**B.M.S. COLLEGE OF ENGINEERING, BENGALURU-19**
**Autonomous Institute, Affiliated to VTU**
DEPARTMENT OF MATHEMATICS
THIRD SEMESTER B.E COURSE(CSE/ISE/CSE-AIDS/CS-IOT/CSE-DS)

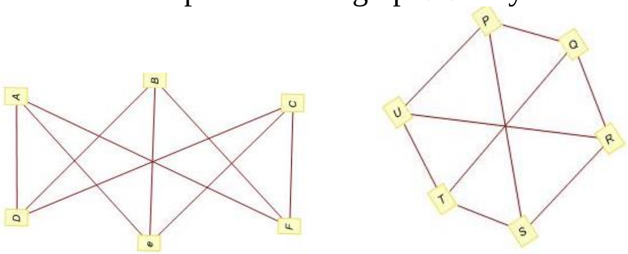
**Course Title: Statistics and Discrete Mathematics** **Course Code 23MA3BSSDM**

## UNIT-1 Graph Theory

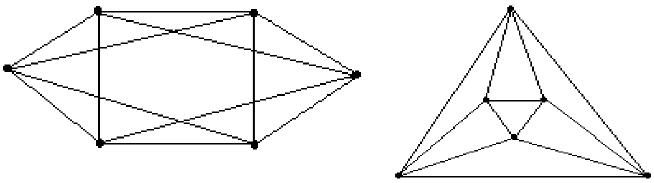
1. Define the following with an example.
   (i) Digraph (ii) complete bipartite graph (iii) 3-D hypercube
2. Suppose that in a group of 5 people: A, B, C, D, and E, the following pairs of people are acquainted with each other- A and C, A and D, B and C, C and D & C and E.
   (a) Draw a graph G to represent this situation. (b) Identity the degree of each vertex.
3. Model the following situation as a graph and write its vertex set and the edge set. Also, write the degree of each vertex. In Netherland, there is a highway from Amsterdam to Breda, another highway from Amsterdam to Cappele aan den IJssel, a highway from Breda to Dordrecht, a highway from Breda to Ede and another one from Dordrecht to Ede, and a highway from Cappele aan den IJssel to Ede.
4. Let P,Q, R,S and T represent 5 cricket teams. Suppose that the teams P,Q,R have played one game with each other and the teams P,S,T have played one game with each other. Represent this situation as a graph and hence determine:
   (i) The teams that have not played with each other.
   (ii) The number of games played by each team.
5. For a graph $G=(V,E)$, what is the largest possible value of $|V|$ if $|E|=35$ and $\deg(v)\ge 3, \forall v \in V$ ?
6. For a graph $G=(V,E)$, what is the largest possible value of $|V|$ if $|E|=19$ and $\deg(v)\ge 4, \forall v \in V$ ?
7. For a graph with n vertices and m edges if $\delta$ is the minimum and $\Delta$ is the maximum of the degrees of the vertices, show that $\delta \le \frac{2m}{n} \le \Delta$.
8. Determine the order of the graph in the following cases.
   (i) G is a cubic graph with 9 edges.
   (ii) G has 10 edges with 2 vertices of degree 4 and all other vertices of degree 3.
   (iii) G is a regular graph with 15 edges.

9. Let G be a graph of order 9 such that each vertex has degree 5 or 6. Prove that at least 5 vertices have degree 6 or at least 6 vertices have degree 5.
10. Show that every simple graph order greater than or equal to 2 must have atleast 2 vertices of the same degree.
11. Define Isomorphism of graphs. Verify that the given two graphs are isomorphic or not11

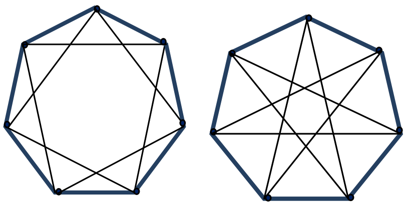
12. Define Isomorphism of the graph. Verify that the given two graphs are isomorphic or not.




