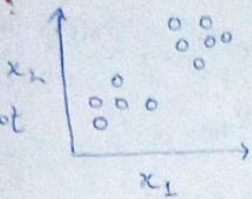


# Week\_3 Machine learning

## Unsupervised Learning

clustering:

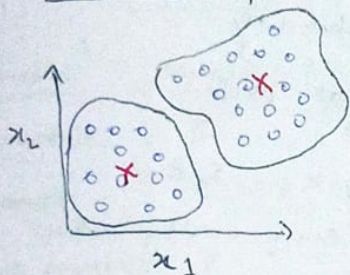
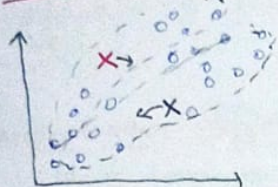
labels are not present.



Training set:  $\{x^{(1)}, x^{(2)}, \dots\}$

- grouping similar news
- DNA analysis
- Astronomical data analysis

### K-means Algorithm



- Take a random guess to where are center of cluster **cluster centroid**
- Recompute the centroid  
It will compute the average location

$\Rightarrow$  Randomly initialize  $K$  cluster centroid  $u_1, u_2, \dots, u_K$

Repeat {  
# Assign points to cluster centroid  
for  $i = 1$  to  $m$

$c^{(i)}$  = index (from 1 to  $K$ ) of cluster centroid closest to  $x^{(i)}$

$\rightarrow \min_k \|x^{(i)} - u_k\|^2$

# Move cluster centroid

for  $k = 1$  to  $K$

$u_k := \text{average (mean) of points assigned to cluster } k$

• Optimization:

Cost function  $\rightarrow$  minimizing it.

$$J(c^{(1)}, \dots, c^{(m)}, u_1, \dots, u_K) = \frac{1}{m} \sum_{i=1}^m \|x^{(i)} - u_{c(i)}\|^2$$

• choosing the value of  $K$

Evaluate K-means based on how well it performs on the later purpose



## • Anomaly detection

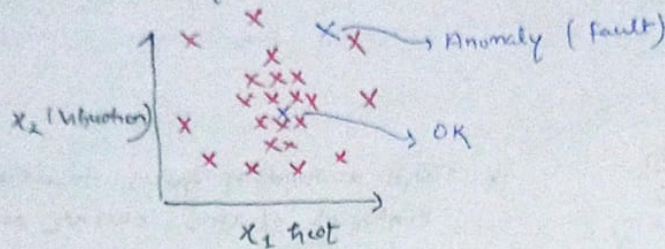
Ex. Aircraft engine features:

$x_1$  = heat generated

$x_2$  = vibration intensity

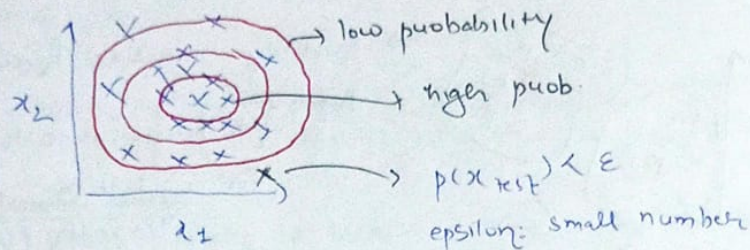
Dataset:  $\{x^{(1)}, x^{(2)}, \dots, x^{(m)}\}$

New engine:  $x_{\text{test}}$



## • Density estimation

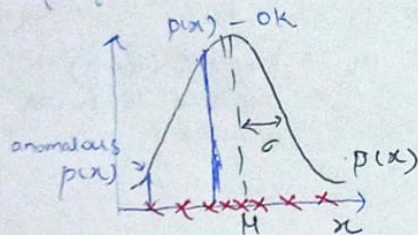
probability of  $x$  being seen in dataset



## • Gaussian (Normal) distribution

Say  $x$  is a number

probability of  $x$  is determined by a Gaussian with mean  $\mu$ , variance  $\sigma^2$



