ECEN 5028: Constrained Control

Homework 5: Nonlinear Model Predictive Control

Exercise 1. [100 points.] Consider the following nonlinear model of the lateral dynamics of a vehicle

$$\begin{bmatrix} \dot{y} \\ \dot{v} \\ \dot{\psi} \\ \dot{\omega} \\ \dot{\delta}_f \\ \dot{\delta}_r \end{bmatrix} = \begin{bmatrix} s\sin(\psi) + v\cos(\psi) \\ -s\omega + \frac{1}{M}(F(\alpha_f)\cos(\delta_f) + F(\alpha_r)\cos(\delta_r)) \\ \omega \\ \frac{1}{J}(F(\alpha_f)\cos(\delta_f)L_f - F(\alpha_r)\cos(\delta_r)L_r) \\ u_1 \\ u_2 \end{bmatrix},$$

where

$$\alpha_f = \delta_f - \arctan\left(\frac{v + L_f\omega}{s}\right)$$
$$\alpha_r = \delta_r - \arctan\left(\frac{v - L_r\omega}{s}\right)$$

are the sideslip angles at the front and rear tire,

$$F(\alpha) = \mu M g \sin(c \arctan(b\,\alpha))$$

is the Pacejka tire force model, and $M = 2041\ kg$, $J = 4964\ kgm^2$, $g = 9.81\ m/s^2$, $L_f = 1.56\ m$, $L_r = 1.64\ m$, $\mu = .8$, $b = 12$, $c = 1.285$, $s = 30\ m/s$ are the system parameters. Perform the following:

a. Given the sampling period $t_s = 0.04\ s$ and a prediction horizon of 30 steps, use NMPC to steer the system from the origin to the target reference $r = [5; 0; 0; 0; 0]$ under the following conditions:

   - Stage cost $l(x,u) = \frac{1}{2}(x-r)^T Q(x-r) + \frac{1}{2}u^T R u$, with $Q = diag([1\ 0\ 1\ 0\ 0\ 0])$ and $R = 0.1 \cdot I_2$;
   - No terminal cost, no constraints.

b. Introduce a quadratic terminal cost $m(x) = \frac{1}{2}(x-r)^T P(x-r)$, where P is obtained by linearizing the system dynamics around the target equilibrium point and then solving the Discrete Algebraic Riccati Equation. Compare the results to the previous point.

c. Introduce the following constraints on the steering angles (no terminal constraints)

$$\delta_f \in [-30°, 30°],\ \ \delta_r \in [-6°, 6°],\ \ u_1 \in [-1, 1],\ \ u_2 \in [-1, 1].$$

d. Introduce the following constraints on the sideslip angles (no terminal constraints)

$$\alpha_f \in [-6^\circ, 6^\circ], \ \ \alpha_r \in [-6^\circ, 6^\circ].$$