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Exercise 1

```
clc;
clear all;
close all;
Ac = [0 1; 0 0];
Bc = [0; 1];
C = [1 0; 0 1; 0 0];
D = [0; 0; 1];
E = [1 0];
F = 0;
yub = [6, 2, 2]';
ylb = [-6, -2, -2]';
C = [C
    -C];
D = [D
    -D];
y = [yub
    -ylb];
ts = 0.1;
```

Part A

```
sys_c = ss(Ac,Bc,[],[]);
sys_d = c2d(sys_c, ts);
A = sys_d.A;
B = sys_d.B;
```

Part B

```
G = null([A - eye(size(A)), B, zeros(size(B)); E, F, -eye(size(E, 1))]);
Gx = G(1:length(A));
Gu = G(length(A)+1 : length(A)+length(F));
Gz = G(length(A)+length(F)+1:end);
```

Dividing with Gz to normalise the values

```
Gx = Gx/Gz;
Gu = Gu/Gz;
Gz = eye(size(Gz));
```

Part C

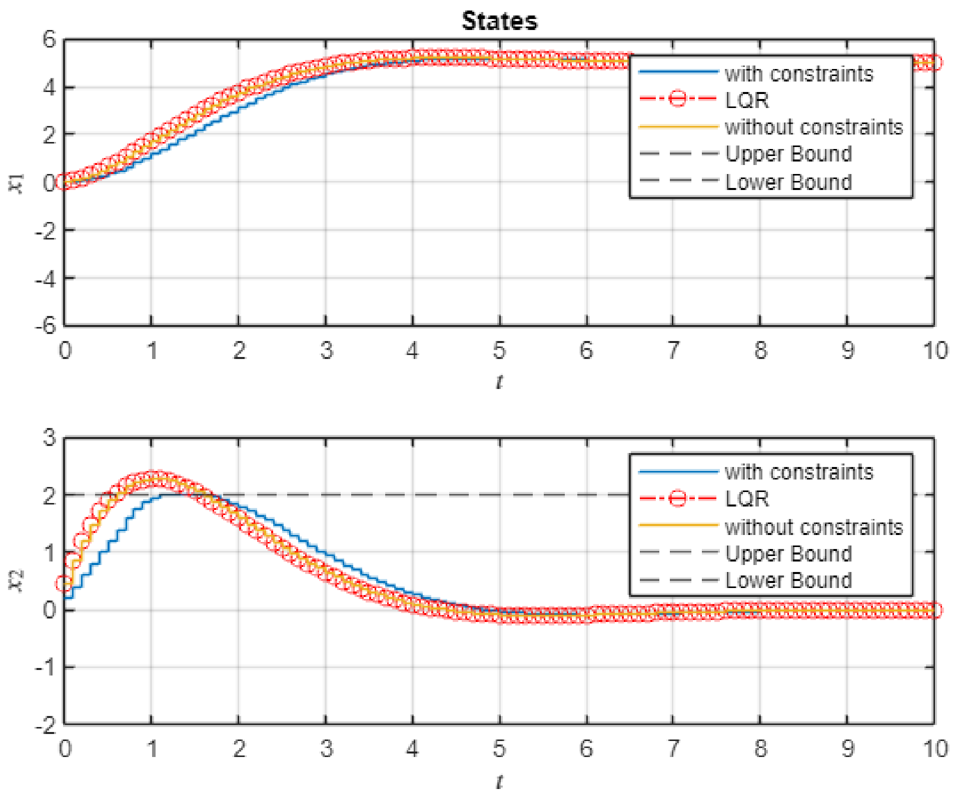
```
x0 = [0;0];
ref = 5;
v = ref;

[n,m] = size(B);
```

```
Q = E'*E;
R = 1;
[K,P,~] = dlqr(A,B,Q,R);

% Prediction Horizon
N = 10;

% Terminal Constraints (None)
% Ox = zeros(1,n);
% Ov = zeros(1,m);
% oy = 0;

% Terminal Constraints (O_infinity)
[Ox,Ov,oy] = calc_Oinf(A,B,C,D,K,y,Gx,Gu);

% Pad Ox with Zeros
Ox = [zeros(length(oy),N*m+(N-1)*n) Ox];
% QP, Sparse Formulation
In = eye(N);
In1 = diag(ones(N-1,1),-1);

H = blkdiag(kron(In,R),kron(eye(N-1),Q),P);
f = zeros(N*(n+m),1);
% fnew = [repmat(R*Gu, N, 1); repmat(Q*Gx, (N-1), 1); zeros(size(P*Gx))];
fnew = [repmat(R*Gu, N, 1); repmat(Q*Gx, (N-1), 1); P*Gx];

E  = [kron(In,B) kron(In1,A)-eye(N*n)];
gx = [-A;zeros((N-1)*n,n)];

Y  = [kron(In,D) kron(In1,C)];
yx = [-C;zeros((N-1)*length(y),n)];
yc = kron(ones(N,1),y);
out = sim('HW_4_Q1.slx');

% Plots
t = out.tout;
x0 = out.x0';
u0 = out.u0';
V0 = out.V0';
x = out.x';
u = out.u';
V = out.V';
x1 = out.x1';
u1 = out.u1';
V1 = out.V1';
```

