Data Visualization COS 701 (FALL 2022)

Problem 6: Random vectors and stereographic projection

a. Consider a set of N random points (say, N = 1000) distributed on the surface of a unit sphere. Construct the stereographic projection of the points on the upper (north) hemisphere with respect to the north pole (0, 0, 1) on the extended x-y plane as discussed in the class. Obtain an expression of the projected coordinates on the plane in terms of its coordinates on the sphere.

b. Find out the radius of the circle (of the projected points) on the x-y plane that originates from points on the sphere having a constant polar angle θ. Present your results with appropriate visualization for a few values of θ.

c. Conversely, given a distribution of random points on the x-y plane, reconstruct the points on the surface of a unit sphere. Assume that the minimum distance between points on the plane is unity.

d. Compare your results with analytical results whenever possible.