

MBA 753 : Causal Inference Methods for Business Analytics

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Agenda

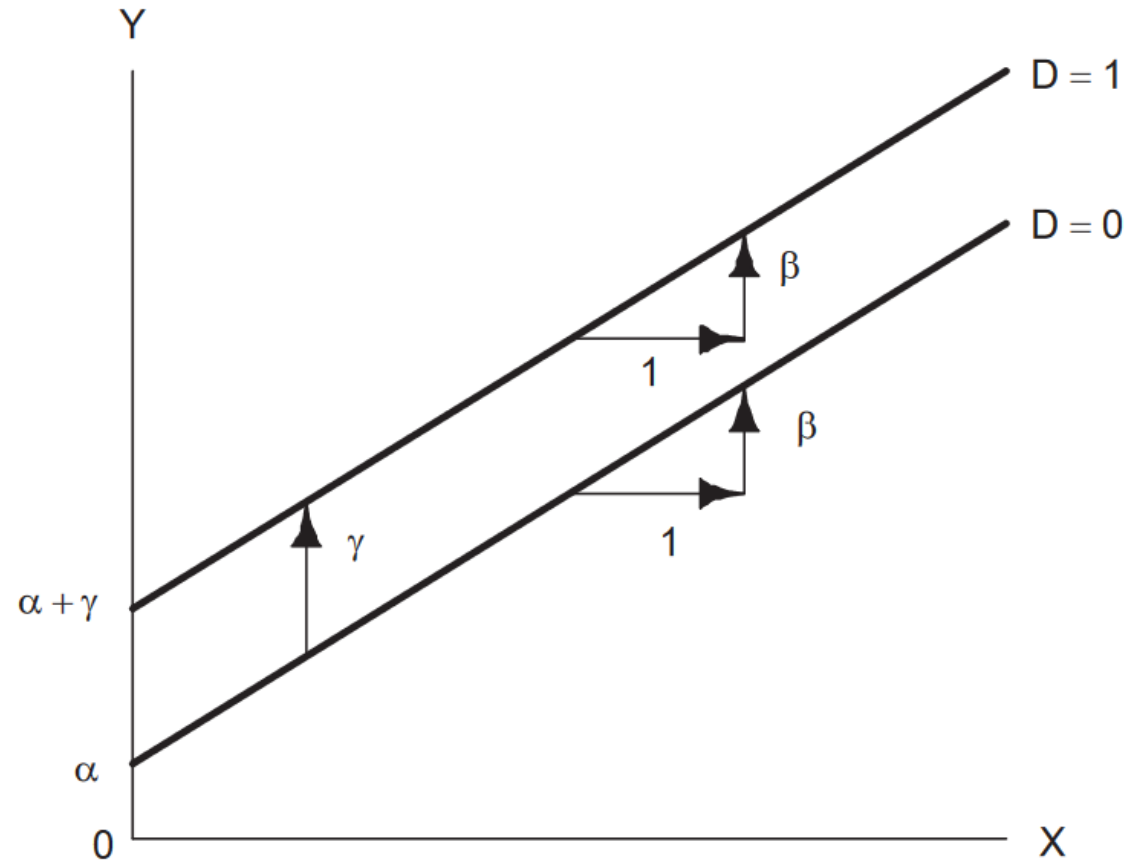
- Categorical predictors and its interpretation
- Interaction effects and its interpretation
- Experiments

Categorical Predictors

$$\gamma = \beta_0 + \beta_1 X + \varepsilon$$

Categorical Predictor Variables

- Categorical independent variables can be incorporated into a regression model by converting them into 0/1 (“dummy”) variables
 - Involves categorical X variable with two levels
 - Assumes only intercept is different
 - Slopes are constant across categories



3 cat \rightarrow 2 DV

Dummy Regressors

- Dummy regressors are easily extended to explanatory variables with more than two categories
 - A variable with m categories has $m - 1$ regressors
 - As with the two-category case, one of the categories is a reference group (coded 0 for all dummy regressors)