$\sigma$  standard deviation

$\sigma^2$  variance

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\sigma^2 = \frac{1}{m} \sum_{i=1}^m (x^{(i)} - \mu)^2$$

$$\mu = \frac{1}{m} \sum_{i=1}^m x^{(i)}$$

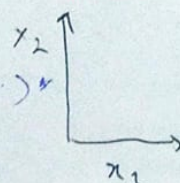
## Algorithm:

Training set:  $\{\vec{x}^{(1)}, \vec{x}^{(2)}, \dots, \vec{x}^{(m)}\}$

Each example  $\vec{x}^{(i)}$  has  $n$  features

$$p(\vec{x}) = p(x_1; \mu_1, \sigma_1^2) * p(x_2; \mu_2, \sigma_2^2) * p(x_3; \dots) * \dots * p(x_n)$$

$$= \prod_{j=1}^n p(x_j; \mu_j, \sigma_j^2)$$



$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$



### Anomaly detection algorithm

1. choose  $n$  features  $x_i$  that you think might be indicative of anomalous ex.
2. fit parameters  $\mu_1, \dots, \mu_n, \sigma_1^2, \dots, \sigma_n^2$

$$\mu_j = \frac{1}{m} \sum_{i=1}^m x_{ij}^{(1)} \quad \sigma_j^2 = \frac{1}{m} \sum_{i=1}^m (x_{ij}^{(1)} - \mu_j)^2$$

3. Give new example  $x$ , compute  $p(x)$

$$p(x) = \prod_{j=1}^n p(x_j; \mu_j, \sigma_j^2)$$

Anomaly if  $p(x) < \epsilon$

### Recommender System:

- Movie Recommendation

| Movies           | Rating    |         |           |          | features           |                   |
|------------------|-----------|---------|-----------|----------|--------------------|-------------------|
|                  | Alice (1) | Bob (2) | Carol (3) | Dave (4) | $x_1$<br>(romance) | $x_2$<br>(action) |
| Love at last     | 5         | 5       | 0         | 0        | 0.9                | 0                 |
| Romance ...      | 5         | ?       | ?         | 0        | 1.0                | 0.01              |
| Cute puppies ... | ?         | 4       | 0         | ?        | 0.99               | 0                 |
| Nonstop Cars ... | 0         | 0       | 5         | 4        | 0.1                | 1.0               |
| Shaved Vs karate | 0         | 0       | 5         | ?        | 0                  | 0.9               |

for user 3: Predict rating for movie  $i$  as

$$w^{(3)} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \quad b^{(3)} = 0 \quad x^{(3)} = \begin{bmatrix} 0.99 \\ 0 \end{bmatrix} \quad \leftarrow \text{just linear regression}$$

$$x^{(3)} = \begin{bmatrix} 0.9 \\ 0 \end{bmatrix}$$

$$x^{(3)} = \begin{bmatrix} 0.99 \\ 0 \end{bmatrix}$$

$$w^{(3)} \cdot x^{(3)} + b^{(3)} = 4.95 \quad \text{approx prediction}$$

Cost function.

$$\min J(w^{(j)}, b^{(j)}) \Rightarrow \frac{1}{2m(j)} \sum_{i: x^{(i,j)}=1} (w^{(j)} \cdot x^{(i)} + b^{(j)} - y^{(i,j)})^2 + \frac{\lambda}{2m(j)} \sum_{k=1}^n (w_k^{(j)})^2$$

regularization term

to avoid overfitting

# • Collaborative filtering Vs Content-based filtering

Recommend items to you based on rating of users who gave similar rating as you

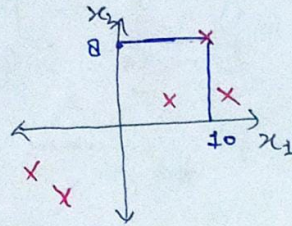
Recommend items to you based on features of user and item to find good match

## • PCA algorithm

coordinates  $x_1 = 10$

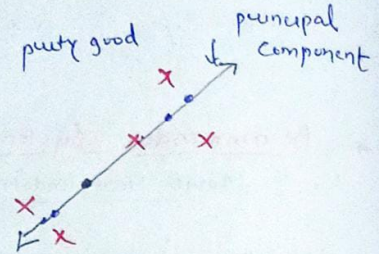
$x_2 = 8$

Can we choose a different axis?

