Dark Energy: Theory, Observations, and Challenges

Anil Kandel

ICTP PWF Holographic Himalaya

Jan 11, 2026

Homogeneous and Isotropic Universe

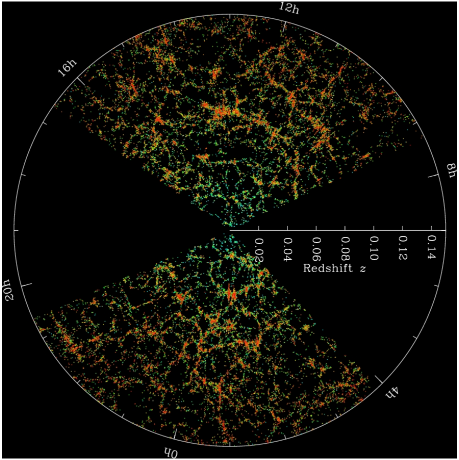


Figure: Sloan Digital Sky Survey.

Friedmann Equations

Einstein Field Equation:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Friedmann Equations (FLRW universe):

1. First Friedmann: $$H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$
2. Second Friedmann: $$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}$$

2011 Nobel Prize in Physics

- 1999: *Riess et al*. and *Perlmutter et al*. discover that the expansion of the Universe is ***ACCELERATED***

OBSERVATIONAL EVIDENCE FROM SUPERNOVAE FOR AN ACCELERATING UNIVERSE AND A COSMOLOGICAL CONSTANT

Adam G. Riess,[1] Alexei V. Filippenko,[1] Peter Challis,[2] Alejandro Clocchiatti,[3] Alan Diercks,[4] Peter M. Garnavich,[2] Ron L. Gilliland,[5] Craig J. Hogan,[4] Saurabh Jha,[2] Robert P. Kirshner,[2] B. Leibundgut,[6] M. M. Phillips,[7] David Reiss,[4] Brian P. Schmidt,[8,9] Robert A. Schommer,[7] R. Chris Smith,[7,10] J. Spyromilio,[6] Christopher Stubbs,[4] Nicholas B. Suntzeff,[7] and John Tonry[11]

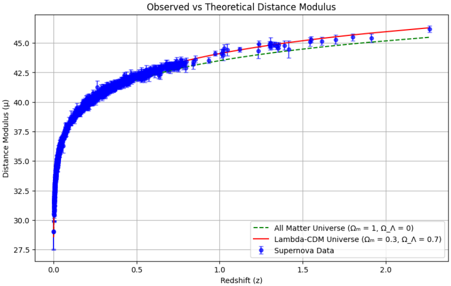
Received 1998 March 13; revised 1998 May 6

MEASUREMENTS OF Ω AND Λ FROM 42 HIGH-REDSHIFT SUPERNOVAE

S. Perlmutter,[1] G. Aldering, G. Goldhaber,[1] R. A. Knop, P. Nugent, P. G. Castro,[2] S. Deustua, S. Fabbro,[3] A. Goobar,[4] D. E. Groom, I. M. Hook,[5] A. G. Kim,[1,6] M. Y. Kim, J. C. Lee,[7] N. J. Nunes,[2] R. Pain,[3] C. R. Pennypacker,[8] and R. Quimby

The Nobel Prize in Physics 2011

Evidence for Dark Energy

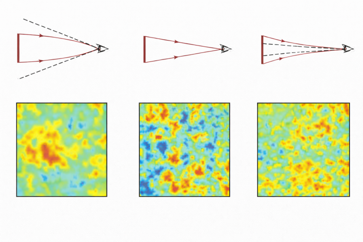


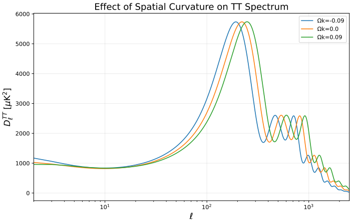
- Type Ia supernova observations show that the cosmic expansion is **accelerating**.
- Ordinary matter and gravity alone would lead to a **decelerating** expansion.
- The ΛCDM model provides the **best fit** to current supernova data.

