Assignment 6

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Find Python Codes from below link

https://github.com/AnilMondedla/Python/Assignment_6

and latex-tikz codes from

https://github.com/AnilMondedla/Python/Assignment_6

1 Examples 1

1.1 Question 13

Prove that the points (2,-2),(8,4),(5,7), and (-1,1) are at the angular points of a rectangle.

1.2 Solution

a quadrilateral to be a rectangle, the opposite sides of the quadrilateral must be equal and the diagonals must be equal as well.

$$\mathbf{A} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{1.2.1}$$

$$\begin{aligned} \|\mathbf{A}-\mathbf{B}\| &= \sqrt{\left(\mathbf{A}-\mathbf{B}\right)^\top\left(\mathbf{A}-\mathbf{B}\right)} && (1.2.2)\\ &= \sqrt{\left(\begin{pmatrix} 2 \\ -2 \end{pmatrix}-\begin{pmatrix} 8 \\ 4 \end{pmatrix}\right)^\top\left(\begin{pmatrix} 2 \\ -2 \end{pmatrix}-\begin{pmatrix} 8 \\ 4 \end{pmatrix}\right)} && (1.2.3)\\ &= \sqrt{\begin{pmatrix} -6 \\ -6 \end{pmatrix}^\top\begin{pmatrix} -6 \\ -6 \end{pmatrix}} && (1.2.4)\\ &= \sqrt{\begin{pmatrix} -6 & -6 \end{pmatrix}\begin{pmatrix} -6 \\ -6 \end{pmatrix}} && (1.2.5)\\ &= \sqrt{\left(36+36\right)}\\ &= \sqrt{\left(72\right)} \end{aligned}$$

$$\begin{aligned} \|\mathbf{C}-\mathbf{D}\| &= \sqrt{\left(\mathbf{C}-\mathbf{D}\right)^\top\left(\mathbf{C}-\mathbf{D}\right)} && (1.2.6)\\ &= \sqrt{\left(\begin{pmatrix} 5 \\ 7 \end{pmatrix}-\begin{pmatrix} -1 \\ 1 \end{pmatrix}\right)^\top\left(\begin{pmatrix} 5 \\ 7 \end{pmatrix}-\begin{pmatrix} -1 \\ 1 \end{pmatrix}\right)} && (1.2.7)\\ &= \sqrt{\begin{pmatrix} 6 \\ 6 \end{pmatrix}^\top\begin{pmatrix} 6 \\ 6 \end{pmatrix}} && (1.2.8)\\ &= \sqrt{\begin{pmatrix} 6 & 6 \end{pmatrix}\begin{pmatrix} 6 \\ 6 \end{pmatrix}} && (1.2.9)\\ &= \sqrt{\left(36+36\right)}\\ &= \sqrt{\left(72\right)} \end{aligned}$$

AB=CD

$$\begin{aligned} \|\mathbf{A}-\mathbf{C}\| &= \sqrt{\left(\mathbf{A}-\mathbf{C}\right)^\top\left(\mathbf{A}-\mathbf{C}\right)} && (1.2.10)\\ &= \sqrt{\left(\begin{pmatrix} 2 \\ -2 \end{pmatrix}-\begin{pmatrix} 5 \\ 7 \end{pmatrix}\right)^\top\left(\begin{pmatrix} 2 \\ -2 \end{pmatrix}-\begin{pmatrix} 5 \\ 7 \end{pmatrix}\right)} && (1.2.11)\\ &= \sqrt{\begin{pmatrix} -3 \\ -9 \end{pmatrix}^\top\begin{pmatrix} -3 \\ -9 \end{pmatrix}} && (1.2.12)\\ &= \sqrt{\begin{pmatrix} -3 & -9 \end{pmatrix}\begin{pmatrix} -3 \\ -9 \end{pmatrix}} && (1.2.13)\\ &= \sqrt{\left(9+81\right)}\\ &= \sqrt{\left(90\right)} \end{aligned}$$

