## **HW13**

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2024年5月22日

题目 1. (9.3.4) 设

$$A = egin{pmatrix} oldsymbol{lpha}_1^ op \ oldsymbol{lpha}_2^ op \ dots \ oldsymbol{lpha}_m^ op \end{pmatrix}$$

则

$$W(A) = span\{\alpha_1, \alpha_2, \cdots, \alpha_m\}$$

记方程 AX = 0 的基础解系为

$$\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \cdots, \boldsymbol{\xi}_r$$

则

$$N(A) = span\{\xi_1, \xi_2, \cdots, \xi_r\}$$

$$(\boldsymbol{\xi_i}, \boldsymbol{\alpha_j}) = 0, \forall 1 \leq i \leq r, 1 \leq j \leq m$$

所以  $orall lpha \in W(A), oldsymbol{\xi} \in N(A)$ 

$$(\boldsymbol{\alpha}, \boldsymbol{\xi}) = (\sum_{i} k_{i} \boldsymbol{\alpha}_{i}, \sum_{j} t_{j} \boldsymbol{\xi}_{j}) = \sum_{i} \sum_{j} k_{i} t_{j} (\boldsymbol{\alpha}_{i}, \boldsymbol{\xi}_{j}) = 0$$

即

$$W(A) \perp N(A)$$

从而 W(A), N(A) 之和为直和。又因为  $\dim(W(A)) + \dim(N(A)) = n$ ,所以

$$\mathbb{R}^n = oldsymbol{W(A)} \oplus oldsymbol{N(A)}$$