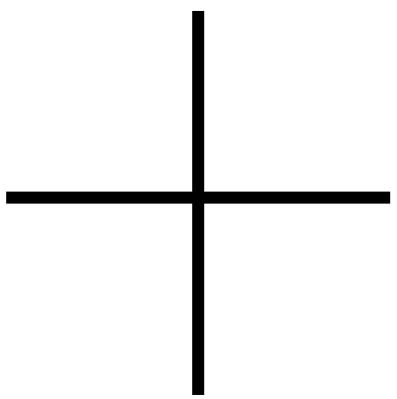
Karnaugh Maps

COMP311 Connor McMahon

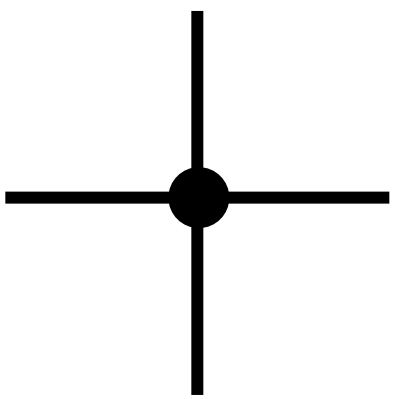
Announcements

- Homework 2 due 2/8
- Lab 1 due 2/14
- Midterm 1 on 2/16

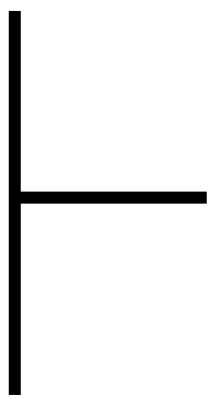
Brief aside: wiring notation



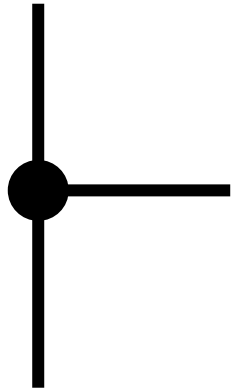
not connected



connected



connected
(T-junction)

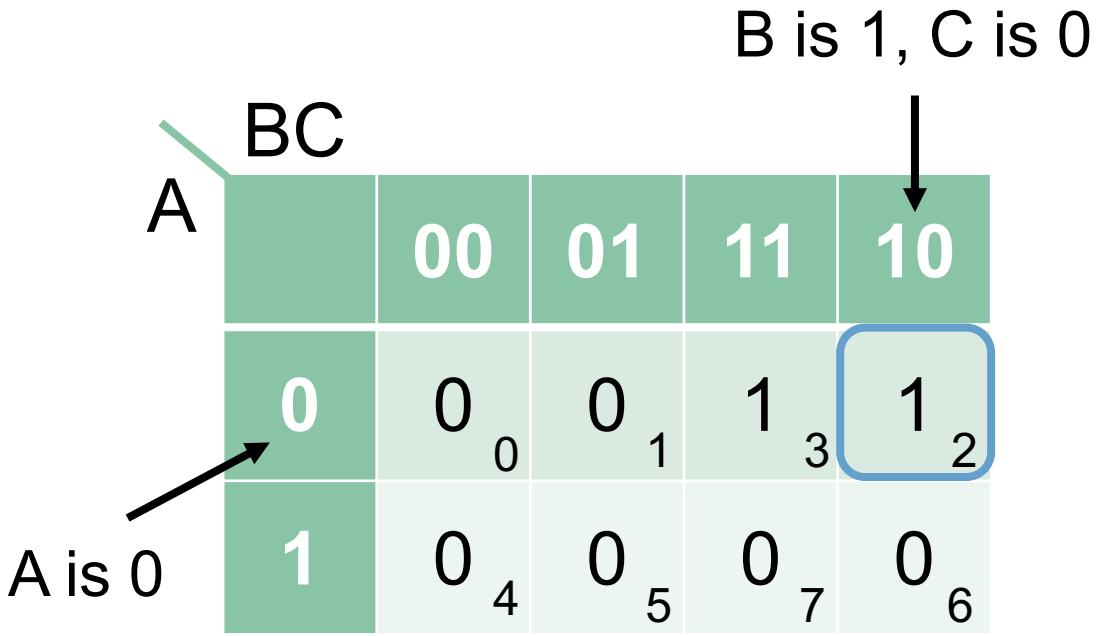


connected

How to read K-map

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

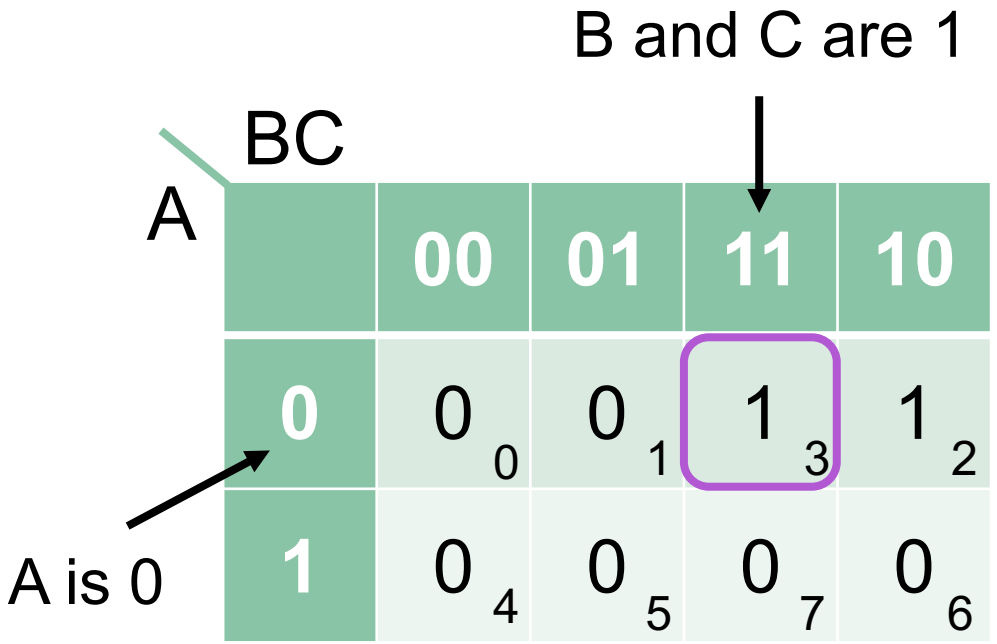
$\bar{A}B\bar{C}$



How to read K-map

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

$\bar{A}BC$



How to read K-map

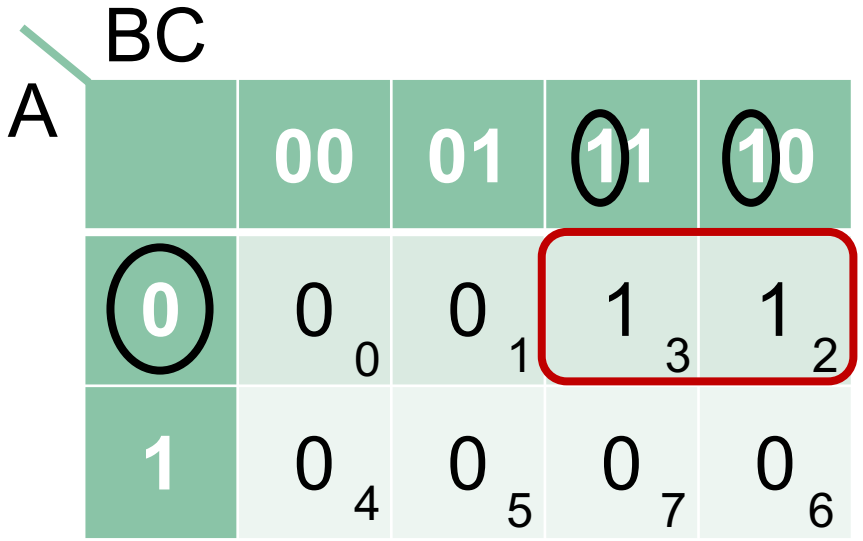
A	B	C	Y	
0	0	0	0	
0	0	1	0	
0	1	0	1	$\bar{A}B\bar{C}$
0	1	1	1	$\bar{A}BC$
1	0	0	0	
1	0	1	0	
1	1	0	0	
1	1	1	0	

A \ BC	00	01	11	10
0	0 0	0 1	1 3	1 2
1	0 4	0 5	0 7	0 6

$$Y = \bar{A}B\bar{C} + \bar{A}BC$$

Simplification using k-map

A	B	C	Y	
0	0	0	0	
0	0	1	0	
0	1	0	1	$\bar{A}B\bar{C}$
0	1	1	1	$\bar{A}BC$
1	0	0	0	
1	0	1	0	
1	1	0	0	
1	1	1	0	



