Maths

UNIT - 1

1] (i) let $P(D) = \frac{13}{52}$ & $P(S) = \frac{13}{52}$

$$P(E) = \frac{13C_1 \times 13C_1}{52C_2} = \frac{13 \times 13}{1326} = \frac{169}{1326} = 0.127$$

$$nC_r = \frac{n!}{(n-r)!\,r!}$$

$\therefore P(E) = 0.127$

(ii) Given,

Total no. of trucks = 12

Trucks don't meet emission = 5

The trucks randomly selected = 4.

$$\therefore P(E) = \frac{5C_4}{12C_4} = \frac{5}{495} = 0.010.$$

2] Given,

$P(t) = 0.6$ $P(m) = 0.3$.

$P(t \cap m) = 0.15$

(a) $P(t \cup m) = P(t) + P(m) - P(t \cap m)$

$= 0.6 + 0.3 - 0.15$

$= 0.9 - 0.15$

$= 0.75$

(b) $P(t \cup m)' = 1 - P(t \cup m)$

$= 1 - 0.75$

$= 0.25$

3] $P(B_1) = 30\% = \frac{30}{100} = 0.3$

$P(B_2) = 45\% = \frac{45}{100} = 0.45$

$P(B_3) = 25\% = \frac{25}{100} = 0.25$

$P(D/B_1) = 2\% = \frac{2}{100} = 0.02$

$P(D/B_2) = 3\% = 0.03$

$P(D/B_3) = 2\% = 0.02$

a) the product drawn is defective from machine B_3

$$P(B_3/D) = \frac{P(B_3) \cdot P(D/B_3)}{P(B_1)P(D/B_1) + P(B_2)P(D/B_2) + P(B_3)P(D/B_3)}$$

$$= \frac{(0.25)(0.02)}{(0.8)(0.02) + (0.45)(0.03) + (0.25)(0.02)}$$

$$= \frac{0.005}{0.016 + 0.014 + 0.005} = \frac{0.005}{0.025}$$

$$= 0.2$$

4]

X	0	1	2	3	4	5	6	7
F	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

(i) K $\Rightarrow \sum_{i=0}^{7} P(x_i) = 1$

$\Rightarrow 0 + K + 2K + 2K + 3K + K^2 + 2K^2 + 7K^2 + K = 1$

$\Rightarrow 9K + 10K^2 = 1$

$\Rightarrow 10K^2 + 9K - 1 = 0$

$$K = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-9 \pm \sqrt{9^2 - 4(10)(-1)}}{2(10)}$$

$$K = \frac{-9 \pm \sqrt{81+40}}{20} = \frac{-9 \pm \sqrt{121}}{20}$$

$$K = \frac{-9+11}{20} \; ; \; K = \frac{-9-11}{20}$$

$$K = \frac{2}{20} \; ; \; K = \frac{-20}{20}$$

$$K = \frac{1}{10} \; , \; -1$$

(ii) Mean $\mu \Rightarrow \sum_{i=0}^{7} p_i x_i$

$\Rightarrow 0(0) + k + 2(2k) + 3(2k) + 4(3k) + 5k^2 + 12k^2 + 49k^2 + 7k$

$\Rightarrow k + 4k + 6k + 12k + 5k^2 + 12k^2 + 49k^2 + 7k$

$\Rightarrow 66k^2 + 30k$

$\Rightarrow 66\left[\frac{1}{10}\right]^2 + 30\left[\frac{1}{10}\right]$

$\Rightarrow \frac{66}{100} + \frac{30}{10} \Rightarrow \frac{66+30}{100} \Rightarrow \frac{96}{100}$

$\Rightarrow 0.66 + 3 \Rightarrow 3.66.$

(ii) variance $\Rightarrow \sum_{i=0}^{7} (p_i(x_i)^2 - \mu^2$

$\Rightarrow \{0(0)^2 + k(1)^2 + 2k(2)^2 + 2k(3)^2 + 3k(4)^2 + k^2(5)^2 + 2k^2(6)^2 + (7k^2+k)(7)^2\} - [3.66]^2$

$\Rightarrow (k + 8k + 18k + 48k + 25k^2 + 72k^2 + 343k^2 + 49k) - 13.396$

$\Rightarrow (440k^2 + 124k) - 13.396$

$\Rightarrow \left\{440\left[\frac{1}{10}\right]^2 + 124\left[\frac{1}{10}\right]\right\} - 13.396$

