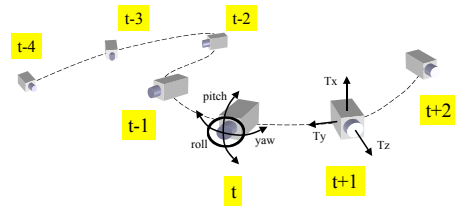


Lecture 22: Camera Motion

Readings: T&V Sec 8.1 and 8.2

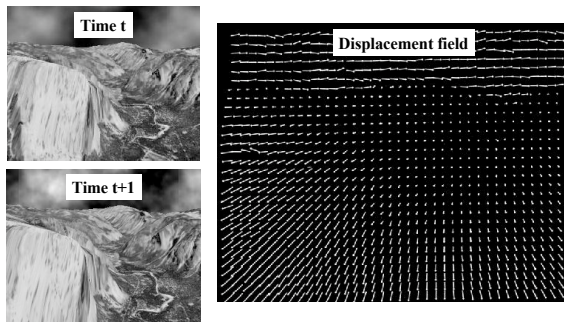
Moving Camera



Camera takes a sequence of images (frames) indexed by time t

From one time to the next, the camera undergoes rotation (roll, pitch, yaw) and translation (t_x, t_y, t_z)

Motion (Displacement) Fields



Motion Field vs Optic Flow

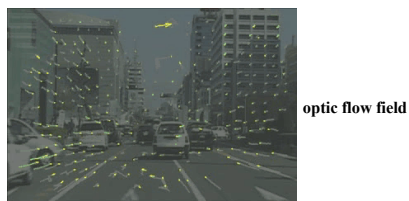
Motion Field: projection of 3D relative velocity vectors onto the 2D image plane.

Optic Flow: observed 2D displacements of brightness patterns in the image.

Motion field is what we want to know.
Optic flow is what we can estimate.

Motion Field vs Optic Flow

Sometimes optic flow is a good approximation to the unknown motion flow.



We can then infer relative motion between the camera and objects in the world.

Warning: Optic Flow \neq Motion Field

Consider a moving light source:



$MF = 0$ since the points on the scene are not moving
 $OF \neq 0$ since there is a moving pattern in the images

Motion Field

We are going to derive an equation relating
3D scene structure and velocity
to the
2D motion flow field.

Motion Field

What is a **Field** anyways?

Image a vector at each point in space. This is a vector field. In 3D space, we will look at the field of 3D velocity vectors induced by camera motion.

In 2D, we will be looking at the projections of those 3D vectors in the image. There will be a 2D flow vector at each point in the image. This is the 2D motion flow field.

Recall : General Projection Equation

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \sim \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Strategy:

- 1) Assume internal params known (set to identity)
- 2) At time t, set $R=I$, $T=0$
- 3) At time t+1, movement generates a relative R and T
simplifying assumption: small motion \rightarrow small rotation
- 4) Compute 3D velocity vector
- 5) Compute 2D velocity vector
a function of X,Y,Z,R,T,f

Displacement of 3D World Point

Time t: 3D position P
Time t+1: 3D position RP+T

3D Displacement = $RP+T - P$

Now consider short time period (like time between two video frames = 1/30 sec). Can assume a small rotation angle in that amount of time. Make a small angle approximation and rewrite displacement. In the limit (infinitesimal time period), we will get a velocity.

Write Rotation matrix in terms of Euler Angles
2.2.1 Euler Angle Transformation

Three principal rotations:

- $\lambda = [0, 0, 1]^T$ $\beta = \psi$
- $\lambda = [0, 1, 0]^T$ $\beta = \theta$
- $\lambda = [1, 0, 0]^T$ $\beta = \phi$

Rotation matrices:

$$R_{\psi} = \begin{bmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{\theta} = \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix}$$

$$R_{\phi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix}$$

Diagram illustrating the three principal rotations: (1) Rotation over yaw angle ψ about z_1 , (2) Rotation over pitch angle θ about y_1 , and (3) Rotation over roll angle ϕ about x_1 . Note that $v_1 = v_2$.

