Digital Logic

COMP311 Connor McMahon

Announcements

- New video + slides on CMOS gates
- Homework 1 due on Thursday
- Homework 2 + Lab 1

Your Turn!

Using the laws that you have just learned, simplify the following Boolean expression: $ABC + AB\bar{C} + \overline{AC}B + ACB + \overline{\overline{AB}\,\overline{C}\,\overline{D}} + \overline{\bar{A}B + \bar{C}}$

Name	OR Form	AND Form
Identity	$\mathrm{A} + 0 = \mathrm{A}$	$\mathrm{A} \cdot 1 = \mathrm{A}$
Null	$\mathrm{A} + 1 = 1$	$\mathrm{A} \cdot 0 = 0$
Idempotent	$\mathrm{A} + \mathrm{A} = \mathrm{A}$	$\mathrm{A} \cdot \mathrm{A} = \mathrm{A}$
Complement	$\mathrm{A} + \overline{\mathrm{A}} = 1$	$\mathrm{A} \cdot \overline{\mathrm{A}} = 0$
Commutative	$\mathrm{A} + \mathrm{B} = \mathrm{B} + \mathrm{A}$	$\mathrm{A} \cdot \mathrm{B} = \mathrm{B} \cdot \mathrm{A}$
Associative	$(\mathrm{A} + \mathrm{B}) + \mathrm{C} = \mathrm{A} + (\mathrm{B} + \mathrm{C})$	$(\mathrm{A} \cdot \mathrm{B}) \cdot \mathrm{C} = \mathrm{A} \cdot (\mathrm{B} \cdot \mathrm{C})$
Distributive	$\mathrm{A} \cdot (\mathrm{B} + \mathrm{C}) = (\mathrm{A} \cdot \mathrm{B}) + (\mathrm{A} \cdot \mathrm{C})$	$\mathrm{A} + (\mathrm{B} \cdot \mathrm{C}) = (\mathrm{A} + \mathrm{B}) \cdot (\mathrm{A} + \mathrm{C})$

Your Turn!

$$ABC + AB\bar{C} + \overline{AC}B + ACB + \overline{\overline{AB}\,\overline{C}\,\overline{D}} + \overline{\bar{A}B + \bar{C}}$$

Your Turn!

$$ABC + AB\bar{C} + \overline{AC}B + ACB + \overline{\overline{AB}\,\overline{C}\,\overline{D}} + \overline{\bar{A}B + \bar{C}}$$

$$AB(C + \bar{C}) + B(\overline{AC} + AC) + AB + C + D + (\overline{\bar{A}B})C$$

