2D Bose gas

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Having in mind the renormalization argument reported in the previous paper, we are going to attempt to obtain the self-consistent equation for the qualitative behaviour of chemical potential for 2D Bose gas.

In the manner similar to the 3D case, we start with introducing g_2, which for 2D case is

$$g_2(E|p) = -\frac{2\pi}{\log\left(d\sqrt{\frac{1}{4}p^2 - E}\right)} \tag{1}$$

Here d is the interaction parameter that for quasi-2D systems is linked to the scattering length by

$$d = Cl_\perp \exp\left[\sqrt{\frac{\pi}{2}}\frac{l_\perp}{a}\right]$$

here C is a numerical constant.

Let us consider the back-of-the envelope estimation of the potential behaviour with only the 2-body term self-consistent equation

$$\mu = \frac{4\pi n_0}{\log(d^2 2\mu)} + ... \tag{2}$$

that can also be presented as

$$\log(d^2 2\mu) = \frac{4\pi n_0}{\mu} \tag{3}$$

One may notice that in a stark contrast to the 3D case there is no sign of instability in the crossover limit as there is a solution for any fixed finite d. It is a hint of a general feature that seems to be the case that unlike the 3D case, for 2D case the few-body terms 'cause' the instability of the high interaction limit.

We have discussed before the g_3 form for 2D case

$$g_3 = \frac{16\pi^2}{\log^2(d\sqrt{2\mu})}\int_0^\Lambda \frac{dkk}{\{2\mu + k^2\}}\frac{G_3(k)}{\log\left(\frac{3}{4}k^2d^2 + 3\mu d^2\right)}$$

with $G_3(p)$ being the solution of the following integral equation

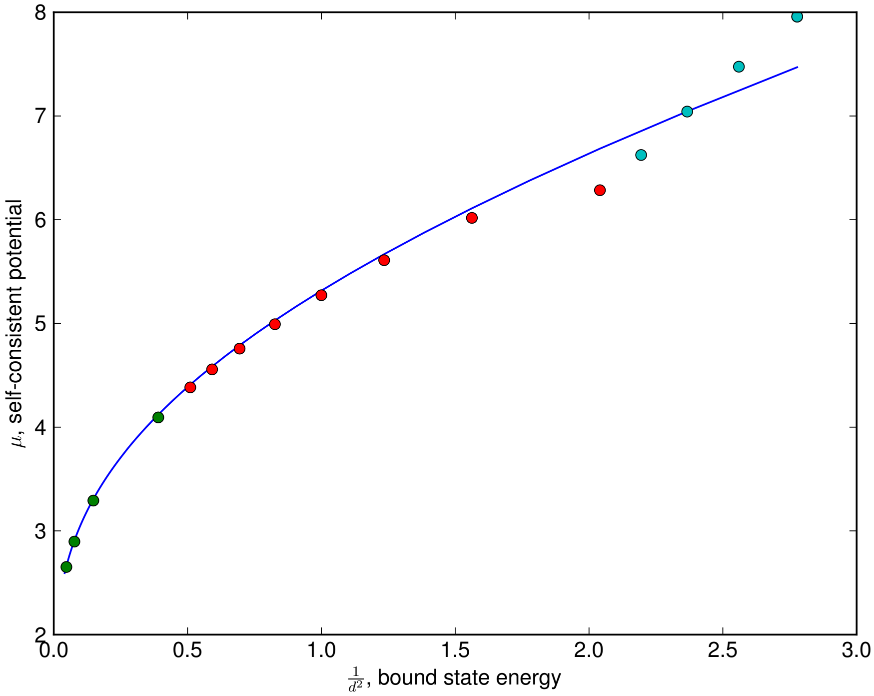
$$G_3(p) = -4\int_0^\Lambda \frac{dkk}{\log\left(\frac{3}{4}k^2d^2 + 3\mu d^2\right)}\left\{\frac{1}{2\mu + k^2} + G_3(k)\right\}\left\{(3\mu + k^2 + p^2)^2 - p^2k^2\right\}^{-\frac{1}{2}}$$