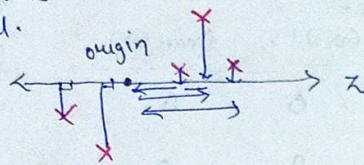


previous features

• Its pretty good



• Its pretty not good choice as our data is more spread.





# Reinforcement learning

position of helicopter  $\longrightarrow$  How to move control sticks

state  $s$   $\longrightarrow$  action  $a$

$x$   $\longrightarrow$   $y$

$\uparrow$   
reward function

Application:

- controlling robots
- factory optimization
- financial (stock) trading
- Playing game

positive reward: helicopter flying well +1  
negative reward: helicopter flying poorly -1000

ex

|       |   |   |     |   |      |
|-------|---|---|-----|---|------|
| \$100 | 0 | 0 | $x$ | 0 | \$40 |
| 1     | 2 | 3 | 4   | 5 | 6    |

Return:  $\Rightarrow 0 + (0.9)0 + (0.9)^2 0 + (0.9)^3 100 = 72.9$

$\Rightarrow R_1 + \gamma R_2 + \gamma^2 R_3 \dots$  until terminal state

Discount factor  $\uparrow$   
 $\gamma = 0.9 \quad 0.99 \quad 0.999$

$\gamma = 0.5$

Return:  $0 + (0.5)0 + (0.5)^2 0 + (0.5)^3 100 = 12.5$

Example of return

|     |              |              |              |              |    |
|-----|--------------|--------------|--------------|--------------|----|
| 100 | 50           | 25           | 12.5         | 6.25         | 40 |
| 100 | $\leftarrow$ | $\leftarrow$ | $\leftarrow$ | $\leftarrow$ | 40 |
|     | 0            | 0            | 0            | 0            |    |

$\gamma = 0.5$

The return depends on the action you take

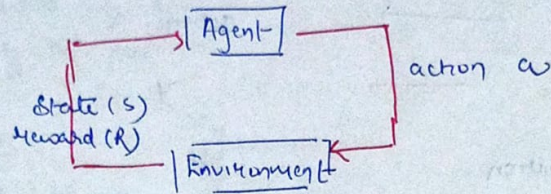
|     |               |               |               |               |     |
|-----|---------------|---------------|---------------|---------------|-----|
| 100 | 2.5           | 5             | 10            | 20            | -40 |
| 100 | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ | 40  |
|     | 0             | 0             | 0             | 0             |     |

state  $(s)$   $\xrightarrow[\pi]{\text{policy}}$  action  $(a)$

|     |              |              |              |               |    |
|-----|--------------|--------------|--------------|---------------|----|
| 100 | 50           | 25           | 12.5         | 40            | 40 |
| 100 | $\leftarrow$ | $\leftarrow$ | $\leftarrow$ | $\rightarrow$ | 40 |
|     | 0            | 0            | 0            | 0             |    |

## Markov Decision Process (MDP)

- This state future depends on where you are present now. not how you reach.

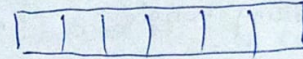


## # Bellman Equation

$Q(s, a) \Rightarrow$  Return if you

- start in state  $s$ .
- take action  $a$  (once)
- then behave optimally after that.

$$R(1)=100 \quad R(2)=0 \quad \dots \quad R(6)=40$$



$s$ : current state

$a$ : current action

$s'$ : state you get to after taking action  $a$

$a'$ : action that you take in state  $s'$

$R(s)$  = Reward of current state

$$Q(s, a) = R(s) + \gamma \max_{a'} Q(s', a')$$

discount factor

$$\left\{ Q(s, a) = R_1 + \gamma [R_2 + \gamma R_3 + \gamma^2 R_4 + \dots] \right\}$$