The ΛCDM Model

- The ΛCDM model is the **standard cosmological model** describing the large-scale evolution of the Universe.
- It assumes a homogeneous and isotropic Universe governed by **General Relativity**.
- The cosmic energy budget consists of:
  - $\sim 4\%$ baryonic matter,
  - $\sim 26\%$ cold dark matter (CDM),
  - $\sim 70\%$ dark energy in the form of a cosmological constant Λ.
- Dark energy (Λ) drives the **late-time accelerated expansion** of the Universe.

CMB Angular Scale in a Flat Universe ($\Omega_k = 0$)





- Flat spatial geometry $\Rightarrow$ **straight photon geodesics**
- $\ell \simeq 220 \ \Rightarrow$ **Flat Universe**

$$\Omega_k = 0$$

- $\ell > 220 \ \Rightarrow$ **Open Universe**

$$\Omega_k > 0$$

- $\ell < 220 \ \Rightarrow$ **Closed Universe**

$$\Omega_k < 0$$

The angular position of the first acoustic peak provides a geometric probe of spatial curvature.

What is Λ

- ΛCDM is the **standard model of cosmology**
- Λ represents **vacuum energy (dark energy)**
- Appears in Einstein equations:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- Energy density and pressure:

$$\rho_\Lambda = \frac{\Lambda}{8\pi G}, \quad p_\Lambda = -\rho_\Lambda$$

- Equation of state:

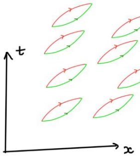
$$w_\Lambda = -1$$

- Causes **late-time accelerated expansion**
- Constant in time: $\rho_\Lambda = \text{const.}$
- Observations (SNe Ia, CMB, BAO) give:

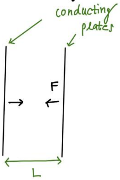
$$\Omega_\Lambda \simeq 0.7$$

Quantum Mechanics Λ

$\Delta E \cdot \Delta t \geq \hbar$



The vacuum is a sea of virtual particle antiparticle particles continously created and destroyed.



The vacuum is not boring.

Boundary condition changes & leads to Force of Attraction.

"Casimir Effect confirms the existence of vacuum Fluctuation".

Momentum of Particles in an Expanding Universe

Massive particles

- Momentum scales as

$$p \propto \frac{1}{a}$$

- Even if the particle is relativistic.
- Quickly,

$$p \to 0, \qquad E \to mc^2$$

- Rest mass energy dominates over kinetic energy.
- On cosmic scales, massive particles effectively "hang around".

Massless particles (photons)

- Energy–momentum relation:

$$E = pc$$

- Momentum scales as

$$p \propto \frac{1}{a}$$

- Hence,

$$E \propto \frac{1}{a}$$

- This leads to **redshift**.
- Photon Pressure : $P = \rho/3$

Equation of State

- **Equation of State (EoS):**
  - Describes the physical relation between pressure and energy density

$$p = w\rho$$

- **Equation of State Parameter:**
  - Defined as the ratio of pressure to energy density

$$w \equiv \frac{p}{\rho}$$

- **Cosmological Fluids:**
  - **Dust (Cold Matter):**

$$p = 0 \quad \Rightarrow \quad w = 0$$

  - **Radiation:**

$$p = \frac{1}{3}\rho \quad \Rightarrow \quad w = \frac{1}{3}$$

  - **Vacuum Energy :**

$$p = -\rho \quad \Rightarrow \quad w = -1$$

Evolution of Energy Density

- Energy density evolution in an expanding Universe

$$\nabla_\mu T^\mu_\nu = 0 \quad \Rightarrow \quad \dot{\rho} + 3H(\rho + p) = 0$$