13. Verify whether the following graphs are isomorphic or not.



14. With proper labeling show that the following graphs are isomorphic.



15. Suppose a committee has seven members, these members meet each day for lunch at a round table. They decide to sit in such a way that every member has different neighbors at each lunch. How many ways can this arrangement last?
16. Suppose that a tree T has two vertices of degree 2, four vertices of degree 3 and three vertices of degree 4. Find the number of leaves in T.
17. Let G be a simple graph with n vertices and m edges where m is at least 3. If $m \geq \frac{(n-1)(n-2)}{2}+2$ , prove that G is Hamiltonian graph.
18. Prove that a connected graph G remains connected even after removing an edge *e* from G if and only if *e* is a part of some cycle in G.

19. Let G be a disconnected graph of even order n with two components each of which is complete. Prove that G has a minimum of $\frac{n(n-2)}{4}$ edges.

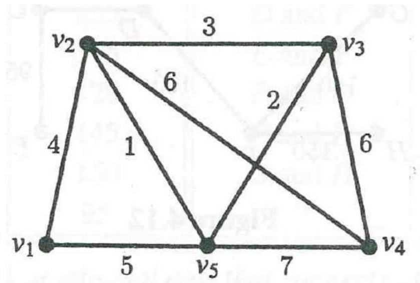
20. If G is a simple graph with n vertices in which the degree of every vertex is at least $(n-1)/2$, prove that G is connected.

21. Show that a connected graph with exactly 2 vertices of odd degree has an Euler trail.

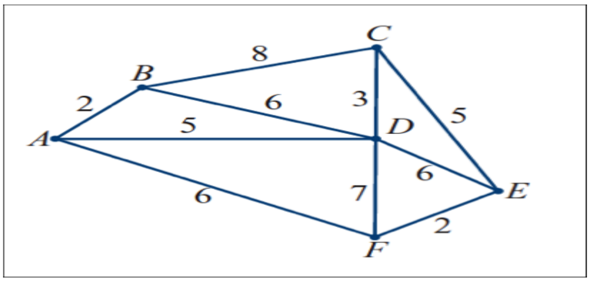
22. Find all the spanning trees of the given graph:

23. For the given graph, draw the following:

(i) Spanning Tree (ii) Edge Disjoint Subgraph (iii)Induced Subgraph

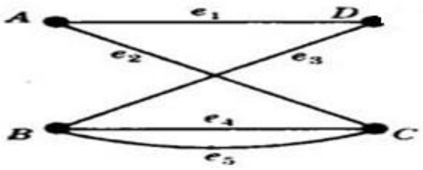



24. For the given graph, draw the following:(i) Spanning Tree (ii) Edge Disjoint Subgraph (iii)Induced Subgraph

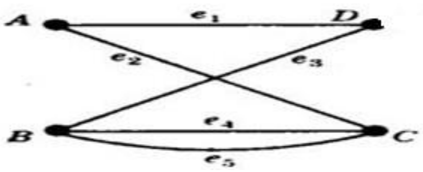


**<u>Matrix Representation of a graph</u>**.

1. For the given graph, write its incidence matrix. Hence, write any three observations on it.




2. For the given graph, write its adjacency matrix. Hence, write any three observations on it.




3. Draw the graph G whose incidence matrix is given and hence obtain the adjacency matrix of the corresponding graph G

$$A(G)=\begin{bmatrix}1&1&0&0&0\\1&0&1&0&0\\0&1&0&1&1\\0&0&1&1&0\\0&0&0&0&1\end{bmatrix}$$

4. For the given adjacency matrix $A(G)$, construct incidence matrix and also write any three observations on incidence matrix.

$$A(G)=\begin{bmatrix}0&0&1&0&1\\0&0&1&0&1\\1&1&0&1&0\\0&0&1&0&0\\1&1&0&0&0\end{bmatrix}$$

5. Draw the graph G whose incidence matrix is given and hence obtain the adjacency matrix of the corresponding graph G.

$$A(G)=\begin{bmatrix}1&0&1&0&0\\1&0&1&0&0\\0&1&0&1&1\\0&0&0&1&0\\0&1&0&0&1\end{bmatrix}$$

6. Obtain the incidence matrix for the graph whose adjacency matrix is given below.

$$X(G)=\begin{array}{c}\\a\\b\\c\\d\\e\end{array}\begin{array}{c}\begin{array}{ccccc}a&b&c&d&e\end{array}\\\begin{bmatrix}0&1&0&1&0\\1&0&1&0&1\\0&1&0&0&1\\1&0&0&0&1\\0&1&1&1&0\end{bmatrix}\end{array}$$

