

# CSA03 – DATA STRUCTURES

## Concept Mapping

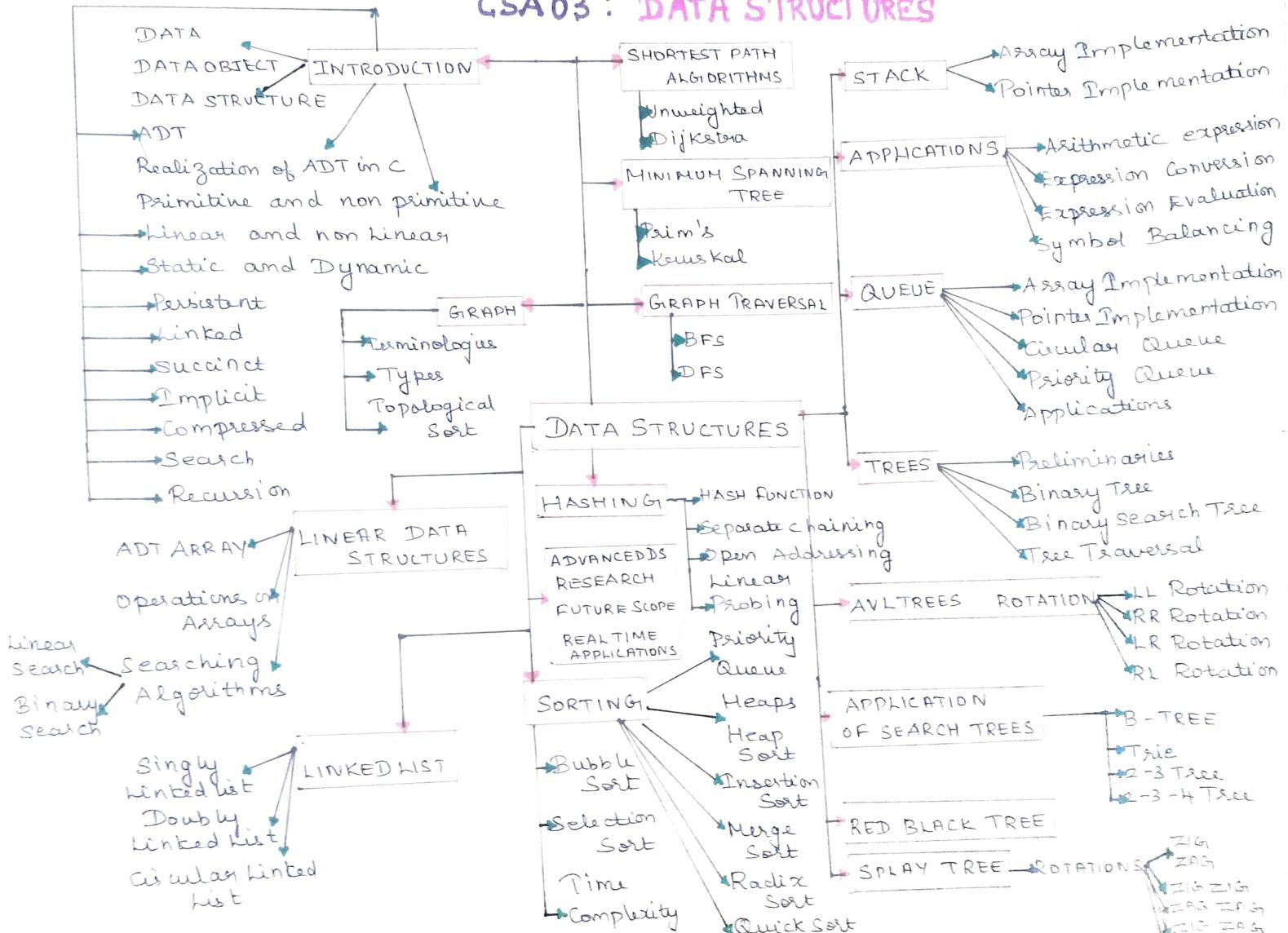
<b>Topic 1:</b> Introduction to Data Structures	1	<ul style="list-style-type: none"><li>➤ Data</li><li>➤ Data Object</li><li>➤ Data Structures</li><li>➤ Abstract Data Types (ADT)</li><li>➤ Realization of ADT in C</li><li>➤ Primitive and Non-Primitive DS</li><li>➤ Linear and Non-Linear DS</li><li>➤ Static and Dynamic DS</li><li>➤ Persistent Data Structures</li><li>➤ Linked Data Structures</li><li>➤ Succinct Data Structures</li><li>➤ Implicit Data Structures</li><li>➤ Compressed Data Structures</li><li>➤ Search Data Structures</li><li>➤ Recursion</li></ul>
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<b>Topic 3:</b> Linked List	3	<ul style="list-style-type: none"><li>➤ Singly Linked List</li><li>➤ Doubly Linked List</li><li>➤ Circularly Linked List</li></ul>
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<b>Topic 12:</b> Hashing	12	<ul style="list-style-type: none"><li>➤ Separate Chaining</li><li>➤ Open Addressing</li><li>➤ Linear Probing</li></ul>
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<b>Topic 15:</b> Quick Sort	15	<ul style="list-style-type: none"><li>➤ Divide &amp; Conquer</li><li>➤ Time Complexity</li></ul>
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# CSA03 – DATA STRUCTURES

## Concept Mapping

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Topic 6: Queue	6	Topic 13: Priority Queue	13
Topic 14: Sorting	14	➤ Heap ➤ Linear Probing	
Topic 7: Trees	7	➤ Insertion Sort ➤ Merge Sort ➤ Radix Sort	
Topic 15: Quick Sort	15	➤ Divide & Conquer ➤ Time Complexity	
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Topic 19: Minimum Spanning Tree (MST)	19	➤ Unweighted Shortest Path ➤ Dijkstra's Algorithm	
Topic 20: Graph Traversal	20	➤ Prim's Algorithm ➤ Kruskal's Algorithm	
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Topic 22: Research Area		➤ Future Scope ➤ Real Time Applications	
Topic 23: Research Area		➤ Research Area	
Topic 24: Stack	4	➤ Separate Chaining ➤ Open Addressing	
Topic 25: Stack Application	5	➤ Arithmetic Expression – Notations ○ Infix Notation ○ Prefix Notation ○ Postfix Notation	

# CSA 03 : DATA STRUCTURES



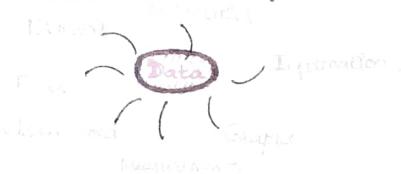
# Topic 1: Introduction to Data Structures

- Data
- Data Object
- Data Structures
- Abstract Data Types (ADT)
- Realization of ADT in C
- Primitive and Non-Primitive DS
- Linear and Non-Linear DS
- Static and Dynamic DS
- Persistent Data Structures
- Linked Data Structures
- Succinct Data Structures
- Implicit Data Structures
- Compressed Data Structures
- Search Data Structures
- Recursion

# TOPIC 1 - INTRODUCTION - DATA STRUCTURES

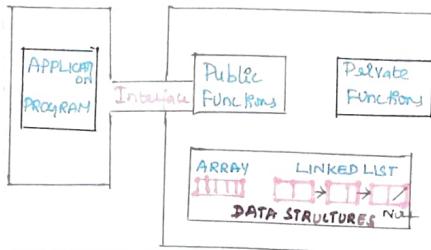
## DATA

- A Piece of Information



→ A header file Contains the ADT declaration

## STATIC AND DYNAMIC



→ Size of the Structure is fixed.

→ Content Cannot be modified without changing memory space.

→ Size of the Structure is not fixed.

→ Content Can be modified during the Operations performed on it.

## IMPLICIT DATA STRUCTURE

- Stores little information
- requires low overhead

## COMPRESSED DATA STRUCTURE

- Interception of Data Structure and Data Compression
- Used for Suffix Array

## SEARCH DATA STRUCTURE

- Efficient retrieval of Specific Items from a set of Items

- Specific record from a database

- String matching
- Text mining
- Text Summarization
- Document clustering
- Language modeling

## RECUSION

→ Technique of making a function call itself.

- ① Used with functions.
- ② Smaller Code Size
- ③ Memorizes When the box becomes true
- ④ Every recursive call needs extra space.

## TERMINOLOGY

- Factorial
- $\text{Factorial}(n) = \begin{cases} 1, & \text{when } n=0 \\ n \times \text{Factorial}(n-1), & \text{when } n>0 \end{cases}$
- $n! = n \times (n-1) \times (n-2) \times \dots \times 1$
- $4! = 4(3!) \dots 4(1) = 24$
- $3! = 3(2) \dots 3(1) = 6$
- $2! = 2(1) = 2$
- $1! = 1(0) = 1$



## Stack

→ Dinner Plates

## Queue

→ Waiting in line

## Graph

→ Google MAP

## DATA OBJECT

- Region of Storage
- Values / group of Values

Instance of Class → Structure

## DATA STRUCTURES

- Storage Used to store and Organize data.

⇒ Way of Organising data into Memory.

⇒ Can be accessed & Updated efficiently.

## ADT

### ABSTRACT DATA TYPE

→ Type for objects with well-defined interface

→ Set of Values & operations

→ to hide how the Operation is performed on the data.