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2.2.1 Euler Angle Transformation

Linear velocity transformation (zyx-convention):

$$\dot{p}^n = R^n(\Theta) \dot{v}_o^b$$

Example:

$$\dot{v}_o^b = R_o^b(\Theta) \dot{v}_o^b$$

where

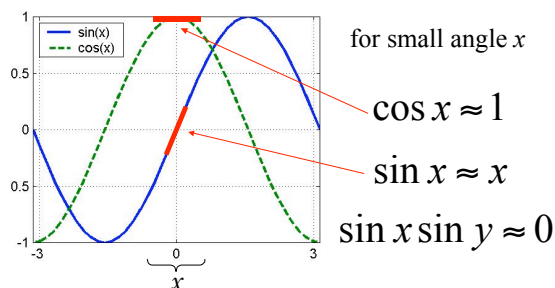
$$R_o^b(\Theta) = R_{z,y,x} = R_{\psi,\theta,\phi}$$

$$R_o^b(\Theta)^{-1} = R_o^b(\Theta) = R_{\psi,\theta,\phi}^T = R_{\phi,\theta,\psi}^T$$

$$R_o^b(\Theta) = \begin{bmatrix} c\psi c\theta & -s\psi c\theta + c\psi s\theta s\phi & s\psi s\theta + c\psi c\theta s\phi \\ s\psi c\theta & c\psi c\theta + s\psi s\theta s\phi & -c\psi s\theta + s\psi c\theta s\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix}$$

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Small Angle Approx



2.2.1 Euler Angle Transformation

Linear velocity transformation (zyx-convention):

Example:

$$\dot{p}^i = R_i^b(\Theta) \dot{v}_0^b$$

$$\dot{v}_0^b = R_b^i(\Theta) \dot{v}_0^i$$

where

$$R_i^b(\Theta) = R_{z,\phi} R_{y,\theta} R_{x,\psi} \quad R_b^i(\Theta)^{-1} = R_b^i(\Theta) = R_{x,\psi}^T R_{y,\theta}^T R_{z,\phi}^T$$

$$R_b^i(\Theta) = \begin{bmatrix} c\psi c\theta & -s\psi c\theta + c\psi s\theta s\phi & s\psi c\theta + c\psi s\theta c\phi \\ s\psi c\theta & c\psi c\theta + s\psi s\theta s\phi & -c\psi c\theta + s\psi s\theta c\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix}$$

Small angle approximation:

$$R \approx \begin{bmatrix} 1 & -\psi & \theta \\ \psi & 1 & -\phi \\ -\theta & \phi & 1 \end{bmatrix} = I + \begin{bmatrix} 0 & -\psi & \theta \\ \psi & 0 & -\phi \\ -\theta & \phi & 0 \end{bmatrix} = I + S$$

3D Velocity

Under small angle approx,

displacement = $RP + T - P$

$$= (I + S)P + T - P$$

$$= SP + T$$

$$S = \begin{bmatrix} 0 & -\theta_z & \theta_y \\ \theta_z & 0 & -\theta_x \\ -\theta_y & \theta_x & 0 \end{bmatrix}$$

Note: $SP = [\theta_x, \theta_y, \theta_z]^T \times P$

In limit, displacement becomes a velocity

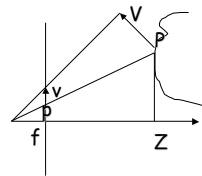
$$\text{3D velocity } V = T + \omega \times P \quad \text{where } \omega = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

linear velocity

angular velocity

Caution: Abuse of notation \Rightarrow we are re-using T as a velocity

3D Relative Velocity



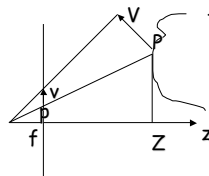
The relative velocity of P wrt camera:

$$V = -T - \omega \times P$$

$$T = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} \quad \omega = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \quad P = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Note: we change signs of the velocity to be consistent with the book. This should not cause concern. It just depends on whether you want to think of the motion as being due to the camera or the scene.

