----|--|
|         | 00 | 01 | 11 | 10 |  |
| 00      | 1  | 1  | 0  | 0  |  |
| 01      | 0  | 1  | 1  | 0  |  |
| 11      | 1  | 1  | 1  | 0  |  |
| 10      | 1  | 1  | 0  | 0  |  |

Q: Is this an essential prime implicant?

Yes!

# Definition: Essential Prime Implicant

A prime implicant with at least one element that is not covered by one or more prime implicants

| AB \ CD |    |    |    |    |  |
|---------|----|----|----|----|--|
|         | 00 | 01 | 11 | 10 |  |
| 00      | 1  | 1  | 0  | 0  |  |
| 01      | 0  | 1  | 1  | 0  |  |
| 11      | 1  | 1  | 1  | 0  |  |
| 10      | 1  | 1  | 0  | 0  |  |

Q: Is this an essential prime implicant?

# Definition: Essential Prime Implicant



A prime implicant with at least one element that is not covered by one or more prime implicants

| CD |    |    |    |    |    |
|----|----|----|----|----|----|
| AB |    | 00 | 01 | 11 | 10 |
|    | 00 | 1  | 1  | 0  | 0  |
|    | 01 | 0  | 1  | 1  | 0  |
|    | 11 | 1  | 1  | 1  | 0  |
|    | 10 | 1  | 1  | 0  | 0  |

Q: Is this an essential prime implicant?

No!

# Definition: Non-Essential Prime Implicant

Prime implicant that has no element that cannot be covered by other prime implicant

|    |    | CD |    |    |    |
|----|----|----|----|----|----|
| AB |    | 00 | 01 | 11 | 10 |
|    | 00 | 1  | 1  | 0  | 0  |
|    | 01 | 0  | 1  | 1  | 0  |
|    | 11 | 1  | 1  | 1  | 0  |
|    | 10 | 1  | 1  | 0  | 0  |

# Formal K-Map Procedure

1. Convert truth table to K-map
2. Include all essential primes
3. Include non-essential primes as needed to completely cover all ones

## Revisiting Q3

### Essential Prime Implicants

Karnaugh map showing the function with variables AB and CD. The map is a 5x5 grid. The columns are labeled CD (00, 01, 11, 10) and the rows are labeled AB (00, 01, 11, 10). The cells contain the function value (0 or 1). Three prime implicants are circled in red, indicating they are essential.

| AB \ CD | 00 | 01 | 11 | 10 |
|---------|----|----|----|----|
| 00      | 0  | 0  | 1  | 1  |
| 01      | 1  | 1  | 1  | 1  |
| 11      | 1  | 1  | 0  | 0  |
| 10      | 0  | 0  | 1  | 1  |



# Revisiting Q3

To cover the remaining ones, we will choose one of these non-essential prime implicants

| AB \ CD |    |    |    |    |  |
|---------|----|----|----|----|--|
|         | 00 | 01 | 11 | 10 |  |
| 00      | 0  | 0  | 1  | 1  |  |
| 01      | 1  | 1  | 1  | 1  |  |
| 11      | 1  | 1  | 0  | 0  |  |
| 10      | 0  | 0  | 1  | 1  |  |
|         |    |    |    |    |  |

| AB \ CD |    |    |    |    |  |
|---------|----|----|----|----|--|
|         | 00 | 01 | 11 | 10 |  |
| 00      | 0  | 0  | 1  | 1  |  |
| 01      | 1  | 1  | 1  | 1  |  |
| 11      | 1  | 1  | 0  | 0  |  |
| 10      | 0  | 0  | 1  | 1  |  |
|         |    |    |    |    |  |

