**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans :- The data is normally distributed

*μ* = 45 minutes , *σ* = 8 minutes

Let X be the amount of time it takes to complete the repair on a customer's car. To finish in one hour you must have X ≤ 50.

Z = (x - *μ)/σ = (X - 45)/8.0*

P (X ≤ 50) = P (Z ≤ (50 - 45)/8.0) = P (Z ≤ 0.625) =0.734

The probability that the service manager cannot meet his commitment

= P (X > 50) = 1 – P (X ≤ 50).

= 1 – 0.734 = 0.266 i.e., Option B is right answer

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans :- The data is normally distributed

*μ* = 38 , *σ* = 6

Let X be the no of employees

1. P(X >44 ) = 1 – P(X ≤ 44)

So, P(X ≤ 44) = P( Z ≤ (44 - 38)/6) = P(Z ≤ 1) = 84.1345 %

P(X >44 ) = 100 – 84.1345 = 15.8655 %

Probabilty that the employee will be greater than age of 44 = 15.86%

So, The probability of number of employees between 38-44 years of age

= P (X<44) - 0.5 = 84.1345 - 0.5 = 34.1345%

Hence we conclude that the statement is “TRUE”

1. Probabilty of employees less than age of 30 = P (X<30).

P (X ≤ 30) = P (Z ≤ (30 - 38)/6) = P (Z ≤ -1.333) = 0.912

So the number of employees with probability 0.912 of them being under age 30

= 0.0912 \* 400=36.48 ( 36 employees ).

Therefore the statement of the question is also “TRUE” .

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans :- As we know that if X ∼ N (µ1, σ12), and Y ∼ N (µ2, σ22 ) are two independent random variables then X + Y ∼ N(µ1 + µ2, σ12 + σ22 ) , and X − Y ∼ N(µ1 − µ2, σ12 + σ22 ) .

Similarly, if Z = aX + bY, where X and Y are as defined above, i.e Z is linear combination of X and Y, then Z ∼ N (aµ1 + bµ2, a2 σ12 + b2 σ22).

Therefore,

2X1~ N (2μ, 4σ2) and

X1+X2 ~ N (µ + µ, σ2 + σ2) ~ N (2 u, 2σ2)

2X1-(X1+X2) = N (4µ,6 σ2)

1. Let X ~ N (100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans :- The Probability of getting value between a and b should be 0.99.

So, The Probability outside the a and b area is 0.01 (i.e., 1 - 0.99).

The Probability towards left from a = -0.005 (i.e., 0.01/2).

The Probability towards right from b = +0.005 (i.e., 0.01/2).

So, since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

Z = (X- μ) / σ

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \* σ + μ = X

Z (-0.005) \*20+100 = -(-2.57) \*20+100 = 151.4

Z (+0.005) \*20+100 = (-2.57) \*20+100 = 48.6

So, option D is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans :- Set+2 Que 5 .ipynb.