3D Relative Velocity



The relative velocity of P wrt camera:

$$V = -T - \omega \times P$$

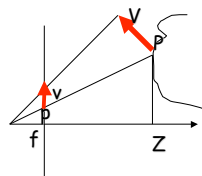
$$T = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} \quad \omega = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \quad P = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$V_x = -T_x - \omega_y Z + \omega_z Y$$

$$V_y = -T_y - \omega_z X + \omega_x Z$$

$$V_z = -T_z - \omega_x Y + \omega_y X$$

3D Relative Velocity



Where are we in this lecture?

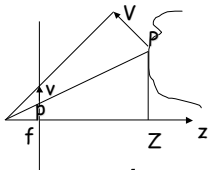
- We just derived an equation relating R, T and to the 3D velocity vector at each scene point.

- We can think of that velocity as a little vector in the scene.

- Now ask, what does the projection of that vector look like in the image? It is a 2D vector. It is one of the vectors that make up the Motion Field!

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Motion Field: the 2D velocity of p



Perspective projection

$$p = \frac{fP}{Z}$$

Taking derivative wrt time:

$$\frac{dp}{dt} = v = \frac{d}{dt} \left(\frac{fP}{Z} \right)$$

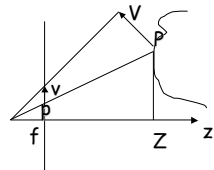
$$\frac{dp}{dt} = v = \frac{f}{Z^2} \left[\frac{dP}{dt} \cdot Z - P \cdot \frac{dZ}{dt} \right]$$

$$\frac{dp}{dt} = v = \frac{f}{Z^2} [V \cdot Z - P \cdot V_z]$$

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Motion Field: the velocity of p



$$\frac{dp}{dt} = v = \frac{f}{Z^2} [V \cdot Z - P \cdot V_z]$$

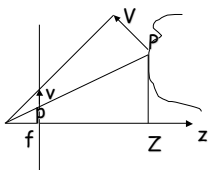
$$p = \frac{fP}{Z} \quad P = \frac{pZ}{f}$$

$$v = f \frac{V}{Z} - p \frac{V_z}{Z}$$

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Motion Field: the velocity of p



$$v = f \frac{V}{Z} - p \frac{V_z}{Z}$$

$$v_x = f \frac{V_x}{Z} - x \frac{V_z}{Z}$$

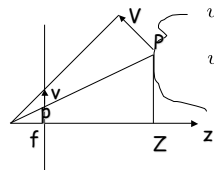
$$v_y = f \frac{V_y}{Z} - y \frac{V_z}{Z}$$

$$v_z = f \frac{V_z}{Z} - f \frac{V_z}{Z} = 0$$

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Motion Field: the velocity of p



$$v_x = f \frac{V_x}{Z} - x \frac{V_z}{Z}$$

$$v_y = f \frac{V_y}{Z} - y \frac{V_z}{Z}$$

$$V_x = -T_x - \omega_y Z + \omega_z Y$$

$$V_y = -T_y - \omega_z X + \omega_x Z$$

$$V_z = -T_z - \omega_x Y + \omega_y X$$

$$v_x = \frac{T_z x - T_x f}{Z} - \omega_y f + \omega_z y + \frac{\omega_x x y}{f} - \frac{\omega_y x^2}{f}$$

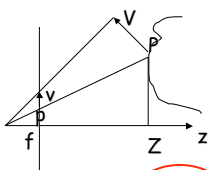
$$v_y = \frac{T_z y - T_y f}{Z} + \omega_x f - \omega_z x - \frac{\omega_y x y}{f} + \frac{\omega_x y^2}{f}$$

THIS IS THE EQUATION WE WANT!!!!

