MBA 753 : Causal Inference Methods for Business Analytics

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Agenda

- Interaction effects
- Types of experiments
- Potential outcomes framework

Interaction Effect and its Interpretation

Interaction Effects

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \varepsilon$$

- In a regression with interaction terms, the main terms should always be included
- Interaction tells us about regression slope differences
- An interaction regression weight tells the direction and extent of change in the slope of one Y- X_1 regression line for each 1-unit increase in the X_2, holding all the other variables in the model constant at 0.

Interpretation

```
Call:
lm(formula = sales ~ youtube * facebook, data = train.data)

Residuals:
   Min     1Q Median     3Q    Max 
-7.438 -0.482  0.231  0.748  1.860 

Coefficients:
                 Estimate Std. Error t value Pr(>|t|)    
(Intercept)      7.90e+00   3.28e-01   24.06   <2e-16 ***
youtube          1.95e-02   1.64e-03   11.90   <2e-16 ***
facebook         2.96e-02   9.83e-03    3.01    0.003 ** 
youtube:facebook 9.12e-04   4.84e-05   18.86   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.18 on 158 degrees of freedom
Multiple R-squared:  0.966,  Adjusted R-squared:  0.966 
F-statistic: 1.51e+03 on 3 and 158 DF,  p-value: <2e-16
```

Causality and Empirical Research

Experiments and Causation

- Cause, Effect, and Causal Relationship
  - Causal relationship exists if
    - Cause preceded the effect
    - Cause was related to the effect – variation in cause related to variation in effect
    - No other plausible alternative explanation

- Experiments can help study causal descriptions and explanations
  - Experiments: a study in which an intervention is deliberately introduced to observe its effects

Types of Experiments

- Randomized experiment: Units are assigned to receive treatment or an alternative condition through a random process
- Quasi experiment: Units are not assigned to conditions randomly
  - Cause is manipulable and occurs before the effect
- Natural experiment: Cause cannot be manipulated
  - Naturally occurring contrasts between treatment and a comparison condition
- Correlational study: Observational study that records size and direction of relationships among variables
  - Structural features of experiments are missing

Regression to Causality

- Regression helps in understanding associations among variables of interest
  - Conditional Expectation Function: $E(Y|X = x) = \beta_0 + \beta_1 X_1 + \ \dots + \beta_p X_p$
  - Regression is causal if CEF is causal
  - CEF is causal when it describes the differences in average **potential outcomes** for a population

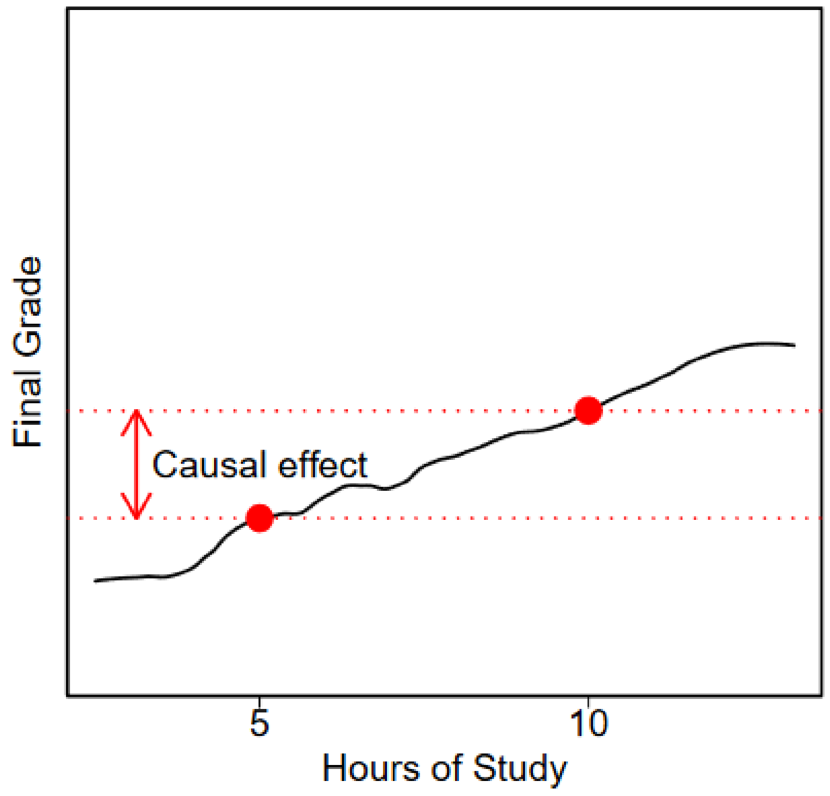
We think of a cause as something that makes a difference, and the difference it makes must be a difference from what would have happened without it.