$$AB(1) + B(1) + AB + C + D + (A + \bar{B})C$$

$$AB + B + AB + C + D + AC + \bar{B}C$$

$$AB + B + C + D + AC + \bar{B}C$$

$$B(A + 1) + C(1 + A) + \bar{B}C + D$$

$$B(1) + C(1) + \bar{B}C + D$$

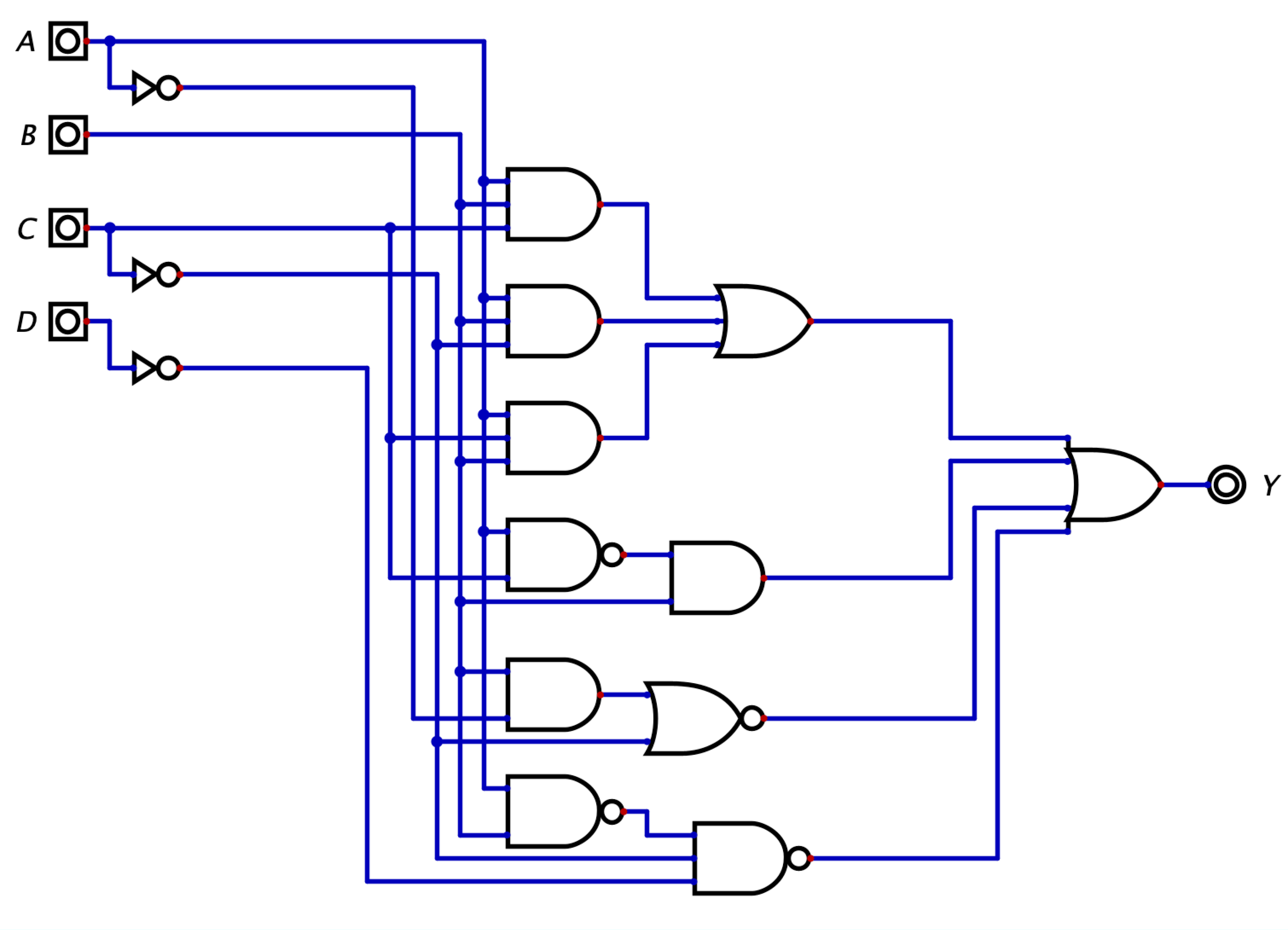
$$B + C + \bar{B}C + D$$

$$B + C(1 + \bar{B}) + D$$

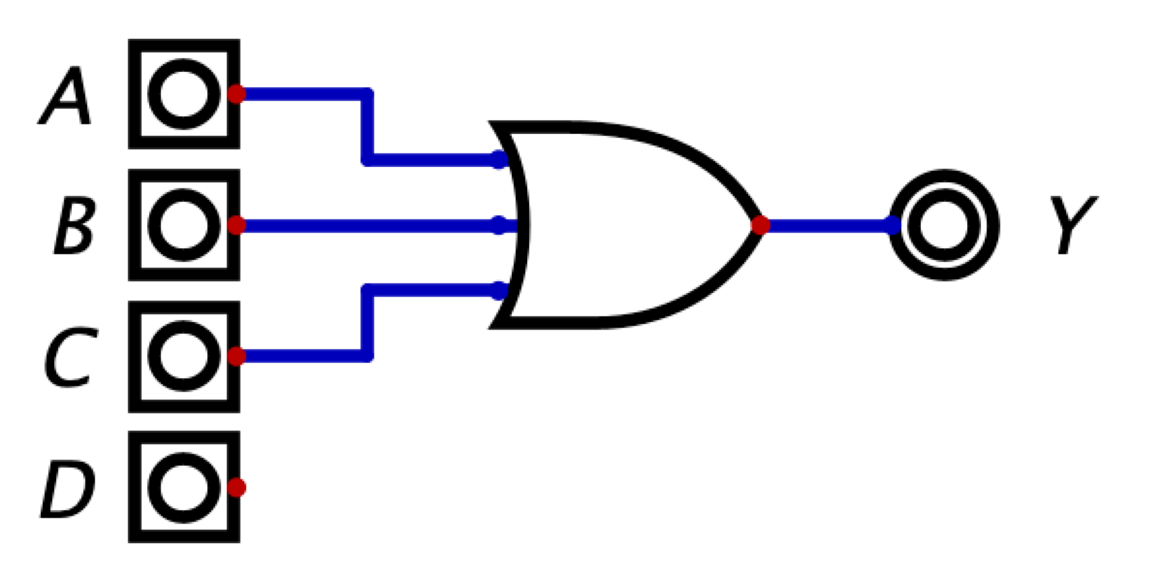
$$B + C(1) + D$$

$$B + C + D$$

Optimizing Our Gates



Optimizing Our Gates



Multiple Output Circuits

A	B	C	Y_1	Y_0
0	0	0	1	1
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	0	1
1	0	1	0	0
1	1	0	0	0
1	1	1	0	1

Multiple Output Circuits

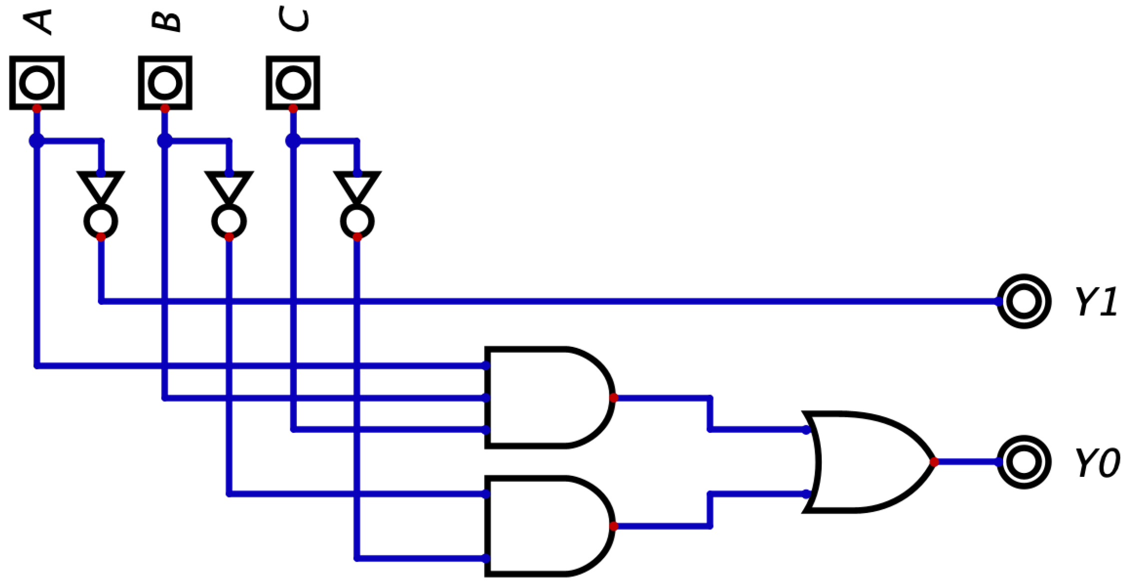
A	B	C	Y_1	Y_0
0	0	0	1	1
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	0	1