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Motion Field: the velocity of p



Translational component

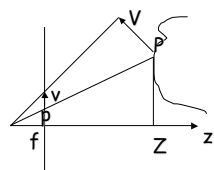
$$v_x = \frac{T_z x - T_x f}{Z} - \omega_y f + \omega_z y + \frac{\omega_x x y}{f} - \frac{\omega_y x^2}{f}$$

$$v_y = \frac{T_z y - T_y f}{Z} + \omega_x f - \omega_z x - \frac{\omega_y x y}{f} + \frac{\omega_x y^2}{f}$$

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Motion Field: the velocity of p



Rotational component

$$v_x = \frac{T_z x - T_x f}{Z} - \omega_y f + \omega_z y + \frac{\omega_x x y}{f} - \frac{\omega_y x^2}{f}$$

$$v_y = \frac{T_z y - T_y f}{Z} + \omega_x f - \omega_z x - \frac{\omega_y x y}{f} + \frac{\omega_x y^2}{f}$$

NOTE: The rotational component is independent of depth Z!

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Special Case I: Pure Translation

$$\omega = 0 \Rightarrow \begin{aligned} v_x &= \frac{T_z x - T_x f}{Z} \\ v_y &= \frac{T_z y - T_y f}{Z} \end{aligned}$$

Assume $T_z \neq 0$

Define: $p_o = \begin{bmatrix} x_o \\ y_o \\ f \end{bmatrix} = \begin{bmatrix} \frac{f T_x}{T_z} \\ \frac{f T_y}{T_z} \\ f \end{bmatrix}$

$$\begin{aligned} v_x &= \frac{T_z x - T_z x_o}{Z} = (x - x_o) \frac{T_z}{Z} \\ v_y &= \frac{T_z y - T_z y_o}{Z} = (y - y_o) \frac{T_z}{Z} \end{aligned}$$

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Special Case I: Pure Translation

$$\omega = 0 \Rightarrow \begin{aligned} v_x &= \frac{T_z x - T_x f}{Z} \\ v_y &= \frac{T_z y - T_y f}{Z} \end{aligned}$$

What if $T_z = 0$?

$$\begin{aligned} v_x &= -f \frac{T_x}{Z} \\ v_y &= -f \frac{T_y}{Z} \end{aligned}$$



All motion field vectors are parallel to each other and inversely proportional to depth!

TIE IN WITH SIMPLE STEREO!

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Special Case I: Pure Translation

$$\begin{aligned} v_x &= (x - x_o) \frac{T_z}{Z} \\ v_y &= (y - y_o) \frac{T_z}{Z} \end{aligned}$$



$T_z > 0$



$T_z < 0$

The motion field in this case is **RADIAL**:

• It consists of vectors passing through $p_o = (x_o, y_o)$

• If:

- $T_z > 0$, (camera moving towards object)
 - the vectors point away from p_o
 - p_o is the **POINT OF EXPANSION**
- $T_z < 0$, (camera moving away from object)
 - the vectors point towards p_o
 - p_o is the **POINT OF CONTRACTION**

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Pure Translation: Properties of the MF

- If $T_z \neq 0$ the MF is **RADIAL** with all vectors pointing towards (or away from) a single point p_o . If $T_z = 0$ the MF is **PARALLEL**.
- The length of the MF vectors is inversely proportional to depth Z . If $T_z \neq 0$ it is also directly proportional to the distance between p and p_o .

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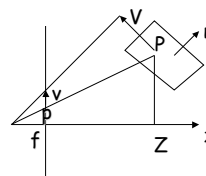
Pure Translation: Properties of the MF

- p_o is the vanishing point of the direction of translation.
- p_o is the intersection of the ray parallel to the translation vector and the image plane.

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Special Case II: Moving Plane

Self-study



Planar surfaces are common in man-made environments

Question: How does the MF of a moving plane look like?

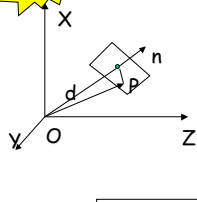
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Self-study

Special Case II: Moving Plane

Points on the plane must satisfy the equation describing the plane.



Let

- \mathbf{n} be the unit vector normal to the plane.
- d be the distance from the plane to the origin.
- **NOTE:** If the plane is moving wrt camera, \mathbf{n} and d are functions of time.