## COLLECTION

→ homogenous

→ heterogeneous

→ same type eg: Array

→ different types eg: Java Stack

→ contains elements of

same type eg: Array

→ contains elements of different types eg: Java Stack

→ utilizes Computer memory efficiently

→ organized by references (Point or pointer)

→ Applications of Linear

→ Applications of Non-Linear

→ Artificial Intelligence

→ Image Processing

→ Software Development

→ Applications of Graph

→ Applications of Tree

→ Applications of Non-Tree

# Topic 2: Array

- ADT Array
- Operations on Arrays
- Searching Algorithms
  - ✓ Linear Search
  - ✓ Binary Search



# Topic 3: Linked List

- Singly Linked List
- Doubly Linked List
- Circularly Linked List

## Topic: 3. Linked List - Single, Doubly & Circular

### Single Linked List (SLL)

- Collection of nodes connected from head node to tail.
- Each node contains two fields.
  - Data
  - Address
- Connection in one direction.

Data      Address      Node  
 { int, char, }      { pointer }  
 { float }      { \*next }

- Linear data structure, in which nodes are created dynamically in memory & linked to the link.

### Operations in SLL

- In insertion, Deletion, Searching, Sorting, Updation & Modification.

### Insert at Begin:

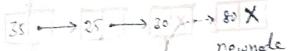
head → head



- Assign  $p = \text{head}$ .
- $\text{newnode} \rightarrow \text{next}$  is equal to  $p$ 's pointer.
- $\text{head}$  is equal to  $\text{newnode}$ .
- $\text{newnode} \rightarrow \text{next} \leftarrow p$ .

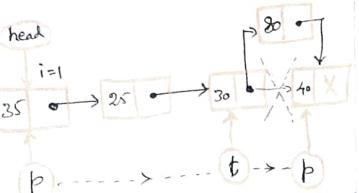
### Insert at End:

head



- Move the pointer from head to tail.
- $p \rightarrow \text{next} = \text{newnode}$ .

### Insert at any position:

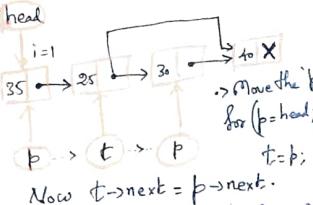


Assume: position = 4 ; ele = 80 (node value)

- Move the pointer  $p$ ' from head to position  $i=4$ .
- Also move  $t$  from head to its previous position.
- Now  $t \rightarrow \text{next} = \text{newnode}$ .
- $\text{newnode} \rightarrow \text{next} = p$ .

APPLICATION:  
FACEBOOK

### Delete at any position:



Assume:  
position = 3

- Move the  $p$ ' to its position.
- for ( $p = \text{head}; i=1; i < pos;$   
 $i++; p = p \rightarrow \text{next}, i++$ )
- Now  $t \rightarrow \text{next} = p \rightarrow \text{next}$ .

### Delete at End:

head



- Move  $t$ ' pointer from head to end node.
- With respect to make  $t'$  to its previous position.
- $t \rightarrow \text{next} = \text{NULL}$ .
- $\text{free}(p); \text{Delete the } p \text{ node}$ .

### Insert at any Position:



- Move  $p$ ' pointer from head to position  $i=3$ .
- Now  $t \rightarrow \text{next} = \text{newnode}$ ;
- $\text{newnode} \rightarrow \text{prev} = t$ ;
- $\text{newnode} \rightarrow \text{next} = p$ .

### Delete at any position:



- Move the pointer from head to position  $i=3$ .
- Now  $t \rightarrow \text{next} = p \rightarrow \text{next}$ ;
- $p \rightarrow \text{next} = t$ ;
- $\text{p} \rightarrow \text{prev} = \text{newnode}$ .

### Circular Single Linked List:



- Move the  $p$ ' from first to last.
- $p \rightarrow \text{next} = \text{head}$ .

### Circular Double Linked List:



- Move  $p$ ' to last node.
- $p \rightarrow \text{next} = \text{head}$ .
- $\text{head} \rightarrow \text{prev} = p$ .

### Insert at End:

head



newnode

- $p \rightarrow \text{next} = \text{last node}$ .
- $\text{p} \rightarrow \text{next} = \text{newnode}$ .
- $\text{newnode} \rightarrow \text{prev} = p$ .

### Delete at Begin:

head



- $p \rightarrow \text{head}$ ;
- $\text{head} = p \rightarrow \text{next}$ ;
- $\text{free}(p)$ ;
- $\text{head} \rightarrow \text{prev} = \text{NULL}$ ;

# Topic 4: Stack

- Array Implementation
- Linked List Implementation

## TOPIC - 4 STACK IMPLEMENTATION

**Stack:** An Abstract Data Type (ADT)  
 \* A linear DS in which the operations are performed at only one end - LIFO

\* Eg: Pile of coins, stack of Trays

\* **PRINCIPLE:** Last in First out LIFO

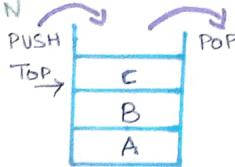
Lastly Inserted element will be removed first

**IMPLEMENTATION:** 2 Types

- ↳ ARRAYS (Fixed size DS)
- ↳ LINKED LIST - (Variable size DS)

**BASIC OPERATION**

- \* PUSH
- \* POP
- \* PEAK
- \* IS FULL
- \* IS EMPTY



**ARRAY IMPLEMENTATION**

**PUSH**: Process of inserting a new data element when stack is empty    TOP = -1

STEPS

- \* check if the stack is full (overflow)
- \* If the stack is full, display stack full and exit  
  [Top = Maxsize - 1]

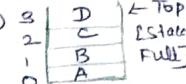
\* If stack is not full increment Top Pop++

+ Add data element to the stack    stack[Top] = Value



**IS FULL** - check if stack is full

If (top == Maxsize - 1)  
    return true;



**IS EMPTY** - check if stack is empty

If (top == -1)  
    return true;



**POP**: Process of removing top element from the stack

**steps**

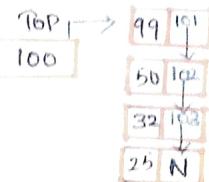
- \* check if stack is empty (underflow)
- \* If empty display stack empty  
    (Top = -1)
- \* If not empty decrements
- \* Top value by 1 (Top = Top - 1)
- Return

**PEEK**: Returns the value of top most element

**steps**

- check if the stack is empty (underflow)
- If empty displays "stack empty"  
    (Top = -1)
- If not empty, return the element  
    which TOP is pointing (Stack[Top])

**LINKED LIST IMPLEMENTATION**



**PUSH**: Process of inserting new element into the stack

\* Create a newNode with given value

\* check whether stack is empty  
    (Top == NULL)

\* If it is Empty then set  
    newNode → next = NULL

\* If it is Not Empty then set  
    newNode → next = Top

\* Finally, set Top = newNode

**POP**: Process of Deleting an Element from a Stack

\* Check whether stack is empty  
    (TOP == NULL)

\* If it is Empty then "stack is empty" Deletion is not possible

\* If it is Not Empty then define  
    a Node pointer 'temp' and  
    set it to Top

\* Then set Top = Pop - next  
Finally delete 'temp' (free(temp))

**PEEK**: Returns the value of top most element

\* check whether stack is empty  
    (Top == NULL)

\* If it is empty then "stack is empty" cannot display

\* If it is Not Empty then check  
    (Temp → next == NULL) and  
    display the element

# Topic 5: Stack Application

- Arithmetic Expression – Notations
  - ✓ Infix Notation
  - ✓ Prefix Notation
  - ✓ Postfix Notation
- Expression Conversion
- Expression Evaluation
- Balancing Symbols

## TOPIC - 5 (STACK APPLICATIONS)

### Arithmetical Expression -

#### Types of Notations:

##### \* Infix Notation - arithmetic

operator appears between operands stack.  
(Op1 operator Op2) (eg:- a+b)

##### \* Prefix Notation - arithmetic

operator appears before operands stack.  
(Polish Notation) (eg:- +ab)

##### \* Postfix Notation - arithmetic

operator appears after operands stack.  
(Reverse Polish Notation) (eg:- abt)

of the operator. If LTR(left to right) associativity, then pop the operator from stack. If RTL(right to left) associativity, then push into the stack.

\* if the input is '(', push it into

\* if the input is ')', pop out all operators until we find '('

Repeat step 1, 2 & 3 till expression has reach '#' [# - Delimiter].

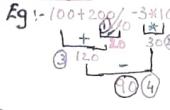
4) Now pop all the remaining operators from the stack and push into the output postfix expression.

5) Exit.

Example: A+(B\*(C+D))/E#

### Precedence of Operators & Associativity.

Operator	Precedence	Value
Exp(^)	Highest	3
* , /	Next High	2
+ , -	Lowest	1

Eg:- 100+200/(3\*10)  


### Algorithms for Infix-Postfix Conversion

1) Scan infix expression from left to right, if it is operand, output it into postfix expression.

2) If the input is an operator and stack is empty, push operator into operator's stack.

3) If the operator's stack is not empty

\* if the precedence of operator is greater than the top most op of stack, push it into stack.

\* if the precedence of operator is less than the top most operator of stack, pop it from the stack until we find a low precedence op.