To be precise $\nu = 0$, gives density evolution

- Solution as a function of scale factor a:

$$\rho(a) \propto a^{-3(1+w)}$$

- **Examples:**
  - **Dust (Matter):**

$$w = 0 \quad \Rightarrow \quad \rho_m \propto a^{-3}$$

  - **Radiation:**

$$w = \frac{1}{3} \quad \Rightarrow \quad \rho_r \propto a^{-4}$$

  - **Vacuum Energy :**

$$w = -1 \quad \Rightarrow \quad \rho_\Lambda = \text{constant} \;!!!$$

Second Friedmann Equation and Cosmic Acceleration

Second Friedmann (Acceleration) Equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

Pure Dust	Radiation	Vacuum / Λ
$P = 0$	$P = \frac{\rho}{3}$	$P = -\rho$
$\rho > 0$	$\rho + 3P > 0$	$\rho + 3P = -2\rho < 0$
$\ddot{a} < 0$	$\ddot{a} < 0$	$\ddot{a} > 0$

$$\Downarrow$$

Negative pressure
drives acceleration

Conservation of Energy in Expanding Universe

- In classical physics, **energy is conserved**:

$$\text{Time translation invariance} \implies \text{Energy conservation}$$

- In an **expanding universe**, spacetime itself evolves with time.
- Example: **Radiation energy density**

$$\rho_r \propto a^{-4} \quad \Rightarrow \quad E_r = \rho_r a^3 \propto a^{-1} \neq \text{constant}$$

- **Takeaways:**
  - Energy conservation requires time-translation symmetry.
  - Expanding universe breaks this symmetry.

- **Cosmological Constant Problem**
  - Quantum Field Theory predicts vacuum energy:

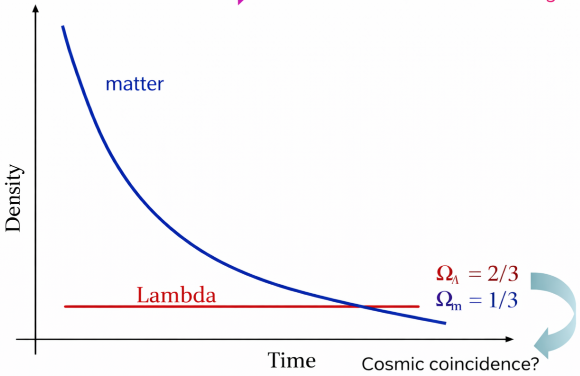
$$\rho_{\text{vac}}^{\text{QFT}} \sim 10^{120}\, \rho_{\Lambda}^{\text{obs}}$$

  - Expected value $\Rightarrow$ 10^{112} ergs/cm^3
  - Observed value $\Rightarrow$ 10^{-8} ergs/cm^3

Fine Tuning Problem.

$\rho_\Lambda/\rho_m \sim 10^{-44}$ at the electroweak scale if $\rho_{0m} \sim \rho_\Lambda$ now.

Initial conditions need fine tuning



Cosmic coincidence?

Dynamical Dark Energy

Scalar Fields

- **A scalar field respects the isotropy of the universe.**
- **The scalar field is taken to be a function of time only, $\phi = \phi(t)$.**
- **The scalar field evolves very slowly.**

Have we detected any scalar field?

$$\Downarrow$$

Higgs Field

Quintessence Model

- **Action** for a scalar field ϕ:

$$S = \int d^4x \sqrt{-g}\, a^3 \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)\right]$$

- **Energy density**:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

- **Pressure**:

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

- **Equation of state**:

$$w_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}$$

(can vary between $-1 < w_\phi < 1$)

Revisiting 2^{nd} Friedmann equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p\right)$$

(ρ = Total energy density, p = Total pressure)

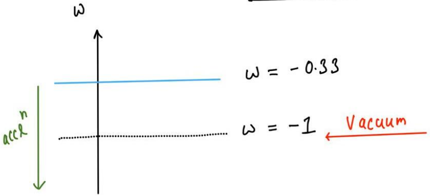
$$\ddot{a} > 0 \Rightarrow \rho + 3p < 0$$

$$\Rightarrow \rho + 3w\rho < 0$$

$$\Rightarrow (1 + 3w) < 0$$

$$\boxed{\therefore\ w_{eff} < -\frac{1}{3}}$$

"Accelerates universe"



Hunting for Equation of State Parameter

- **Dark energy: dynamical or constant?**

$\rightarrow$ Number to characterize how much dark energy density evolves.