	D_1	D_2
Blue Collar	1	0
Professional	0	1
White Collar	0	0

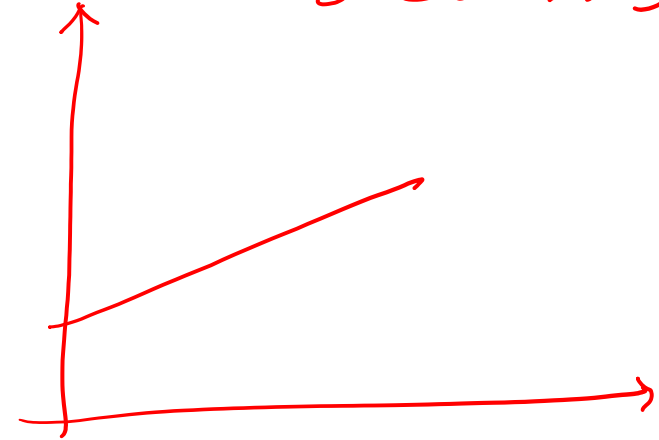
reference \rightarrow
group

2 cat \rightarrow 1 dummy var

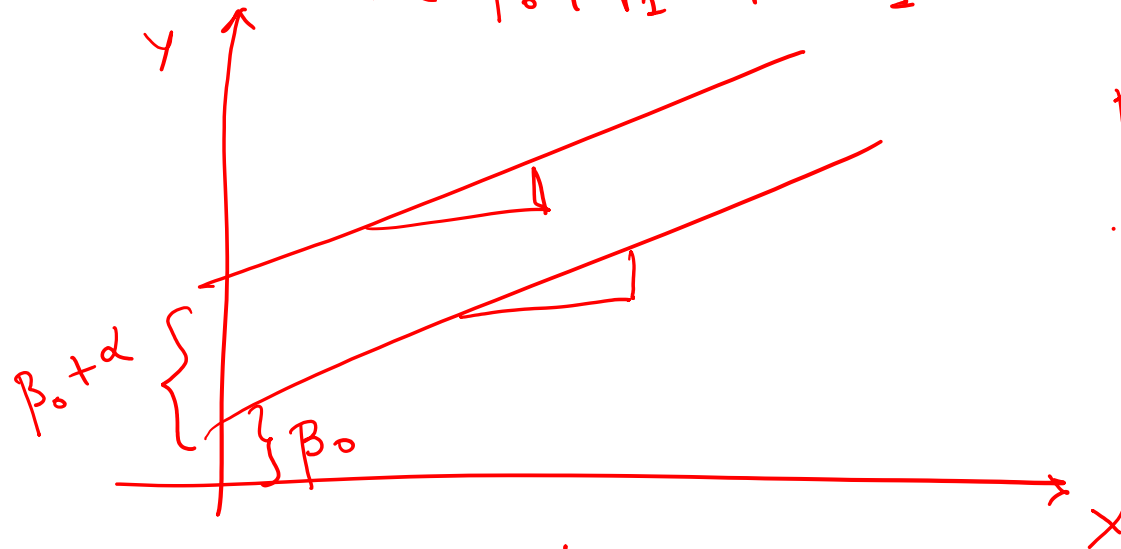
$X \rightarrow D_1 \quad \underline{Y} = \beta_0 + \beta_1 X + \varepsilon$

