Bayes Classifier (Optimal)

P(ω_j | X = x) = P(X = x | ω_j) * P(ω_j) / P(X=x)

Given , $\omega_j \in$ {0,1}
$P(\omega_j = 0) = 0.95$
Posterior are normally distributed as :
$P(X = x|\omega_j = 0)$ ~ N(0,1)
$P(X = x|\omega_j = 1)$ ~ N(1,1)

(Optimal) Bayes Classifier under 0-1 loss function is one that choose class ω_j which maximizes posterior probability P(ω_j | X = x)

Import required libraries

In [1]:

```
import numpy as np
import matplotlib.pyplot as plt
```

Fixed variables

In [23]:

```
N = 10000
p0 = 0.95
mu_0, mu_1 = 0., 1.
var_0 = var_1 =  sigma = 1.
```

In [3]:

```
def normal(mean, var, X):
    return np.exp( -(X-mean)**2 / ( 2* var**2 ) )/ np.sqrt(2*np.pi*var)

def prior(omega_j):
    if omega_j == 0 :
        return p0
    return (1 - p0)

def class_conditional(X,omega_j):
    if omega_j == 0 :
        y = normal(mu_0,var_0,X)
    else :
        y = normal(mu_1,var_1,X)
    return y

def posterior_prob(X, omega_j) :
    return class_conditional(X,omega_j)*prior(omega_j)

def predict_class(X):
    boolean = posterior_prob(X,1) > posterior_prob(X,0)
    if boolean :
        return 1
    return 0
```

In [21]:

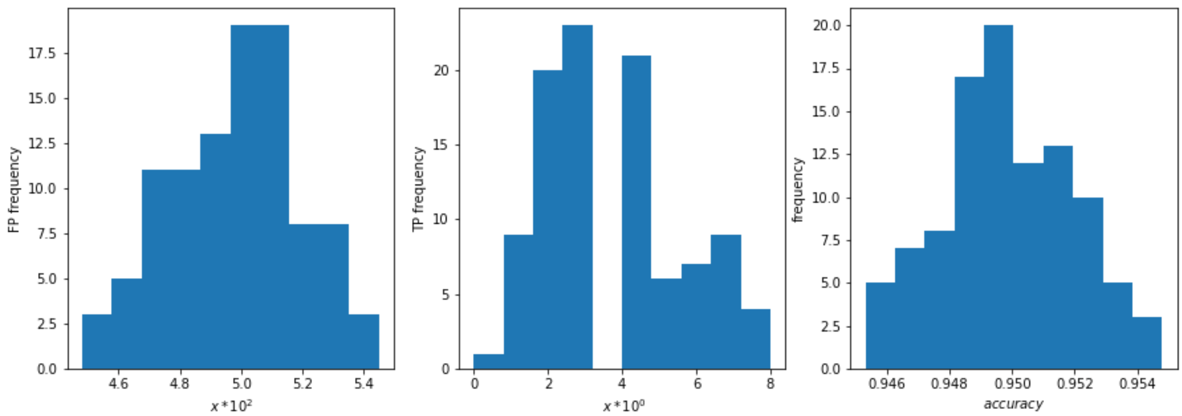
```
# Generate dummy data

def one_simulation():
    global N
    data = []
    true_labels = []
    for i in range(N):
        label = np.random.choice([0,1], p=[p0,1-p0])
        if label == 0 :
            x = np.random.normal(mu_0,var_0)
        else :
            x = np.random.normal(mu_1, var_1)
        data.append(x)
        true_labels.append(label)

    # Test our Classifier
    pred_labels = []
    for i in range(N):
        pred = predict_class(data[i])
        pred_labels.append(pred)

    # Q-1 False Positive FP
    FP = [1 if ( pred_labels[i] == 0  and true_labels[i] == 1 ) else 0 for i in range(N)]
    FP = sum(FP)

    # Q-2 True Positive TP
    TP = [1 if (pred_labels[i] == 1 and true_labels[i]==1) else 0 for i in range(N) ]
    TP = sum(TP)

    # Q-3 Accuracy
    acc = [1. if pred_labels[i] == true_labels[i] else 0 for i in range(N)]
    acc = sum(acc)/N

    return FP, TP, acc

def n_simulation(n):
    res = [ [], [], []]
    for i in range(n):
        FP,TP,acc = one_simulation()
        res[0].append(FP)
        res[1].append(TP)
        res[2].append(acc)
    return np.array(res)
```

