Naive Bayes

Naïve Bayes is a **probabilistic classifier** based on **Bayes' Theorem**. It assumes that features are **conditionally independent**, which is why it is called "naïve." Despite this assumption, it performs well in real-world applications like **spam detection, sentiment analysis, and document classification**.

🔍 Types of Naïve Bayes Classifiers

1. **Gaussian Naïve Bayes**: Assumes features follow a normal distribution.
2. **Multinomial Naïve Bayes**: Used for **text classification** (e.g., spam filtering, sentiment analysis).
3. **Bernoulli Naïve Bayes**: Suitable for **binary feature values** (e.g., word presence/absence in text).

Day-7 Naive Bayes:

1. The Naive Bayes classifier is a supervised machine learning algorithm that is used for classification task such as text Classification. They use principles of probability to perform classification tasks.

Naive Bayes:

feature vector x?

dictionary
— a
aadv...
⋮
buy....
⋮
zyxnergy

↑ 10,000 = n ↓ use these words as feature set

$$X = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \qquad x \in \{0,1\}^n$$

$x_i = \mathbb{1}\{(\text{word } i \text{ appears in e-mail})\}$

Model of $p(x/y)$, $p(y)$

$2^{10,000}$ possible value of x

$2^{10,000} - 1$

we are going to Assume

x_i's are conditionally independent given y.

$$P(x_1 \dots x_{10,000} | y) = p(x_1/y)\, p(x_2|x_1, y)\, p(x_3|x_1, x_2, y) \dots$$

assume;

$$= p(x_1/y)\, p(x_2/y)\, p(x_3/y) \dots\dots\dots p(x_{10,000}/y)$$

sometimes called naive assumption

Not true in Mathematical sense

$$= \prod_{i=1}^{n} p(x_i|y)$$

Parameters:

$$\phi_{j|y=1} = p(x_j = 1 \mid y = 1)$$

$$\phi_{j|y=0} = p(x_j = 1 \mid y = 0)$$

$$\phi_y = p(y = 1)$$

Joint likelihood

$$\mathcal{L}(\phi_y, \phi_{j|y}) = \prod_{i=1}^{n} p(x^{(i)}, y^{(i)}; \phi_y, \phi_{j|y})$$

After Solving

MLE:

$$\phi_y = \frac{\sum_{i=1}^{m} 1\{y^{(i)} = 1\}}{m}$$ fraction of spam emails

$$\phi_{j|y=1} = \frac{\sum_{i=1}^{m} 1\{x_j = 1, y^{(i)} = 1\}}{\sum_{i=1}^{m} 1\{y^{(i)} = 1\}}$$

Indicator function notation