

Number Theory CP Questions (ACM Style)

Instructions

Sample input/output formats are standard ACM: each test case on a new line, output as specified. You may add multiple test cases as needed.

1. Division

Problem: Given two integers A and B , find the quotient and remainder when A is divided by B .

Input: Two integers A and B ($-10^9 \leq A, B \leq 10^9$, $B \neq 0$).

Output: Print the quotient and remainder separated by a space.

Sample Input: 17 5

Sample Output: 3 2

2. The Division Algorithm

Problem: Given an integer N and a positive integer d , find unique integers q and r such that $N = dq + r$, where $0 \leq r < d$.

Input: Two integers N and d ($-10^9 \leq N \leq 10^9$, $1 \leq d \leq 10^9$).

Output: Print q and r separated by a space.

Sample Input: -7 3

Sample Output: -3 2

3. Modular Arithmetic

Problem: Given a number A and a modulus M , compute $A \bmod M$.

Input: Two integers A and M ($-10^{18} \leq A \leq 10^{18}$, $1 \leq M \leq 10^9$).

Output: Print $A \bmod M$.

Sample Input: 123456789012345678 1000000007

Sample Output: 643499475

4. Modular Arithmetic (Greatest Common Divisor)

Problem: Given two integers A and B , find their greatest common divisor (GCD) using the Euclidean algorithm.

Input: Two integers A and B ($1 \leq A, B \leq 10^{12}$).

Output: Print their GCD.

Sample Input: 48 18

Sample Output: 6

5. Modular Arithmetic (Least Common Multiple)

Problem: Given two integers A and B , find their least common multiple (LCM).

Input: Two integers A and B ($1 \leq A, B \leq 10^{12}$).

Output: Print their LCM.

Sample Input: 12 15

Sample Output: 60

6. Modular Arithmetic (GCD Linear Combination)

Problem: Given two integers A and B , find integers x and y such that $Ax + By = \gcd(A, B)$.

Input: Two integers A and B ($1 \leq A, B \leq 10^6$).

Output: Print x and y separated by a space.

Sample Input: 30 12

Sample Output: 1 -2

7. Solving Linear Congruence

Problem: Given integers a , b , and m , solve the linear congruence $ax \equiv b \pmod{m}$ for integer x , or print -1 if there is no solution.

Input: Three integers a , b , and m ($1 \leq a, b, m \leq 10^6$).

Output: Print the smallest non-negative solution x ($0 \leq x < m$), or -1 if no solution exists.

Sample Input: 4 8 12

Sample Output: 2

8. Fermat's Little Theorem

Problem: Given integers a and prime p , compute $a^{p-1} \bmod p$ and verify Fermat's Little Theorem.

Input: Two integers a and p ($1 \leq a < p \leq 10^9$, p is prime).

Output: Print $a^{p-1} \bmod p$.

Sample Input: 3 7

Sample Output: 1

9. Euler's Theorem

Problem: Given integers a and n ($1 \leq a < n \leq 10^6$), where $\gcd(a, n) = 1$, compute $a^{\phi(n)} \bmod n$ using Euler's Theorem ($\phi(n)$ is Euler's totient function).

Input: Two integers a and n .

Output: Print $a^{\phi(n)} \bmod n$.

Sample Input: 3 10

Sample Output: 1

10. Check Divisibility

Problem: Given two integers A and B , determine whether A is divisible by B .

Input: Two integers A and B ($-10^9 \leq A, B \leq 10^9$, $B \neq 0$).

Output: Print YES if A is divisible by B , otherwise print NO.

Sample Input: 10 5

Sample Output: YES

11. Find Modular Multiplicative Inverse

Problem: Given two integers a and m , find an integer x such that $ax \equiv 1 \pmod{m}$. If no such x exists, output -1 .

Input: Two integers a and m ($1 \leq a, m \leq 10^6$).

Output: Print the modular inverse x ($0 \leq x < m$), or -1 if it does not exist.

Sample Input: 3 11

Sample Output: 4

12. Count Numbers Coprime to n

Problem: Given an integer n , count the number of integers between 1 and n (inclusive) that are coprime with n .

Input: One integer n ($1 \leq n \leq 10^6$).

Output: Print the count of integers coprime to n .

Sample Input: 6

Sample Output: 2

13. Find All Divisors

Problem: Given an integer n , print all its positive divisors in ascending order.

Input: One integer n ($1 \leq n \leq 10^9$).

Output: Print all divisors of n separated by spaces in ascending order.

Sample Input: 12

Sample Output: 1 2 3 4 6 12

14. Solve System of Two Linear Congruences

Problem: Given two congruences: $x \equiv a_1 \pmod{m_1}$ and $x \equiv a_2 \pmod{m_2}$, where m_1 and m_2 are coprime, find the smallest non-negative integer x satisfying both congruences.

Input: Four integers a_1, m_1, a_2, m_2 ($1 \leq a_1, a_2 < m_1, m_2 \leq 10^6$, $\gcd(m_1, m_2) = 1$).

Output: Print the smallest non-negative solution x ($0 \leq x < m_1 m_2$).

Sample Input: 2 3 3 5

Sample Output: 8

15. Large Modular Exponentiation

Problem: Given integers a , b , and m , compute $a^b \bmod m$ efficiently.

Input: Three integers a, b, m ($1 \leq a, m \leq 10^9$, $0 \leq b \leq 10^9$).

Output: Print the value of $a^b \bmod m$.

Sample Input: 2 10 1000

Sample Output: 24

16. Prime Factorization

Problem: Given an integer n , print all of its prime factors in ascending order.

Input: One integer n ($2 \leq n \leq 10^9$).

Output: Print the prime factors separated by spaces in ascending order.

Sample Input: 60

Sample Output: 2 2 3 5

17. Test for Primality

Problem: Given an integer n , determine whether it is prime.

Input: One integer n ($2 \leq n \leq 10^9$).

Output: Print YES if n is prime, otherwise print NO.

Sample Input: 17

Sample Output: YES

18. Sum of GCDs

Problem: Given an integer n , compute the sum $\sum_{k=1}^n \gcd(n, k)$.

Input: One integer n ($1 \leq n \leq 10^6$).

Output: Print the sum of GCDs.

Sample Input: 6

Sample Output: 14
