

Program – 10

AIM: To implement a program to calculate and verify DSA given in DSS.

Introduction and Theory

The Digital Signature Algorithm (DSA) is a Federal Information Processing Standard for digital signatures, based on the mathematical concept of modular exponentiations and the discrete logarithm problem.

The DSA algorithm works in the framework of public-key cryptosystems and is based on the algebraic properties of the modular exponentiations, together with the discrete logarithm problem (which is considered to be computationally intractable). Messages are signed by the signer's private key and the signatures are verified by the signer's corresponding public key. The digital signature provides message authentication, integrity and non-repudiation.

The first part of the DSA algorithm is the public key and private key generation, which can be described as:

- Choose a prime number q , which is called the prime divisor.
- Choose another prime number p , such that $p-1 \bmod q = 0$. p is called the prime modulus.
- Choose an integer g , such that $1 < g < p$, $g^{q-1} \bmod p = 1$ and $g = h^{((p-1)/q)} \bmod p$. q is also called g 's multiplicative order modulo p .
- Choose an integer, such that $0 < x < q$.
- Compute y as $g^x \bmod p$.
- Package the public key as $\{p, q, g, y\}$.
- Package the private key as $\{p, q, g, x\}$.

The second part of the DSA algorithm is the signature generation and signature verification, which can be described as:

To generate a message signature, the sender can follow these steps:

- Generate the message digest h , using a hash algorithm like SHA1.
- Generate a random number k , such that $0 < k < q$.
- Compute r as $(g^k \bmod p) \bmod q$. If $r = 0$, select a different k .
- Compute i , such that $k \cdot i \bmod q = 1$. i is called the modular multiplicative inverse of k modulo q .
- Compute $s = i \cdot (h + r \cdot x) \bmod q$. If $s = 0$, select a different k .
- Package the digital signature as $\{r, s\}$.

Program – 10

To verify a message signature, the receiver of the message and the digital signature can follow these steps:

- Generate the message digest h , using the same hash algorithm.
- Compute w , such that $s \cdot w \bmod q = 1$. w is called the modular multiplicative inverse of s modulo q .
- Compute $u_1 = h \cdot w \bmod q$.
- Compute $u_2 = r \cdot w \bmod q$.
- Compute $v = (((g^{u_1}) \cdot (y^{u_2})) \bmod p) \bmod q$.
- If $v == r$, the digital signature is valid.

Code

```
1 // #include<iostream>
2 // #include<string>
3 // #include<utility>
4 // #include<cmath>
5 #include<bits/stdc++.h>
6
7 using namespace std;
8
9 long long int power(long long int x, long long int y, long long int
10 p)
11 {
12     long long int res = 1; // Initialize result
13
14     x = x % p; // Update x if it is more than or
15                // equal to p
16
17     while (y > 0)
18     {
19         // If y is odd, multiply x with result
20         if (y & 1)
21             res = (res*x) % p;
22
23         // y must be even now
24         y = y>>1; // y = y/2
25         x = (x*x) % p;
26     }
27     return res;
28 }
29
30
31 long long Hash(string S, int p)
32 {
33     long long hash = 1;
34     int prev = 1;
35     for (int i = 0; i < S.length(); i++)
36     {
37         hash = (hash + int(S[i])*prev % p);
38         prev = int(S[i]);
39     }
40     return hash;
41 }
```

