Binary Trees

Computing Laboratory

http://www.isical.ac.in/~dfslab

Binary trees

(Recursive) Definition

A *binary tree* over a domain D is either:

- the empty set (called an *empty binary tree*); or
- a 3-tuple $\langle S_1, S_2, S_3 \rangle$ where
    - $S_1 \in D$, (called the *root*) and
    - S_2 and S_3 are *binary trees* over D (called the *left* and *right* subtree resp.)

Binary tree properties

- *Height:* h
- *Number of nodes:* n
- *Number of leaves:* l

Binary tree traversals

- *Preorder*
- *Inorder*
- *Postorder*

Binary tree implementation

Conventional implementation:

```
typedef struct tnode {
    DATA d;
    struct tnode *left, *right;
    struct tnode *parent; // optional
} TNODE;
```

- One malloc per node
- Nodes may be scattered all over the heap area

Binary tree implementation

Alternative implementation:

```
typedef struct tnode {
    DATA d;
    int left, right;
    int parent; //optional
} TNODE;
```

- One initial malloc and reallocs as needed
- All nodes located within the same array

Binary tree implementation

Alternative implementation:

```
typedef struct tnode {
    DATA d;
    int left, right;
    int parent; //optional
} TNODE;
```

- One initial malloc and reallocs as needed
- All nodes located within the same array

Initially:

root = -1

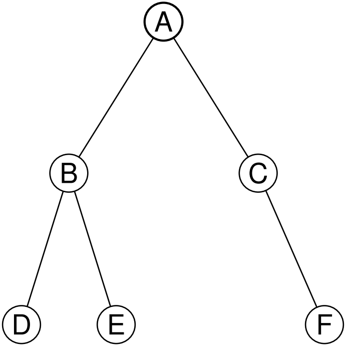
	DATA	left	right
free ⟶ 0	—	1	-1
1	—	2	-1
2	—	3	-1
3	—	4	-1
⋮		⋮	
n-1	—	-1	-1

Binary tree implementation

Alternative implementation:

root = 0

	DATA	left	right
0	A	1	2
1	B	3	4
2	C	-1	5
3	D	-1	-1
4	E	-1	-1
5	F	-1	-1
free ⟶ 6	—	7	-1
⋮		⋮	
n-1	—	-1	-1



Problems – I

For the following problems, some test cases (example trees) are available from the course home page. You may adapt the following code snippet to read in a tree and store it in an array (using the Alternative Implementation).

```
scanf("%u", &numNodes);
if (NULL == (tree = (NODE *) malloc(numNodes * sizeof(NODE)))) {
    fprintf(stderr, "Out of memory\n");
    exit(1);
}
for (node = tree, i = 0; i < numNodes; node++, i++)
    scanf("%d %d %d", &(node->data), &(node->left), &(node->right));
```

Problems – II

1. Using the code fragment given in the previous slide, write a `read_tree()` function with the following prototype.

   ```
   int read_tree(TREE *t);
   ```

   Modify your structure by adding a `parent` field, and modify your `read_tree()` function so that the parent field is filled in while reading the tree, without requiring a separate pass.

2. Given a binary tree with integer-valued nodes, and a *target* value, determine whether there exists a root-to-leaf path in the tree such that the sum of all node values along that path equals the target.
   Modify your program to consider *all* paths, not just root-to-leaf paths.

3. Given a binary tree stored in an array (as in the Alternative Implementation), and the indices of two nodes in the tree, find the index of the node that is the lowest common ancestor of the given nodes.

Problems – III

4. Write a program to print a binary tree, rotated anti-clockwise by 90° on the screen. For example, for the tree on slide 7, your output should like something like:

```
         F
    C
A
         E
    B
         D
```

Now, try to write a program that will print the same tree in the following format:

```
    ----A----
    |       |
  --B--     C--
  |   |       |
  D   E       F
```