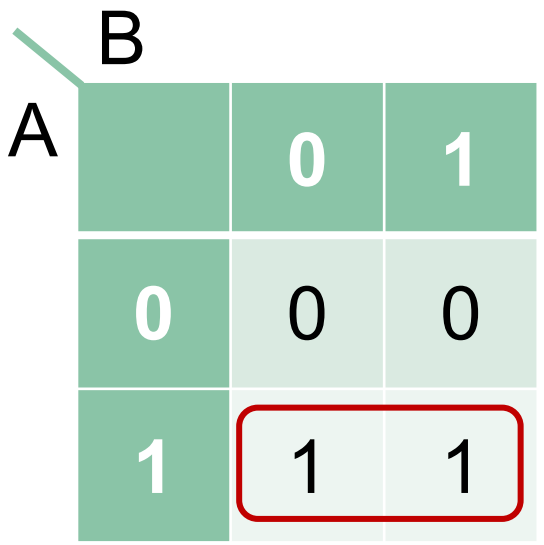
The literals that the boxes have in common are A = 0 and B = 1

$$Y = \bar{A}B$$

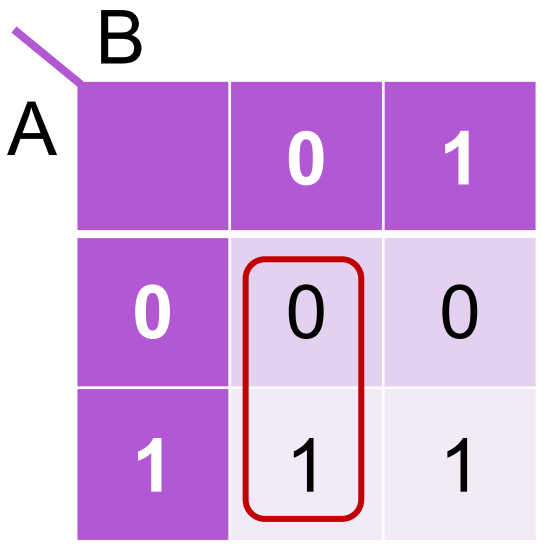
K-Map Rules

K-Map Rules

All squares in each circle must contain 1’s



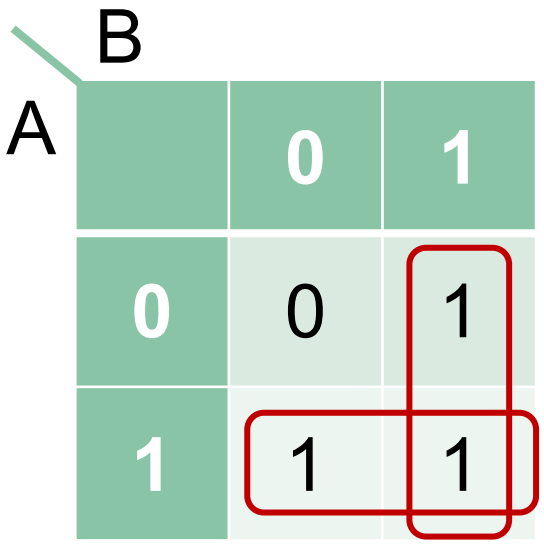
Right



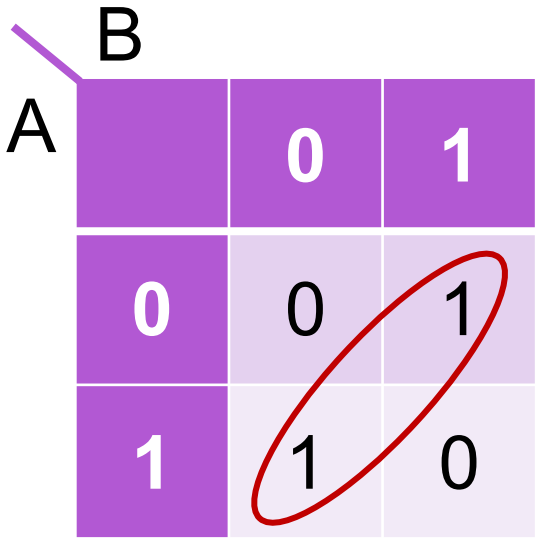
Wrong

K-Map Rules

Groups may be horizontal or vertical, but not diagonal



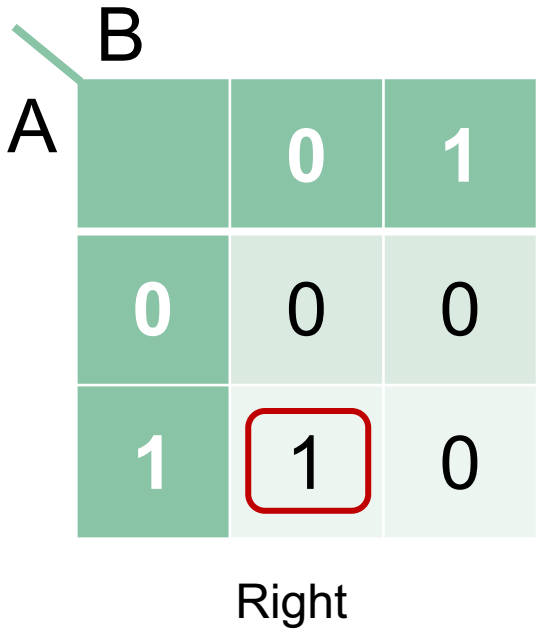
Right



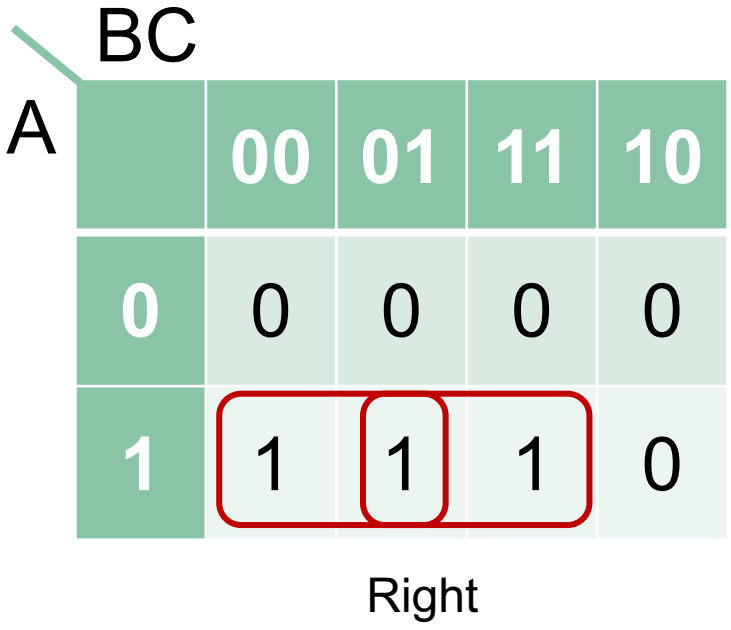
Wrong

K-Map Rules

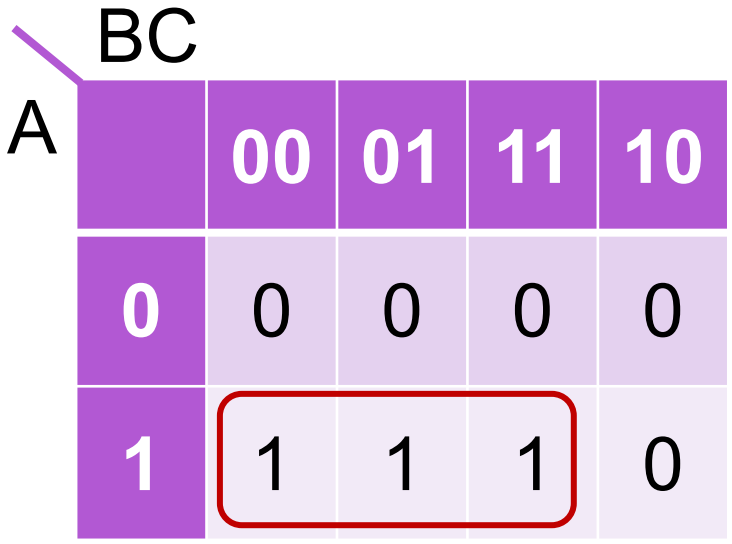
Groups must contain 2^n cells where n = 0, 1, 2, etc.