# Recall: Sum of Products

SOP

$$Y = ABC + AB\bar{C}\bar{D} + D + B\bar{C}$$

Not SOP

$$Y = A(B + C)$$

$$Y = (A + D)(B + C)$$

$$Y = \overline{AB + C}$$

$$Y = \overline{AB}D$$

# Simplify the Following Boolean Equation

$$Y = \overline{A + \bar{B} + C + \bar{D}} + ABC\bar{D} + BC + \overline{\bar{A}\bar{B}} + \overline{\bar{C} + \bar{D}}$$

# Simplify the Following Boolean Equation

$$Y = \overline{A + \bar{B} + C + \bar{D}} + ABC\bar{D} + BC + \overline{\bar{A}\bar{B}} + \overline{\bar{C} + \bar{D}}$$

$$Y = \bar{A}\bar{B}\bar{C}D + ABC\bar{D} + BC + AB(C + D)$$

$$Y = \bar{A}\bar{B}\bar{C}D + ABC\bar{D} + BC + ABC + ABD$$

$$Y = \bar{A}\bar{B}\bar{C}D + ABC\bar{D} + BC + ABD$$

$$Y = \bar{A}\bar{B}\bar{C}D + BC + ABD$$

Is this the most simplified equation?

We can always plug it into a K-Map to check!

# Using a K-Map to Simplify an Equation

$$Y = \bar{A}\bar{B}\bar{C}D + BC + ABD$$

| AB \ CD | CD |    |    |    |
|---------|----|----|----|----|
|         | 00 | 01 | 11 | 10 |
| 00      | 0  | 1  | 3  | 2  |
| 01      | 4  | 5  | 7  | 6  |
| 11      | 12 | 13 | 15 | 14 |
| 10      | 8  | 9  | 11 | 10 |

|    | A | B | C | D | Y |
|----|---|---|---|---|---|
| 0  | 0 | 0 | 0 | 0 |   |
| 1  | 0 | 0 | 0 | 1 |   |
| 2  | 0 | 0 | 1 | 0 |   |
| 3  | 0 | 0 | 1 | 1 |   |
| 4  | 0 | 1 | 0 | 0 |   |
| 5  | 0 | 1 | 0 | 1 |   |
| 6  | 0 | 1 | 1 | 0 |   |
| 7  | 0 | 1 | 1 | 1 |   |
| 8  | 1 | 0 | 0 | 0 |   |
| 9  | 1 | 0 | 0 | 1 |   |
| 10 | 1 | 0 | 1 | 0 |   |
| 11 | 1 | 0 | 1 | 1 |   |
| 12 | 1 | 1 | 0 | 0 |   |
| 13 | 1 | 1 | 0 | 1 |   |
| 14 | 1 | 1 | 1 | 0 |   |
| 15 | 1 | 1 | 1 | 1 |   |

# Using a K-Map to Simplify an Equation

$$Y = \bar{A}\bar{B}\bar{C}D + BC + ABD$$

|    |    | CD              |                 |                 |                 |
|----|----|-----------------|-----------------|-----------------|-----------------|
| AB |    | 00              | 01              | 11              | 10              |
|    | 00 | 0 <sub>0</sub>  | 0 <sub>1</sub>  | 0 <sub>3</sub>  | 0 <sub>2</sub>  |
|    | 01 | 0 <sub>4</sub>  | 1 <sub>5</sub>  | 1 <sub>7</sub>  | 1 <sub>6</sub>  |
|    | 11 | 0 <sub>12</sub> | 1 <sub>13</sub> | 1 <sub>15</sub> | 1 <sub>14</sub> |
|    | 10 | 0 <sub>8</sub>  | 0 <sub>9</sub>  | 0 <sub>11</sub> | 0 <sub>10</sub> |