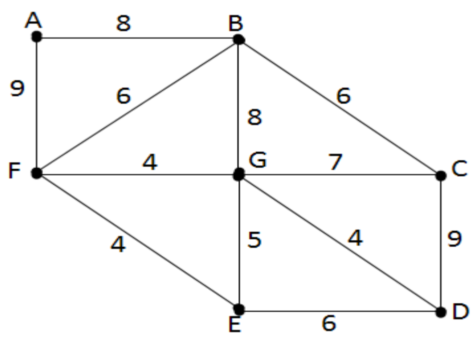
# Kruskal's algorithm

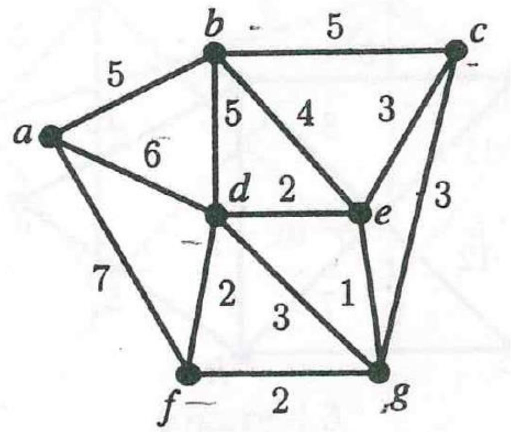
1. Eight cities A,B,C,D,E,F,G,H are required to be connected by a new railway network. The possible tracks and the cost involved to lay them (in crores of rupees) are summarized in the Table:

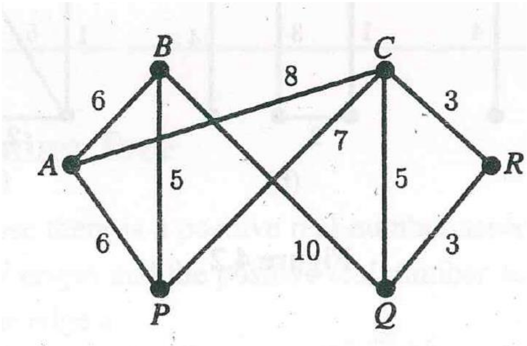
| Track between | Cost | Track between | Cost |
|---|---|---|---|
| | | C and E | 95 |
| A and B | 155 | D and F | 100 |
| A and D | 145 | E and F | 150 |
| A and G | 120 | F and G | 140 |
| B and C | 145 | F and H | 150 |
| C and D | 150 | G and H | 160 |

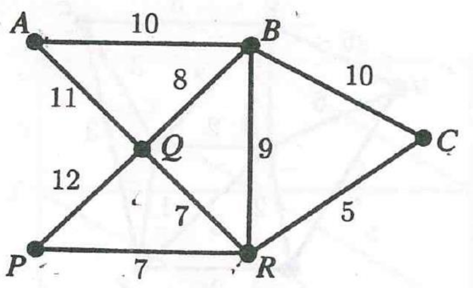
Determine a railway network of minimal cost that connects all these cities using Kruskal's algorithm.

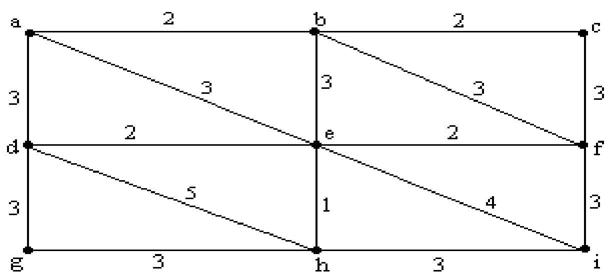
2. Apply Kruskal's algorithm to find a minimal spanning tree for the following weighted graphs and hence find its weight.