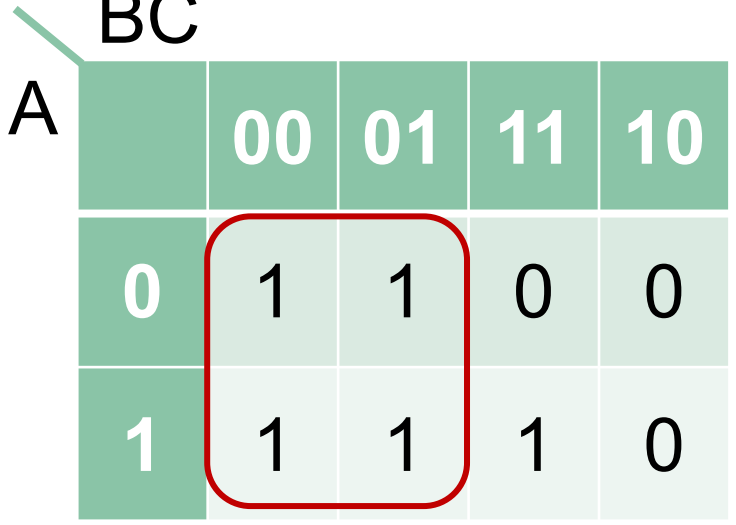
Right



Right



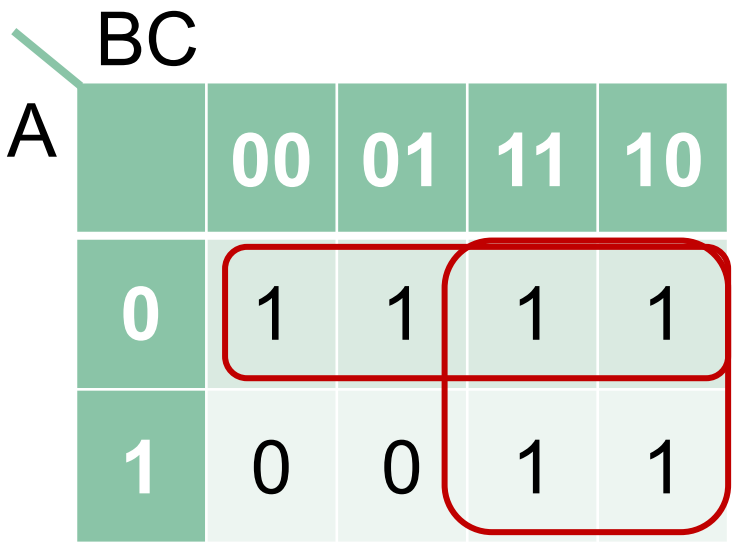
Wrong



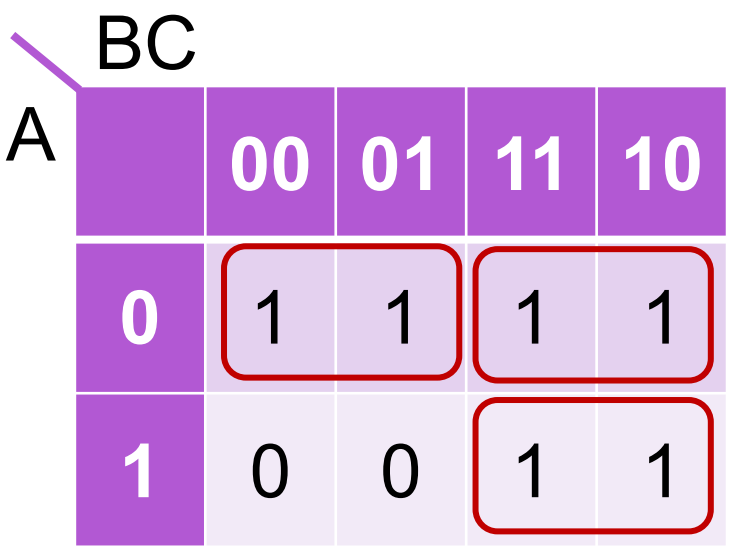
Right

K-Map Rules

Each group must be as large as possible



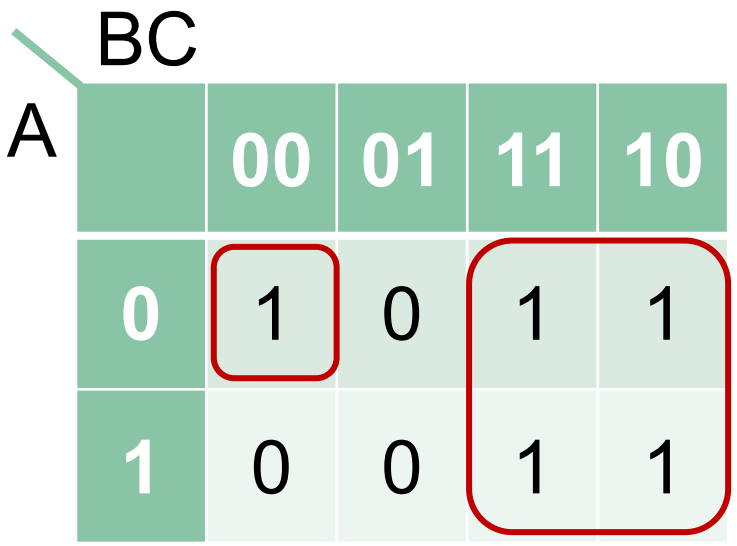
Right



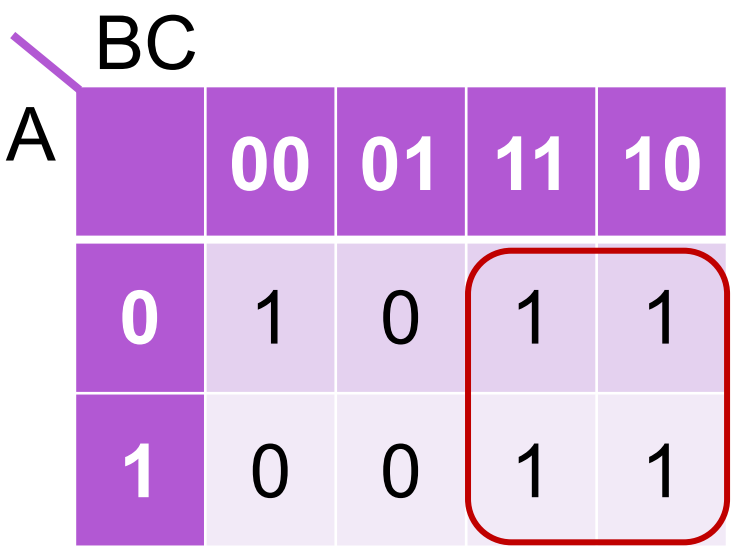
Wrong

K-Map Rules

Each cell containing a 1 must be in at least one group



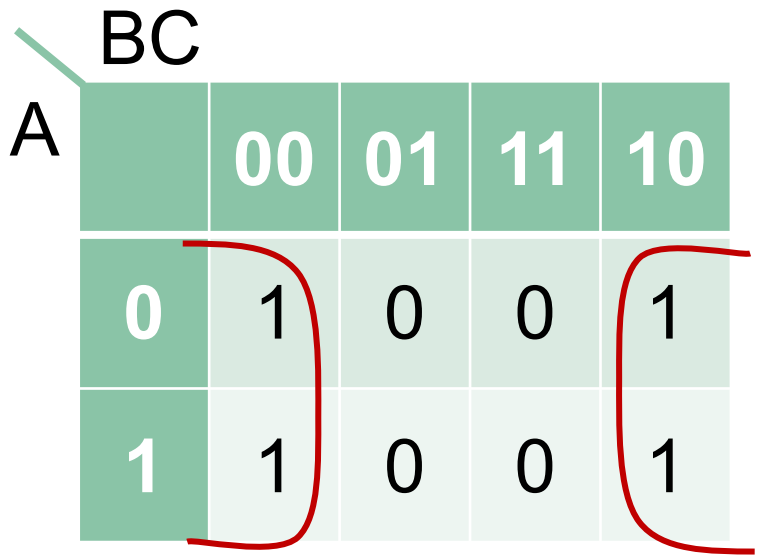
Right



Wrong

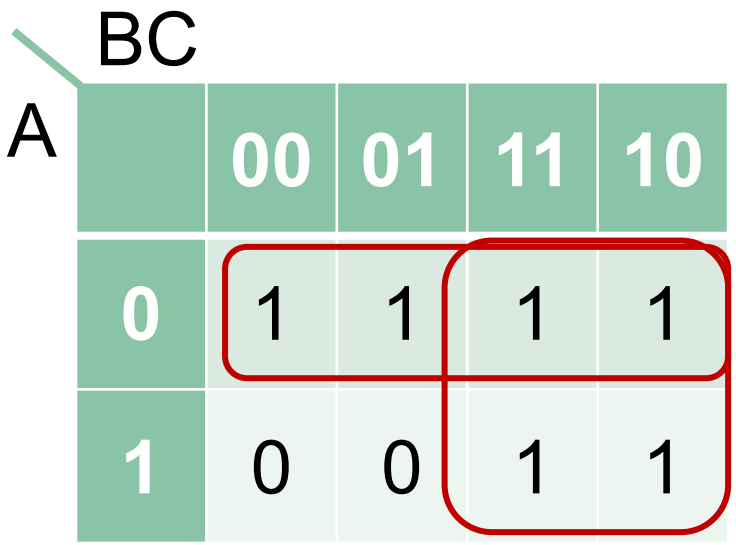
K-Map Rules