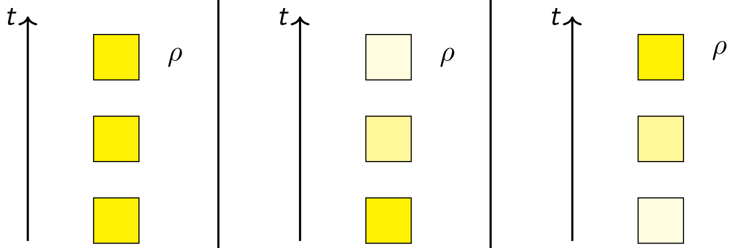
$$\rho \propto a^{-3(1+w)}$$

$$\longrightarrow \quad w = -1 \quad (\rho = \text{constant})$$

$$\longrightarrow \quad w > -1 \quad (\rho \propto a^{-3(1+w)}, \rho_{\mathrm{DE}} \text{ decreasing})$$

$$\longrightarrow \quad w < -1 \quad (\rho \propto a^{-3(1+w)}, \rho_{\mathrm{DE}} \text{ increasing})$$

ΛCDM	Quintessence	Phantom
$\rho =$ constant over time	$\rho \downarrow$ over time but not as fast as $\rho_m \propto a^{-3}$	$\rho \uparrow$ over time leads to Big Rip



Latest DESI DR2 Observation

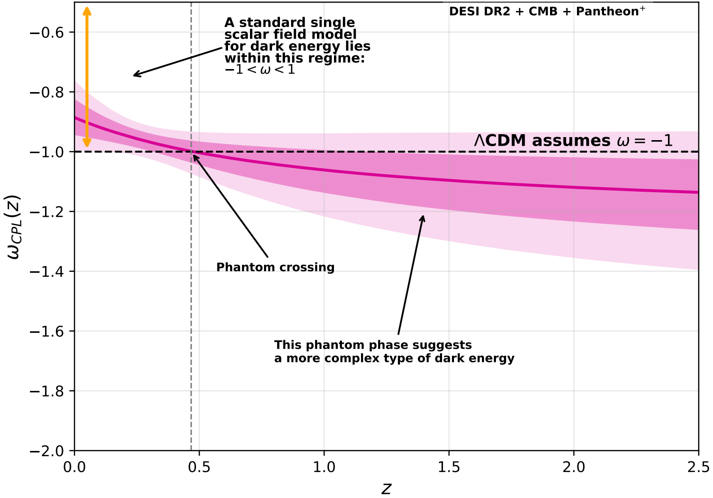
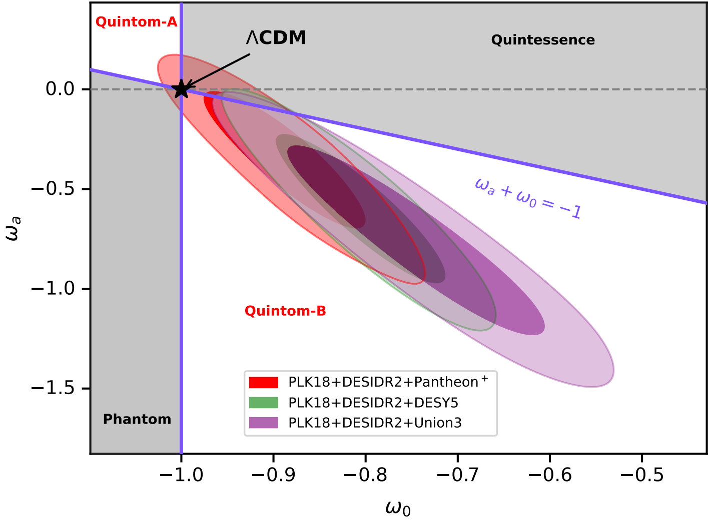