$\underline{Y} = \beta_0 + \cancel{\beta_1} \alpha D_1 + \varepsilon$

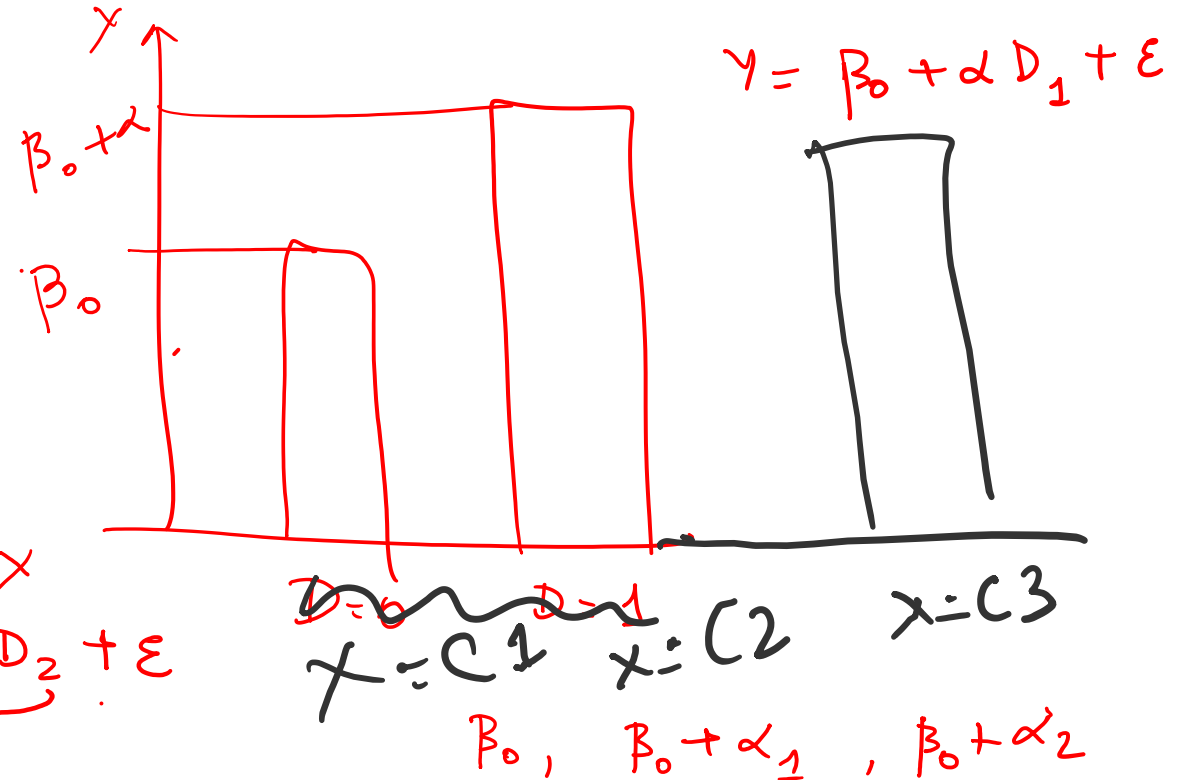
3 cat \rightarrow 2 DV



$Y = \beta_0 + \beta_1 X + \alpha D_1 + \varepsilon$



$\rightarrow Y = \beta_0 + \underbrace{\alpha_1 D_1 + \alpha_2 D_2}_{\text{}} + \varepsilon$