Groups may wrap around the table

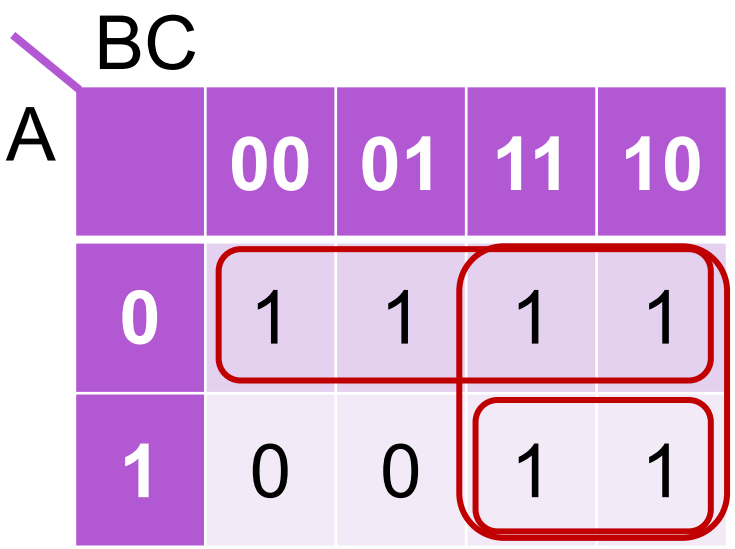


K-Map Rules

Every circle should contain at least one unique 1



Right



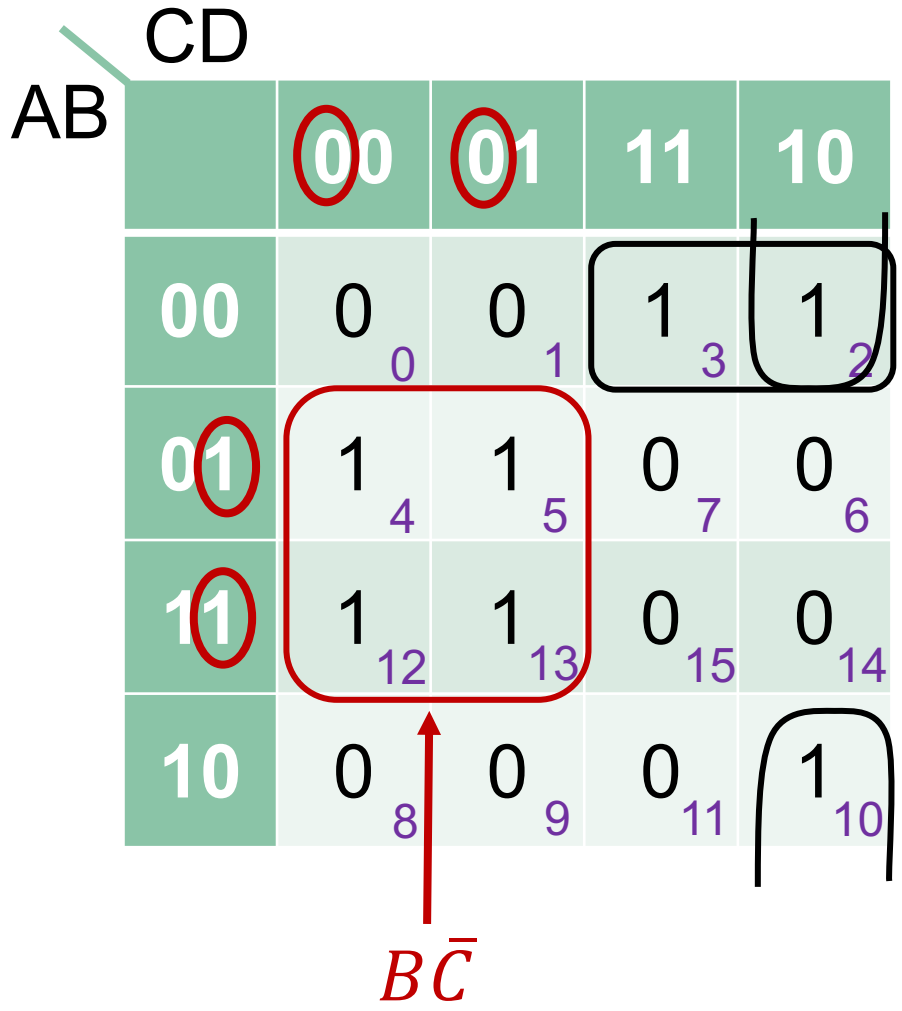
Wrong

4-variable K-Map

AB \ CD	00	01	11	10
00	0 $_0$	0 $_1$	1 $_3$	1 $_2$
01	1 $_4$	1 $_5$	0 $_7$	0 $_6$
11	1 $_{12}$	1 $_{13}$	0 $_{15}$	0 $_{14}$
10	0 $_8$	0 $_9$	0 $_{11}$	1 $_{10}$

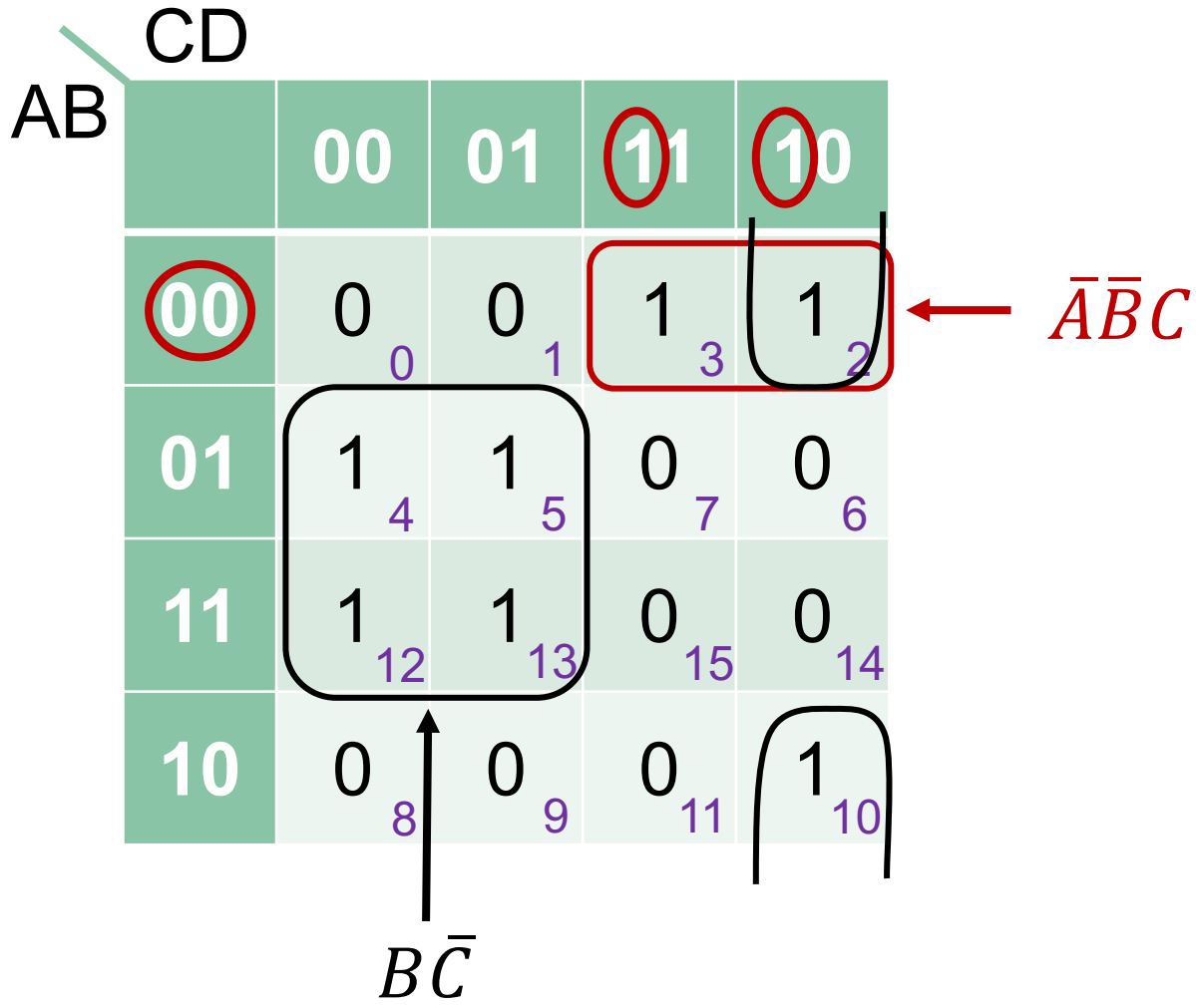
	A	B	C	D	Y
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	0