– David Lewis, Causation, 1973

Potential Outcomes Model

Counterfactuals

- X, is understood to cause Y, if the value for Y would have been different for a different value of X
- Example: Imagine we knew the grade a particular individual would receive for different amounts of study time:
  - Each point on the line represents a potential outcome (the hypothetical outcome associated with each value of our causal factor)
  - Causal effects are defined in terms of **potential outcomes**



Source: Mix Tape by Scott Cunningham

Potential Outcome

- Potential outcome: difference in the outcomes between the two states of the world
  - Actual state where the person did something
  - Counterfactual state where the person did something else

- **Causal inference:** The process of drawing conclusions about features/properties of the full set of **potential outcomes** on the basis of some **observed outcomes.**

Notation and Terminology

- Treatment: Causal variable of interest
  - Defined for binary case, but we can (and will) generalize to continuous treatments

$$D_i\text{: }indicator\ of\ treatment$$

$$D_i = \begin{cases} 1\ if\ unit\ i\ received\ the\ treatment \\ 0 \qquad\qquad\qquad\qquad\qquad otherwise \end{cases}$$

- Outcome: Y_i: Observed outcome variable of interest for unit i

Notation and Terminology

- Potential Outcomes
  - potential outcomes are fixed attributes for each i and represent the outcome that would be observed hypothetically if i were treated/untreated

$$Y_i = \begin{cases} Y_{1i} \; if \; D_i = 1 \\ Y_{01} if \; D_i = 0 \end{cases}$$

$$Y_i = D_i Y_{1i} + (1 - D_i) Y_{0i}$$

  - $Y_{0i} \; and \; Y_{1i}$ are potential outcomes (counterfactuals)
  - Only one outcome is observed, the other is counterfactual

Notation and Terminology

- Causal Effect
  - For each unit i, the causal effect of the treatment on the outcome is defined as the difference between its two potential outcomes:
  - τ_i is the difference between two hypothetical states of the world
    - One where i receives the treatment
    - One where i does not receive the treatment
  - Fundamental problem of Causal Inference: We cannot observe both potential outcomes (Y_{1i}, Y_{0i}) for the same unit i
    - **How do we calculate $\boldsymbol{\tau_i}$?**

Characteristics

- Stable Unit Treatment Value Assumption (SUTVA): Conditional independence assumption
  - Causal variable of interest has to be independent of potential outcomes so that groups are truly comparable
  - Potential outcomes for unit i are unaffected by treatment assignment for unit j
- Average Treatment Effect (ATE): $E(\tau) = E(Y_{1i} - Y_{0i}) = E(Y_{1i}) - E(Y_{0i})$
  - Both potential outcomes are required to calculate ATE
  - ATE is only estimable

ATE Illustration

Imagine a population with 4 units, where we observe both potential outcomes for each unit:

i	Y_i	D_i	Y_{1i}	Y_{0i}	τ_i
1	5	1	5	2	3
2	2	1	2	1	1
3	0	0	1	0	1
4	1	0	1	1	0

$$\begin{aligned} \tau_{\text{ATE}} &\equiv E[Y_{1i} - Y_{0i}] \\ &= \frac{1}{N}\sum_{i=1}^{N}(Y_{1i} - Y_{0i}) \\ &= E[Y_{1i}] - E[Y_{0i}] \end{aligned}$$

$$\textbf{ATE} = \frac{3+1+1+0}{4} = \frac{5+2+1+1}{4} - \frac{2+1+0+1}{4} = 1.25$$

ATE Estimation

- Intuition: Make comparison across units using Y_i
  - Compare the average observed outcome under treatment to the average observed outcome under control
  - True ATE = 1.25
  - Difference in means =
  - In this example at least, difference in means ≠ ATE

i	Y_i	D_i	Y_{1i}	Y_{0i}	τ_i
1	5	1	5	?	?
2	2	1	2	?	?
3	0	0	?	0	?
4	1	0	?	1	?

