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(a) (b) If A is invertible.

then $R_2 \leftarrow R_2 - VA^{-1}R_1$

$$\begin{bmatrix} I & 0 \\ -VA^{-1} & I \end{bmatrix}\begin{bmatrix} A & U \\ V & C \end{bmatrix} = \begin{bmatrix} A & U \\ 0 & -VA^{-1}U + C \end{bmatrix}$$

$$C_2 \leftarrow C_2 - UA^{-1}C_1$$

$$\begin{bmatrix} I & 0 \\ -VA^{-1} & I \end{bmatrix}\begin{bmatrix} A & U \\ V & C \end{bmatrix}\begin{bmatrix} I & -UA^{-1} \\ 0 & I \end{bmatrix}$$

$$= \begin{bmatrix} A & 0 \\ 0 & -VA^{-1}U + C \end{bmatrix}$$

$$\Rightarrow P^{-1} = \begin{bmatrix} I & -UA^{-1} \\ 0 & I \end{bmatrix}\begin{bmatrix} A & 0 \\ 0 & -VA^{-1}U+C \end{bmatrix}\begin{bmatrix} I & 0 \\ -VA^{-1} & I \end{bmatrix}$$

$$= \begin{bmatrix} A & UA^{-1}(-VA^{-1}U+C) \\ 0 & -VA^{-1}U+C \end{bmatrix}\begin{bmatrix} I & 0 \\ -VA^{-1} & I \end{bmatrix}$$

$$= \begin{bmatrix} A - UA^{-1}(-VA^{-1}U+C)(VA^{-1}) & UA^{-1}(-VA^{-1}U+C) \\ -(-VA^{-1}U+C)VA^{-1} & -VA^{-1}U+C \end{bmatrix}$$

(c) Since both (a) & (b) ~~are~~ ~~are~~ given inverse of P so they must be same

$a_{11} = \overset{(b)}{A - UC^{-1}V} = \overset{\text{from part (a)}}{A - UA^{-1}(-VA^{-1}U + C)VA^{-1}}$

$\Rightarrow UC^{-1}V = UA^{-1}(-VA^{-1}U + C)VA^{-1}$

$a_{22} = \overset{(b)}{UA^{-1}(-VA^{-1}U + C)} = (A - UC^{-1}V)(-UC^{-1})$

$a_{33} = -(-VA^{-1}U + C)A^{-1} = -VC^{-1}(A - UC^{-1}V)$

$a_{44} = -VA^{-1}U + C = \underline{-VC^{-1}(A - UC^{-1}V)(-VC^{-1}) + C}$

On some simplification we can get required expression