4-variable K-Map



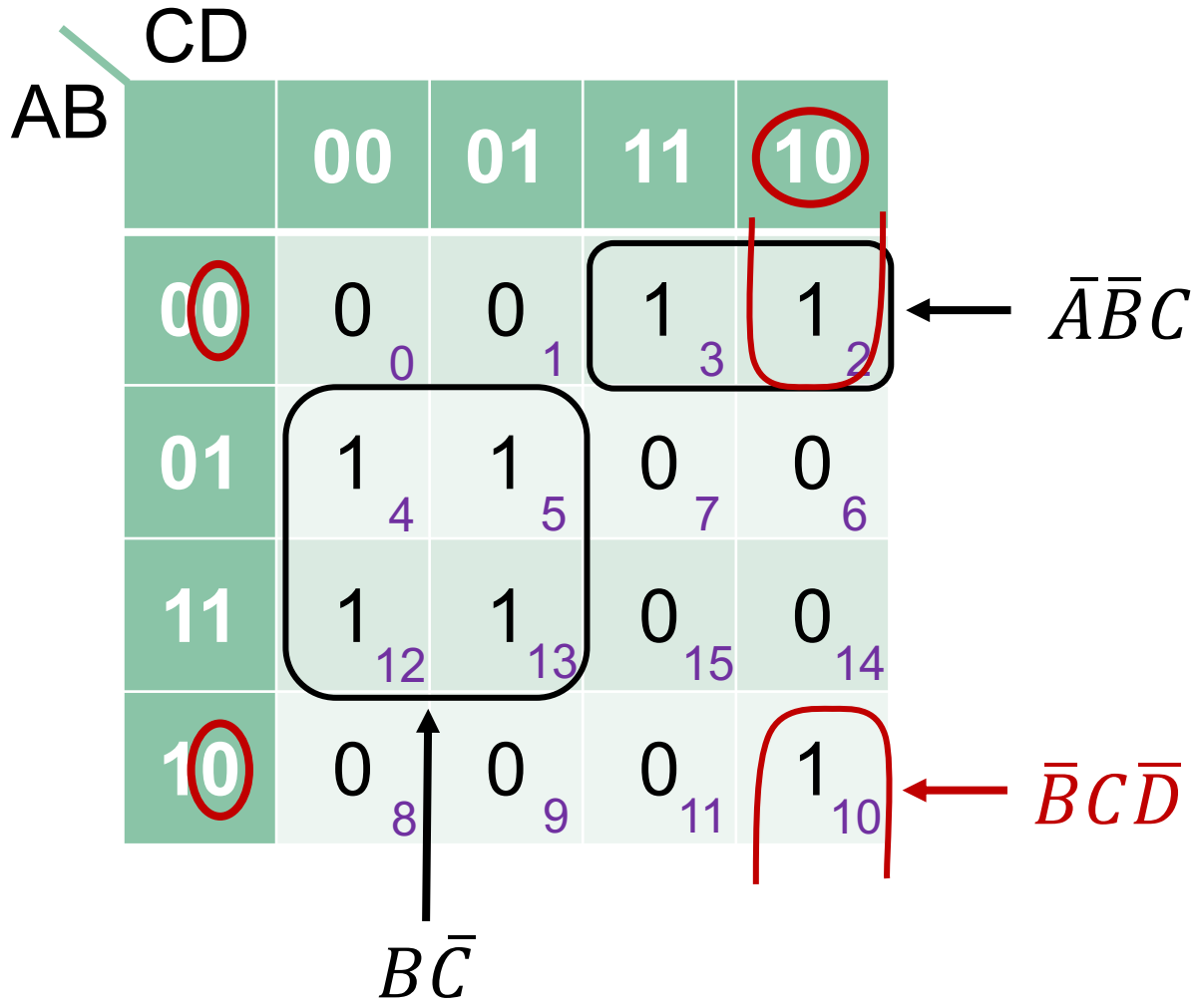
	A	B	C	D	Y
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	0

4-variable K-Map



	A	B	C	D	Y
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	0

4-variable K-Map



	A	B	C	D	Y
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	0

4-variable K-Map

AB \ CD	00	01	11	10
00	0 (0)	0 (1)	1 (3)	1 (2)
01	1 (4)	1 (5)	0 (7)	0 (6)
11	1 (12)	1 (13)	0 (15)	0 (14)
10	0 (8)	0 (9)	0 (11)	1 (10)

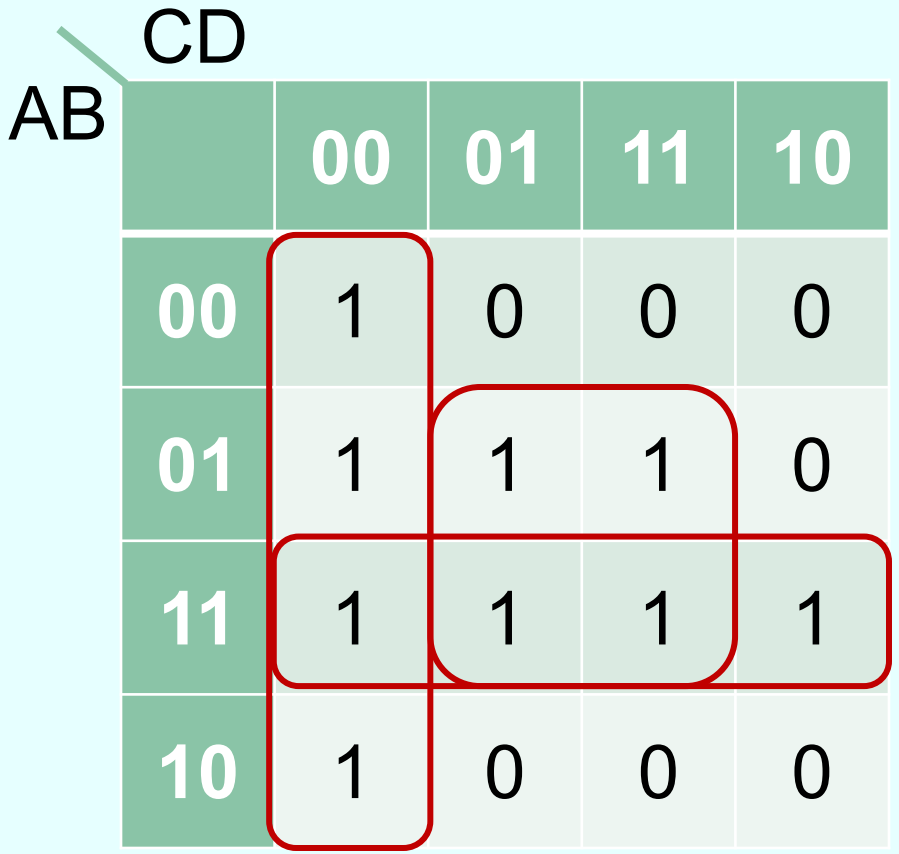
$$Y = B\bar{C} + \bar{A}\bar{B}C + \bar{B}C\bar{D}$$