\* if the precedence of operator is equal, then check the associativity

Op	stack	Output Postfix Exp	Action
A	A		Add A into o/p exp
+	A		Push '+' into stack.
(	(		Push '(' into stack
+	(		Add B into o/p exp.
B	(B		Push '*' into stack.
*	(B*		Add C into o/p exp
C	(B*C		Push '*' into stack.
+	(B*C*		+ operator has less Precedence than * & + add to o/p exp. Push '+'
+	(B+C*		+ operator has less Precedence than * & + add to o/p exp. Push '+'
D	(B+C*D		Add D into o/p exp
+	(B+C*D+		+ has come, so pop + & add it to o/p exp
)	(B+C*		-
/	(B+C*D/		/ has higher Precedence than +, so push '/' to stack
E	(B+C*D+E		Add E into o/p exp
#	(B+C*D+E#		#- delimiter, so pop all operators from the stack one by one and add it to o/p expression
+	(B+C*D+E/+		+ has higher Precedence than /, so push '+' to stack

Output: ABC\*D+E/+

### Evaluating Arithmetic Expressions

#### Algorithm Steps:

1) Scan the input string from left-right

2) Read 1 character at a time until it encounters the delimiter '#'

3) If the character is an opening symbol, push it onto the stack.

4) If the character is a closing symbol, and if the stack is empty then, later, push the result to stack.

5) If it is a closing symbol & if it has a corresponding opening symbol in the stack, Pop it from the stack.

Otherwise, error - Mismatched Symbol.

6) At the end of file, if the stack is not empty, report as "Unbalanced Symbol". Otherwise, report as "Balanced Symbol".

Example: (5+3)\*(8-2)  $\Rightarrow$  Infix

1) Convert into postfix expression.

$$5 \ 3 \ + \ 8 \ 2 \ - \ *$$

2) Evaluate the postfix expression.

Value 1 = Pop(1) 3  
 Value 2 = Pop(8) 8  
 result = value1 \* value2  
 Push(8)

Value 1 = Pop(5) 5  
 Value 2 = Pop(8) 8  
 result = value1 + value2  
 Push(8)

Value 1 = Pop(8) 8  
 Value 2 = Pop(8) 8  
 result = value1 - value2  
 Push(8)

Value 1 = Pop(5) 5  
 Value 2 = Pop(3) 3  
 result = value1 \* value2  
 Push(15)

Value 1 = Pop(15) 15  
 Value 2 = Pop(8) 8  
 result = value1 / value2  
 Push(1.875)

Value 1 = Pop(1.875) 1.875  
 Value 2 = Pop(2) 2  
 result = value1 - value2  
 Push(1.875)

Value 1 = Pop(1.875) 1.875  
 Value 2 = Pop(1) 1  
 result = value1 / value2  
 Push(1.875)

Value 1 = Pop(1.875) 1.875  
 Value 2 = Pop(1) 1  
 result = value1 - value2  
 Push(0)

Value 1 = Pop(0) 0  
 Value 2 = Pop(1) 1  
 result = value1 + value2  
 Push(1)

Value 1 = Pop(1) 1  
 Value 2 = Pop(1) 1  
 result = value1 \* value2  
 Push(1)

Value 1 = Pop(1) 1  
 Value 2 = Pop(1) 1  
 result = value1 - value2  
 Push(0)

Value 1 = Pop(0) 0  
 Value 2 = Pop(1) 1  
 result = value1 + value2  
 Push(1)

Value 1 = Pop(1) 1  
 Value 2 = Pop(1) 1  
 result = value1 \* value2  
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Value 1 = Pop(1) 1  
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Value 1 = Pop(1) 1  
 Value 2 = Pop(1) 1  
 result = value1 \* value2  
 Push(1)

Value 1 = Pop(1) 1  
 Value 2 = Pop(1) 1  
 result = value1 - value2  
 Push(0)

Value 1 = Pop(0) 0  
 Value 2 = Pop(1) 1  
 result = value1 + value2  
 Push(1)

Value 1 = Pop(1) 1  
 Value 2 = Pop(1) 1  
 result = value1 \* value2  
 Push(1)

Note:- if input is '0' (end), empty the stack.

### Bottom-up Parsing

#### Algorithm Steps:

1) Read 1 character at a time until it encounters the delimiter '#'

2) If the character is an opening symbol, push it onto the stack.

3) If the character is a closing symbol, and if the stack is empty then, later, push the result to stack.

4) If it is a closing symbol & if it has a corresponding opening symbol in the stack, Pop it from the stack.

Otherwise, error - Mismatched Symbol.

5) At the end of file, if the stack is not empty, report as "Unbalanced Symbol". Otherwise, report as "Balanced Symbol".

Example: (a+b) #  $\Rightarrow$  Balanced Symbol.

Step Read character Stack Step Read character Stack

1 ( ( ( ( ( (

3 + + + + + +

5 ) ) ) ) ) )

(a+b) #  $\Rightarrow$  Balanced Symbol.

Example: ((a+b) #  $\Rightarrow$  Unbalanced Symbol.

Step Read character Stack Step Read character Stack

1 ( ( ( ( ( (

3 + + + + + +

5 ) ) ) ) ) )

((a+b) #  $\Rightarrow$  Unbalanced Symbol.

Step Read character Stack Step Read character Stack

1 ( ( ( ( ( (

3 + + + + + +

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((a+b) #  $\Rightarrow$  Unbalanced Symbol.

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Step Read character Stack Step Read character Stack

1 ( ( ( ( ( (

3 + + + + + +

5 ) ) ) ) ) )

((a+b) #  $\Rightarrow$  Unbalanced Symbol.

Step Read character Stack Step Read character Stack

1 ( ( ( ( ( (

3 + + + + + +

5 ) ) ) ) ) )

((a+b) #  $\Rightarrow$  Unbalanced Symbol.

Step Read character Stack Step Read character Stack

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((a+b) #  $\Rightarrow$  Unbalanced Symbol.

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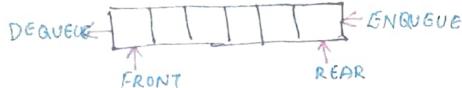
3 +

# Topic 6: Queue

- Array Implementation
- Linked List Implementation
- Circular Queue
- Applications

## TOPIC-6 (QUEUE)

- Queue ADT: Linear Data Structure
- \* Insertion : Rear End - ENQUEUE
- \* Deletion : Front - DEQUEUE
- \* First In First Out (FIFO)



→ Real Time Example:  
Bill Counter, Bank Counter, etc.

→ Applications:  
Printer, CPU task scheduling, etc.

→ Implementations:

- \* Array
- \* Linked List

→ Basic Operations :

\* Enqueue

\* Dequeue

\* Display

→ Implementation - Using Array.

Fixed Size DS.

Enqueue :

\* Before inserting, check whether Queue is full (Overflow Condition)

\* If full, display Queue Overflow. full  
Rear:N

\* If not full, element is added to the end of the Array at rear side.

Note: - N = maximum size of Queue.

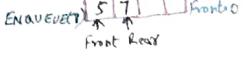
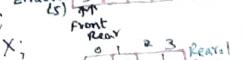
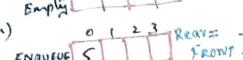
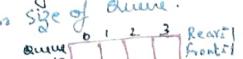
if (rear > maxSize)

Print ("Overflow")

{ Rear = Rear + 1;

Queue [Rear] = X;

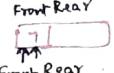
}



Dequeue:

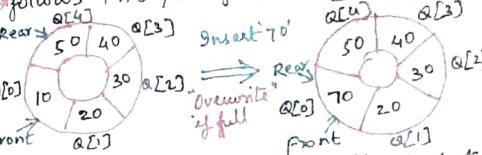
Note: - Before deleting, check whether Queue is empty (Underflow Condition)

- \* If empty, display Queue Underflow. Front: -1 (empty)
- \* If not empty, element in the front is deleted from the Queue and front is pointed to the next element in the Queue.
- DEQUEUE(S) [5 7] Front:front+1
- if (front == -1)  
Print ("Underflow")  
else { Front = Front + 1;  
}
- }



Front Rear

Circular Queue: Last node will be pointing to the first node Ring Buffer



Disadvantage:  
\* When the Circular Queue is full, it starts overwriting the existing values when insert.

→ Basic Operations: - Enqueue & Dequeue.

Priority Queue: Elements will be assigned by a 'priority' & will be processed based on order.  
\* Higher order priority element is processed before lower priority element.

→ Types:

\* Ascending Priority Queue

\* Descending Priority Queue

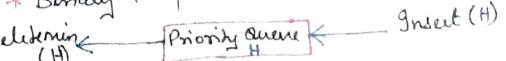
→ Basic Operations: Insert & Delete (Enqueue) (Delete)

→ Implementation:-

\* Linked List

\* Binary Search Tree (BST)

\* Binary Heap.



Double-Ended Queue (DEQUE)

Both insertion and deletion operations are performed at both the ends.



Queue Applications:

- \* Operating Systems - Job Scheduling
- \* CPU Scheduling
- \* Printer Spooler.

Advantages over Arrays:  
Dynamic Size, Easy to insert and delete.

Drawbacks

\* Random access not allowed.

\* Utilization of memory space is more.

