ECEN 5028: Constrained Control

Homework 4: Linear Model Predictive Control

Exercise 1. [60 points.] Consider the dynamics of a double integrator

$$\begin{aligned}\dot{x} &= A_c x + B_c u,\\ y &= Cx + Du,\\ z &= Ex + Fu,\end{aligned}$$

where $y \in \mathcal{Y}$ are the constrained outputs, z is the tracking output, and

$$A_c = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad F = \begin{bmatrix} 0 \end{bmatrix}.$$

The constraint set is

$$\mathcal{Y} = [-6,\ 6] \times [-2,\ 2] \times [-2,\ 2].$$

We wish to drive the system to a desired steady-state reference v using Model Predictive Control to solve the following constrained optimal control problem:

$$\min_{\xi,\mu} ||\xi_N - G_x v||_P^2 + \sum_{i=0}^{N-1} ||\xi_i - G_x v||_Q^2 + ||\mu_i - G_u v||_R^2 \tag{1a}$$

$$\text{s.t. } \xi_0 = x, \tag{1b}$$

$$\xi_{i+1} = A\xi_i + B\mu_i, \quad i \in [0,\ N-1] \tag{1c}$$

$$C\xi_i + D\mu_i \in \mathcal{Y}, \qquad i \in [0,\ N-1] \tag{1d}$$

$$(\xi_N, v) \in \mathcal{T}, \tag{1e}$$

t,V(1,:) where (G_x, G_u) are the steady-state gains associated to the reference v, $N = 10$ is the prediction horizon, $Q = E^T E$ and $R = 1$ are the stage cost matrices, $P > 0$ is the solution to the discrete algebraic Riccati equation, and $\mathcal{T} = O_\infty^\epsilon$ is the maximal output admissible set associated to the LQR controller $u = G_u v - K(x - G_x v)$, with $K = (R + B^T P B)^{-1} B^T P A$.

Perform the following:

a. Given A_c, B_c, and the sampling period $t_s = 0.1\ s$, use any method to compute the discrete-time matrices A, B such that $x_{k+1} = Ax_k + Bu_k$;

b. Compute G_x and G_u using the same method as in Homework 3.

c. Find the sparse representation of the QP matrices for the OCP (1) (note that v is *not* an optimization variable).

d. Given the initial condition $x_0 = 0$ and target reference $r = 5$, implement MPC **without** constraints (i.e. omit (1d) and (1e)) and compare its behavior to the LQR controller.

e. Implement MPC **with** constraints and verify constraint satisfaction for each y.

Exercise 2. [40 points.] Consider the two input lane change maneuver (Ex 2.) from Homework 3 with identical parameters. We wish to solve the same constrained control problem using MPC. To this end, perform the following:

a. Given the initial condition $x_0 = 0$ and target reference $r = [5\ 0]^T$, implement an MPC using the OCP (1) with a horizon length of $N = 70$. Verify that all the constraints are satisfied and compare the results to your command governor from the previous Homework.

b. Reduce the horizon length to $N = 30$. What happens when you try to run the MPC? Why does this not produce a desirable result?

c. Using the same horizon length of $N = 30$, replace the step input with a ramp input with a slope of 10 m/s that saturates at your desired reference. Why do you think this fixes the issue you discovered in the previous question? What could be a more systematic way of implementing such a method?