Dummy Regression Model

$$Y_i = \alpha + \beta X_i + \gamma_1 D_{i1} + \gamma_2 D_{i2} + \varepsilon_i$$

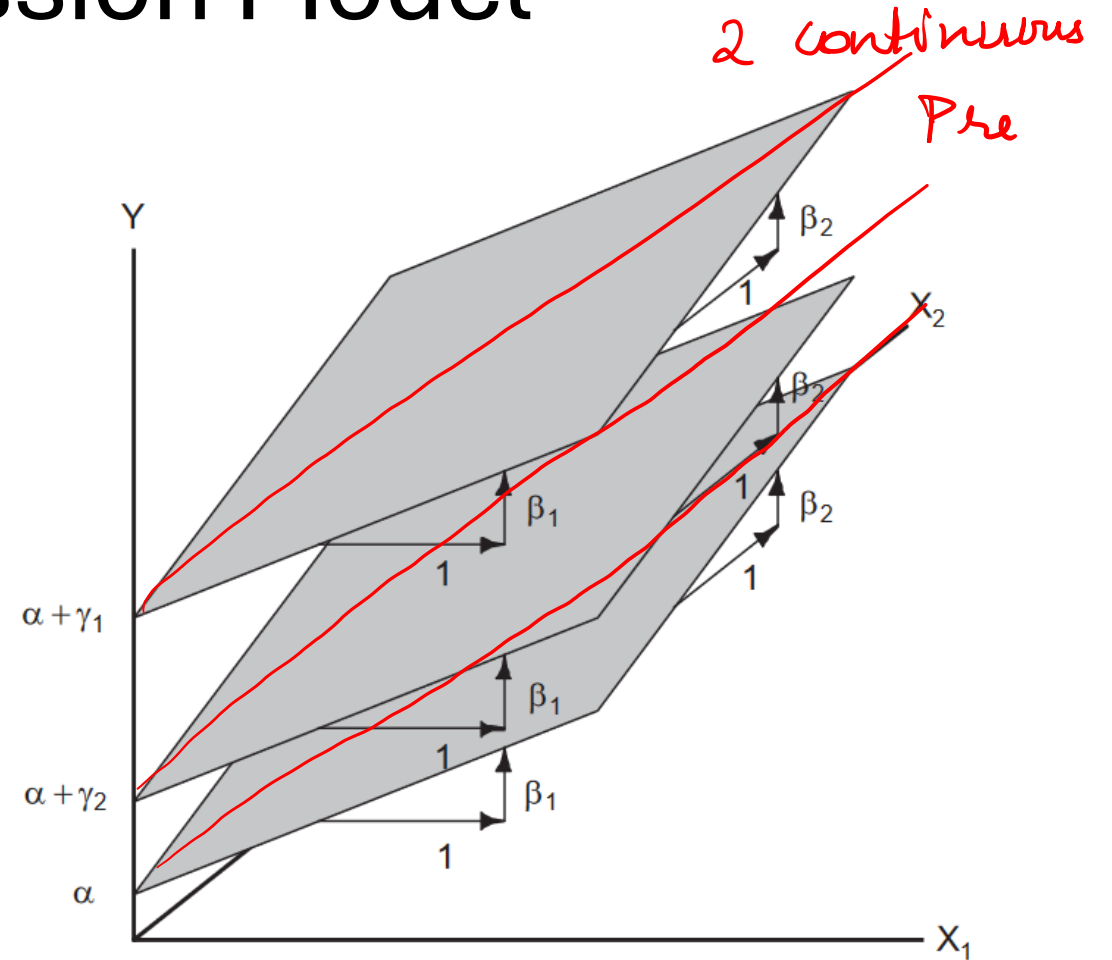
Handwritten notes: "1 cat" above $\gamma_1 D_{i1}$ and "2 DR" below $\gamma_2 D_{i2}$. The coefficient β is circled in red.

