HW13

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题目 1. (9.3.4) 设

$$A = \begin{pmatrix} \boldsymbol{\alpha}_1^\top \\ \boldsymbol{\alpha}_2^\top \\ \vdots \\ \boldsymbol{\alpha}_m^\top \end{pmatrix}$$

则

$$\boldsymbol{W(A)} = span\{\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \cdots, \boldsymbol{\alpha}_m\}$$

记方程 $\boldsymbol{AX = 0}$ 的基础解系为

$$\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \cdots, \boldsymbol{\xi}_r$$

则

$$\boldsymbol{N(A)} = span\{\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \cdots, \boldsymbol{\xi}_r\}$$

$$(\boldsymbol{\xi_i, \alpha_j}) = 0, \forall 1 \leq i \leq r, 1 \leq j \leq m$$

所以 $\forall \boldsymbol{\alpha} \in \boldsymbol{W(A)}, \boldsymbol{\xi} \in \boldsymbol{N(A)}$

$$(\boldsymbol{\alpha, \xi}) = (\sum_i k_i \boldsymbol{\alpha_i}, \sum_j t_j \boldsymbol{\xi_j}) = \sum_i \sum_j k_i t_j (\boldsymbol{\alpha_i}, \boldsymbol{\xi_j}) = 0$$

即

$$\boldsymbol{W(A)} \perp \boldsymbol{N(A)}$$

从而 $\boldsymbol{W(A)}, N(A)$ 之和为直和。又因为 $\dim(W(A)) + \dim(N(A)) = n$，所以

$$\mathbb{R}^n = \boldsymbol{W(A)} \oplus \boldsymbol{N(A)}$$