Terminologies

- Average treatment effect on the treated (ATT)

$$E(\tau_i|D_i = 1) = E(Y_{1i} - Y_{0i}|D_i = 1)$$

- Average treatment effect on the untreated (ATU)

$$E(\tau_i|D_i = 0) = E(Y_{1i} - Y_{0i}|D_i = 0)$$

Estimation of ATE

- For a given sample, one obvious estimator of the ATE is the difference in group means (DIGM)

$$\text{Let } D_i = 1 \ \forall \ i \ \in \{1, \dots, m\} \ \& \ D_i = 0 \ \forall \ i \ \in \{m+1, \dots, n\}$$

$$DIGM = \frac{1}{m} \sum_{i=1}^{m} Y_i - \frac{1}{n-m} \sum_{i=m+1}^{n} Y_i$$

- Is DIGM an unbiased estimator of ATE?

Estimation of ATE

$$DIGM = \frac{1}{m}\sum_{i=1}^{m} Y_i - \frac{1}{n-m}\sum_{i=m+1}^{n} Y_i$$

We know $Y_i = D_i Y_{1i} + (1-D_i)Y_{0i}$ & $\tau_i = Y_{1i} - Y_{0i}$

$$\therefore \; DIGM = \frac{1}{m}\sum_{i=1}^{m}(\tau_i + Y_{0i}) - \frac{1}{n-m}\sum_{i=m+1}^{n} Y_{0i}$$

$$= \frac{1}{m}\sum_{i=1}^{m}\tau_i + \frac{1}{m}\sum_{i=1}^{m} Y_{0i} - \frac{1}{n-m}\sum_{i=m+1}^{n} Y_{0i}$$

$$= E(\tau_i \mid D_i = 1) + \left[E(Y_{0i} \mid D_i = 1) - E(Y_{0i} \mid D_i = 0)\right]$$

$$DIGM = ATT + \text{Selection Bias}$$

Estimation of ATE

i	Y_i	D_i	Y_{1i}	Y_{0i}	τ_i
1	5	1	5	2	3
2	2	1	2	1	1
3	0	0	1	0	1
4	1	0	1	1	0

$$\tau_{\textsf{ATE}} = \frac{3+1+1+0}{4} = 1.25$$

$$\tau_{\textsf{ATT}} = \frac{3+1}{2} = 2$$

$$\textsf{Selection bias} = \frac{2+1}{2} - \frac{0+1}{2} = 1$$

$$\tau_{\textsf{ATT}} + \textsf{Bias} = 2+1 = 3 = \textsf{DIGM}$$

Estimation of ATE

- DIGM is the unbiased estimator of ATE if
  - $\tau_{ATT} = \tau_{ATE}$
  - There is no selection bias i.e. $E(Y_{0i}|D_i = 1) = E(Y_{0i}|D_i = 0)$

- Selection Bias
  - Selection into treatment is often associated with potential outcomes
  - Selection bias can be positive or negative
  - (In general) Do not believe causal arguments based on simple differences between groups!

Solution for Selection Bias

- We need to know more about counterfactuals that we do not observe
    - Make assumptions about how certain units come to be “selected” for treatment

- Treatment Assignment: mechanism that determines which units are selected for treatment
    - **Random assignment**
    - Selection on observable characteristics - matching
    - Selection on unobservable characteristics – DID, RDD, IV