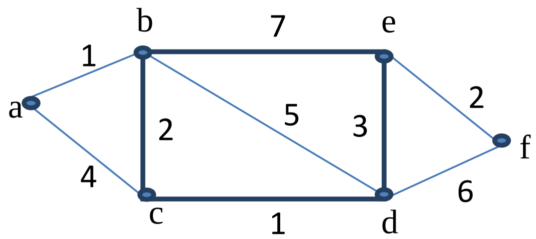


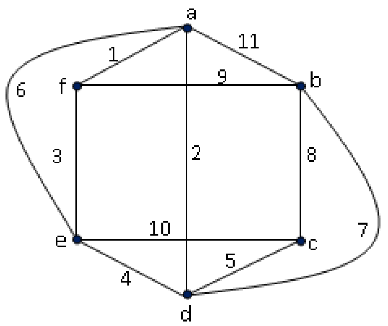



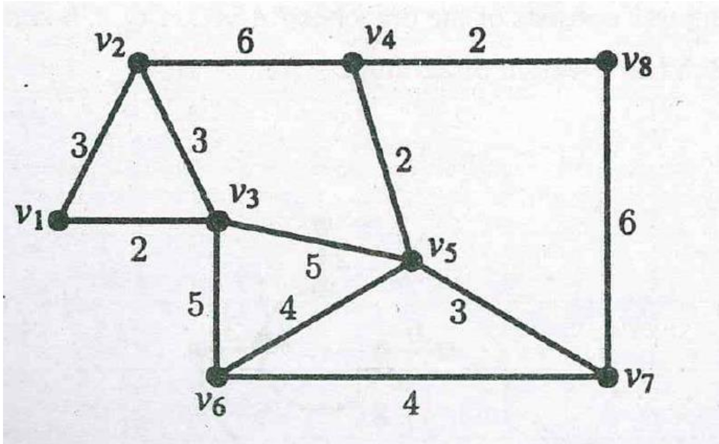



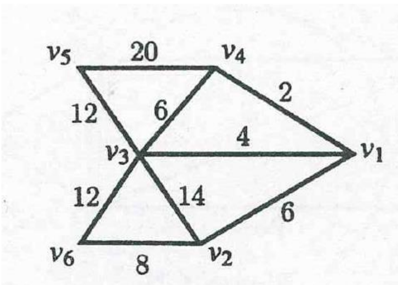





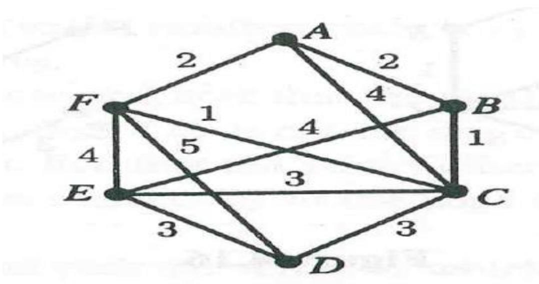
a
1
11
6
f
9
b
3
2
8
e
10
c
7
5
4
d

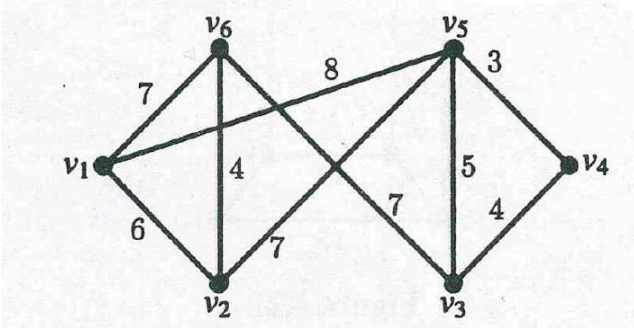
v2
6
v4
2
v8
3
3
2
v1
v3
6
2
5
v5
5
4
3
v6
4
v7