$$\|\mathbf{B}-\mathbf{D}\| = \sqrt{\left(\mathbf{B}-\mathbf{D}\right)^{\top}\left(\mathbf{B}-\mathbf{D}\right)} \tag{1.2.14}$$
$$= \sqrt{\left(\begin{pmatrix}8\\4\end{pmatrix}-\begin{pmatrix}-1\\1\end{pmatrix}\right)^{\top}\left(\begin{pmatrix}8\\4\end{pmatrix}-\begin{pmatrix}-1\\1\end{pmatrix}\right)} \tag{1.2.15}$$
$$= \sqrt{\begin{pmatrix}9\\3\end{pmatrix}^{\top}\begin{pmatrix}9\\3\end{pmatrix}} \tag{1.2.16}$$
$$= \sqrt{\begin{pmatrix}9 & 3\end{pmatrix}\begin{pmatrix}9\\3\end{pmatrix}} \tag{1.2.17}$$
$$= \sqrt{\left(81+9\right)}$$
$$= \sqrt{\left(90\right)}$$

AC=BD

$$\|\mathbf{A}-\mathbf{D}\| = \sqrt{\left(\mathbf{A}-\mathbf{D}\right)^{\top}\left(\mathbf{A}-\mathbf{D}\right)} \tag{1.2.18}$$
$$= \sqrt{\left(\begin{pmatrix}2\\-2\end{pmatrix}-\begin{pmatrix}-1\\1\end{pmatrix}\right)^{\top}\left(\begin{pmatrix}2\\-2\end{pmatrix}-\begin{pmatrix}-1\\1\end{pmatrix}\right)} \tag{1.2.19}$$
$$= \sqrt{\begin{pmatrix}3\\-3\end{pmatrix}^{\top}\begin{pmatrix}3\\-3\end{pmatrix}} \tag{1.2.20}$$
$$= \sqrt{\begin{pmatrix}3 & -3\end{pmatrix}\begin{pmatrix}3\\-3\end{pmatrix}} \tag{1.2.21}$$
$$= \sqrt{\left(9+9\right)}$$
$$= \sqrt{\left(18\right)}$$

$$\|\mathbf{B}-\mathbf{C}\| = \sqrt{\left(\mathbf{B}-\mathbf{C}\right)^{\top}\left(\mathbf{B}-\mathbf{C}\right)} \tag{1.2.22}$$
$$= \sqrt{\left(\begin{pmatrix}8\\4\end{pmatrix}-\begin{pmatrix}5\\7\end{pmatrix}\right)^{\top}\left(\begin{pmatrix}8\\4\end{pmatrix}-\begin{pmatrix}5\\7\end{pmatrix}\right)} \tag{1.2.23}$$
$$= \sqrt{\begin{pmatrix}3\\-3\end{pmatrix}^{\top}\begin{pmatrix}3\\-3\end{pmatrix}} \tag{1.2.24}$$
$$= \sqrt{\begin{pmatrix}3 & -3\end{pmatrix}\begin{pmatrix}3\\-3\end{pmatrix}} \tag{1.2.25}$$
$$= \sqrt{\left(9+9\right)}$$
$$= \sqrt{\left(18\right)}$$

AD=BC

Therefore, point A, B, C and D are the angular points of a rectangle.

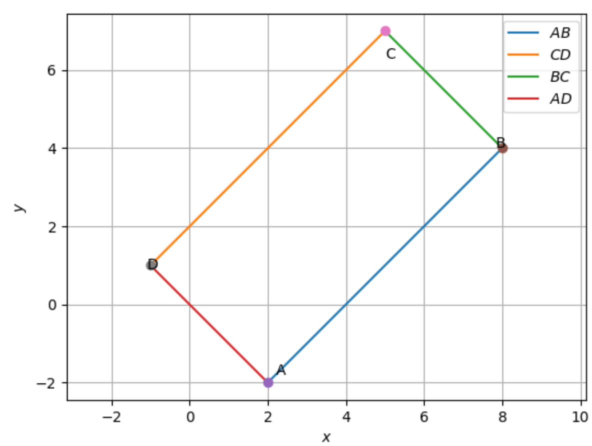

Fig. 0