HW3

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题目 1. (2.2.9) 证明

$$[\mathbf{a}, \mathbf{b}, \mathbf{c} \times \mathbf{d}] + [\mathbf{b}, \mathbf{c}, \mathbf{a} \times \mathbf{d}] + [\mathbf{c}, \mathbf{a}, \mathbf{b} \times \mathbf{d}] = \mathbf{0}$$

解答. 因为对于任意三维空间向量 $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$ 有：

$$
\begin{aligned}
(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) &= \mathbf{a} \cdot \mathbf{b} \times (\mathbf{c} \times \mathbf{d}) \\
&= \mathbf{a} \cdot ((\mathbf{b} \cdot \mathbf{d})\mathbf{c} - (\mathbf{b} \cdot \mathbf{c})\mathbf{d}) \\
&= (\mathbf{b} \cdot \mathbf{d})(\mathbf{a} \cdot \mathbf{c}) - (\mathbf{b} \cdot \mathbf{c})(\mathbf{a} \cdot \mathbf{d})
\end{aligned}
$$

所以

$$
\begin{aligned}
& [\mathbf{a}, \mathbf{b}, \mathbf{c} \times \mathbf{d}] + [\mathbf{b}, \mathbf{c}, \mathbf{a} \times \mathbf{d}] + [\mathbf{c}, \mathbf{a}, \mathbf{b} \times \mathbf{d}] \\
=\ & (\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) + (\mathbf{b} \times \mathbf{c}) \cdot (\mathbf{a} \times \mathbf{d}) + (\mathbf{c} \times \mathbf{a}) \cdot (\mathbf{b} \times \mathbf{d}) \\
=\ & (\mathbf{b} \cdot \mathbf{d})(\mathbf{a} \cdot \mathbf{c}) - (\mathbf{b} \cdot \mathbf{c})(\mathbf{a} \cdot \mathbf{d}) + \\
& (\mathbf{c} \cdot \mathbf{d})(\mathbf{b} \cdot \mathbf{a}) - (\mathbf{c} \cdot \mathbf{a})(\mathbf{b} \cdot \mathbf{d}) + \\
& (\mathbf{a} \cdot \mathbf{d})(\mathbf{c} \cdot \mathbf{b}) - (\mathbf{a} \cdot \mathbf{b})(\mathbf{c} \cdot \mathbf{d}) \\
=\ & \mathbf{0}
\end{aligned}
$$

$\square$