The self-consistent system of equations that sums up our approach is then

$$\mu = n_0 g_2(n_0, -\mu) + n_0^2 g_3(n_0, -\mu)$$

$$n = n(n_0, -\mu)$$

Let us see what the numerical implementation yeilds us



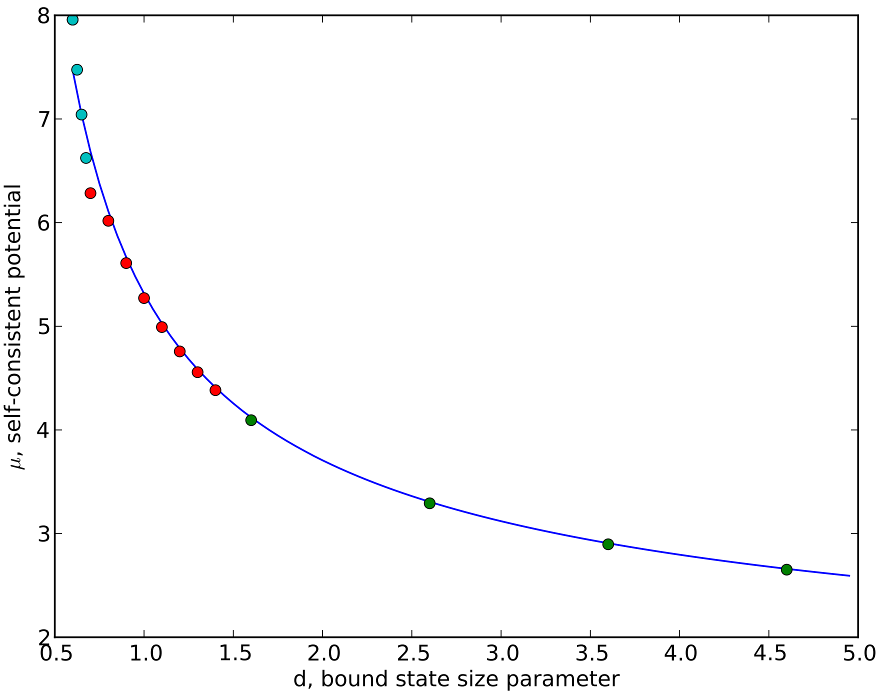
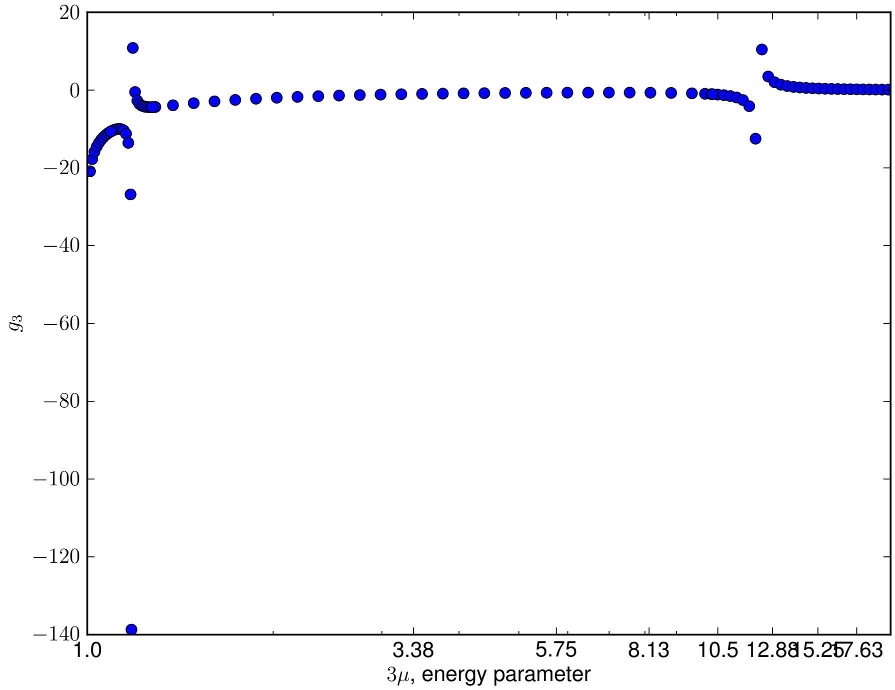


Figure 1: The $\mu(d)$ behaviour with d(lower plot) and $1/d^2$(upper plot) on the horizontal axis. The blue lines are the solution one gets with dropping the g_3 term. The filled dots represent the exact solution of the system with green dot region reproducing the 2-particle term solution with little discrepancy, the red dot region showing the qualitatively different behaviour (saturation) caused by the 3-particle term, and the blue region showing the breaking of the g_3 term and the zone of instability and the significant complex addition.