- This gives three parallel regression lines

Blue Collar: $Y_i = (\alpha + \gamma_1) + \beta X_i + \varepsilon_i$

Professional: $Y_i = (\alpha + \gamma_2) + \beta X_i + \varepsilon_i$

White Collar: $Y_i = \alpha + \beta X_i + \varepsilon_i$



Interaction Effect and its Interpretation

Interaction Effect

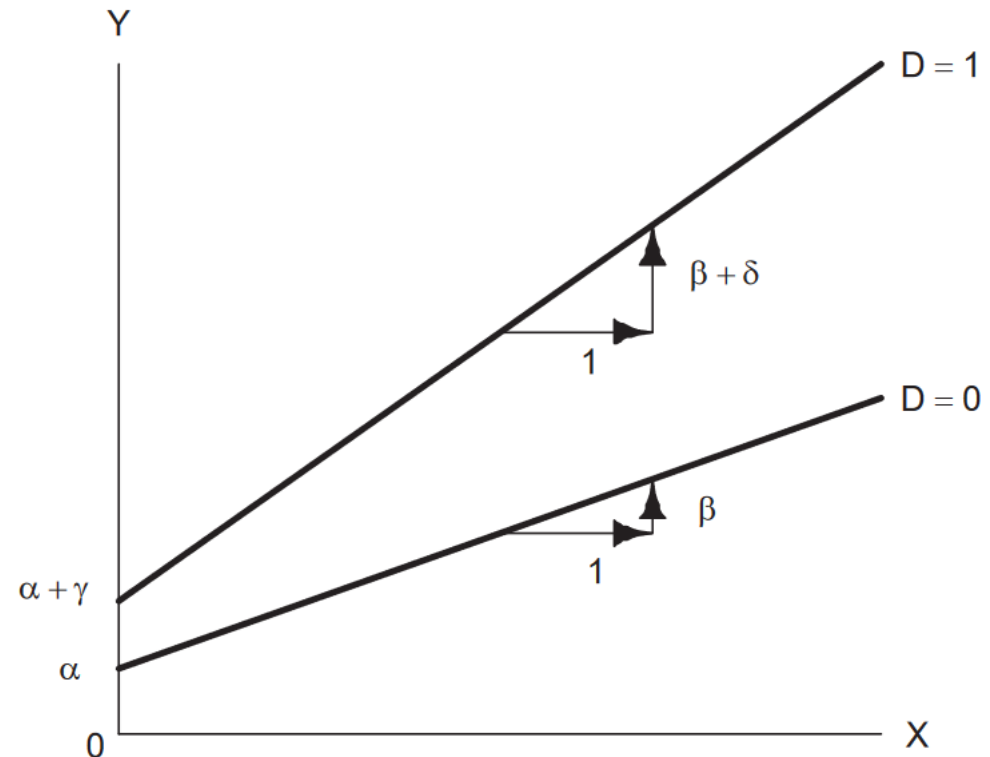
2 Predictors Age & gender

- Two predictor variables “interact” when the partial effect of one variable depends on the value of another variable

- For example, testing whether age effects are different for men (coded 1) and women (coded 0) ← *reference*
- Separate models cannot test for differences among groups
- Testing for differences in slope

$$Y_i = \alpha + \beta X_i + \gamma D_i + \delta(X_i D_i) + \varepsilon_i$$

$$\text{income}_i = \alpha + \beta \text{age}_i + \gamma \text{men}_i + \delta(\text{age}_i \times \text{men}_i) + \varepsilon_i$$



Interaction Interpretation

- When the interaction effect is significant

$$y = \alpha + \beta x + \gamma D + \delta (x \cdot D) + \epsilon$$

- The unique partial effects (for example that of age and gender) are no longer interpretable just by themselves

$$\beta_1 \neq \beta_2 \quad \beta \neq \gamma$$

- Omitting interaction effects can lead to erroneous conclusions

Causality and Empirical Research

Experiments and Causation

- Cause, Effect, and Causal Relationship
 - Causal relationship exists if
 - Cause preceded the effect
 - Cause was related to the effect – variation in cause related to variation in effect
 - No other plausible alternative explanation
- Experiments can help study causal descriptions and explanations
 - Experiments: a study in which an intervention is deliberately introduced to observe its effects

Types of Experiments

- Randomized experiment: Units are assigned to receive treatment or an alternative condition through a random process
- Quasi experiment: Units are not assigned to conditions randomly
 - Cause is manipulable and occurs before the effect
- Natural experiment: Cause cannot be manipulated
 - Naturally occurring contrasts between treatment and a comparison condition
- Correlational study: Observational study that records size and direction of relationships among variables
 - Structural features of experiments are missing

Regression to Causality

- Regression helps in understanding associations among variables of interest
 - Conditional Expectation Function: $E(Y|X = x) = \beta_0 + \beta_1 X_1 + \dots + \beta_p X_p$
 - Regression is causal if CEF is causal
 - CEF is causal when it describes the differences in average **potential outcomes** for a population

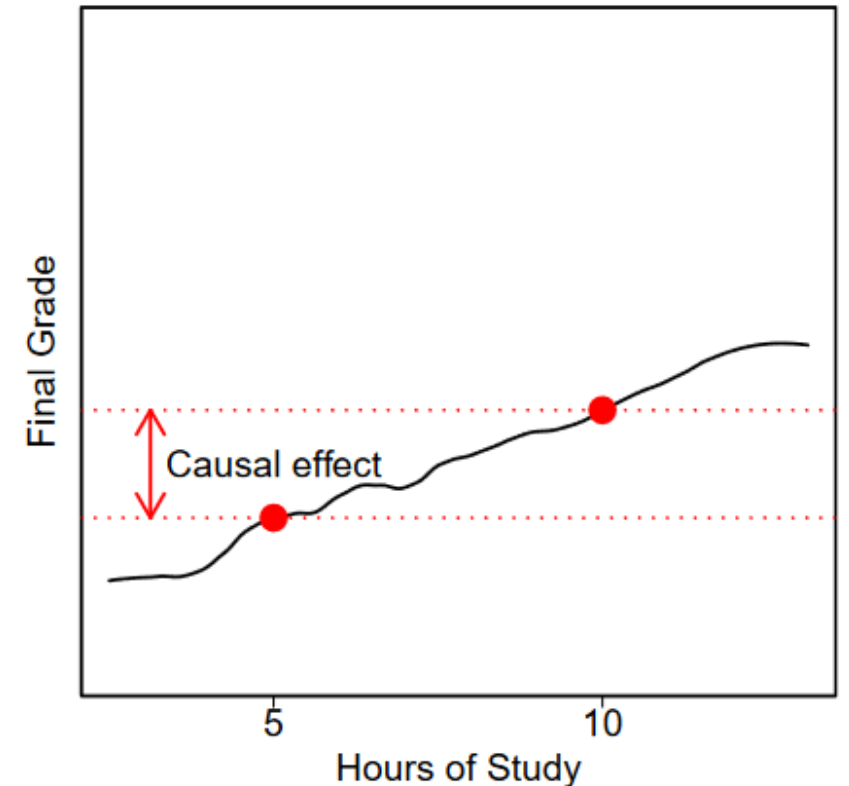
We think of a cause as something that makes a difference, and the difference it makes must be a difference from what would have happened without it.