	A	B	C	D	Y
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	0

Q1: Reading a K-Map

AB \ CD	00	01	11	10
00	1	0	0	0
01	1	1	1	0
11	1	1	1	1
10	1	0	0	0

Q1: Reading a K-Map

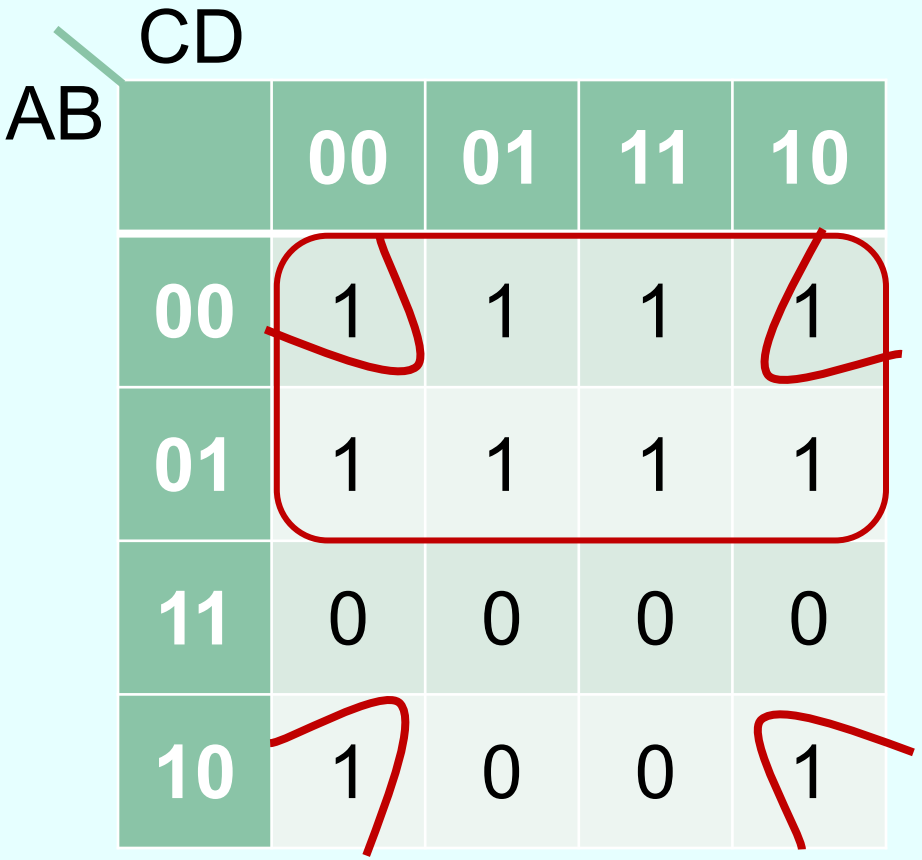


$$Y = \bar{C}\bar{D} + BD + AB$$

Q2: Reading a K-Map

AB \ CD	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	0	0	0	0
10	1	0	0	1

Q2: Reading a K-Map

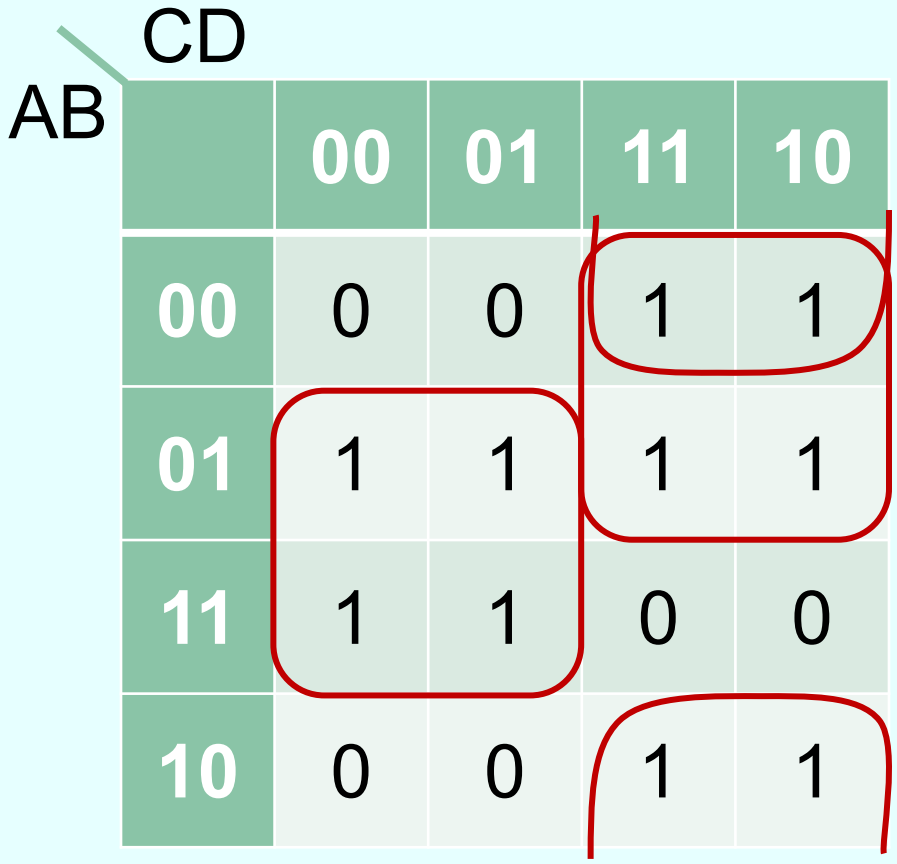


$$Y = \bar{A} + \bar{B}\bar{D}$$

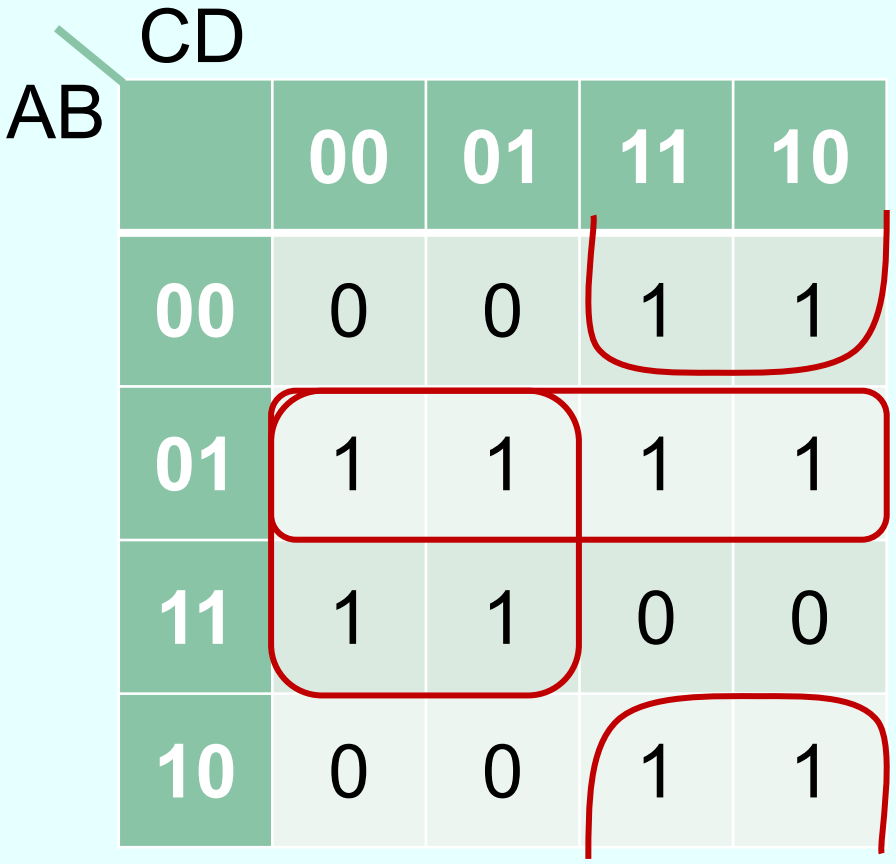
Q3: Reading a K-Map

AB \ CD	00	01	11	10
00	0	0	1	1
01	1	1	1	1
11	1	1	0	0
10	0	0	1	1

Q3: Reading a K-Map



$$Y = B\bar{C} + \bar{B}C + \bar{A}C$$



$$Y = B\bar{C} + \bar{B}C + \bar{A}B$$

Definition: Implicant

Any product term that is a 1 for a given Boolean Equation (i.e. any group of 1s)

A \ BC	00	01	11	10
0	0	0	1	0
1	0	1	1	0

Implicants

$\bar{A}BC$

$A\bar{B}C$

ABC

BC

AC

Definition: Prime Implicant

An implicant that is not a subset of any other implicant

AB \ CD	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	1	1	1	0
10	1	1	0	0

Q: Is this a prime implicant?

Definition: Prime Implicant

An implicant that is not a subset of any other implicant

AB \ CD	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	1	1	1	0
10	1	1	0	0

Q: Is this a prime implicant?

Yes!

Definition: Prime Implicant

An implicant that is not a subset of any other implicant

AB \ CD	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	1	1	1	0
10	1	1	0	0

Q: Is this a prime implicant?

Definition: Prime Implicant

An implicant that is not a subset of any other implicant

AB \ CD	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	1	1	1	0
10	1	1	0	0

Q: Is this a prime implicant?

No, it can be covered by another group

Definition: Prime Implicant

An implicant that is not a subset of any other implicant

AB \ CD	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	1	1	1	0
10	1	1	0	0

Q: Is this a prime implicant?

Definition: Prime Implicant

An implicant that is not a subset of any other implicant

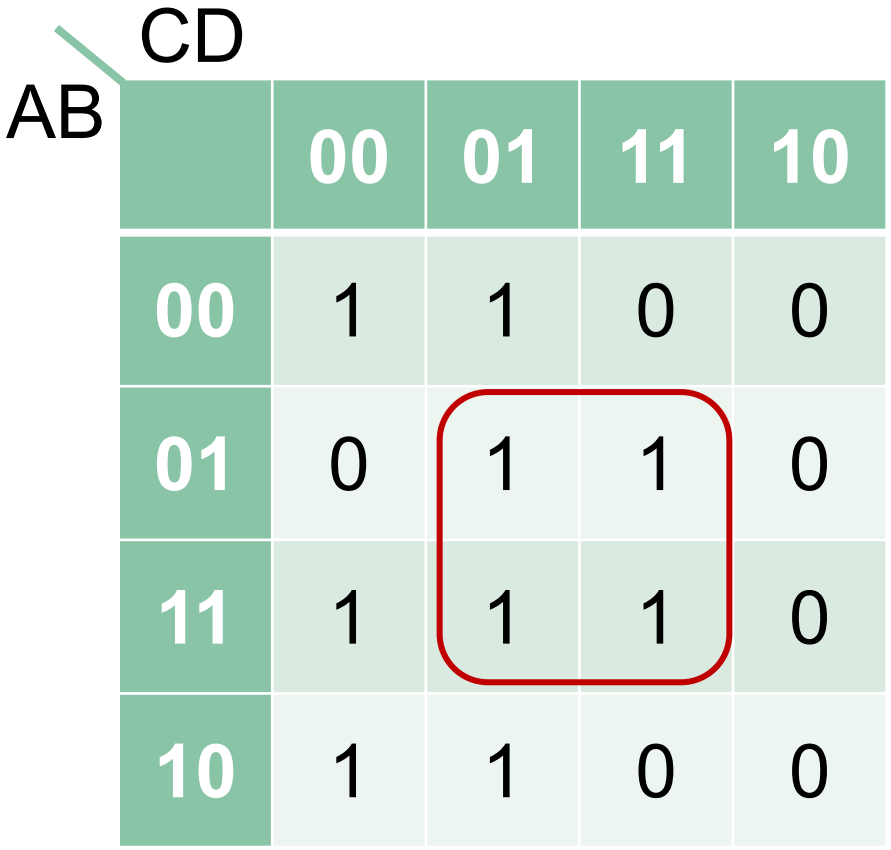
AB \ CD	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	1	1	1	0
10	1	1	0	0

Q: Is this a prime implicant?

No, it can be covered by another group

Definition: Essential Prime Implicant

A prime implicant with at least one element that is not covered by one or more prime implicants (i.e. we must use this group in our final solution in order to cover all 1s)



AB \ CD	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	1	1	1	0
10	1	1	0	0

Q: Is this an essential prime implicant?

Definition: Essential Prime Implicant

A prime implicant with at least one element that is not covered by one or more prime implicants

AB \ CD	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	1	1	1	0
10	1	1	0	0

Q: Is this an essential prime implicant?

Yes!