Program – 10

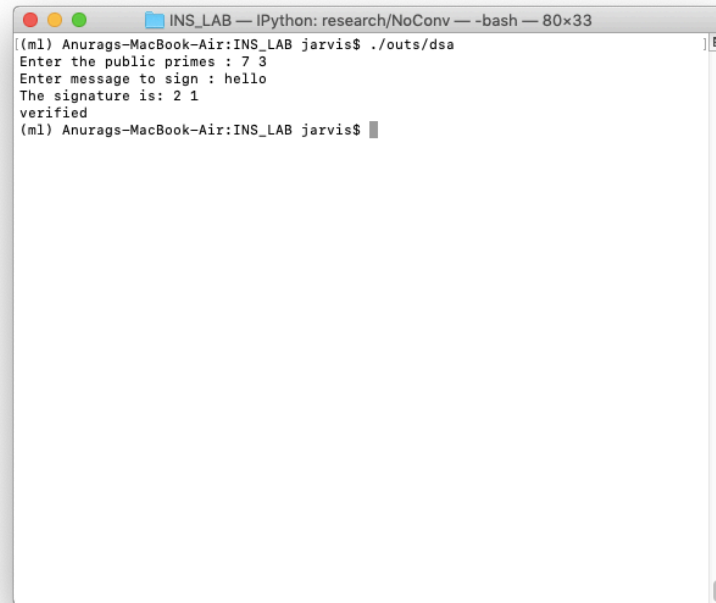
```
42
43 long long int modInverse(long long int a, long long int m)
44 {
45     long long int m0 = m;
46     long long int y = 0, x = 1;
47
48     if (m == 1)
49         return 0;
50
51     while (a > 1)
52     {
53         // q is quotient
54         long long int q = a / m;
55         long long int t = m;
56
57         // m is remainder now, process same as
58         // Euclid's algo
59         m = a % m, a = t;
60         t = y;
61
62         // Update y and x
63         y = x - q * y;
64         x = t;
65     }
66
67     // Make x positive
68     if (x < 0)
69         x += m0;
70
71     return x;
72 }
73
74 class DSA
75 {
76 private:
77     long long int x;
78     long long int g;
79     long long int y;
80     long long int p;
81     long long int q;
82 public:
83     DSA(int , int );
84     pair<long long int, long long int> sign(string);
85     bool verify(string, pair<long long int, long long int>);
86
87 };
88
89 DSA::DSA(int P, int Q)
90 {
91     x = rand() % 100 + 1;
92     p = P;
93     q = Q;
94     g = power(2, (p-1)/q, p);
95     y = power(g, x, p);
96 }
97
98 pair<long long int, long long int> DSA::sign(string s)
```

Program – 10

```
99 {
100     long long int s1 = 0;
101     long long int s2 = 0;
102     long long int r;
103     do
104     {
105         r = rand() % q + 1;
106         s1 = power(g, r, p) % q;
107         long long k = modInverse(r, q);
108         s2 = (k * (Hash(s, p) + x * s1)) % q;
109     }while(s1 == 0 || s2 == 0);
110     return make_pair(s1, s2);
111 }
112 bool DSA::verify(string s, pair<long long int, long long int> sign)
113 {
114     long long int h = Hash(s, p);
115     long long int w, u1, u2, v;
116     w = modInverse(sign.second, p);
117     u1 = (h * w);
118     // cout << u1 << endl;
119     u2 = (sign.first * w);
120     // cout << u2 << endl;
121     v = ((power(g, u1, p) * power(y, u2, p))%p)%q;
122     // cout << v << endl;
123     return (abs(v) == sign.first);
124 }
125
126 int main()
127 {
128     long long int p, q;
129     cout << "Enter the public primes : ";
130     cin >> p >> q;
131     cout << "Enter message to sign : ";
132     string s;
133     cin >> s;
134     DSA D(p, q);
135     pair<long long int, long long int> sin = D.sign(s);
136     cout << "The signature is: " << sin.first << ' ' << sin.second
137 << endl;
138     if (D.verify(s, sin))
139         cout << "verified" << endl;
140     else
141         cout << "Rejected" << endl;
142 }
```

Program – 10

Results and Outputs:

A screenshot of a macOS terminal window titled 'INS_LAB — IPython: research/NoConv — -bash — 80x33'. The terminal shows the execution of a script './outs/dsa' in a directory 'INS_LAB'. The output of the script is as follows:

```
((ml) Anurags-MacBook-Air:INS_LAB jarvis$ ./outs/dsa
Enter the public primes : 7 3
Enter message to sign : hello
The signature is: 2 1
verified
((ml) Anurags-MacBook-Air:INS_LAB jarvis$
```

Findings and Learnings:

1. We have implemented DSA and verified it.