DAA

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ALM-03

Q) What is the Ford-Fulkerson algorithm, and how does it work in solving network flow problems? Prepare a case study.

A) The Ford-Fulkerson algorithm is a method used solve the maximum flow problem in a network. It helps in finding the maximum possible flow from a source node to sink node within a flow network. This algorithm is fundamental in fields like operations research, computer networks and logistics where optimizing resource flow is critical.

How it Works:-

1) Starting with an initial flow of zero.
2) Iteratively finding augmenting paths from source to sink where more flow can be pushed.
3) Increasing the flow along the augmenting path until no more augmenting paths can be found.

This algorithm relies on using residual capacities to find these paths.

Example Walkthrough :-

Consider a network with nodes and directed edges where each edge has a specific capacity. Let's go through a basic example.

1) Network setup:

- Let there be 4 nodes in network labeled as S, A, B and T
- The edges and their capacities are:
  - S → A : 10
  - S → B : 5
  - A → B : 15
  - A → T : 10
  - B → T : 10

Step-1 :- Start with a flow of zero. Identify an augmenting path from S to T that has available capacity.

Step-2 :- Suppose we find the path S → A → T with a minimum capacity of 10. Increase the flow along with path by 10.

Step-3 : Update the residual capacities, reducing the available capacity on S → A and A → T by 10.

Step-4 : Search for another argumenting path, such S → B → T with available capacity of 5.

Step-5 : Increase the flow along this path by 5 and update residual capacities.

After no more argumenting paths are available, the algorithm terminates with maximum flow of 15 units.

Case study: Application in Traffic Routing.

In real-world scenarios, the Ford-Fulkerson algorithm can be used to maximize the traffic flow in a road network. Suppose we have a network of intersections and roads with specified capacities for vehicle flow. By applying the Ford-Fulkerson algorithm, traffic planners can identify the optimal routing planners and paths and manage peak flows efficiently, reducing congestion.

Example:-

Consider a network with following setup:

- Nodes : S, A and T
- Edges and Capacities
  - S → A : Capacity = 8
  - A → T : Capacity = 5

Maximum flow from S to T.

Step by step execution:-

Initial setup:-

- Total Flow: Start with a flow of 0.

Finding an Augmenting path:-

*Identify a path from s to T. The only possible path is

- $S \rightarrow A \rightarrow T$

* Determine the minimum capacity in this path. The capacities along this path are:

- $S \rightarrow A : 8$
- $A \rightarrow T : 5$

*The minimum capacity is 5. So, we can send 5 units of flow along this path.

* Update the capacities in residual network.

- $S \rightarrow A : 3 (8 - 5 = 3)$
- $A \rightarrow T : 0 (5 - 5 = 0)$

Flow Update After iteration 1.

Total Flow : 5.

Since there are no more paths with available capacity from s to T, the algorithm stops.

Final Result:-

Maximum Flow 5 units.

Summary:-

In this simple example, the Ford - Fulkerson algorithm finds a maximum flow of 5 units from s to T.