Recap

Summary

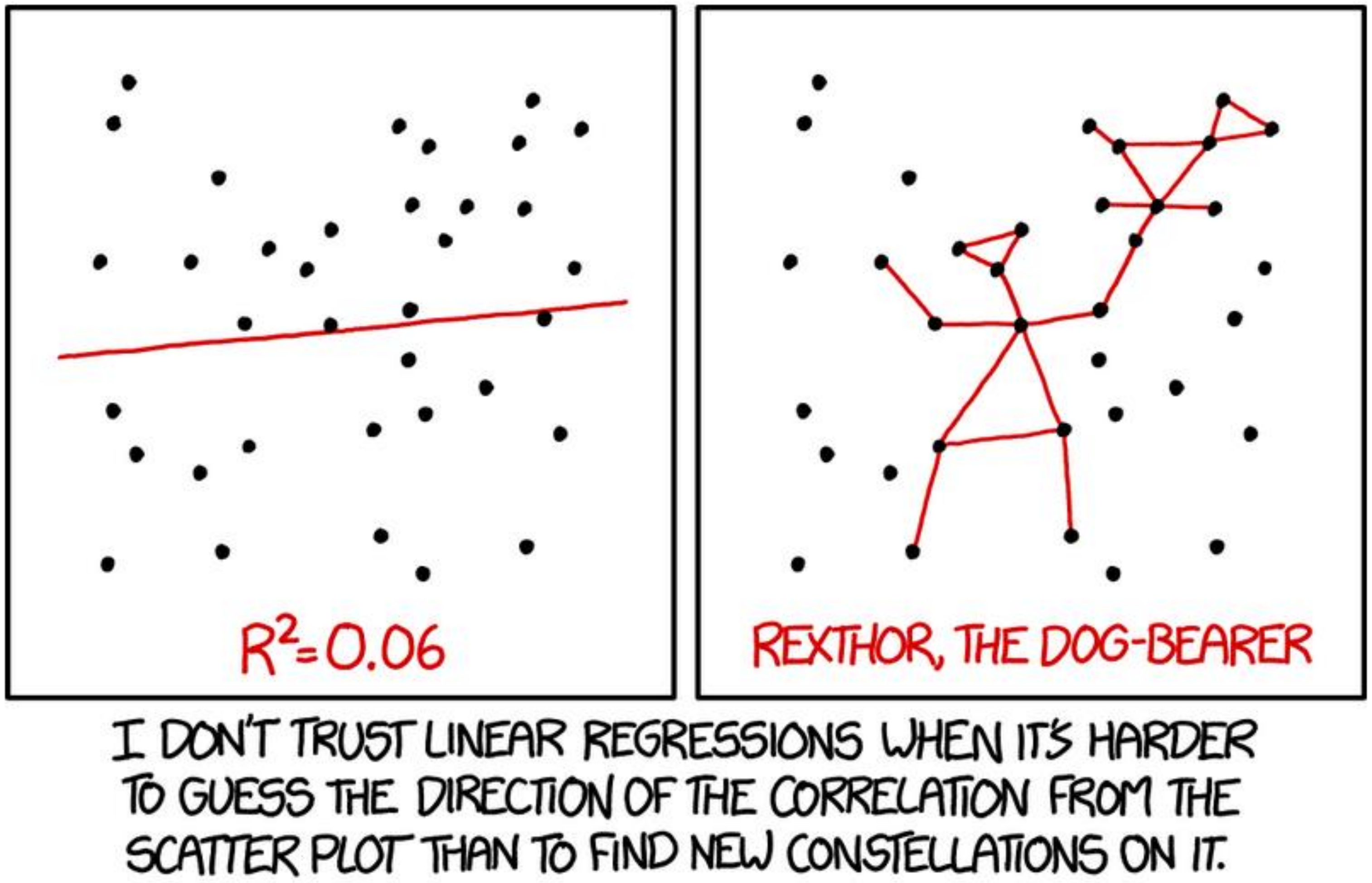
- If interaction effects are significant, the partials effects are not interpreted.
- Causality is defined by potential outcomes, not by observed outcomes
- The difference in means is only an unbiased estimator for the ATE when there is no selection bias

- Objectives achieved:
  - Can interpret dummy regressors and interaction effects
  - Can understand potential outcomes framework and define causal effect
  - Can estimate ATE

References

- Scott Cunningham, Causal Inference: The Mix Tape, Yale University Press.
- Angrist, J. D., & Pischke, J. S. (2009). *Mostly harmless econometrics: An empiricist's companion*. Princeton university press.
- Pearl, J., Glymour, M., & Jewell, N. P. (2016). *Causal inference in statistics: A primer*. John Wiley & Sons.

Thank You ☺



R²=0.06
REXTHOR, THE DOG-BEARER
I DON'T TRUST LINEAR REGRESSIONS WHEN IT'S HARDER TO GUESS THE DIRECTION OF THE CORRELATION FROM THE SCATTER PLOT THAN TO FIND NEW CONSTELLATIONS ON IT.