HW7

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题目 1. (破案问题) 从给定事实中可以得到以下逻辑语句

1. $Victim = A, Murder(A) \vee Murder(B) \vee Murder(C)$
2. $\forall x(Murder(x) \Rightarrow Hate(x, Victim))$
3. $\forall x(Hate(A, x) \Rightarrow \neg Hate(C, x))$
4. $\forall x(\neg(x = B) \Rightarrow Hate(A, x))$
5. $\forall x(Richer(A, x) \Rightarrow Hate(B, x))$
6. $\forall x(Hate(A, x) \Rightarrow Hate(B, x))$
7. $\neg\exists x\forall y Hate(x, y)$
8. $\forall x(Murder(x) \Rightarrow Richer(Victim, x))$
9. $\neg(A = B)$

下面将其化为子句集

$$
\begin{aligned}
&(Victim = A) \wedge (Murder(A) \vee Murder(B) \vee Murder(C)) \wedge (\forall x(Murder(x) \Rightarrow Hate(x, Victim)))\wedge \\
&(\forall x(Hate(A, x) \Rightarrow \neg Hate(C, x))) \wedge (\forall x(\neg(x = B) \Rightarrow Hate(A, x)))\wedge \\
&(\forall x(Richer(A, x) \Rightarrow Hate(B, x))) \wedge (\forall x(Hate(A, x) \Rightarrow Hate(B, x)))\wedge \\
&(\neg\exists x\forall y Hate(x, y)) \wedge (\forall x(Murder(x) \Rightarrow Richer(Victim, x))) \wedge (\neg(A = B)) \\
\equiv&(Victim = A) \wedge (Murder(A) \vee Murder(B) \vee Murder(C)) \wedge (\forall x(\neg Murder(x) \vee Hate(x, Victim)))\wedge \\
&(\forall x(\neg Hate(A, x) \vee \neg Hate(C, x))) \wedge (\forall x((x = B) \vee Hate(A, x)))\wedge \\
&(\forall x(\neg Richer(A, x) \vee Hate(B, x))) \wedge (\forall x(\neg Hate(A, x) \vee Hate(B, x)))\wedge \\
&(\neg\exists x\forall y Hate(x, y)) \wedge (\forall x(\neg Murder(x) \vee Richer(Victim, x))) \wedge (\neg(A = B)) \\
\equiv&(Victim = A) \wedge (Murder(A) \vee Murder(B) \vee Murder(C)) \wedge (\forall y(\neg Murder(y) \vee Hate(y, Victim)))\wedge \\
&(\forall w(\neg Hate(A, w) \vee \neg Hate(C, w))) \wedge (\forall v((v = B) \vee Hate(A, v)))\wedge \\
&(\forall t(\neg Richer(A, t) \vee Hate(B, t))) \wedge (\forall r(\neg Hate(A, r) \vee Hate(B, r)))\wedge \\
&(\forall m\exists n\neg Hate(m, n)) \wedge (\forall z(\neg Murder(z) \vee Richer(Victim, z))) \wedge (\neg(A = B))
\end{aligned}
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消去存在量词后得

$$
\begin{aligned}
&(Victim = A)(Murder(A) \vee Murder(B) \vee Murder(C)) \wedge (\forall y(\neg Murder(y) \vee Hate(y, Victim))) \\
&\wedge(\forall w(\neg Hate(A, w) \vee \neg Hate(C, w))) \wedge (\forall v((v = B) \vee Hate(A, v))) \\
&\wedge(\forall t(\neg Richer(A, t) \vee Hate(B, t))) \wedge (\forall r(\neg Hate(A, r) \vee Hate(B, r))) \\
&\wedge(\forall m Hate(m, f(m))) \wedge (\forall z(\neg Murder(z) \vee Richer(Victim, z))) \wedge (\neg(A = B))
\end{aligned}
$$

转为前束形并略去全称量词得

$$
\begin{aligned}
&(Victim = A) \wedge (Murder(A) \vee Murder(B) \vee Murder(C)) \wedge (\neg Murder(y) \vee Hate(y, Victim)) \\
&\wedge(\neg Hate(A, w) \vee \neg Hate(C, w)) \wedge ((v = B) \vee Hate(A, v)) \\
&\wedge(\neg Richer(A, t) \vee Hate(B, t)) \wedge (\neg Hate(A, r) \vee Hate(B, r)) \\
&\wedge(Hate(m, f(m))) \wedge (\neg Murder(z) \vee Richer(Victim, z)) \wedge (\neg(A = B))
\end{aligned}
$$

已经化为合取范式，消去合取符号并子句变量标准化，把 $Victim$ 换为 A 后得

$$Murder(A) \vee Murder(B) \vee Murder(C)$$
$$\neg Murder(y) \vee Hate(y, A)$$
$$\neg Hate(A, w) \vee \neg Hate(C, w)$$
$$(v = B) \vee Hate(A, v)$$
$$\neg Richer(A, t) \vee Hate(B, t)$$
$$\neg Hate(A, r) \vee Hate(B, r)$$
$$Hate(m, f(m))$$
$$\neg Murder(z) \vee Richer(A, z)$$
$$\neg(A = B)$$

设凶手为 u，将 $\neg Murder(u) \vee ANSWER(u)$ 加入子句集中。再考虑事实“没有人比自己富有”，$\forall x(\neg Richer(x, x))$，化为子句即 $\neg Richer(s, s)$

$$Murder(A) \vee Murder(B) \vee Murder(C) \quad (1)$$
$$\neg Murder(y) \vee Hate(y, A) \quad (2)$$
$$\neg Hate(A, w) \vee \neg Hate(C, w) \quad (3)$$
$$(v = B) \vee Hate(A, v) \quad (4)$$
$$\neg Richer(A, t) \vee Hate(B, t) \quad (5)$$
$$\neg Hate(A, r) \vee Hate(B, r) \quad (6)$$
$$Hate(m, f(m)) \quad (7)$$
$$\neg Murder(z) \vee Richer(A, z) \quad (8)$$
$$\neg(A = B) \quad (9)$$
$$\neg Murder(u) \vee ANSWER(u) \quad (10)$$
$$\neg Richer(s, s) \quad (11)$$

取置换 $\sigma = \{A/v\}$，由 (4),(9) 得 $Hate(A, A)(12)$。

取置换 $\sigma = \{A/w\}$，由 (12),(3) 得 $\neg Hate(C, A)(13)$。

取置换 $\sigma = \{C/y\}$，由 (13),(2) 得 $\neg Murder(C)(14)$。

由 (13),(1) 得 $Murder(B) \vee Murder(A)(15)$。

取置换 $\sigma = \{A/z, A/s\}$，由 (8),(11) 得 $\neg Murder(A)$(16)。

由 (16),(15) 得 $Murder(B)$(17)。

取置换 $\sigma = \{B/u\}$，由 (10),(17) 得 $ANSWER(B)$(18)。

所以凶手是 B。