The QP formulation is taken from the code provided in class, F matrix is formulated with N reps of R*Gu, N-1 reps of Q*Gx and one rep of P*Gx.

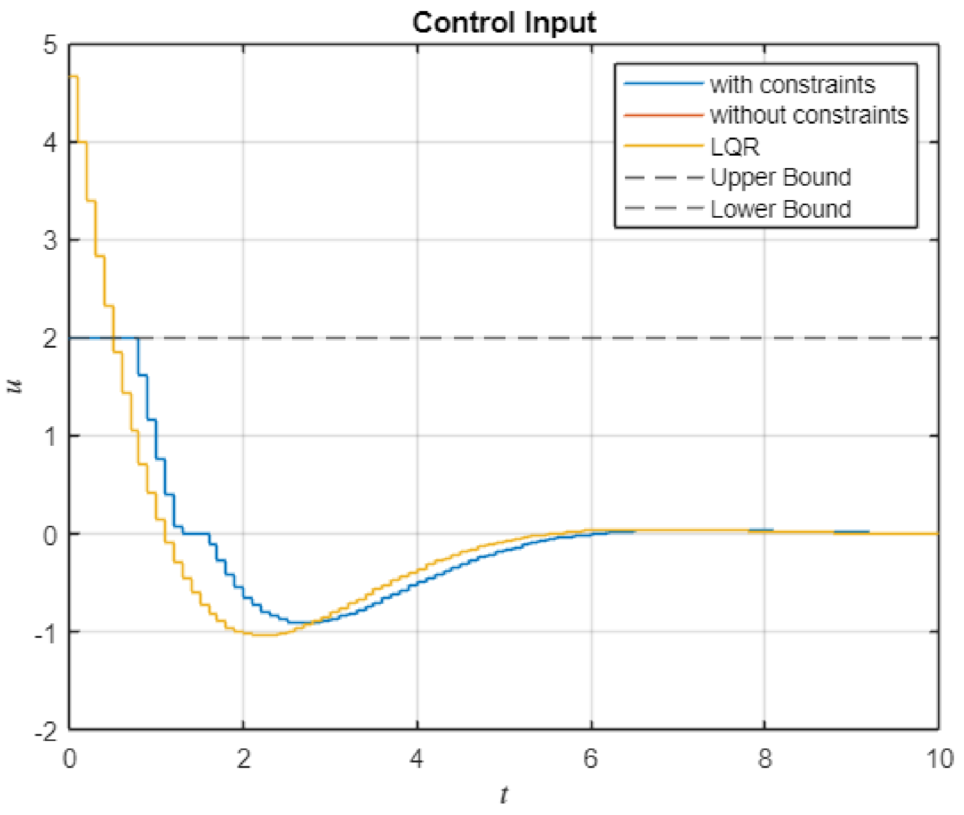
Part D & E

```
figure
subplot(2,1,1)
stairs(t,x(1,:)); hold on;
stairs(t,x0(1,:), '-.OR', 'LineWidth', 0.5);
stairs(t,x1(1,:));
yline(yub(1), '--');
yline(ylb(1), '--'); hold off; grid on;
xlabel('$t$','Interpreter',"latex")
ylabel('$x_1$','Interpreter',"latex")
legend('with constraints','LQR','without constraints','Upper Bound', 'Lower Bound')
title('States')

subplot(2,1,2)
stairs(t,x(2,:)); hold on;
stairs(t,x0(2,:), '-.OR', 'LineWidth', 0.5);
stairs(t,x1(2,:));
yline(yub(2), '--');
yline(ylb(2), '--'); hold off; grid on;
xlabel('$t$','Interpreter',"latex")
ylabel('$x_2$','Interpreter',"latex")
legend('with constraints','LQR', 'without constraints','Upper Bound', 'Lower Bound')
```