In [22]:

```
def find_scale(L):
    scale = 0
    minL = min([ e for e in L if e != 0] )
    while True :
        if minL < 1 :
            break
        scale += 1
        minL = minL/10
    return scale
```

In [18]:

```
fig, axes = plt.subplots(nrows=1,ncols=3, figsize=(15,5))

FP, TP, acc = n_simulation(100)

fp_scale , tp_scale  = find_scale(FP)-1, find_scale(TP)-1
FP = FP/(10**fp_scale)
TP = TP/(10**tp_scale)

axes[0].hist(FP)
axes[0].set_xlabel(' $ x * 10^%d$'%fp_scale)
axes[0].set_ylabel('FP frequency')


axes[1].hist(TP)
axes[1].set_xlabel(' $ x * 10^%d$'%tp_scale)
axes[1].set_ylabel('TP frequency')

axes[2].hist(acc)
axes[2].set_xlabel('$ accuracy $ ')
axes[2].set_ylabel('frequency')

plt.show()

FP = np.mean(FP)*(10**fp_scale)
TP = np.mean(TP)*(10**tp_scale)
acc_mean = np.mean(acc)
print(f"False Positive FP in every %d samples : %d"%(N,round(FP)))
print(f"True  Positive TP in every %d samples : %d"%(N,round(TP)))
print(f"Accuracy : %f"%acc_mean)
```




```
False Positive FP in every 10000 samples : 499
True  Positive TP in every 10000 samples : 4
Accuracy : 0.949849
```

Neyman Pearson Classifier

Threshold : α

$$h_{NP}(X) = \begin{cases} 1 \text{ if } f_1(X)/f_0(X) > K \\ 0 \text{ otherwise} \end{cases}$$

where K is chosen s.t.

$P(f_1(X)/f_0(X) <= K | X \in C_0) = 1 - \alpha$

For binary classification problem where distribution are 1-D gaussian, on simplication we get τ s.t. our classifier become this

$$h_{NP}(X) = \begin{cases} 1 \text{ if } X > \tau \\ 0 \text{ otherwise} \end{cases}$$

where τ = $\sigma * \phi^{-1}(1-\alpha) + \mu_0$

Import required libraries

In [24]:

```
import numpy as np
from scipy.stats import norm
import matplotlib.pyplot as plt
```

Generate dataset

In [25]:

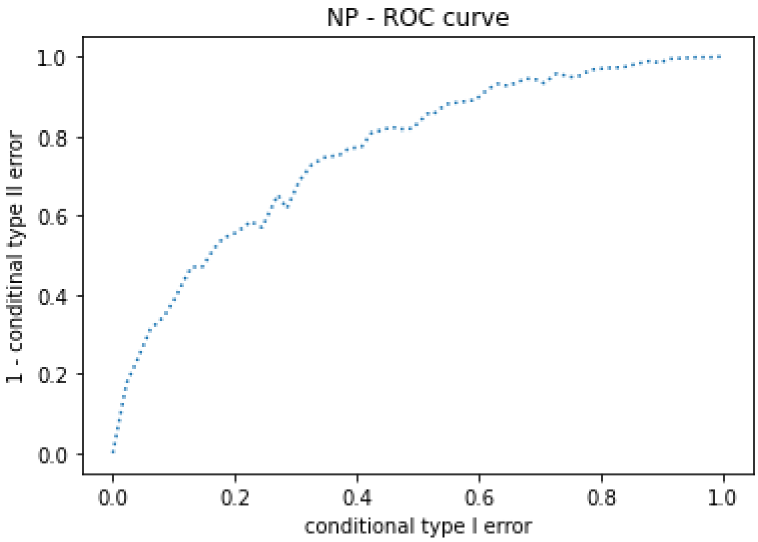
```
# Global variables
mu_0, mu_1 = 0, 1
var_0 = var_1 = sigma =  1
p0 = 0.95
N = 10000

def generate_data() :
    data = []
    true_labels = []
    for i in range(N):
        z_i = np.random.choice([0,1],p=[p0,1-p0])
        true_labels.append(z_i)
        mu = mu_0 if z_i == 0 else mu_1
        var = var_0 if z_i == 0 else var_1
        point = np.random.normal(mu,var)
        data.append(point)
    return data, true_labels
```

