STOCHASTIC REDUCED ORDER MODEL

May 16, 2018

Stochastic Reduce Order Model (SROM) focus on minimizing the error in distribution, moments and correlation of random variables for the given samples. This class focus on solving an optimization problem to estimate the probability law (p) associated with samples. The objective function is:

$$\begin{aligned} &\min_p \sum_{u=1}^{3} \alpha_u e_u(p) \\ &\sum_{k=1}^{m} p_k = 1 \quad \& \quad 0 \leq p_k \leq 1,\ k = 1, \dots, m. \end{aligned} \tag{1}$$

where, α_1, α_2 and α_3 are the weights corresponding to error in distribution ($e_1(p)$), error in moments ($e_2(p)$) and error in correlation ($e_3(p)$), which can be defined as per Grigoriu (2009):

$$\begin{aligned} e_1(p) &= \sum_{i=1}^{d} \sum_{k=1}^{m} w_F(x_{k,i}; i)(\hat{F}_i(x_{k,i}) - F_i(x_{k,i}))^2 \\ e_2(p) &= \sum_{i=1}^{d} \sum_{r=1}^{q} w_\mu(r; i)(\hat{\mu}(r; i) - \mu(r; i))^2 \\ e_3(p) &= \sum_{i,j=1,\dots,d; j>i} w_r(i,j)(\hat{r}(i,j) - r(i,j))^2 \end{aligned} \tag{2}$$

Here, F and $\hat{F}$ denote the marginal distribution of $\mathbf{X}$ and $\hat{\mathbf{X}}$ (reduced order model). Similarly, μ and $\hat{\mu}$ are marginal moments and r and $\hat{r}$ are correlation matrix of $\mathbf{X}$ and $\hat{\mathbf{X}}$. This class only consider first and second order moment about origin, i.e. q=2. And, 'm' is number of samples and 'd' is number of random variables.

Table 1: Variables defined for SROM class in UQpy

	Symbols	Variables	Required	Optional
samples	x_k	Input	*	
pdf_type		Input	*	
moments	$\mu(r)$	Input	*	
weights_error	α_u	Input		*
weights_distribution	w_F	Input		*
default_weights_distribution		Input		*
weights_moments	w_μ	Input		*
default_weights_moments		Input		*
weights_correlation	w_r	Input		*
default_weights_correlation		Input		*
properties		Input		*
pdf_params		Input	*	
correlation	r	Input		*
p_.x	p	Output		

Input file and Variables

- SROM
  Option: 'Yes' or 'No'
  'Yes' as input will apply the SROM on the generated samples.

- Moments ($\mu(r)$)
  Two rows containing the first and second order about origin of all random variables.

- Correlation (r)
  Correlation matrix of random variables.

- Weights_error (α_u)
  Weights associated with error in distribution, moments and correlation. Size of array is fixed for this variable.
  Default: weights_errors = [1, 0.2, 0]

- Weights_distribution (w_F)
  An array of dimension N×d, where 'N' is number of samples and 'd' is number of random variables. It contain weights associated with different samples.
  Default: weights_distribution = N×d dimensional array with all elements equal to 1.

- Default_weights_distribution
  Default weights associated with samples for every random variable is 1. This list modify the weights associated with all samples of each random variable.
  Example: default_weights_distribution = [2, 1, 3] will modify $w_F = \begin{bmatrix} 2w_F(:,1) & w_F(:,2) & 3w_F(:,3) \end{bmatrix}$

- Weights_moments (w_μ)
  An array of dimension 2xd, where 'd' is number of random variables. It contain weights associated with moments.
  Default: weights_moments = Square of reciprocal of elements of moments.

- Default_weights_moments
  This list modify the weights associated with moments of each random variable.
  Example: default_weights_moments = [2, 1, 3] will modify $w_\mu = \begin{bmatrix} 2w_\mu(1) & w_\mu(2) & 3w_\mu(3) \end{bmatrix}$

- Weights_correlation (w_r)
  An array of dimension d×d, where 'd' is number of random variables. It contain weights associated with correlation of random variables.
  Default: weights_correlation = d×d dimensional array with all elements equal to 1.

- Default_weights_correlation
  This list modify the weights associated with each element of correlation matrix.

- Correlation (r)
  Correlation matrix between random variables.

Bibliography

Grigoriu, M. (2009). Reduced order models for random functions. application to stochastic problems. *Applied Mathematical Modelling*, 33(1):161 – 175.