Then:

$$\mathbf{n}^T \cdot \mathbf{P} = d$$

where $\mathbf{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$ $\mathbf{P} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$

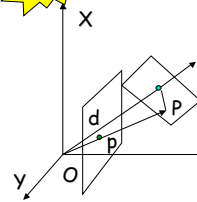
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Self-study

Special Case II: Moving Plane

Let $\mathbf{p} = \begin{bmatrix} x \\ y \\ f \end{bmatrix}$ be the image of \mathbf{P}



Using the pinhole projection equation:

$$\mathbf{p} = \frac{f\mathbf{P}}{Z} \Rightarrow \mathbf{P} = \frac{pZ}{f}$$

Using the plane equation: $\mathbf{n}^T \cdot \mathbf{P} = d \Rightarrow \mathbf{n}^T \cdot \frac{pZ}{f} = d$

Solving for Z : $Z = \frac{fd}{n_x x + n_y y + n_z f}$

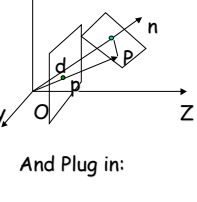
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Self-study

Special Case II: Moving Plane

Now consider the MF equations:



$$v_x = \frac{T_z x - T_x f - \omega_y f + \omega_z y + \frac{\omega_x x y}{f} - \frac{\omega_y x^2}{f}}{Z}$$

$$v_y = \frac{T_z y - T_y f + \omega_x f - \omega_z x - \frac{\omega_y x y}{f} + \frac{\omega_x y^2}{f}}{Z}$$

And Plug in:

$$Z = \frac{fd}{n_x x + n_y y + n_z f}$$

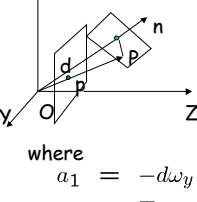
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Special Case II: Moving Plane

The MF equations become:



$$v_x = \frac{1}{fd}(a_1 x^2 + a_2 x y + a_3 f x + a_4 f y + a_5 f^2)$$

$$v_y = \frac{1}{fd}(a_1 x y + a_2 y^2 + a_6 f y + a_7 f x + a_8 f^2)$$

where

$a_1 = -d\omega_y + T_z n_x$	$a_2 = d\omega_x + T_z n_y$
$a_3 = T_z n_z - T_x n_x$	$a_4 = d\omega_z - T_x n_y$
$a_5 = -d\omega_y - T_x n_z$	$a_6 = T_z n_z - T_y n_y$
$a_7 = -d\omega_z - T_y n_x$	$a_8 = d\omega_x - T_y n_z$

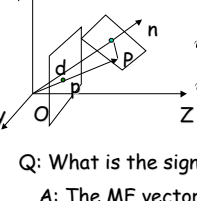
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Special Case II: Moving Plane

MF equations:



$$v_x = \frac{1}{fd}(a_1 x^2 + a_2 x y + a_3 f x + a_4 f y + a_5 f^2)$$

$$v_y = \frac{1}{fd}(a_1 x y + a_2 y^2 + a_6 f y + a_7 f x + a_8 f^2)$$

Q: What is the significance of this?

A: The MF vectors are given by low order (second) polynomials.

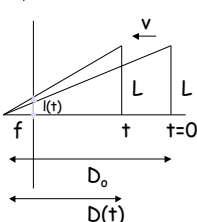
- Their coeffs. a_1 to a_8 (only 8!) are functions of \mathbf{n} , d , T and ω .
- That is, can estimate 8 param global flow rather than 2 params per pixel! Huge savings in time and robustness.

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Self-study

Application: Time to Collision



An object of height L moves with constant velocity \mathbf{v} :

- At time $t=0$ the object is at:
 - $D(0) = D_0$
- At time t it is at
 - $D(t) = D_0 - vt$
- It will crash into the camera at time:
 - $D(t) = D_0 - vt = 0$
 - $\tau = D_0/v$