Types:

- \* Circular Queue
- \* Priority Queue
- \* DEQUE

# Topic 7: Trees

- Preliminaries
- Binary Tree
- Binary Search Tree
- ✓ Tree Traversal



# Topic 8: AVL Trees

- Single Rotations
  - ✓ Left-Left (LL)
  - ✓ Right-Right (RR)
- Double Rotations
  - ✓ Left-Right (LR)
  - ✓ Right-Left (RL)



# Topic 9: Applications of Search Trees

- B-Tree
- TRIE
- 2-3 Tree
- 2-3-4 Tree

## TOPIC-9 (APPLICATIONS OF SEARCH TREES - B-TREE - TRIE - 2-3-4 Tree)

### TRIE

→ All the search trees are used to store the collection of numerical values but they are not suitable for storing the collection of words or strings.

→ TRIE data structures makes retrieval of a string from the collection of strings more easily.

→ Term 'TRIE' comes from the word retrieved key values

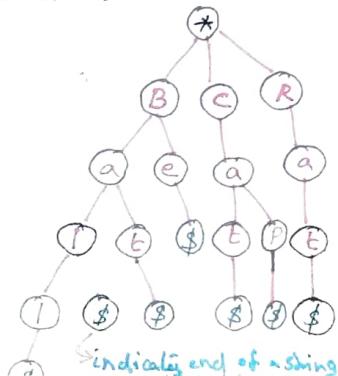
→ It is a special kind of tree that is used to store the dictionary.

→ It is a fast & efficient way for dynamic spell checking.

→ Every node in Trie can have one or a number of children.

→ All the children of a node are alphabetically ordered. If any two strings have a common prefix then they will have the same ancestors.

Eg: Consider the following list of strings to construct Trie Cat, Bat, Ball, Rat, Cap & Be



### B-TREE (2-3 Tree, 2-3-4 Tree)

→ Self Balancing Search Tree.

→ Disk access time is very high compared to main memory access time.

→ The main idea of using B-tree is to reduce the number of disk accesses.

→ Time complexity for Insert, Delete - O(log n)

→ All leaves are at the same level.

→ m order can have m children  $2^m - 1$

### Types:

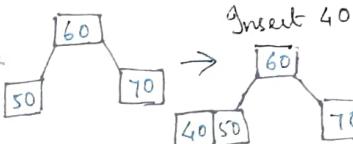
2-3, 2-3-4, 2-3-4-5, ... and so on.

### 2-3 Tree.

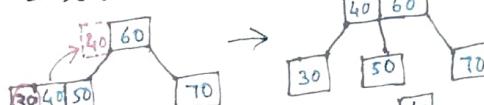
→ B tree of order 3

→ Each node has either 2 or 3 children & almost 2 key values.

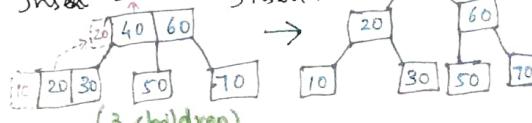
Ex: Insert 50, 60, 70, 40, 30, 20  
Insert 50      Insert 60      Insert 70



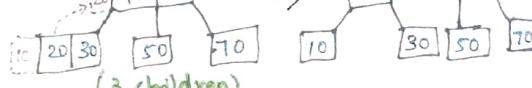
### Insert 30



### Insert 20



### Insert 10



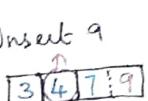
### 2-3-4 Tree

→ B tree of order 4

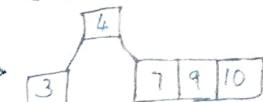
→ Each node has either 2/3/4 children & almost 3 key values.

Ex: Insert 3, 7, 4, 9, 10, 0, 5, 6, 8, 2  
Insert 3      Insert 7      Insert 4

Insert 9



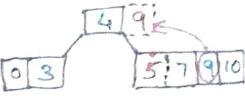
Insert 10



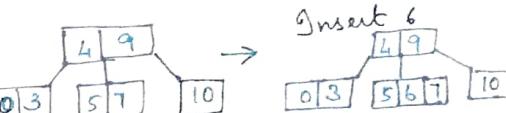
Insert 0



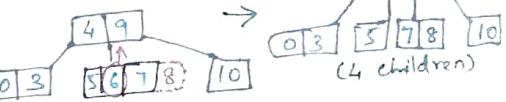
Insert 5



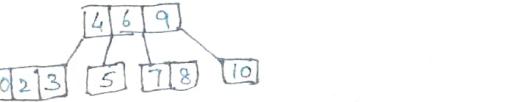
Insert 6



Insert 8



Insert 2



indicates end of a string

# Topic 10: Red- Black Tree

- Properties
- Operations

# TOPIC: 10 RED

## RED-BLACK TREE;

BST in which every node is colored either **RED** or **BLACK**

### Properties

- BST → New node - **RED**
- Root Node - **BLACK**
- No 2 consecutive **RED** nodes
- All paths same no of **BLACK** nodes
- Leaf node - **BLACK**

Insertion Need to satisfy properties if not, perform operations to make it **RED-BLACK** Tree

1. Recolor
2. Rotations
3. Rotations followed by Recolor

### STEPS

- check Pile is empty
- Empty insert Newnode as **Root (BLACK)**
- Not empty insert New node as a **Leaf node (RED)**
- Parent (New Node) **BLACK** exit
- Parent (NewNode) **RED** check Color of

parent node's sibling of New Node

Sibling **BLACK** or **NULL**  
Rotate and Recolor it

Sibling **RED** Recolor repeat  
the steps until tree becomes **RED-BLACK** Tree

Example: Create **RED-BLACK** Tree by inserting 8, 18, 15, 15,  
17, 25, 40, 80

## Insert 8

⑧ **NewNode BLACK**

## Insert 18

Tree Not empty  
New Node **RED**

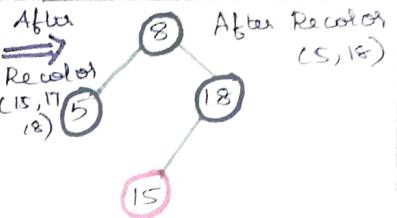
## Insert 5

Tree Not empty  
New Node **RED**

## Insert 15

Tree not empty  
New Node **RED**  
18, 15 (**RED**)  
(consecutive)  
New Node Parent  
Sibling **RED** (5)  
Recolor  
Note: Dont Recolor  
if Parent - Root  
(**BLACK**)

## BLACK TREE



After Recolor  
(5, 18)

## Insert 17

Tree not empty  
New Node **Red**  
15, 17 **RED** (consecutive)  
New Node Parent  
Sibling **NULL**  
Rotate & Recolor

After Left Rotation

15

17

8

5

18

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# Topic 11: Splay Trees

- Zig Rotations
- Zag Rotations
- Zig-Zig Rotations
- Zag-Zag Rotations
- Zig-Zag Rotations
- Zag-Zig Rotations

## TOPIC: 11

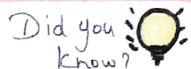
### Splay Tree

Splay tree is a self-adjusted binary search tree in which every operation on an element rearranges the tree so that element is placed at the root position of the tree.

Splaying an element in the process of bringing it to the root position by performing suitable rotation operation.

### Rotation in Splay Tree

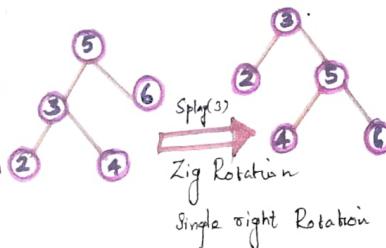
- \* Zig Rotation
- \* Zag Rotation
- \* Zig-Zig Rotation
- \* Zag-Zag Rotation
- \* Zig-Zag Rotation
- \* Zag-Zig Rotation



**Splay Net** [Facebook]

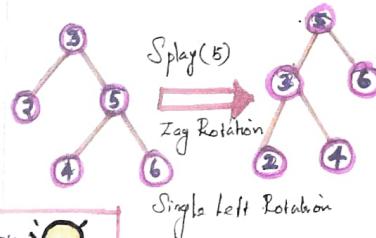
### Zig Rotation:

The Zig Rotation in a Splay tree is a single right rotation in AVL tree rotation. In Zig rotation every node moves one position to the right from its current position.



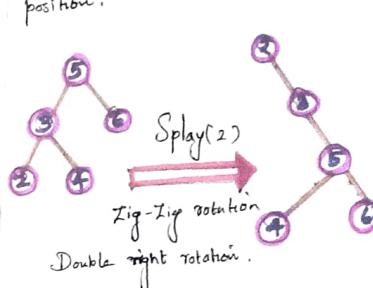
### Zag Rotation:

The Zag Rotation in a Splay tree is similar to the single left rotation in AVL tree rotations. In Zag rotation every one moves one position to the left from its current position.



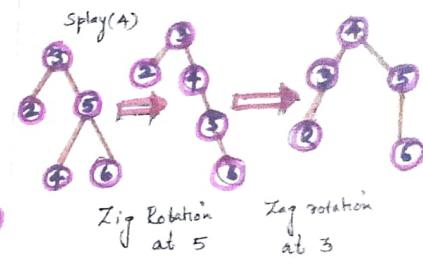
### Zig-Zig Rotation:

The Zig-Zig rotation in a Splay tree is a double Zig rotation. In Zig-Zig rotation every node moves two positions to the right from its current position.