Definition: Essential Prime Implicant

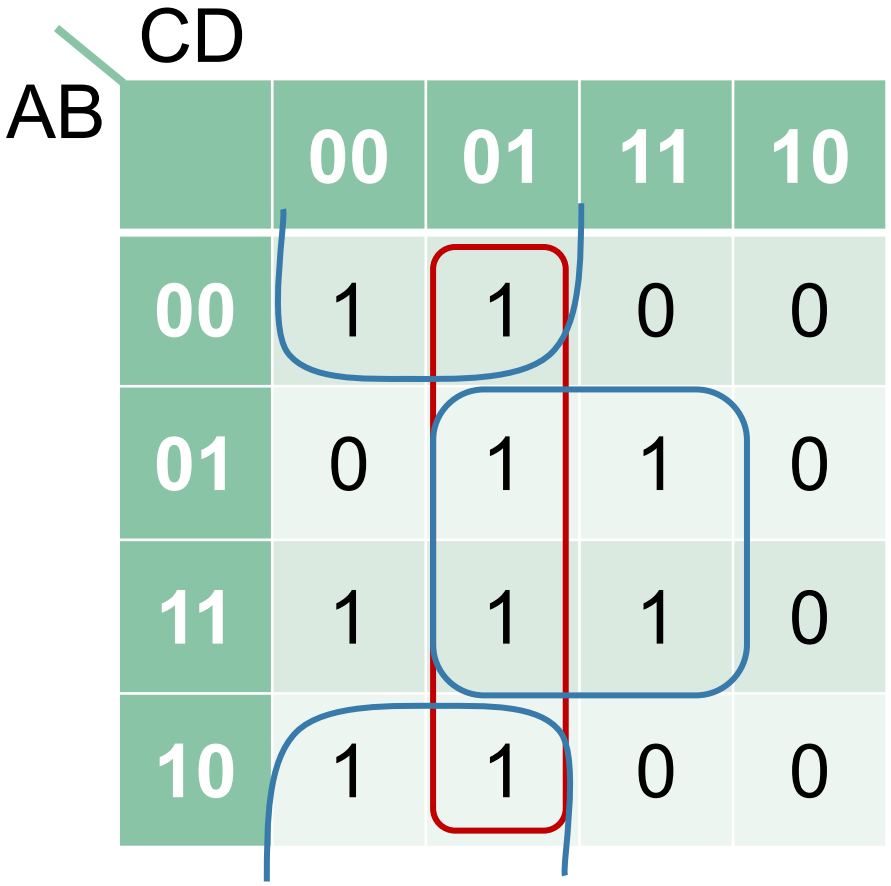
A prime implicant with at least one element that is not covered by one or more prime implicants

AB \ CD	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	1	1	1	0
10	1	1	0	0

Q: Is this an essential prime implicant?

Definition: Essential Prime Implicant

A prime implicant with at least one element that is not covered by one or more prime implicants



Q: Is this an essential prime implicant?

No!

Definition: Non-Essential Prime Implicant

Prime implicant that has no element that cannot be covered by other prime implicant

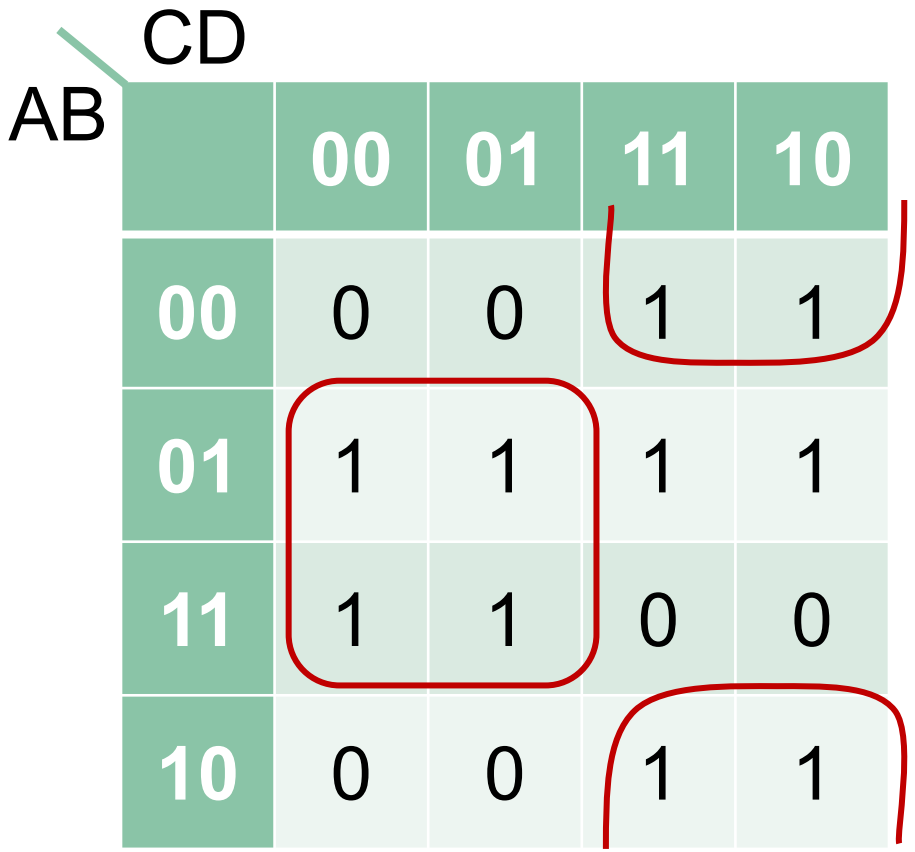
AB \ CD	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	1	1	1	0
10	1	1	0	0

Formal K-Map Procedure

1. Convert truth table to K-map
2. Include all essential primes
3. Include non-essential primes as needed to completely cover all ones

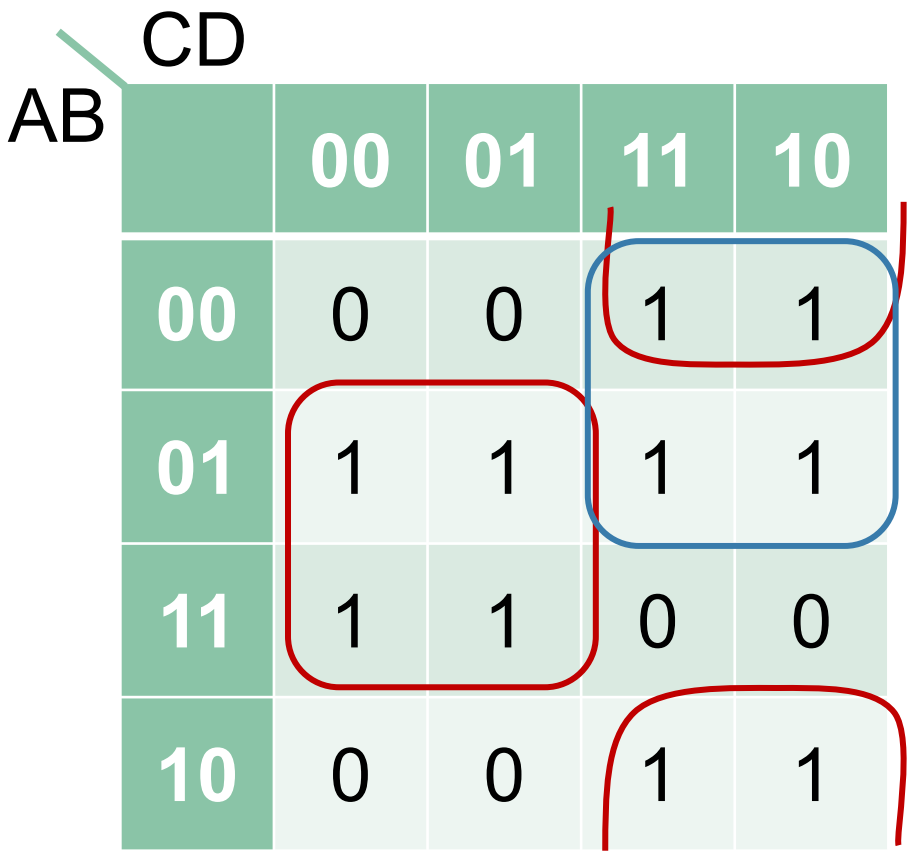
Revisiting Q3

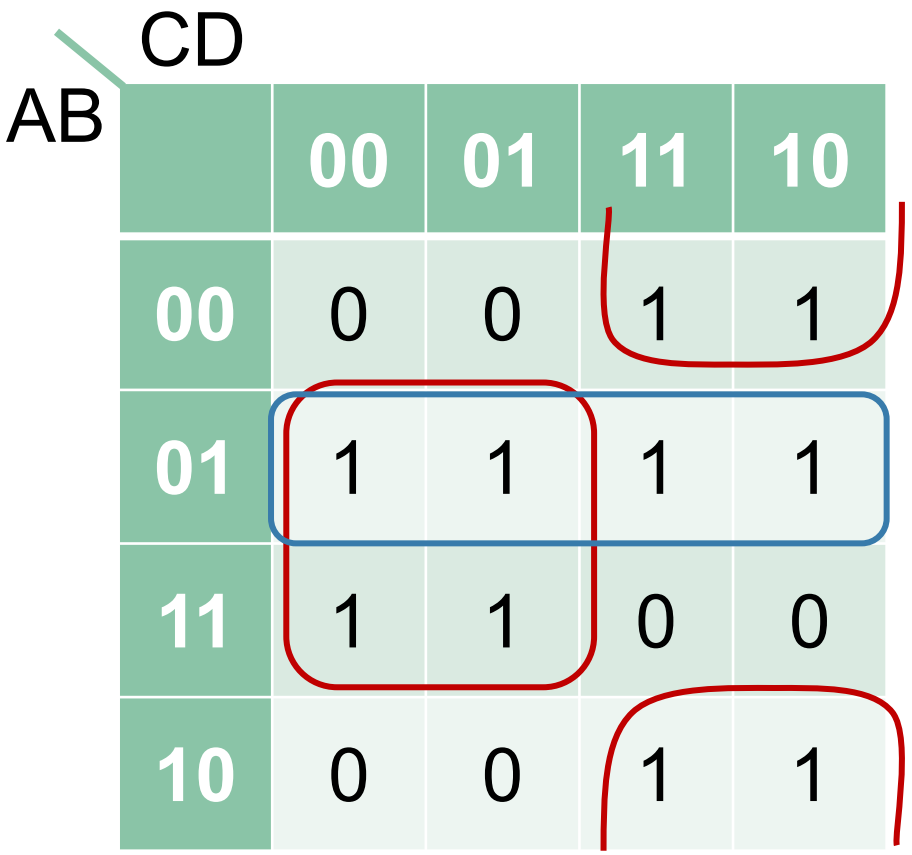
Essential Prime Implicants



Revisiting Q3

To cover the remaining ones, we will choose one of these non-essential prime implicants