1	0	1	0	0
1	1	0	0	0
1	1	1	0	1

$$Y_1 = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC$$

$$Y_1 = \bar{A}$$

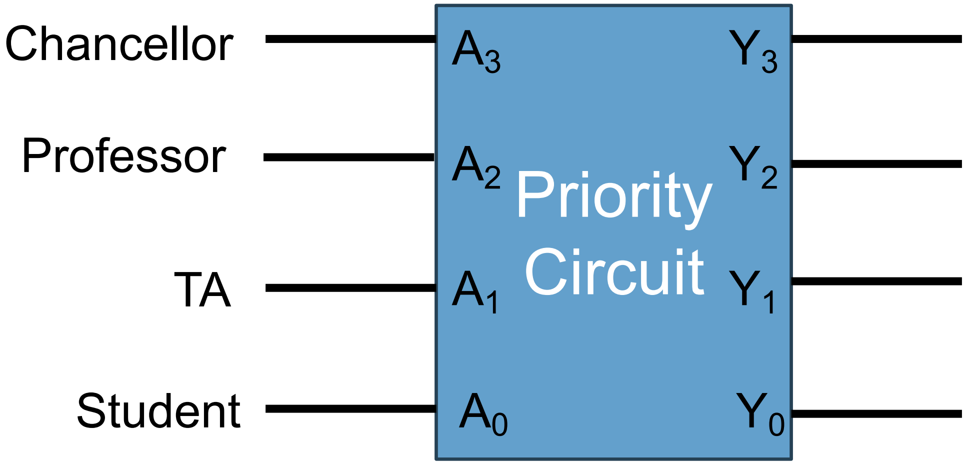
$$Y_0 = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + ABC$$

$$Y_0 = \bar{B}\bar{C} + ABC$$



Priority Circuit

I want to design a circuit that will determine who can reserve a classroom space. Anyone who wants to reserve the room will assert their input (set their input to 1). The order of priority for room reservations is the chancellor, professors, TAs, and students. Assume that only one person from each category will submit a request at a given time. The requestor with the highest priority will get the room reservation.