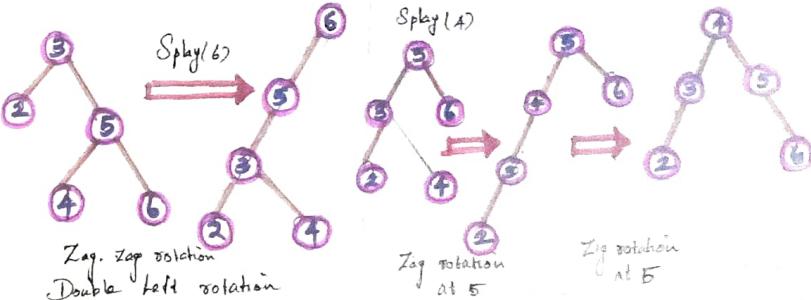
### Zig-Zag Rotation:

The Zig-Zag rotation in a Splay tree is a sequence of Zig-Zag rotation where every node moves one position to the right followed by one position to the left from its current position.



### Zag-Zig Rotation:

The Zag-Zig rotation in a Splay tree is a double Zag rotation followed by Zig rotation. In Zag-Zig rotation every node moves one position to the left followed by one position to the right from its current position.



## Topic: 21

### Splay tree

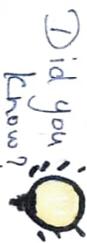
Splay tree is a self-adjusted binary search tree in which every operation on an element rearranges the tree so that element is placed at the root position of the tree.

Splaying an element in the process of bringing it to the root position by performing suitable rotation operation.

### Rotation in Splay Tree

- \* Zig Rotation
- \* Zag Rotation
- \* Zig-Zig Rotation
- \* Zag-Zag Rotation
- \* Zag-Zig Rotation

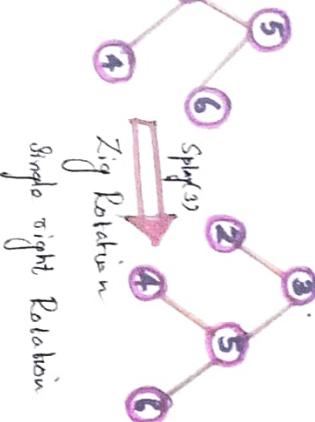
- \* Splay(3)
- \* Splay(2)
- \* Splay(1)
- \* Splay(6)
- \* Splay(5)



### SplayNet [Facebook]

### Zig Rotation

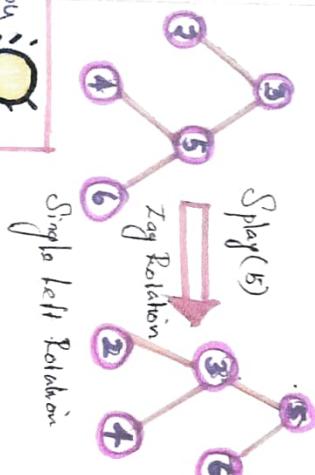
The Zig Rotation in a Splay tree is in Avl tree rotation. In Zig rotation every node moves one position to the right from its current position.



### Zig Rotation

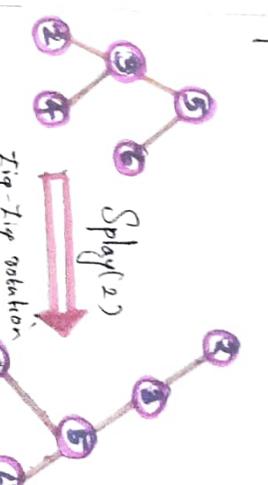
The Zig Rotation in a Splay tree is similar to the Single Left Rotation in Avl tree rotations.

In zig rotation every one moves one position to the left from its current position.



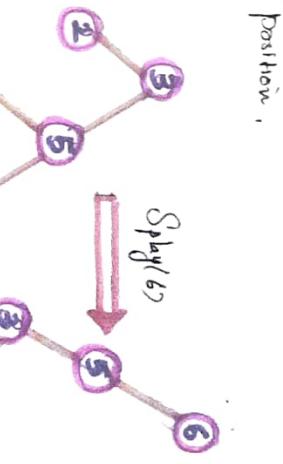
### Zig-Zig Rotation

The Zig-Zig rotation in a Splay tree is a double zig rotation of zig-zag rotation every node moves one position to the right followed by one position to the left from its current position.



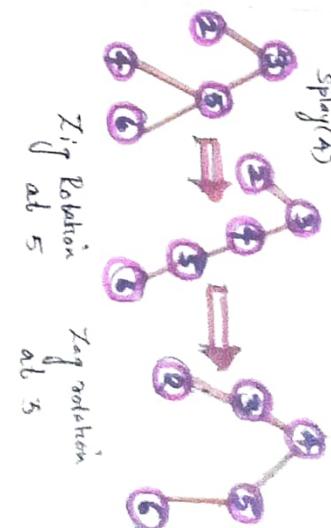
### Zig-Zig Rotation

In a splay tree is a double zig-zag rotation. In zig-zig rotation every node moves two position moves one position to the left followed by one position to the right from its current position.



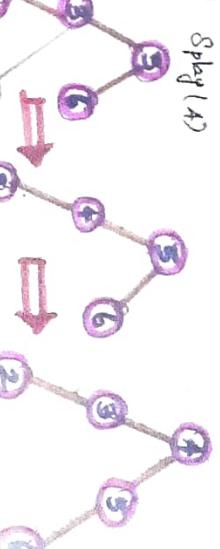
### Zig-Zag Rotation

The Zig-Zag rotation in a Splay tree is a sequence of zig-zag rotation every node moves one position to the right followed by one position to the left followed by one position to the right from its current position.



### Zig-Zag Rotation

The zig-zag rotation in a splay tree is a sequence of zig-zag rotation followed by zig rotation. In zig-zag rotation every one moves one position to the left followed by one position to the right from its current position.



### Zig-Zag Rotation

Zig-Zag rotation

Double left rotation

### Zig Rotation

Zig rotation

at 5

# Topic 12: Hashing

- Hash Function
- Separate Chaining
- Open Addressing
- Linear Probing

## Hashing

Hash Table Data Structure - Arrays of keys

Key - Strings with associated value.

Hashing - Implementation of Hash Table.

Hash Function - Mapping each Key into

Hash Table index with size  $\{1, 2, 3, \dots\}$

H3izo - Hash Table Size

Example:

Index	Value	Key
0	10	John
1	0	Mark
2	1	David
3	0	Marc

H3izo = 4

Hash Function - Types

+ Division Method:

$$\text{Hash}(k) = k \% \text{TableSize} \quad (0 \leq k < \text{TableSize})$$

$$Ex - \text{hash}(92) = 92 \bmod 10 \quad (\text{TableSize} = 10)$$

+ Mod-Binaries Method: Key is squared and mod part of result forms index.

$$Ex - \text{hash}(3101) = 3101 * 3101 = 9616201 \Rightarrow 162$$

+ Digit-Folding Method: Key is divided into separate parts & combine all by using some operations.

$$Ex - \text{hash}(12465512) = 12 + 4 + 6 + 5 + 12 = 79$$

Collision: Hash function returns same for two or more than one record.

hash key for more than one record.

$$Ex - \text{hash}(42) = 42 \% 10 = 2$$

$$\text{hash}(37) = 37 \% 10 = 7$$

$$\text{hash}(75) = 75 \% 10 = 5$$

$$\text{hash}(12) = 12 \% 10 = 2$$

TableSize = 10

10 : 75

2 : 42

↳ collision occurs

Collision Resolution Techniques are

Used to reduce Collisions.

## TOPIC-12 (HASHING - HASH FUNCTION - SEPARATE CHAINING - OPEN ADDRESSING)

Collision Resolution Techniques

Linear Probing

Address =  $(k \% \text{TableSize}) \% \text{TableSize}$

Up to key  $k \rightarrow$  TableSize;  $i \rightarrow$  Iteration

Ex: Input Keys = {23, 32, 20, 28, 67}

TableSize = 10

Address = 23, 32, 20, 28, 67

Index = 2, 3, 1, 2, 6

1, 2, 3, 4, 5, 6, 7, 8, 9, 10

23, 32, 20, 28, 67, 23, 32, 20, 28, 67

2, 3, 4, 5, 6, 7, 8, 9, 10

23, 32, 20, 28, 67, 23, 32, 20, 28, 67

3, 4, 5, 6, 7, 8, 9, 10

23, 32, 20, 28, 67, 23, 32, 20, 28, 67

4, 5, 6, 7, 8, 9, 10

23, 32, 20, 28, 67, 23, 32, 20, 28, 67

5, 6, 7, 8, 9, 10

23, 32, 20, 28, 67, 23, 32, 20, 28, 67

6, 7, 8, 9, 10

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7, 8, 9, 10

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8, 9, 10

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# Topic 13: Priority Queue

- Heap
- Heap Sort

## TOPIC-13 (PRIORITY QUEUE, HEAP & HEAP SORT)

### Priority Queue:

\* Processing of an object based on priority.

### Priority Heap

#### Implementation:

→ Linked List

→ Binary Search Tree

→ Binary Heap



Enqueue Dequeue

### Binary Heap : Heap DS

Created using binary tree.

Heapify: process of creating binary heap DS either in Min. Heap or Max. Heap.