$$Y = BD + BC$$

|    | A | B | C | D | Y |
|----|---|---|---|---|---|
| 0  | 0 | 0 | 0 | 0 | 0 |
| 1  | 0 | 0 | 0 | 1 | 0 |
| 2  | 0 | 0 | 1 | 0 | 0 |
| 3  | 0 | 0 | 1 | 1 | 0 |
| 4  | 0 | 1 | 0 | 0 | 0 |
| 5  | 0 | 1 | 0 | 1 | 1 |
| 6  | 0 | 1 | 1 | 0 | 1 |
| 7  | 0 | 1 | 1 | 1 | 1 |
| 8  | 1 | 0 | 0 | 0 | 0 |
| 9  | 1 | 0 | 0 | 1 | 0 |
| 10 | 1 | 0 | 1 | 0 | 0 |
| 11 | 1 | 0 | 1 | 1 | 0 |
| 12 | 1 | 1 | 0 | 0 | 0 |
| 13 | 1 | 1 | 0 | 1 | 1 |
| 14 | 1 | 1 | 1 | 0 | 1 |
| 15 | 1 | 1 | 1 | 1 | 1 |

# Recall: Don't Cares

Priority Circuit Truth Table

| A <sub>3</sub> | A <sub>2</sub> | A <sub>1</sub> | A <sub>0</sub> | Y <sub>3</sub> | Y <sub>2</sub> | Y <sub>1</sub> | Y <sub>0</sub> |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 0              | 0              | 0              | 0              | 0              | 0              | 0              | 0              |
| 0              | 0              | 0              | 1              | 0              | 0              | 0              | 1              |
| 0              | 0              | 1              | X              | 0              | 0              | 1              | 0              |
| 0              | 1              | X              | X              | 0              | 1              | 0              | 0              |
| 1              | X              | X              | X              | 1              | 0              | 0              | 0              |

A “don't care” value (X)  
indicates that the input  
does not affect the  
output

# Output Values Can Also Be Don't Care!

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | X |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

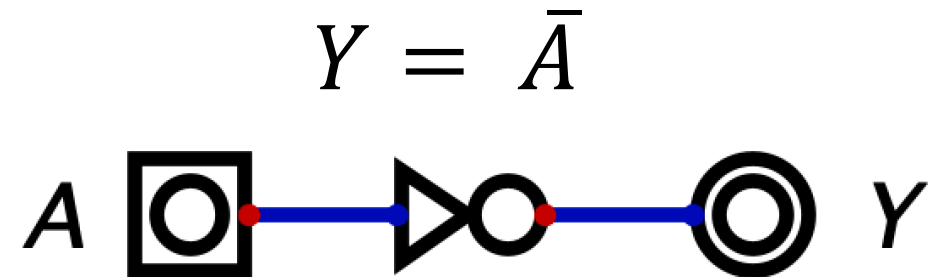
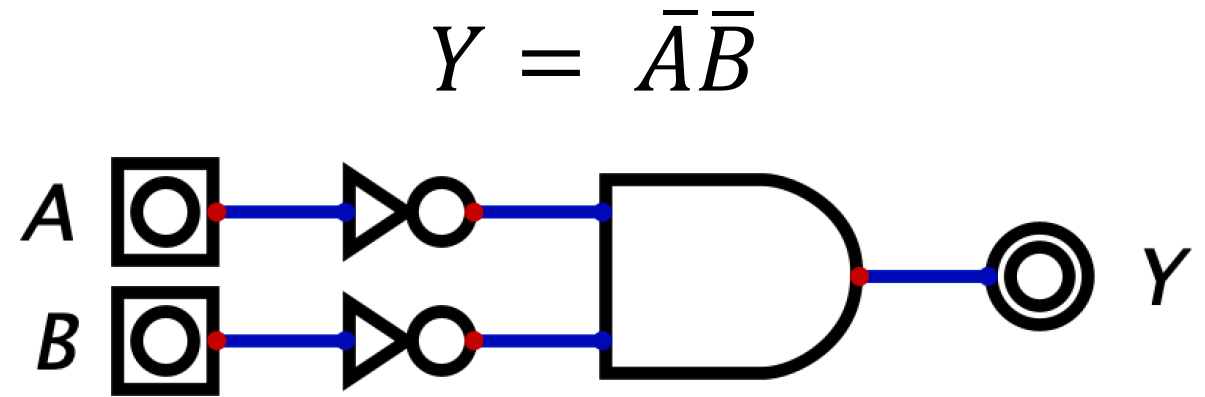
← When A and B are 1,  
the output can be 0  
or 1!