Figure: Dynamical Dark Energy

w0-wa Plane



Quintom-A
ΛCDM
Quintessence
0.0
−0.5
−1.0
−1.5
ω_a
$\omega_a + \omega_0 = -1$
Quintom-B
PLK18+DESIDR2+Pantheon$^+$
PLK18+DESIDR2+DESY5
PLK18+DESIDR2+Union3
Phantom
−1.0
−0.9
−0.8
−0.7
−0.6
−0.5
ω_0

Data

Baryon Acoustic Oscillations: 13 BAO measurements from DESI DR2 in the redshift range $0.295 \leq z \leq 2.33$ *M. A. Karim et al., DESI DR2, arXiv:2503.14738 (2025).*

Cosmic Chronometers: 15 Hubble parameter measurements in the redshift range $0.1791 \leq z \leq 1.965$ *M. Moresco et al., JCAP 2012(07), 053*, *M. Moresco et al., MNRAS Lett. 450, L16 (2015)*, and *M. Moresco et al., JCAP 2016(05), 014*

Pantheon^{+}: 1701 light curves of 1550 SNe Ia, $0.01 \leq z \leq 2.26$. *We exclude very low-z events ($z < 0.01$) where peculiar velocities dominate, leaving 1590 light curves.* *D. Brout et al., Astrophys. J. 938(2), 110 (2022).*

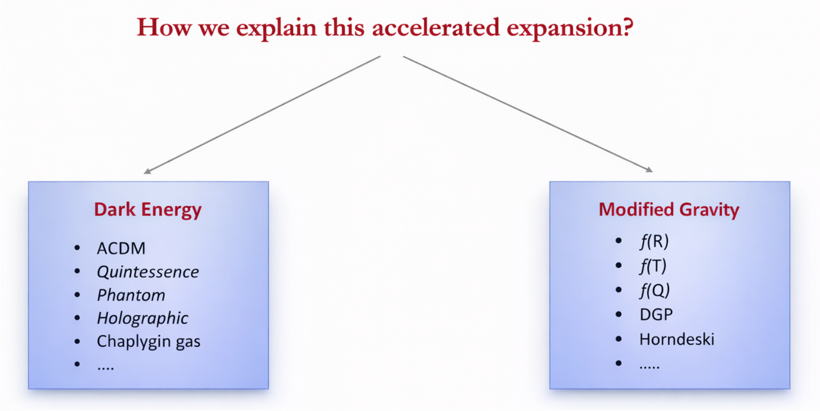
DES-SN5Y 1829 SNe Ia, $0.025 \leq z \leq 1.12$ (1635 DES + 194 low-z anchors). *DES Collaboration et al., Astrophys. J. Lett. 973(1), L14 (2024)*

Union3: 2087 SNe Ia, $0.050 \leq z \leq 2.26$, homogenized across 24 surveys. *D. Rubin et al., Astrophys. J. 986(2), 231 (2025)*

CMB Distance Priors uses derived parameters (e.g., R, l_A, $\Omega_b h^2$) and Planck 18 uses derived parameters (e.g., ω_b, ω_{cb}, $D_M(z_*)/r_d$) *L. Chen, Q.-G. Huang, and K. Wang, JCAP* ***2019****(02), 028.*, and *E. Aubourg et al., Phys. Rev. D 92(12), 123516 (2015).*

ΛCDM Model Fails !!!

Beyond Λ: Alternative Models



Thank you for your attention.

Any Questions?