#### Types:

→ Max. Heap (parent > child)

→ Min. Heap (parent < child)

#### 3 properties:

→ Shape property

↳ Complete Binary Tree

→ Heap property

↳ Max. Heap or Min. Heap

#### Operations

Build, Insert, Delete.

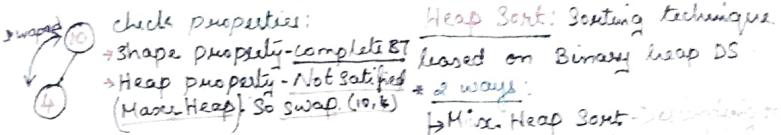
### Example: Build Binary

Heap by inserting {4, 10, 3, 5, 1} tree 1.

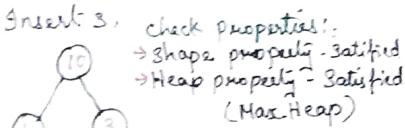
#### Insert 4:



swap  
Yes



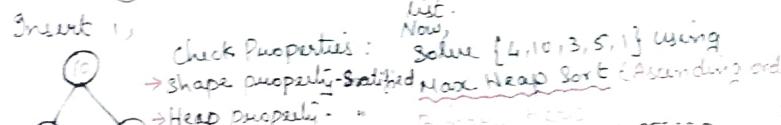
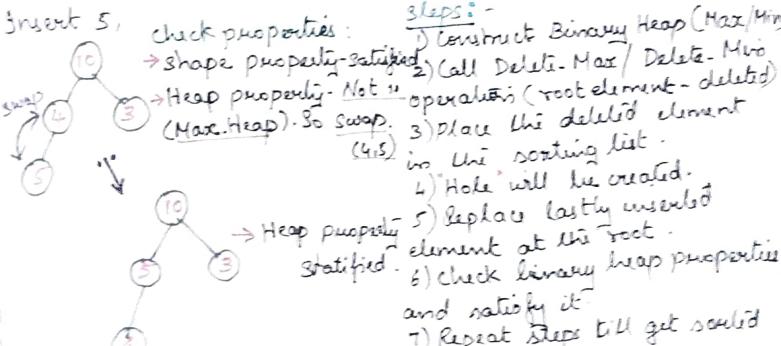
Heap Sort: Sorting technique based on Binary Heap DS  
→ 2 ways:  
↳ Min. Heap Sort - Descending order  
↳ Max. Heap Sort - Ascending order



### Max. Heap Sort:

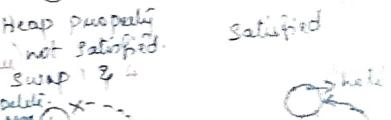
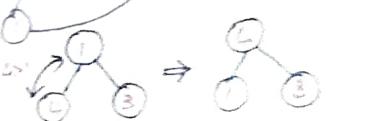
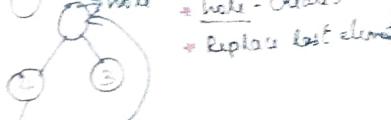
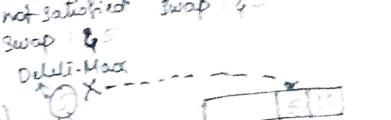
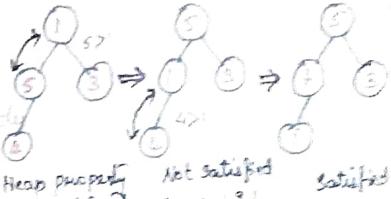
uses Delete-Max operation.

Example: Heap Sort - {4, 10, 3, 5, 1}



→ hole will be created.  
→ lastly inserted element will be replaced.

In Max. Heap, maximum element will be at root.  
In Min. Heap, minimum element will be at root.



# Topic 14: Sorting

- Insertion Sort
- Merge Sort
- Radix Sort

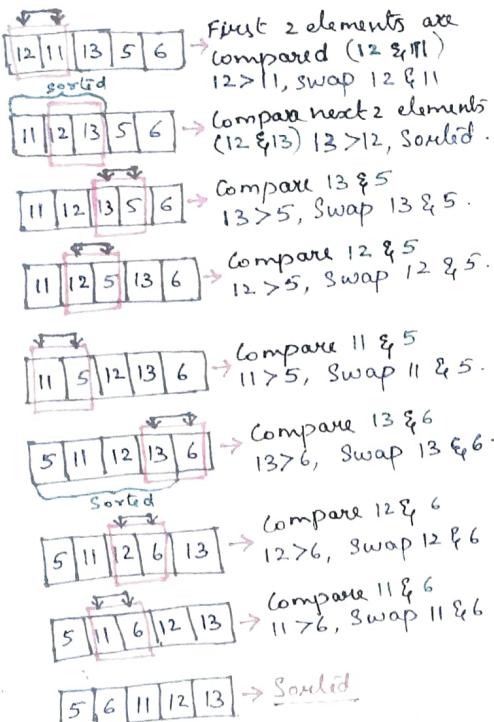
## TOPIC-14 (INSERTION SORT, MERGE SORT, RADIX SORT)

### Insertion Sorting:

- \* Simple
- \* Efficient for small data values.

Example: Sort 12, 11, 13, 5, 6 Using

Insertion sort.



Best Case Time Complexity -  $O(n)$

Worst Case Time Complexity -  $O(n^2)$

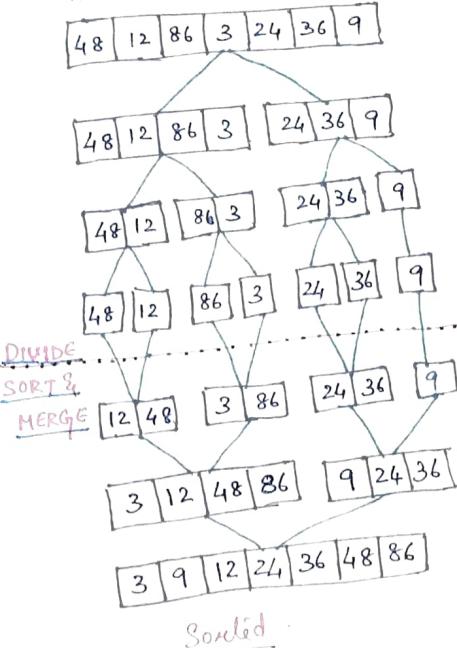
Average Case Time Complexity -  $O(n^2)$

### Merge Sort:

- \* Divide & Conquer Method
- \* Recursively dividing the array into 2 halves, sort & then merge

Example: - Sort 48, 12, 86, 3, 24, 36, 9

Using merge sort.



Time Complexity -  $O(n \log n)$

### Radix Sort:

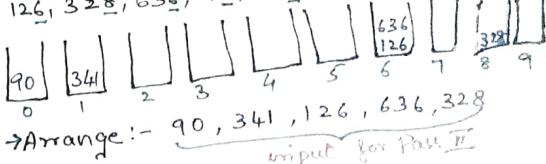
- \* Non Comparative integer sorting algorithm
- \* Digit-by-digit sorting (Sorting done from least significant digit)

Example: - Sort 126, 328, 636, 90, 341

Using Radix Sort.

Pass I: Consider the 1's place and keep it in respective buckets (0-9)

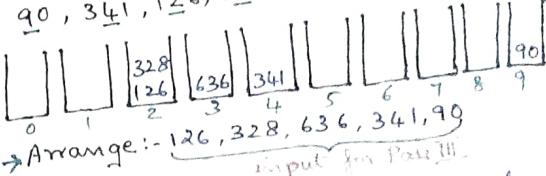
126, 328, 636, 90, 341



→ Arrange: - 90, 341, 126, 636, 328  
input for Pass II

Pass II: Consider the 10's place and keep it in respective buckets (0-9)

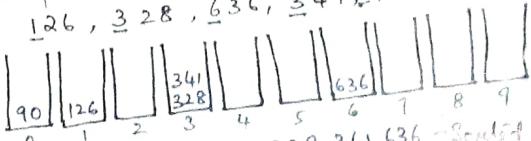
90, 341, 126, 636, 328



→ Arrange: - 126, 328, 636, 341, 90  
input for Pass III

Pass III: Consider the 100's place and keep it in respective buckets (0-9)

126, 328, 636, 341, 90



→ Arrange: - 90, 126, 328, 341, 636 - Sorted!

Note: - No. of digits = No. of Passes

Time Complexity -  $O(dn)$   
( $n \rightarrow$  array size,  $d \rightarrow$  no. of digits)

# Topic 15: Quick Sort

- Divide & Conquer
- Time Complexity

## TOPIC - 15 (QUICK SORT)

### Quick Sort:

\* Divide & Conquer Method

Divide - partition the array into 2 sub-arrays

Conquer - recursively, sort 2 subarrays.

Combine - combine the already sorted array.

\* choosing a pivot element [Mean, Median, First, Last]

Steps :-

Step 1: choose a pivot element from the list.

Step 2: Define 2 variables,  $i \rightarrow i$  &  $j \rightarrow$  last value

Step 3: Increment  $i$  until  $\text{list}[i] > \text{pivot}$ , then stop

Step 4: Decrement  $j$  until  $\text{list}[j] < \text{pivot}$ , then stop

Step 5: If  $i \leq j$ , then swap  $\text{list}[i]$  and  $\text{list}[j]$

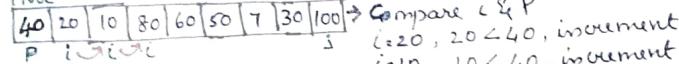
Step 6: Repeat steps 3, 4, 5 until  $i > j$ .