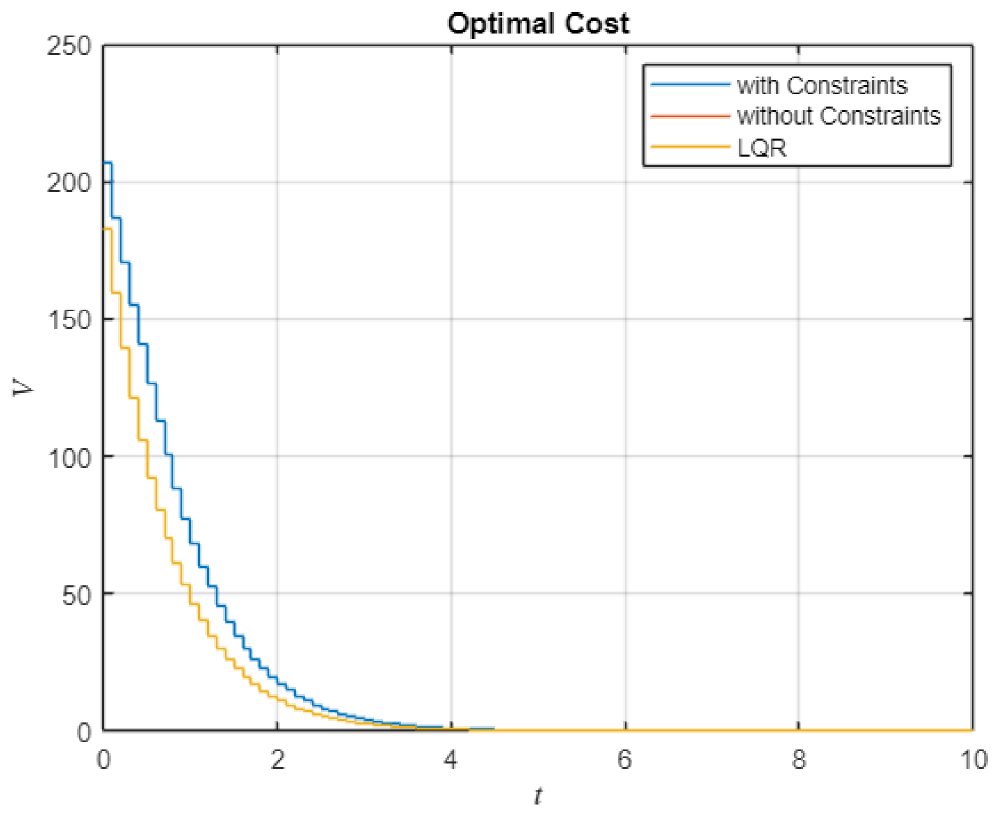
We can see that step response for LQR and unconstrained is exactly same resulting in cover lapping of both the responses. In addition both unconstrained as well as constrained MPC response our states reach the desired position but the step response of unconstrained MPC violates the state constrained for velocity, whereas the constrained MPC does not violates state bounds and also it is faster compared to the step response of constrained MPC.

```
figure
stairs(t,u); hold on;
stairs(t,u1);
stairs(t,u0);
yline(yub(3), '--');
yline(ylb(3), '--'); hold off; grid on;
xlabel('$t$','Interpreter',"latex")
ylabel('$u$','Interpreter',"latex")
legend('with constraints','without constraints','LQR','Upper Bound', 'Lower Bound')
title('Control Input')
```