The output that is 1 will tell us who gets the room reservation (only one output will be 1 at any time)

Create the truth table for this circuit

A_3	A_2	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0				
0	0	0	1				
0	0	1	0				
0	0	1	1				
0	1	0	0				
0	1	0	1				
0	1	1	0				
0	1	1	1				
1	0	0	0				
1	0	0	1				
1	0	1	0				
1	0	1	1				
1	1	0	0				
1	1	0	1				
1	1	1	0				
1	1	1	1				

Create the truth table for this circuit

A_3	A_2	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	0	1	1	0	0	1	0
0	1	0	0	0	1	0	0
0	1	0	1	0	1	0	0
0	1	1	0	0	1	0	0
0	1	1	1	0	1	0	0
1	0	0	0	1	0	0	0
1	0	0	1	1	0	0	0
1	0	1	0	1	0	0	0
1	0	1	1	1	0	0	0
1	1	0	0	1	0	0	0
1	1	0	1	1	0	0	0
1	1	1	0	1	0	0	0
1	1	1	1	1	0	0	0

Find the simplified SOP equation for each output

Find the simplified SOP equation for each output

A_3	A_2	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	0	1	1	0	0	1	0
0	1	0	0	0	1	0	0
0	1	0	1	0	1	0	0
0	1	1	0	0	1	0	0
0	1	1	1	0	1	0	0
1	0	0	0	1	0	0	0
1	0	0	1	1	0	0	0
1	0	1	0	1	0	0	0
1	0	1	1	1	0	0	0
1	1	0	0	1	0	0	0
1	1	0	1	1	0	0	0
1	1	1	0	1	0	0	0
1	1	1	1	1	0	0	0

$Y_3 =$

$Y_2 =$

$Y_1 =$

$Y_0 =$

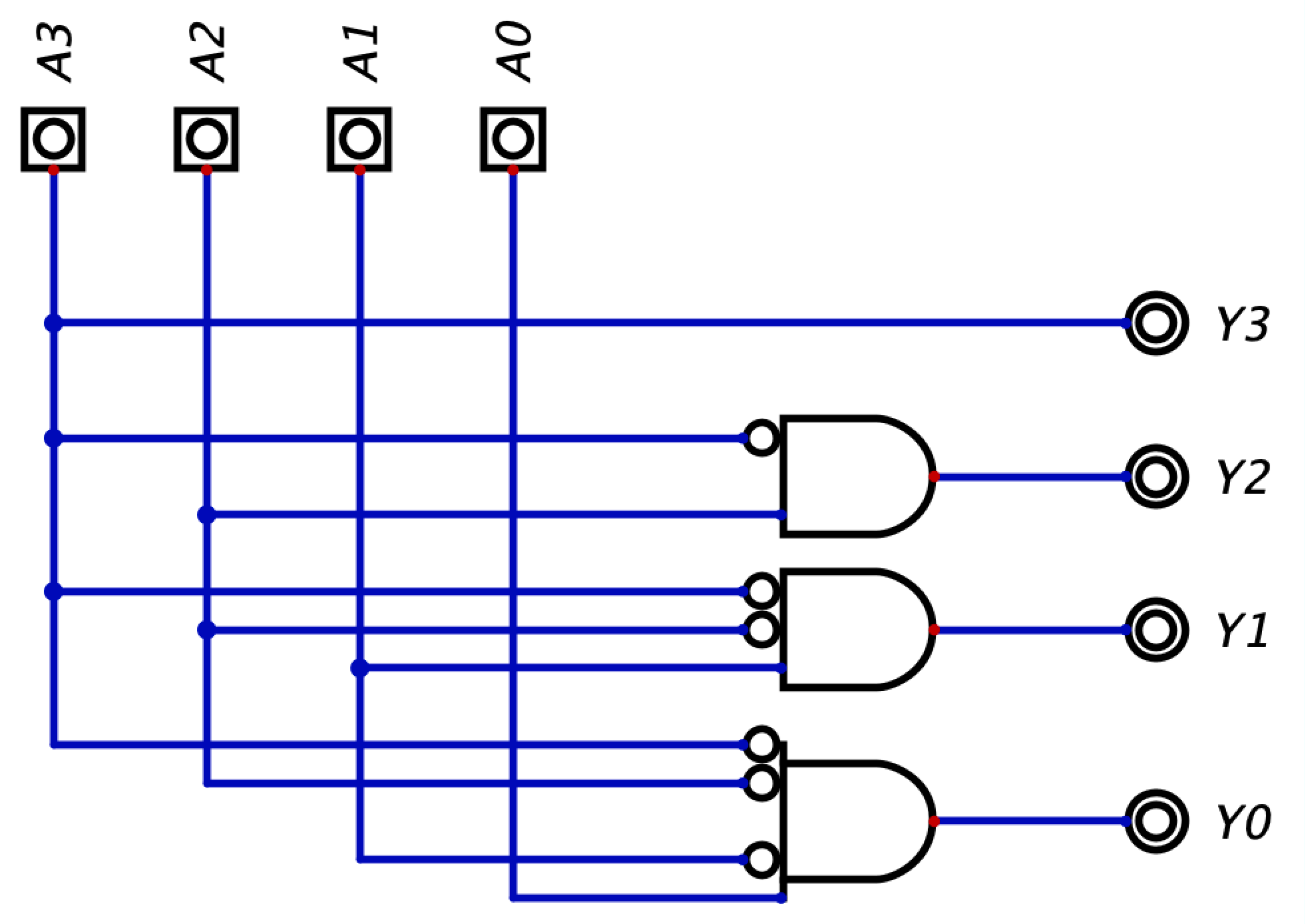
Find the simplified SOP equation for each output