v5
20
v4
2
12
6
v3
4
v1
12
14
6
v6
8
v2

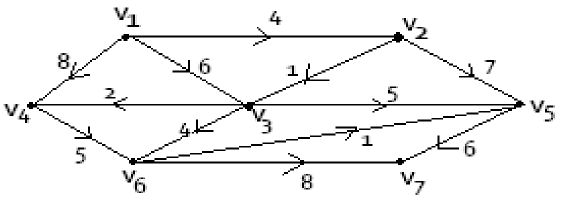
A
2
2
4
F
1
4
B
4
5
1
E
3
C
3
3
D

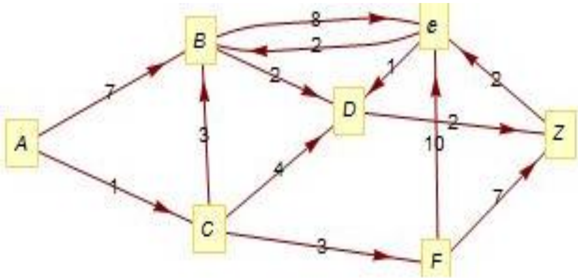
v6
v5
3
7
8
v1
4
5
v4
7
4
6
7
v2
v3

**Dijkstra's**

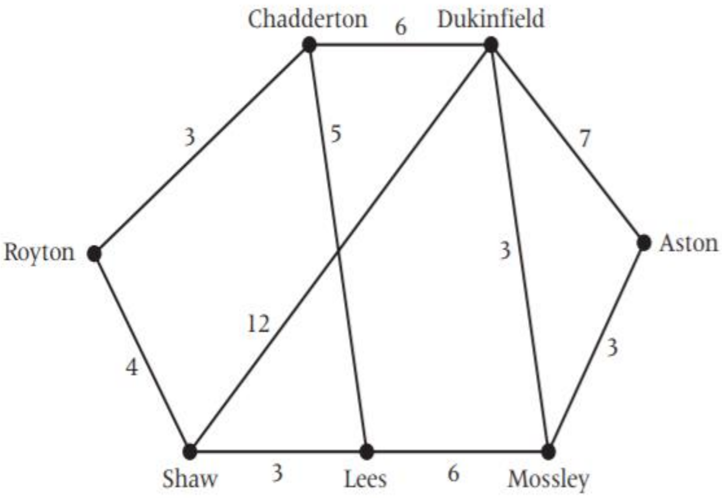
1. Apply Dijkstra's algorithm to find the shortest path and its weight from vertex $v_1$ to vertex $v_5$ from the weighted, directed network shown below.



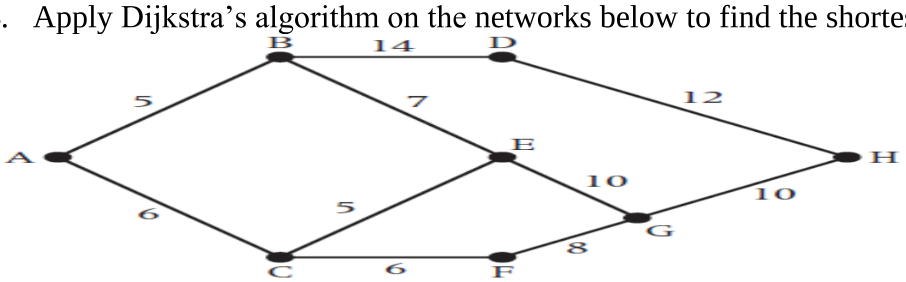
2. Apply Dijkstra's algorithm to find the shortest path and its weight from vertex A to vertex Z from the weighted, directed network



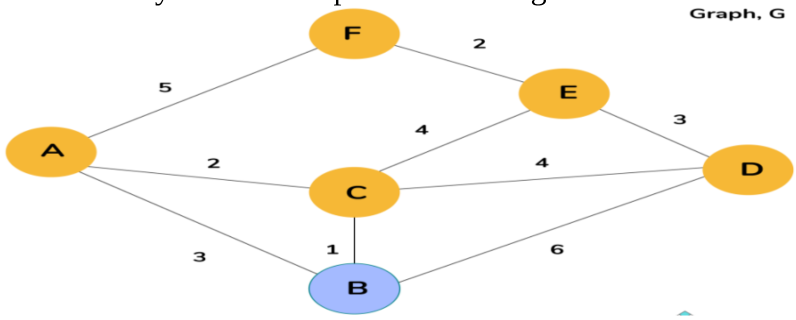
3. The diagram below shows roads connecting villages near to Royton. The numbers on each arc represent the distance, in miles, along each road. Leon lives in Royton and works in Ashton. Use Dijkstra's algorithm to find the minimum distance for Leon's journey to work.




4. Apply Dijkstra's algorithm on the networks below to find the shortest distance from A to H.



5. Consider the map below. The cities have been selected and marked from alphabets A to F and every edge has a cost associated with it. We need to travel from Bengaluru (Vertex B) to all other places. Identify the shortest paths from Bengaluru to other destinations.

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