$\Rightarrow \left[\frac{440}{100} + \frac{124}{10}\right] - 13.396.$

$\Rightarrow (4.4 + 12.4) - 13.396 \Rightarrow 16.8 - 13.396$

$\Rightarrow 3.404.$

5]

$$f(x) = \begin{cases} k(1-x^2), & 0<x<1 \\ 0, & \text{otherwise} \end{cases}$$

a) Find k :

we know,

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{0}^{1} k(1-x^2)\,dx = 1$$

$$k\int_{0}^{1}(1-x^2)\,dx = 1$$

$$k\left[x-\frac{x^3}{3}\right]_0^1 = 1$$

$$k\left[1-\frac{1}{3}\right]-\left[0-\frac{0}{3}\right] = 1$$

$$k\left[\frac{3-1}{3}\right] = 1$$

$$k\left(\frac{2}{3}\right) = 1$$

$$\therefore \boxed{k = \frac{3}{2}}$$

(ii) Mean $\Rightarrow \int_{-\infty}^{\infty} x f(x)\,dx$

$$\Rightarrow k\int_0^1 x(1-x^2)\,dx$$

$$\Rightarrow k\int_0^1 (x-x^3)\,dx$$

$$\Rightarrow k\left[\frac{x^2}{2}-\frac{x^4}{4}\right]_0^1$$

$$\Rightarrow k\left[\frac{1}{2}-\frac{1}{4}\right]-\left[\frac{0}{2}-\frac{0}{4}\right]$$

$$\Rightarrow \frac{3}{2}\left[\frac{4-2}{8}\right] \Rightarrow \frac{3}{\cancel{2}}\left[\frac{\cancel{2}}{8}\right] \Rightarrow \frac{3}{8}$$

(iii) variance. $\int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$

$$K\int_0^1 x^2(1-x^2)\,dx - \left(\frac{3}{8}\right)^2$$

$$K\int_0^1 (x^2 - x^4)\,dx - \left(\frac{3}{8}\right)^2$$

$$K\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1 - \left(\frac{9}{64}\right)$$

$$K\left[\frac{1}{3} - \frac{1}{5}\right] - \frac{9}{64} \Rightarrow K\left[\frac{5-3}{15}\right] - \frac{9}{64}$$

$$\Rightarrow \frac{3}{\cancel{2}}\left[\frac{\cancel{2}}{15}\right] - \frac{9}{64} \Rightarrow \frac{3}{15} - \frac{9}{64}$$

$$\Rightarrow 0.2 - 0.141$$

$$\Rightarrow 0.059$$

(iv) $P(0.3 < x < 0.6)$.

$$\int_{0.3}^{0.6} K(1-x^2)\,dx = \frac{3}{2}\int_{0.3}^{0.6}(1-x^2)\,dx$$

$$= \frac{3}{2}\left[x - \frac{x^3}{3}\right]_{0.3}^{0.6}$$

$$= \frac{3}{2}\left\{\left[0.6 - \frac{(0.6)^3}{3}\right] - \left[0.3 - \frac{(0.3)^3}{3}\right]\right\}$$

$$= \frac{3}{2}\left\{\left[0.6 - \frac{0.216}{3}\right] - \left[0.3 - \frac{0.027}{3}\right]\right\}$$

$$= \frac{3}{2}\left\{\left(\frac{1.8 - 0.216}{3}\right) - \left(\frac{0.9 - 0.027}{3}\right)\right\}$$

$$= \frac{3}{2}\left[\frac{1.584}{3} - \frac{0.873}{3}\right]$$

$$= \frac{3}{2}\left[0.528 - 0.291\right] = \frac{3}{2}\left[0.237\right]$$

$$= 0.356$$

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Unit - 2

2a) Given, 800 families.
$n = 5$ since there are 5 children in each family

$p = \frac{1}{2}$ (boys)
$q = \frac{1}{2}$ (girls)

Probability of having boy $p = \frac{1}{2}$ (success)
" " " girl $q = \frac{1}{2}$ (failure)

(i) P(having 3 boys)

$$P(x=3) = nC_r \, p^r \, q^{n-r}$$

$$= 5C_3 \, p^3 \, q^{5-3} = 10 (0.5)^3 (0.5)^2$$

$$= 10(0.25)(0.125) = 10(0.031)$$

$$= 0.31$$

[800 × 0.31 = 248 boys] expected

(ii) P (having 5 girls)
i.e., no boys

$$P(x=0) = 5C_0 \, (p)^0 \, (q)^5$$

$$= 1(0.5)^5$$

$$= 0.03125$$

Expected no. of families having 5 girls

$$= 800 \times 0.03125$$

$$= 25$$

(iv) P(atleast one boy)

$$P(x \geq 1) = 1 - P(x < 1)$$

$$= 1 - P(x=0).$$

$$= 1 - 0.03125$$

$$= 0.969.$$

Expected no. of families having atleast 1 boy

$$= 800 \times 0.969$$

$$= 775$$

(iii) P(having either 2 or 3 boys)

$P(x=2) + P(x=3)$

$= 5C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^3 + 5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$