Both LQR (yellow line) and unconstrained MPC (orange line) violates the control input bounds but the constrained MPC (blue line) does not violates the control input bounds.

```
figure
stairs(t,V(1,:))
hold on; grid on
stairs(t,V1(1,:))
stairs(t,V0(1,:))
xlabel('$t$','Interpreter',"latex")
ylabel('$V$','Interpreter',"latex")
legend('with Constraints','without Constraints','LQR')
title('Optimal Cost')
```



Both constrained and unconstrained MPC have higher costs than LQR. Initially, unconstrained MPC has higher cost due to it's quick response and constrained MPC has lower cost due to slow response but after t=4 when both responses reach the desired states they have same cost, this can be corellated (goes parallel) with step response.

Auxiliary Functions

```
function [Ox,Ov,oy] = calc_Oinf(A_ol,B_ol,C_ol,D_ol,K,y, Gx, Gu)

% Steady-State Manifold
% G = null([A_ol-eye(size(A_ol)) B_ol]);
% Gx = G(1:size(A_ol,2),:);
% Gu = G(size(A_ol,2)+1:end,:);

% Closed Loop System
A = A_ol-B_ol*K;
B = B_ol*(Gu+K*Gx);
C = C_ol-D_ol*K;
D = D_ol*(Gu+K*Gx);

% Steady-State Constraints
eps = 0.05;
Hss = [0*C C*Gx+D];
hss = (1-eps)*y;
```

```
% 0-step Constraint Admissible Set
H0 = [C D];
h0 = y;

Oinf = Polyhedron([Hss;H0],[hss;h0]);

while isempty(h0)~=1
    % Plot Polyhedron
    % Oinf.plot
    % xlabel('$x_1$','Interpreter','latex')
    % ylabel('$x_2$','Interpreter','latex')
    % zlabel('$v$','Interpreter','latex')
    % axis equal
    % pause(0.01)

    % Update Constraints
    H1 = H0*[A B; zeros(size(B')) eye(size(B,2))];
    h1 = h0;

    % Eliminate Redundant Rows
    [H1,h1] = elim_redundancies(Oinf,H1,h1);

    % Update the Polyhedron
    Oinf = Polyhedron([Oinf.A; H1],[Oinf.b; h1]);

    % Reinitialize Iteration
    H0 = H1;
    h0 = h1;
end
% Oinf Constraints
Ox = Oinf.A(:,1:2);
Ov = Oinf.A(:,3);
oy = Oinf.b;
end
```

```
function [H_nred,h_nred] = elim_redundancies(poly,H,h)
% Nonredundant Constraints
H_nred = [];
h_nred = [];

for i = 1:length(h)
    % Check Redundancy for each individual row
    f = -H(i,:)';

    A = [poly.A
         H((1:length(h))~=i,:)];

    b = [poly.b
```

```
          h((1:length(h))~=i,:)];

    % Solve LP
    [~,fval,flag]=linprog(f,A,b,[],[],[],
[],optimoptions('linprog','Display','none'));

    if flag == -3 || -fval>h(i)
        % Constraint is not redundant
        H_nred = [H_nred;H(i,:)];
        h_nred = [h_nred;h(i)];
    end

end

end
```