

Because $E[\text{constant}] = \text{constant}$, (and f and \hat{f} are constants), $E[\epsilon] = 0$, and $\text{Var}(\epsilon) = E[(\epsilon - E[\epsilon])^2]$. You get the expected results.

$$\begin{aligned} E(Y - \hat{Y})^2 &= E[f(X) + \epsilon - \hat{f}(X)]^2 \\ &= E[(f(X) - \hat{f}(X)) + \epsilon]^2 \\ &= E\left[(f(X) - \hat{f}(X))^2 + 2\epsilon(f(X) - \hat{f}(X)) + \epsilon^2\right] \\ &= E\left[(f(X) - \hat{f}(X))^2\right] + E\left[2\epsilon(f(X) - \hat{f}(X))\right] + E[\epsilon^2] \\ &= (f(X) - \hat{f}(X))^2 + 0 + E[\epsilon^2] \end{aligned}$$