Step 7: If  $i > j$  (crossed), then swap crossed index with pivot element.

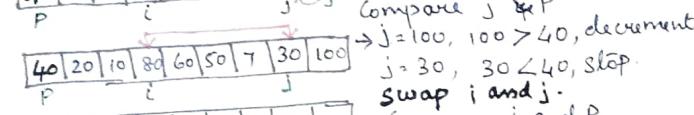
\* Partition Result  $\rightarrow$  left element  $<$  pivot  $<$  right element

Step 8: Choose another pivot element & repeat the steps till gets sorted.

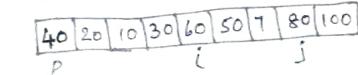
Example: Sort 40, 20, 10, 80, 60, 50, 7, 30, 100 Using Quick sort. pivot ( $P$ )  $\rightarrow$  40,  $i \rightarrow 20$ ,  $j \rightarrow 100$  & consider first

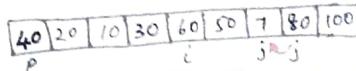
 Compare  $i$  &  $P$   
 $i = 20, 20 < 40$ , increment  
 $i = 10, 10 < 40$ , increment

  $i = 80, 80 > 40$ , stop

 Compare  $j$  &  $P$   
 $j = 100, 100 > 40$ , decrement  
 $j = 30, 30 < 40$ , stop  
 Swap  $i$  and  $j$ .

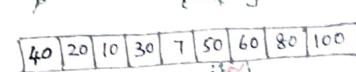
 Compare  $i$  and  $P$   
 $i = 30, 30 < 40$ , increment  
 $i = 60, 60 > 40$ , stop



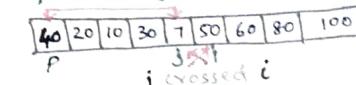
 Compare  $j$  and  $P$   
 $j = 80, 80 > 40$ , decrement  
 $j = 7, 7 < 40$ , stop  
 Swap  $i$  and  $j$

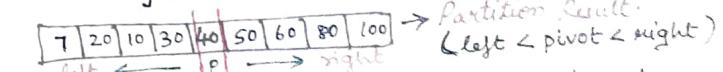


Compare  $i$  and  $P$   
 $i = 7, 7 < 40$ , increment  
 $i = 50, 50 > 40$ , stop

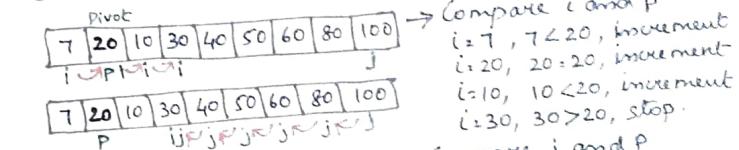
 Compare  $j$  and  $P$   
 $j = 60, 60 > 40$ , decrement  
 $j = 50, 50 > 40$ , decrement

$j = 7, 7 < 40$ , stop  
 Swap  $j$  and  $P$  ( $i > j$ , crossed)

  $j$  crossed  $i$

 Partition Result:  
 $(\text{left} < \text{pivot} < \text{right})$

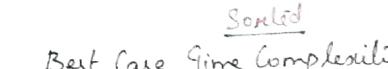
choose another pivot element ( $P = 20$ ),  $i \rightarrow 7$ ,  $j \rightarrow 100$

 Pivot  
 $i = 20, 20 < 20$ , increment  
 $i = 10, 10 < 20$ , increment  
 $i = 30, 30 > 20$ , stop

Compare  $j$  and  $P$   
 $j = 100, 100 > 20$ , decrement  
 $j = 80, 80 > 20$ , decrement  
 $j = 60, 60 > 20$ , decrement  
 $j = 50, 50 > 20$ , decrement

$j = 40, 40 > 20$ , decrement  
 $j = 30, 30 > 20$ , decrement  
 $j = 10, 10 < 20$ , stop

Swap  $j$  and  $P$  ( $i > j$ , crossed)



Sorted

Best Case Time Complexity -  $O(n \log n)$

Worst Case Time Complexity -  $O(n^2)$

Average Case Time Complexity -  $O(n \log n)$

# Topic 16: Sorting

- Bubble Sort
- Selection Sort
- Time Complexity

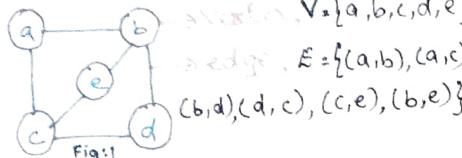


# Topic 17: Graph

- Terminologies
- Types
- Topological Sort

# TOPIC - 17 (GRAPH - TERMINOLOGIES - TYPES - TOPOLOGICAL SORT)

Graphs:  $G = (V, E)$



Terminologies: (Refer Fig 1)

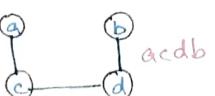
Adjacent vertices connected by an edge  $a \rightarrow b, c; c \rightarrow a, e, d$

Length  $\rightarrow$  No. of edges.

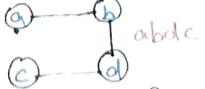
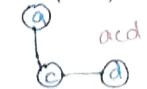
Degree  $\rightarrow$  No. of adjacent vertices

Degree of  $a=2$ , Degree of  $b=3$

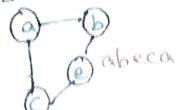
Path  $\rightarrow$  Sequence of vertices.



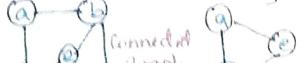
Simple path  $\rightarrow$  No repeated vertices.



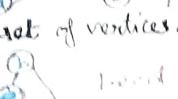
Cycle  $\rightarrow$  Last & first vertex - Same.



Connected Graph  $\rightarrow$  2 vertices connected by some path.



SubGraph  $\rightarrow$  Subset of vertices, edges



Connected Components - max. connected Subgraph.

Directed Graph (Digraph):

specify directions in edges.

Undirected Graph:

No directed edges.

Weighted Graph:

have weights in edges

Complete Graph:

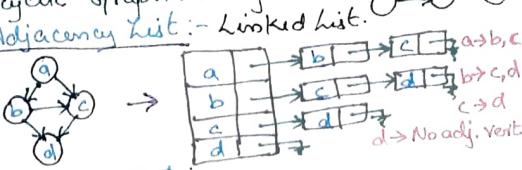
all vertices are connected to each other (Undirected Graph)

Strongly Connected Graph:

all vertices are connected in both directions (Directed Graph)

Acyclic Graph: No cycles.

Adjacency List: - Linked list.



Adjacency Matrix:

	1	2	3	4
a	0	1	1	0
b	1	0	1	1
c	2	1	0	0
d	2	1	0	0

\* 4 Vertices  
\* 4x4 Matrices

Topological Sort: used in Directed Acyclic Graph (DAG) - directed & No cycles.

\* Sorting - DAG.

\* if  $u \rightarrow v$ , then  $v$  appears after  $u$ .

Algorithm steps:

$\rightarrow$  Compute indegrees of all vertices

$\rightarrow$  Find vertex  $U$  with indegree 0, place in ordered list.

$\rightarrow$  Remove  $U$  and its edges ( $U, V$ )

$\rightarrow$  Update indegree of remaining vertices

$\rightarrow$  Repeat till all vertices are in ordered list.

Example:

Adjacency Matrix:

	a	b	c	d
a	0	1	1	0
b	0	0	1	1
c	0	0	0	1
d	0	0	0	0

$\text{indegree}[a] = 0$ ,  $\text{indegree}[b] = 1$   
 $\text{indegree}[c] = 2$ ,  $\text{indegree}[d] = 2$

Vertex	1	2	3	4
a	0			
b	1			
c	2			
d	2			

\* Enqueue  $a$   
\* Dequeue  $a$  from list.  
\* Remove adj. edges of  $a \rightarrow b, c, d$ .

Vertex	1	2	3	4
a	0	0	0	0
b	1	0	0	0
c	2	1	0	0
d	2	1	0	0

\* Enqueue  $ab$   
\* Dequeue  $ab$  from list.  
\* Remove adj. edges of  $b \rightarrow c, d$ .

Vertex	1	2	3	4
a	0	0	0	0
b	1	0	0	0
c	2	1	0	0
d	2	1	0	0

\* Enqueue  $c$   
\* Enqueue  $c$   
\* Dequeue  $c$  from list.  
\* Remove adj. edges of  $c \rightarrow d$ .

Vertex	1	2	3	4
a	0	0	0	0
b	1	0	0	0
c	2	1	0	0
d	2	1	0	0

\* Enqueue  $c$   
\* Dequeue  $c$  from list.  
\* Remove adj. edges of  $c \rightarrow d$ .  
\* Enqueue  $d$   
\* Dequeue  $d$  from list.  
\* No adj. edges of  $d$ .  
\* Searched.

# Topic 18: Shortest Path Algorithms

- Unweighted Shortest Path
- Dijkstra's Algorithm

# TOPIC-18 SHORTEST PATH ALGORITHMS - UNWEIGHTED SHORTEST PATH Dijkstra's Algorithm

## Shortest Path Algorithms

→ Finds minimum cost from source to other vertex.

