



End Term (Odd) Semester Examination December 2024

Roll No...2419010.....

Name of the Course and semester: Bachelor of Technology, I Semester

Name of the Paper: Engineering Mathematics-I

Paper Code: TMA-101

Time: 3 hours

Maximum Marks: 100

Note:

- All the questions are compulsory.
- Answer any two sub questions from a, b and c in each main question.
- Total marks for each question is 20 (twenty).
- Each sub-question carries 10 marks.

Q1.

(2x10=20 Marks) (CO1)

- a. Using elementary transformations, find the inverse of the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

- b. Test the consistency of the following system of linear equations:

$$2x + 3y + 4z = 11, x + 5y + 7z = 15, 3x + 11y + 13z = 25.$$

If consistent, find the solution.

- c. Find the eigen values and corresponding eigen vectors for the matrix

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

Q2.

(2x10=20 Marks) (CO2)

- a. Verify Lagrange's mean value theorem for the following functions: ✓

(i) $f(x) = 2x^2 - 10x + 29$ in $[2, 7]$

(ii) $f(x) = \sqrt{x^2 - 4}$ in $[2, 4]$

- b. State and verify Rolle's theorem for the function $f(x) = (x-a)^m (x-b)^n$ in the interval $[a, b]$, where m and n are positive integers.

- c. Evaluate the following limits:

(i) $\lim_{x \rightarrow 0} \frac{\log(\sin 2x)}{\log(\sin x)}$

(ii) $\lim_{x \rightarrow 0} \left(\frac{1}{x} \right)^{\tan x}$

Q3.

(2x10=20 Marks) (CO3)

- a. If $u = \tan^{-1}(x^2 + 2y^2)$, show that

(i) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$

(ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2 \sin u \cos 3u$

- b. If $y = (\sin^{-1} x)^2$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$.

- c. Find the point upon the plane $ax + by + cz = p$ at which the function $f(x, y, z) = x^2 + y^2 + z^2$ has a minimum value and find this minimum value of f .



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Q4.

(2x10=20 Marks) (CO4)

a. Prove that $\beta(l, m) = \frac{\Gamma(l)\Gamma(m)}{\Gamma(l+m)}$.

b. Sketch the region bounded by the curves $y = x^2$ and $x + y = 2$. Express the area of the given region as a double integral in two different ways and evaluate the area using any one way.

c. Evaluate $\int_0^{\log 2} \int_0^x \int_0^{x+y} e^{x+y+z} dx dy dz$.

Q5.

(2x10=20 Marks) (CO5)

a. Apply Green's Theorem to evaluate $\int_C (2x^2 - y^2)dx + (x^2 + y^2)dy$, where C is the boundary of the

area enclosed by the x -axis and the upper half of the circle $x^2 + y^2 = 1$.

b. Evaluate $\oint_C \vec{F} \cdot d\vec{r}$ by Stoke's Theorem, where $\vec{F} = y^2\hat{i} + x^2\hat{j} - (x+z)\hat{k}$ and C is the boundary of

triangle with vertices at $(0,0,0)$, $(1,0,0)$ and $(1,1,0)$.

c. Prove that $(y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$ is both solenoidal and irrotational.