A_3	A_2	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	0	1	1	0	0	1	0
0	1	0	0	0	1	0	0
0	1	0	1	0	1	0	0
0	1	1	0	0	1	0	0
0	1	1	1	0	1	0	0
1	0	0	0	1	0	0	0
1	0	0	1	1	0	0	0
1	0	1	0	1	0	0	0
1	0	1	1	1	0	0	0
1	1	0	0	1	0	0	0
1	1	0	1	1	0	0	0
1	1	1	0	1	0	0	0
1	1	1	1	1	0	0	0

$$Y_3 = A_3$$
$$Y_2 = \overline{A_3}A_2$$
$$Y_1 = \overline{A_3}\,\overline{A_2}A_1$$
$$Y_0 = \overline{A_3}\,\overline{A_2}\,\overline{A_1}A_0$$

Draw the Schematic Diagram

Design the priority circuit



$$Y_3 = A_3$$

$$Y_2 = \overline{A_3}A_2$$

$$Y_1 = \overline{A_3}\,\overline{A_2}A_1$$

$$Y_0 = \overline{A_3}\,\overline{A_2}\,\overline{A_1}A_0$$

Don't Cares in the Priority Circuit

Whenever $A_3 = 1$,
$Y_3 = 1$
$Y_2 = 0$
$Y_1 = 0$
$Y_0 = 0$
It does not matter what A_2, A_1, and A_0 are

We can say that we "don't care" about the values of A_2, A_1, and A_0

A_3	A_2	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	0	1	1	0	0	1	0
0	1	0	0	0	1	0	0
0	1	0	1	0	1	0	0
0	1	1	0	0	1	0	0
0	1	1	1	0	1	0	0
1	0	0	0	1	0	0	0
1	0	0	1	1	0	0	0
1	0	1	0	1	0	0	0
1	0	1	1	1	0	0	0
1	1	0	0	1	0	0	0
1	1	0	1	1	0	0	0
1	1	1	0	1	0	0	0
1	1	1	1	1	0	0	0

Don’t Cares in the Priority Circuit

Whenever $A_3 = 1$,
$Y_3 = 1$
$Y_2 = 0$
$Y_1 = 0$
$Y_0 = 0$
It does not matter what A_2, A_1, and A_0 are

We can say that we “don’t care” about the values of A_2, A_1, and A_0

A_3	A_2	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	0	1	1	0	0	1	0
0	1	0	0	0	1	0	0
0	1	0	1	0	1	0	0
0	1	1	0	0	1	0	0
0	1	1	1	0	1	0	0
1	X	X	X	1	0	0	0