### TYPES:

→ Single Source Shortest Path (SSSP)

→ All Pairs shortest Path (APSP)

→ Shortest Path between all pairs of vertices.

→ Floyd Warshall Johnson

## Single Source Shortest Path

→ Unweighted

Known → Visited (known)

Not Visited (not known)

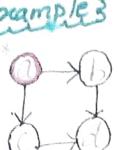
dV → Distance from source initially edge of weight  $\infty$

PV → Actual Path

### Steps:

1. Source Node - 'S', Enqueue
2. Dequeue 'S', is known as 1 and find its adjacency vertex.
3. Distance, dV → Source vertex  
distance increment by 1 & enqueue the vertex.
4. Repeat from step 2 until queue becomes empty.

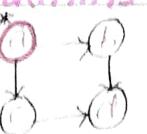
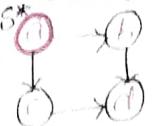
Initial configuration



A → Source Node

Enqueue :

Dequeue :



V	Known	dV	PV
a	1	0	0
b	0	$\infty$	0
c	0	$\infty$	0
d	0	$\infty$	0

Enqueue : a

Dequeue : a

V	Known	dV	PV
a	0	0	0
b	0	$\infty$	0
c	0	$\infty$	0
d	0	$\infty$	0

Dijkstra's Algorithm (Weighted)

→ Single Source shortest path algorithm

→ Similar to unweighted SSSP, Needs to calculate the distance using weights.

Example



# Topic 19: Minimum Spanning Tree (MST)

- Prim's Algorithm
- Kruskal's Algorithm

## Minimum Spanning Tree (MST)

Spanning Tree - Connected Graph,  $\delta$  No cycles

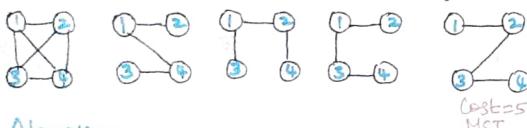
Subgraph with all Vertices

Minimum Spanning Tree - Spanning tree

With minimum cost.

$n^{n-2}$  Spanning tree will obtain for  $n$  nodes/ vertex

Example: 4 Nodes [ $n^{n-2} = 4^2 = 16$  Spanning trees]



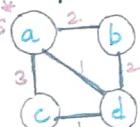
Algorithms:-

- \* Prim's Algorithm
- \* Kruskal's Algorithm

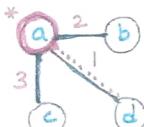
Prim's Algorithm:-

- ↳ Use Greedy Techniques
- ↳ Based on Vertices.

Example:-

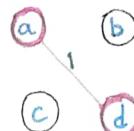


V	KNOWN	dv	Pv
a	0	0	0
b	0	2	0
c	0	2	0
d	0	2	0

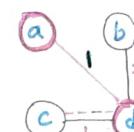


$a \rightarrow d$  (0) minimum distance  
 $d \rightarrow$  KNOWN (1), path from 'a'

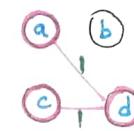
## TOPIC-19 MINIMUM SPANNING TREE - PRIM'S & KRUSKAL'S ALGORITHM



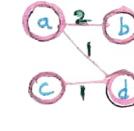
V	KNOWN	dv	Pv
a	1	0	0
b	0	2	a
c	0	3	a
d	1	1	a



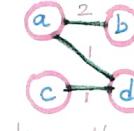
V	KNOWN	dv	Pv
a	1	0	0
b	0	2	a
c	0	1	d
d	1	1	a



V	KNOWN	dv	Pv
a	1	0	0
b	0	2	a
c	1	1	d
d	1	1	a



V	KNOWN	dv	Pv
a	1	0	0
b	1	2	a
c	1	1	d
d	1	1	a



V	KNOWN	dv	Pv
a	1	0	0
b	1	2	a
c	1	1	d
d	1	1	a

Minimum Spanning Tree

Minimum Cost:

$$C_{a,b} + C_{a,d} + C_{c,d} = 2+1+1 = 4$$

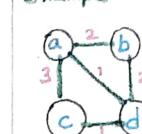
Kruskal's Algorithm:-

- ↳ Use Greedy Techniques.
- ↳ Based on Edges.

Steps:

- 1) Remove all loops & parallel edges (high cost).
- 2) Arrange all edges in Ascending Order.
- 3) Add the edges which has the least cost.

Example:



Edge	dv
(a,b)	2
(a,c)	3
(a,d)	1
(b,d)	2
(c,d)	1

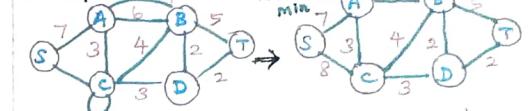
Edge	dv
(a,b)	1
(a,c)	2
(a,d)	3
(b,d)	2
(c,d)	1

Sort edges in Ascending order

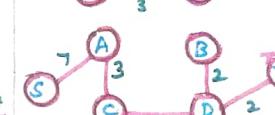
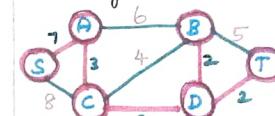
Minimum Spanning Tree  
Minimum cost:

$$C_{a,b} + C_{a,d} + C_{c,d} = 2+1+1 = 4$$

Example:-



→ Arrange remaining edges in Ascending order.



Minimum Spanning Tree

$$\text{Minimum Cost: } C_{S,A} + C_{A,C} + C_{C,D} + C_{D,B} + C_{D,T} = 7+3+3+2+2 = 17$$

Applications:-

Google Maps



# Topic 20: Graph Traversal

- Breadth First Search
- Depth First Search

## TOPIC-20 (BREADTH FIRST SEARCH, DEPTH FIRST SEARCH)

Breadth First Search (BFS)

\* Breadth Breadth-wise

\* Go Source (empty) Queue Application:

\* FIFO Non-visited require front elem. to be more

\* All vertices Algorithm to visited list

\* Minimum Spanning Tree Enqueue Adj. vertices to queue (0)

\* Shortest path Adj. vertices to queue (0)

Visited & not visited

Algorithm steps:

1) Create Queue-size (No of vertices)

2) Enqueue any one (source) Vertex - Visited list

3) Enqueue front item from Queue to Visited list.

Also, enqueue its adjacent

vertices to Queue.

4) Repeat the steps till Queue

is empty.

Example:

0 → 3 → Visited

0 → 2 → Visited

0 → 1 → Visited

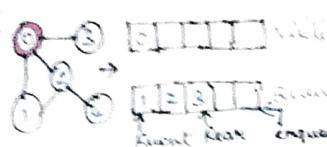
0 → 4 → Visited

0 → Visited

Enqueue 0 → Visited

Adjacent vertices (0) → 1, 2, 3

Queue



Front rear enqueue

Enqueue 1 → queue

Adj. vertices (1) → queue (0)

Visited & not visited

Depth First Search (DFS)

\* Visited Depth-wise top-bottom

\* Visited concept (450)

Applications:

\* Topological sorting

\* Detect cycles in the graph

\* Minimum Spanning Tree

\* Shortest path

\* Algorithm steps

1) Create stack - size (No of vertices)

2) Insert any one (source) vertex - Visited list

3) Push top item from stack to Visited list

4) Adjacent vertices (0) → 1, 2, 3

5) Push top item from stack to Visited list

6) Adjacent vertices (1) → 2, 3

7) Adjacent vertices (2) → 3

8) Adjacent vertices (3) → 0

9) Adjacent vertices (4) → 0

10) Adjacent vertices (5) → 0

11) Adjacent vertices (6) → 0

12) Adjacent vertices (7) → 0

13) Adjacent vertices (8) → 0

14) Adjacent vertices (9) → 0

15) Adjacent vertices (10) → 0

16) Adjacent vertices (11) → 0

17) Adjacent vertices (12) → 0

18) Adjacent vertices (13) → 0

19) Adjacent vertices (14) → 0

20) Adjacent vertices (15) → 0

21) Adjacent vertices (16) → 0



Pop '1' (stack), Push → 2, 3, 4, 5

Adjacent vertices (2) → 3, 4, 5

Pop '2' (stack), Push → 3, 4, 5

Adjacent vertices (3) → 4, 5

Pop '3' (stack), Push → 4, 5

Adjacent vertices (4) → 5

Pop '4' (stack), Push → 5

Adjacent vertices (5) → None

Pop '5' (stack), Push → 3, 4

(No adjacent vertices) stack only

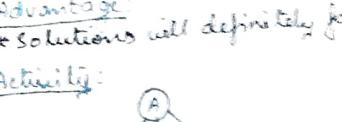
0, 1, 2, 3, 4, 5 → Visited

stack empty

Advantage:

\* Solutions will definitely found

Activity:



Pop Top item '1' & move to Visited list

Adjacent vertices (2) → 3, 4

Pop Top item '2' & move to Visited list

Adjacent vertices (3) → 4

Pop Top item '3' & move to Visited list

Adjacent vertices (4) → None

Pop Top item '4' & move to Visited list

Adjacent vertices (5) → None

Visited & not visited



BFS : ABCDEF

DFS : ADFCEB

# Topic 21: Advanced Data Structures

- Advanced Data Structures
- Future Scope
- Real Time Applications
- Research Area