Recall: Sum of Products

SOP

$$Y = ABC + AB\bar{C}\overline{D} + D + B\bar{C}$$

Not SOP

$$Y = A(B + C)$$

$$Y = (A + D)(B + C)$$

$$Y = \overline{AB + C}$$

$$Y = \overline{AB}D$$

Simplify the Following Boolean Equation

$$Y = \overline{A + \overline{B} + C + \overline{D}} + ABC\overline{D} + BC + \overline{\overline{AB} + \overline{C + D}}$$

Simplify the Following Boolean Equation

$$Y = \overline{A + \bar{B} + C + \bar{D}} + ABC\bar{D} + BC + \overline{\overline{AB} + \overline{C + D}}$$

$$Y = \bar{A}B\bar{C}D + ABC\bar{D} + BC + AB(C + D)$$

$$Y = \bar{A}B\bar{C}D + ABC\bar{D} + BC + ABC + ABD$$

$$Y = \bar{A}B\bar{C}D + ABC\bar{D} + BC + ABD$$

$$Y = \bar{A}B\bar{C}D + BC + ABD$$

Is this the most simplified equation?

We can always plug it into a K-Map to check!

Using a K-Map to Simplify an Equation

$$Y = \bar{A}B\bar{C}D + BC + ABD$$

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

	A	B	C	D	Y
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

Using a K-Map to Simplify an Equation

$$Y = \bar{A}B\bar{C}D + BC + ABD$$

AB \ CD	00	01	11	10
00	0 (0)	0 (1)	0 (3)	0 (2)
01	0 (4)	1 (5)	1 (7)	1 (6)
11	0 (12)	1 (13)	1 (15)	1 (14)
10	0 (8)	0 (9)	0 (11)	0 (10)

$$Y = BD + BC$$

	A	B	C	D	Y
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	0
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	1
14	1	1	1	0	1
15	1	1	1	1	1

Recall: Don’t Cares

Priority Circuit Truth Table

A_3	A_2	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	X	0	0	1	0
0	1	X	X	0	1	0	0
1	X	X	X	1	0	0	0

A “don’t care” value (X) indicates that the input does not affect the output

Output Values Can Also Be Don't Care!

A	B	Y
0	0	1
0	1	X
1	0	0
1	1	0

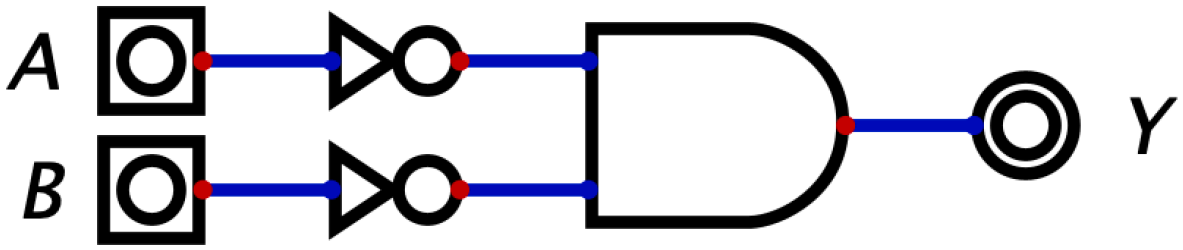
When A and B are 1, the output can be 0 or 1!

Output Values Can Also Be Don't Care!

There are two possible implementations of this truth table

A	B	Y
0	0	1
0	1	X
1	0	0
1	1	0

$$Y = \bar{A}\bar{B}$$



$$Y = \bar{A}$$

Don’t Cares in K-Maps

We can treat a don’t care value as either 0 or 1. An X can be circled if it helps us cover the 1s with fewer or larger circles, but they do not have to be circled if they are not helpful.

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	1	X	0
11	0	1	X	0
10	0	0	0	0

Don’t Cares in K-Maps

We can treat a don’t care value as either 0 or 1. An X can be circled if it helps us cover the 1s with fewer or larger circles, but they do not have to be circled if they are not helpful.

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	1	X	0
11	0	1	X	0
10	0	0	0	0

If we group the 1s on their own, we have the equation $Y = B\bar{C}D$

If we group the 1s with the X’s, we have the equation $Y = BD$

Don’t Cares in K-Maps

The left equation requires more gates, so it would be better to use the right equation.

$Y = B\bar{C}D$ $\qquad$ $Y = BD$

For the input combinations that produce a don’t care output in the k-map, the circuit would output a 0 using the left implementation and a 1 using the right implementation.

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	1	X	0
11	0	1	X	0
10	0	0	0	0

i.e. when A = 0, B = 1, C = 1, D = 1 -> Y = 0 for the left and 1 for the right

and when A = 1, B = 1, C = 1, D = 1 -> Y = 0 for the left and 1 for the right

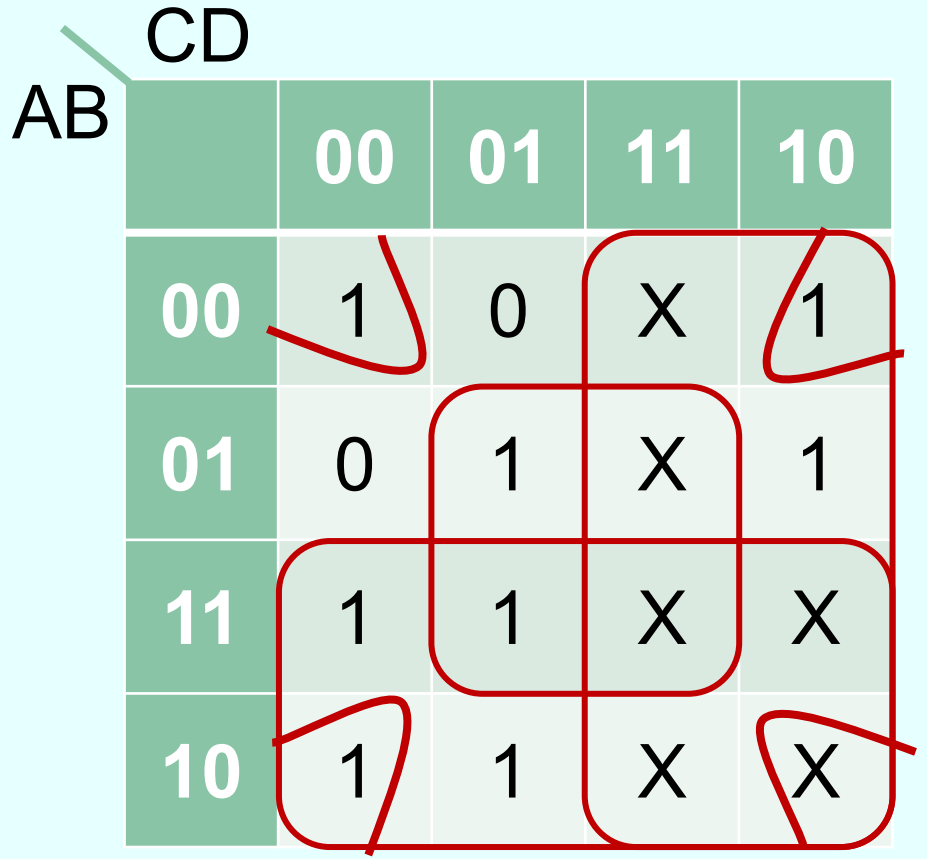
Don’t Cares in K-Maps

We can treat a don’t care value as either 0 or 1. An X can be circled if it helps us cover the 1s with fewer or larger circles, but they do not have to be circled if they are not helpful.

AB \ CD	00	01	11	10
00	1	0	X	1
01	0	1	X	1
11	1	1	X	X
10	1	1	X	X

Don’t Cares in K-Maps

We can treat a don’t care value as either 0 or 1. An X can be circled if it helps us cover the 1s with fewer or larger circles, but they do not have to be circled if they are not helpful.



$$Y = A + C + \overline{B}\,\overline{D} + BD$$