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Time to Collision

The image of the object has size $l(t)$:

$$l(t) = \frac{fL}{D(t)}$$

Taking derivative wrt time:

$$l'(t) = \frac{dl(t)}{dt} = fL \frac{d(1/D(t))}{dt}$$

$$l'(t) = fL \frac{-1}{D^2(t)} \frac{d(D(t))}{dt}$$

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Time to Collision

$$l'(t) = fL \frac{-1}{D^2(t)} \frac{d(D(t))}{dt}$$

$$D(t) = D_o - vt$$

$$\frac{d(D(t))}{dt} = -v$$

$$l'(t) = fL \frac{v}{D^2(t)}$$

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Time to Collision

$$l'(t) = fL \frac{v}{D^2(t)}$$

$$l(t) = \frac{fL}{D(t)}$$

And their ratio is:

$$\frac{l(t)}{l'(t)} = \frac{fL}{D(t)} \frac{D^2(t)}{fLv} = \frac{D(t)}{v} = \tau$$

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Time to Collision

$$l'(t) = fL \frac{v}{D^2(t)}$$

$$l(t) = \frac{fL}{D(t)}$$

Can be directly measured from image

And time to collision:

$$\tau = \frac{l(t)}{l'(t)}$$

Can be found, without knowing L or D_o or v !!

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Application: Decomposition of Vehicle Flow Field into Rotational and Translational Components

Motivation

- Estimate steering angles
- Computation of scene structure simplifies when rotational motion is removed

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Review: Flow Due to Self-Motion

Note: I've changed sign again!

Flow:

$$u' - u = f\omega_Y - v\omega_Z + \frac{u^2}{f}\omega_Y - \frac{uv}{f}\omega_X + f\frac{T_X}{Z} - u\frac{T_Z}{Z}$$

$$v' - v = -f\omega_X + u\omega_Z - \frac{v^2}{f}\omega_X + \frac{uv}{f}\omega_Y + f\frac{T_Y}{Z} - v\frac{T_Z}{Z}$$

Rotation Translation

Rotation component Translation component

Approach

Use prior knowledge of car's likely motion to simplify the problem.
Assume we know approx direction of translation (T_x, T_y, T_z)
[e.g. if driving forward, we choose (0,0,1)]
Assume roll angle of motion (w_z) is 0
Problem reduces to solving for pitch and yaw angles (w_x and w_y)

$$\begin{aligned} u' - u &= f\omega_y - \cancel{v\omega_z} + \frac{u^2}{f}\omega_y - \frac{uv}{f}\omega_x + f\frac{T_x}{Z} - \cancel{v\frac{T_z}{Z}} \\ v' - v &= -f\omega_x + \cancel{u\omega_z} - \frac{v^2}{f}\omega_x + \frac{uv}{f}\omega_y + f\frac{T_y}{Z} - \cancel{u\frac{T_z}{Z}} \end{aligned}$$

known

Approach (continued)

For each observed flow vector, form a linear constraint on w_x and w_y by taking the dot product of the flow equation with a new vector that is constructed to "annihilate" the translation component of flow.

This annihilation vector is $\begin{pmatrix} fT_y - vT_z \\ -fT_x + uT_z \end{pmatrix}$ Note: these are all known values!

Verify:

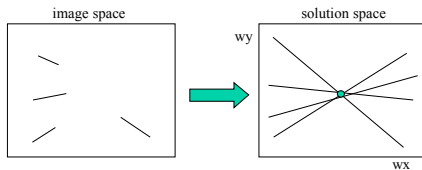
$$\begin{pmatrix} f\frac{T_x}{Z} - u\frac{T_z}{Z} \\ f\frac{T_y}{Z} - v\frac{T_z}{Z} \end{pmatrix} \cdot \begin{pmatrix} fT_y - vT_z \\ -fT_x + uT_z \end{pmatrix} = 0$$

Approach (continued)

For each observed flow vector, we annihilate the translation component to form one linear constraint of the form $(a_i w_x + b_i w_y + c_i) = 0$.

Given the whole set of n observed flow vectors, we seek w_x and w_y that simultaneously satisfy the set of n linear equations

Geometric intuition:



Use Robust Estimator

Solving using least-squares will be sensitive to outliers (grossly incorrect data). Instead, we use a robust estimator (Random Sample Consensus – RANSAC).