Don’t Cares in the Priority Circuit

Whenever $A_3 = 0$ and $A_2 = 1$
$Y_3 = 0$
$Y_2 = 1$
$Y_1 = 0$
$Y_0 = 0$
It does not matter what A_1, and A_0 are

We can say that we “don’t care” about the values of A_1 and A_0

A_3	A_2	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	0	1	1	0	0	1	0
0	1	0	0	0	1	0	0
0	1	0	1	0	1	0	0
0	1	1	0	0	1	0	0
0	1	1	1	0	1	0	0
1	X	X	X	1	0	0	0

Don't Cares in the Priority Circuit

Whenever $A_3 = 0$ and $A_2 = 1$
$Y_3 = 0$
$Y_2 = 1$
$Y_1 = 0$
$Y_0 = 0$
It does not matter what A_1, and A_0 are

We can say that we "don't care" about the values of A_1 and A_0

A_3	A_2	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	0	1	1	0	0	1	0
0	1	X	X	0	1	0	0
1	X	X	X	1	0	0	0

Don’t Cares in the Priority Circuit

Whenever $A_3 = 0$, $A_2 = 0$, and $A_1 = 1$
$Y_3 = 0$
$Y_2 = 0$
$Y_1 = 1$
$Y_0 = 0$
It does not matter what A_0 is

We can say that we “don’t care” about the value A_0

A_3	A_2	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	0	1	1	0	0	1	0
0	1	X	X	0	1	0	0
1	X	X	X	1	0	0	0

Don’t Cares in the Priority Circuit

Whenever A_3 = 0, A_2 = 0, and A_1 = 1
Y_3 = 0
Y_2 = 0
Y_1 = 1
Y_0 = 0
It does not matter what A_0 is

We can say that we “don’t care” about the value A_0

A_3	A_2	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	X	0	0	1	0
0	1	X	X	0	1	0	0
1	X	X	X	1	0	0	0

Boolean Equations from Don't Cares

A_3	A_2	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	X	0	0	1	0
0	1	X	X	0	1	0	0
1	X	X	X	1	0	0	0

$$Y_3 = A_3$$
$$Y_2 = \overline{A_3}A_2$$
$$Y_1 = \overline{A_3}\,\overline{A_2}A_1$$
$$Y_0 = \overline{A_3}\,\overline{A_2}\,\overline{A_1}A_0$$

Transforming the inputs to don't cares makes finding the simplified SOP equations much easier!

Definitions

- Literal
  - A single variable. May be complemented.
  - Ex1: A
  - Ex2: $\bar{A}$
- Product Term
  - AND of literals
  - Ex: $A\bar{B}C$
  - Not a product term: $A\overline{BC}$

Minterm

Product term in which all variables appear once

Minterm	Not a Minterm
$\bar{A}\bar{B}\bar{C}$	A
$\bar{A}BC$	$\bar{A}C$
$\bar{A}B\bar{C}$	BC
ABC	

Example assumes that our only variables are A, B, and C

Minterm

A	B	C	Minterm	Minterm name
0	0	0	$\bar{A}\bar{B}\bar{C}$	m_0
0	0	1	$\bar{A}\bar{B}C$	m_1
0	1	0	$\bar{A}B\bar{C}$	m_2
0	1	1	$\bar{A}BC$	m_3
1	0	0	$A\bar{B}\bar{C}$	m_4
1	0	1	$A\bar{B}C$	m_5
1	1	0	$AB\bar{C}$	m_6
1	1	1	ABC	m_7

Implementation Using Minterms

A	B	C	Y	Minterm	Minterm Name
0	0	0	1	$\bar{A}\bar{B}\bar{C}$	m_0
0	0	1	0	$\bar{A}\bar{B}C$	m_1
0	1	0	1	$\bar{A}B\bar{C}$	m_2
0	1	1	0	$\bar{A}BC$	m_3
1	0	0	0	$A\bar{B}\bar{C}$	m_4
1	0	1	1	$A\bar{B}C$	m_5
1	1	0	0	$\bar{A}\bar{B}C$	m_6
1	1	1	1	ABC	m_7