– David Lewis, Causation, 1973

Potential Outcomes Model

Counterfactuals

- X, is understood to cause Y, if the value for Y would have been different for a different value of X
- Example: Imagine we knew the grade a particular individual would receive for different amounts of study time:
 - Each point on the line represents a potential outcome (the hypothetical outcome associated with each value of our causal factor)
 - Causal effects are defined in terms of **potential outcomes**



Source: Mix Tape by Scott Cunningham

Potential Outcome

- Potential outcome: difference in the outcomes between the two states of the world
 - Actual state where the person did something
 - Counterfactual state where the person did something else
- **Causal inference:** The process of drawing conclusions about features/properties of the full set of **potential outcomes** on the basis of some **observed outcomes**.

Notation and Terminology

- Treatment: Causal variable of interest
 - Defined for binary case, but we can (and will) generalize to continuous treatments

D_i : indicator of treatment

$$D_i = \begin{cases} 1 & \text{if unit } i \text{ received the treatment} \\ 0 & \text{otherwise} \end{cases}$$

- Outcome: Y_i : Observed outcome variable of interest for unit i

Notation and Terminology

- Potential Outcomes

- potential outcomes are fixed attributes for each i and represent the outcome that would be observed hypothetically if i were treated/untreated

$$Y_i = \begin{cases} Y_{1i} & \text{if } D_i = 1 \\ Y_{0i} & \text{if } D_i = 0 \end{cases}$$

$$Y_i = D_i Y_{1i} + (1 - D_i) Y_{0i}$$

- Y_{0i} and Y_{1i} are potential outcomes (counterfactuals)
- Only one outcome is observed, the other is counterfactual

Notation and Terminology

- Causal Effect
 - For each unit i , the causal effect of the treatment on the outcome is defined as the difference between its two potential outcomes:
 - τ_i is the difference between two hypothetical states of the world
 - One where i receives the treatment
 - One where i does not receive the treatment
 - Fundamental problem of Causal Inference: We cannot observe both potential outcomes (Y_{1i}, Y_{0i}) for the same unit i
 - **How do we calculate τ_i ?**

Recap

Summary

- MLR – fitting the best regression space
- Partial effects are estimated assuming *ceteris paribus*
- A categorical predictor with m groups will have $m-1$ regressors
- Interaction effects - when effect of one predictor depends on the values of the other
- Objectives achieved:
 - Can understand and interpret “effects” in a MLR model with dummy variables and interactions
 - Can identify different types of experiments

References

- Stock, J. H., Watson, M. W., & Wooldridge, J. M. Introductory Econometrics: A Modern Approach (4th Edition International).
- Neter J, Kutner MH, Nachtsheim CJ, Wasserman W. Applied linear statistical models. McGraw Hill Education.
- Scott Cunningham, Causal Inference: The Mix Tape, Yale University Press.
- Angrist, J. D., & Pischke, J. S. (2009). *Mostly harmless econometrics: An empiricist's companion*. Princeton university press.

Thank You 😊

