Dark Energy Parametrization

December 8, 2025

1 Introduction

$$E(z)^2 = \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda f_{\mathrm{DE}}(z).$$

where, $E(z)^2 = H^2(z)/H_0^2$ is the dimensionless Hubble function, H_0 is the present-day Hubble constant.

$$f_{\mathrm{DE}}(z) = \exp\left[3\int_0^z \frac{1+\omega(z')}{1+z'}\,dz'\right].$$

For $\omega = -1$, $f_{\mathrm{DE}} = 1$ and the model reduces to oΛCDM.

$$E(z)^2 = \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda.$$

In the special case of a spatially flat Universe ($\Omega_k = 0$), we get ΛCDM

$$E(z)^2 = \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda.$$

Parameterization	$\omega(z)$	$f_{DE}(z)$
ωCDM	ω_0	$(1+z)^{3(1+\omega_0)}$
Logarithmic	$\omega_0 + \omega_a \log(1+z)$	$(1+z)^{3(1+\omega_0)} e^{\frac{3}{2}\omega_a (\log(1+z))^2}$
CPL	$\omega_0 + \frac{z}{1+z}\,\omega_a$	$(1+z)^{3(1+\omega_0+\omega_a)} e^{-\frac{3\omega_a z}{1+z}}$
BA	$\omega_0 + \frac{z(1+z)}{1+z^2}\,\omega_a$	$(1+z)^{3(1+\omega_0)} (1+z^2)^{\frac{3\omega_a}{2}}$
JBP	$\omega_0 + \frac{z}{(1+z)^2}\,\omega_a$	$(1+z)^{3(1+\omega_0)} e^{\frac{3\omega_a z^2}{2(1+z)^2}}$