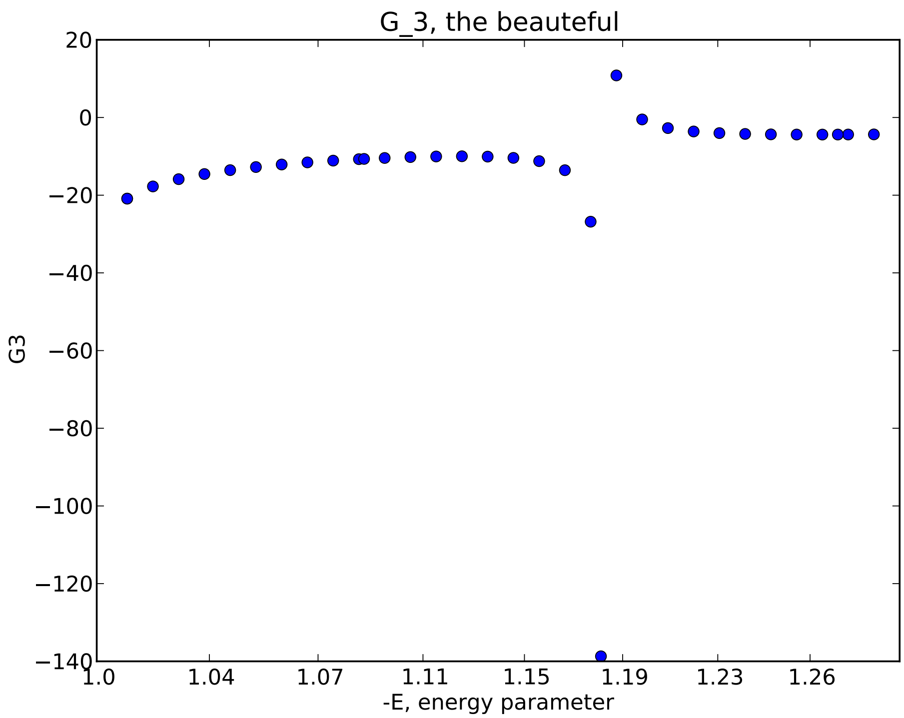
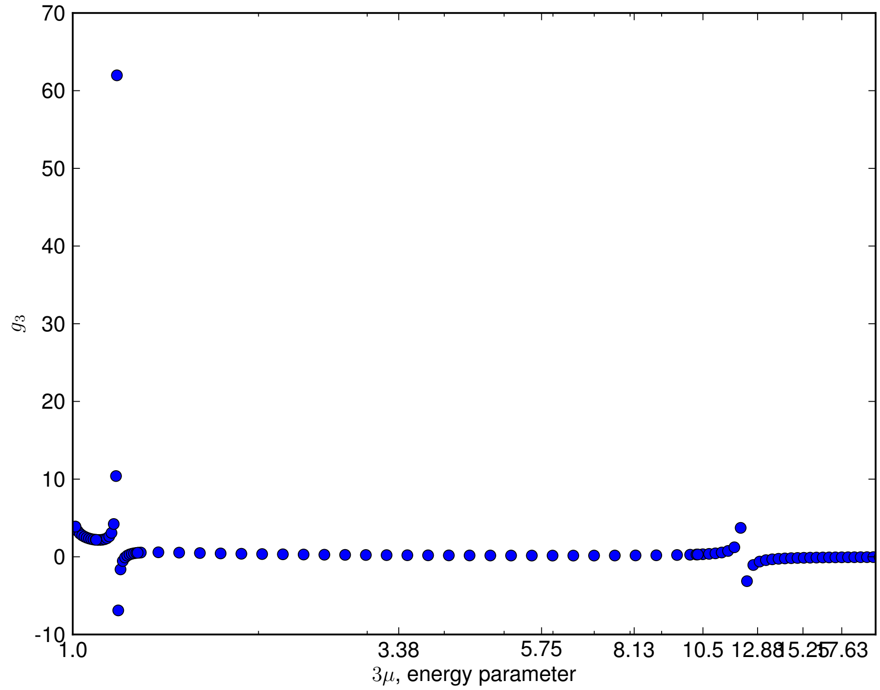


Figure 2: The $G_3(0)$ as a function of $-E$ parameter, has two poles and is depicted here.



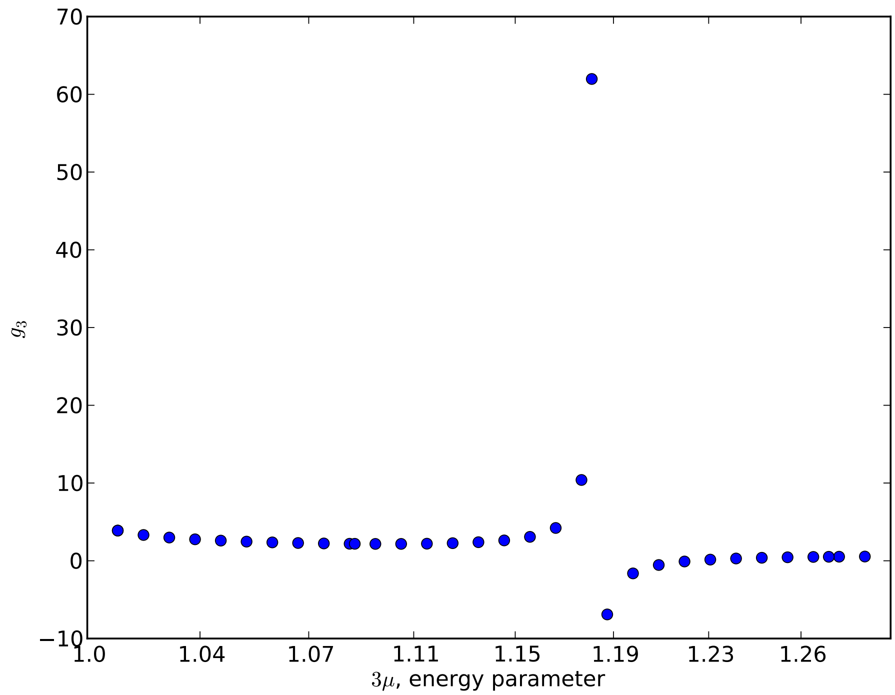


Figure 3: The g_3 as a function of -3μ parameter, has two poles and is depicted here.