Implementation Using Minterms

A	B	C	Y	Minterm	Minterm Name
0	0	0	1	$\bar{A}\bar{B}\bar{C}$	m_0
0	0	1	0	$\bar{A}\bar{B}C$	m_1
0	1	0	1	$\bar{A}B\bar{C}$	m_2
0	1	1	0	$\bar{A}BC$	m_3
1	0	0	0	$A\bar{B}\bar{C}$	m_4
1	0	1	1	$A\bar{B}C$	m_5
1	1	0	0	$AB\bar{C}$	m_6
1	1	1	1	ABC	m_7

$$F = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \mathrm{A}\bar{B}C + ABC$$

$$F(A,B,C) = \sum(m_0, m_2, m_5, m_7)$$

Sum of Products

- When two or more products (AND) are summed (OR) together

Sum of Products	Not Sum of Products
$Y = AB + A\bar{C}$	$Y = (A + B)(C + A)$

Canonical Sum of Products

- A sum of products form in which each product contains all literals

Canonical Sum of Products Form $F = \bar{X}\bar{Y}\bar{Z} + \bar{X}Y\bar{Z} + X\bar{Y}Z + XYZ$

Simplified Sum of Products Form $F = \bar{X}\bar{Z} + XZ$

When simplifying Boolean expressions, we usually want to find the simplified sum of products form!

Maxterm

A	B	C	Maxterm	Maxterm Name
0	0	0	$A + B + C$	M_0
0	0	1	$A + B + \bar{C}$	M_1
0	1	0	$A + \bar{B} + C$	M_2
0	1	1	$A + \bar{B} + \bar{C}$	M_3
1	0	0	$\bar{A} + B + C$	M_4
1	0	1	$\bar{A} + B + \bar{C}$	M_5
1	1	0	$\bar{A} + \bar{B} + C$	M_6
1	1	1	$\bar{A} + \bar{B} + \bar{C}$	M_7

Maxterm

A	B	C	F	Maxterm	Maxterm Name
0	0	0	1	$A + B + C$	M_0
0	0	1	1	$A + B + \bar{C}$	M_1
0	1	0	1	$A + \bar{B} + C$	M_2
0	1	1	0	$A + \bar{B} + \bar{C}$	M_3
1	0	0	1	$\bar{A} + B + C$	M_4
1	0	1	0	$\bar{A} + B + \bar{C}$	M_5
1	1	0	1	$\bar{A} + \bar{B} + C$	M_6
1	1	1	1	$\bar{A} + \bar{B} + \bar{C}$	M_7

$$F = (\mathrm{A} + \bar{B} + \bar{C})(\bar{A} + \mathrm{B} + \bar{C})$$

$$F = \Pi(M_3, M_5)$$

X	Y	Z	F	$\bar{F}$	Minterm	Maxterm
0	0	0	1	0	$\bar{A}\bar{B}\bar{C}$	$A + B + C$
0	0	1	1	0	$\bar{A}\bar{B}C$	$A + B + \bar{C}$
0	1	0	1	0	$\bar{A}B\bar{C}$	$A + \bar{B} + C$
0	1	1	0	1	$\bar{A}BC$	$A + \bar{B} + \bar{C}$
1	0	0	1	0	$A\bar{B}\bar{C}$	$\bar{A} + B + C$
1	0	1	0	1	$A\bar{B}C$	$\bar{A} + B + \bar{C}$
1	1	0	1	0	$\bar{A}\bar{B}C$	$\bar{A} + \bar{B} + C$
1	1	1	1	0	ABC	$\bar{A} + \bar{B} + \bar{C}$

$$\bar{F} = \bar{A}BC + A\bar{B}C$$

$$F = \overline{\bar{A}BC + A\bar{B}C}$$

$$F = (\overline{\bar{A}BC})(\overline{A\bar{B}C})$$

$$F = (A + \bar{B} + \bar{C})(\bar{A} + B + \bar{C})$$

Find the Boolean equation of this truth table in each of the following forms.

X	Y	Z	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

1. Canonical SOP form
2. Simplified SOP form
3. Canonical POS form

Find the Boolean equation of this truth table in each of the following forms.

X	Y	Z	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

1. Canonical SOP form

$$F = \bar{X}\bar{Y}\bar{Z} + \bar{X}\bar{Y}Z + XY\bar{Z} + XYZ$$

2. Simplified SOP form

$$F = \bar{X}\bar{Y} + XY$$

3. Canonical POS form

$$F = (X + \bar{Y} + Z)(X + \bar{Y} + \bar{Z})(\bar{X} + Y + Z)(\bar{X} + Y + \bar{Z})$$