# Output Values Can Also Be Don't Care!

There are two possible implementations of this truth table

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | X |
| 1 | 0 | 0 |
| 1 | 1 | 0 |



# Don't Cares in K-Maps

We can treat a don't care value as either 0 or 1. An X can be circled if it helps us cover the 1s with fewer or larger circles, but they do not have to be circled if they are not helpful.

| AB \ CD | CD |    |    |    |    |
|---------|----|----|----|----|----|
|         |    | 00 | 01 | 11 | 10 |
| 00      | 0  | 0  | 0  | 0  | 0  |
| 01      | 0  | 1  | X  | 0  |    |
| 11      | 0  | 1  | X  | 0  |    |
| 10      | 0  | 0  | 0  | 0  | 0  |

# Don't Cares in K-Maps

We can treat a don't care value as either 0 or 1. An X can be circled if it helps us cover the 1s with fewer or larger circles, but they do not have to be circled if they are not helpful.

| AB \ CD |    |    |    |    |  |
|---------|----|----|----|----|--|
|         | 00 | 01 | 11 | 10 |  |
| 00      | 0  | 0  | 0  | 0  |  |
| 01      | 0  | 1  | X  | 0  |  |
| 11      | 0  | 1  | X  | 0  |  |
| 10      | 0  | 0  | 0  | 0  |  |

If we group the 1s on their own, we have the equation  $Y = B\bar{C}D$

If we group the 1s with the X's, we have the equation  $Y = BD$

# Don't Cares in K-Maps

The left equation requires more gates, so it would be better to use the right equation.

$$Y = B\bar{C}D$$

$$Y = BD$$

For the input combinations that produce a don't care output in the k-map, the circuit would output a 0 using the left implementation and a 1 using the right implementation.

| AB \ CD | CD |    |    |    |
|---------|----|----|----|----|
|         | 00 | 01 | 11 | 10 |
| 00      | 0  | 0  | 0  | 0  |
| 01      | 0  | 1  | X  | 0  |
| 11      | 0  | 1  | X  | 0  |
| 10      | 0  | 0  | 0  | 0  |

i.e. when  $A = 0, B = 1, C = 1, D = 1 \rightarrow Y = 0$  for the left and 1 for the right

and when  $A = 1, B = 1, C = 1, D = 1 \rightarrow Y = 0$  for the left and 1 for the right

# Don't Cares in K-Maps

We can treat a don't care value as either 0 or 1. An X can be circled if it helps us cover the 1s with fewer or larger circles, but they do not have to be circled if they are not helpful.

|    |    | CD |    |    |    |    |
|----|----|----|----|----|----|----|
|    |    |    | 00 | 01 | 11 | 10 |
| AB |    |    | 00 | 01 | 11 | 10 |
|    | 00 | 1  | 0  | X  | 1  |    |
|    | 01 | 0  | 1  | X  | 1  |    |
|    | 11 | 1  | 1  | X  | X  |    |
|    | 10 | 1  | 1  | X  | X  |    |

# Don't Cares in K-Maps



We can treat a don't care value as either 0 or 1. An X can be circled if it helps us cover the 1s with fewer or larger circles, but they do not have to be circled if they are not helpful.

| AB \ CD | CD |    |    |    |
|---------|----|----|----|----|
|         | 00 | 01 | 11 | 10 |
| 00      | 1  | 0  | X  | 1  |
| 01      | 0  | 1  | X  | 1  |
| 11      | 1  | 1  | X  | X  |
| 10      | 1  | 1  | X  | X  |

$$Y = A + C + \bar{B}\bar{D} + BD$$