In [26]:

```
def NP_classifier(tau):

    global N

    data, y_true = generate_data()
    predictions = []

    for i in range(N):
        X = data[i]
        if X > tau :
            predictions.append(1)  # class 1
        else :
            predictions.append(0) # class 0

    # Calculate Type I and Type II errors

    # Type-I : P(h_NP(X) = 1 | X ∈ C_0 )
    num_type_1 = [1  for i in range(N)  if y_true[i]==0 and predictions[i]==1 ]
    den_type_1 = [1 for i in range(N) if y_true[i]==0 ]
    type_1 = sum(num_type_1)/sum(den_type_1)

    # Type-II : P(h_NP(X) = 0 | X ∈ C_1 )
    num_type_2 = [1 for i in range(N) if y_true[i]==1 and predictions[i]==0]
    den_type_2 = [1 for i in range(N) if y_true[i]==1]
    type_2 = sum(num_type_2)/sum(den_type_2)

    return type_1, type_2
```

In [27]:

```
alphas = np.linspace(0,1,50)

ROC = [ [], []]
for alpha in alphas :
    tau = sigma*norm.ppf(1 - alpha) + mu_0  # reference used : https://youtu.be/B1g4FN9eAuA
    type_1, type_2 = NP_classifier(tau)
    ROC[0].append(type_1)
    ROC[1].append(1-type_2)

plt.plot(ROC[0],ROC[1],linestyle=':')

plt.title('NP - ROC curve')
plt.xlabel('conditional type I error')
plt.ylabel('1 - conditinal type II error')
plt.show()
```



Min Max Classifier

A min-max classifier is a classifier in which risk is made independent of priors by choosing suitable threshold τ

Mathematically , In binary classification , such τ will satisfy the follwing condition :

τ = $\underset{t:t\in R}{\text{argmin}}$ max P(error)

on simplication we get τ satisfies follwing condition

$\int_{\tau}^{\inf} f_0(x)dx$ = $\int_{-\inf}^{\tau} f_0(x)dx$

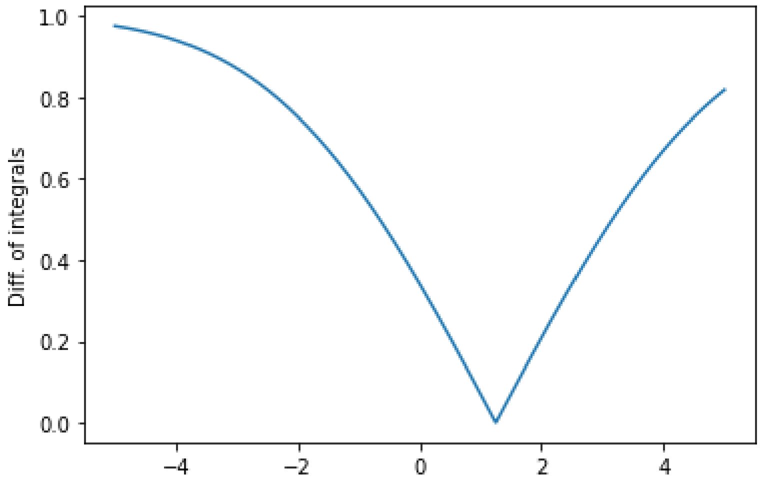
Import required libraries

In [28]:

```
import numpy as np
from scipy.stats import norm
import matplotlib.pyplot as plt
```

In [29]:

```
mu_0, mu_1 = 0, 1
var_0 = var_1 = sigma = 1

taus = np.linspace(-2,2,1000)
errors = []
x = np.linspace(-5,5,1000)
for tau in taus :
    integral_1 = 1- norm.cdf(tau,mu_0,var_0)
    integral_2 = norm.cdf(tau,mu_1, var_1)
    diff = abs(integral_2 - integral_1)
    errors.append(diff)


plt.plot(x,errors)
plt.ylabel('Diff. of integrals')
plt.show()

tau_opt = x[np.argmin(errors)]  # Optimal Tau
print("Optimal Tau : %f"%tau_opt)
```




```
Optimal Tau : 1.246246
```

In [30]:

```
# Make predictions

def one_experiment() :
    data, y_true = generate_data()
    predictions = []
    for i in range(N):
        X = data[i]
        if X > tau_opt :
            predictions.append(1)  # class 1
        else :
            predictions.append(0) # class 0

    accuracy = [ 1 if predictions[i] == y_true[i] else 0 for i in range(N)]
    acc = sum(accuracy)/len(accuracy)*100
    return acc

def n_experiments(n):
    acc_list = []
    for i in range(n):
        acc = one_experiment()
        acc_list.append(acc)
    return acc_list
```

In [31]:

```
acc_list = n_experiments(100)
plt.xlabel('Accuracy')
plt.hist(acc_list)
plt.show()
```