$= 10\left(\frac{1}{4}\right)\left(\frac{1}{8}\right) + 10\left(\frac{1}{8}\right)\left(\frac{1}{4}\right)$

$= 0.625$

Expected no. of families having either 2 or 3 boy

$= 800 \times 0.625$

$= 500.$

UNIT-2

1]

b] Bank received 6 bad checks per day

$\lambda = 6$

(i) $P(y) = \dfrac{e^{-\lambda} \lambda^{r}}{r!}$

$r = 4$

$P(x=4) = \dfrac{e^{-6} \lambda^{4}}{4!} \Rightarrow \dfrac{0.002478 \cdot 6^{4}}{4 \times 3 \times 2 \times 1} = \dfrac{3.211488}{24}$

$= 0.133$

(ii) $P(x=10)$ 2 days. $\lambda = 6(2) = 12$

$P = \dfrac{e^{-12} \lambda^{10}}{10!} \Rightarrow \dfrac{0.0000061442 \times 6917364224}{3628800}$

$\Rightarrow \dfrac{380433.288439}{3628800} \Rightarrow 0.1048$

(iii) $P(x=0) \Rightarrow \dfrac{e^{-\lambda} \lambda^{0}}{0!} = \dfrac{e^{-6} 6^{0}}{0!} = 0.00247$

2]

x	0	1	2	3	4	5	6
f(x)	13	25	52	58	32	16	4

Binomial Distribu

$N = 200$ $n = 6$.

$\mu = \dfrac{\Sigma f_i x_i}{\Sigma f_i} = \dfrac{25 + 104 + 174 + 128 + 80 + 24}{200}$

$= \dfrac{535}{200} = 2.675$

$\boxed{\mu = np}$

$2.675 = 6P$

$P = \dfrac{2.675}{6} = 0.44$ $\boxed{p = 0.44}$ $\boxed{q = 0.56}$

$P(x=0) \Rightarrow N \; {}^nC_r \, p^r \, q^{n-r}$

$\Rightarrow 200 \; 6C_0 \, (0.44)^0 \, (0.56)^6$

$\Rightarrow 200(1)(1)(0.03084)$

$\Rightarrow 6.1681$

$P(x=1) \Rightarrow 200 \; 6C_1 \, (0.44)^1 \, (0.56)^5$

$\Rightarrow 29.079$

$P(x=2) \Rightarrow 200 \cdot 6C_2 \, (0.44)^2 \, (0.56)^4$

$\Rightarrow 57.119$

$P(x=3) \Rightarrow 200 \cdot 6C_3 \, (0.44)^3 \, (0.56)^3$

$\Rightarrow 59.839$

$P(x=4) \Rightarrow 200 \cdot 6C_4 \, (0.44)^4 \, (0.56)^2$

$\Rightarrow 35.262$

$P(x=5) \Rightarrow 200 \cdot 6C_5 \, (0.44)^5 \, (0.56)^1$

$\Rightarrow 11.082$

$P(x=6) \Rightarrow 200 \cdot 6C_6 \, (0.44)^6 \, (0.56)^0$

$\Rightarrow 1.451$

3] Poisson Distribution

x	0	1	2	3	4	5
$f(x)$	142	156	69	27	5	1

$$\lambda = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{156 + 138 + 81 + 20 + 5}{400} = \frac{400}{400}$$

$N = 400 \qquad \lambda = 1 \qquad e^{-1} = 0.368$

$P(x=0) \Rightarrow N \times \frac{e^{-\lambda} \lambda^r}{r!} \Rightarrow 400 \times \frac{e^{-1} 1^0}{0!} \Rightarrow 147.151$

$P(x=1) \Rightarrow 400 \times \frac{e^{-1} 1^1}{1!} \Rightarrow 147.151$

$P(x=2) \Rightarrow 400 \times \frac{e^{-1} 1^2}{2!} \Rightarrow 73.575$

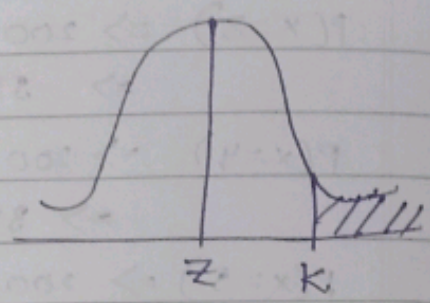
$P(x=3) \Rightarrow 400 \times \frac{e^{-1} 1^3}{3!} \Rightarrow 24.525$

$P(x=4) \Rightarrow 400 \times \frac{e^{-1} 1^4}{4!} \Rightarrow 6.131$

$P(x=5) \Rightarrow 400 \times \frac{e^{-1} 1^5}{5!} \Rightarrow 1.226$

P(x)	0	1	2	3	4	5
expected frequency.	147.151	147.151	73.575	24.525	6.131	1.226

4] a) $P(z > K) = 0.2946$



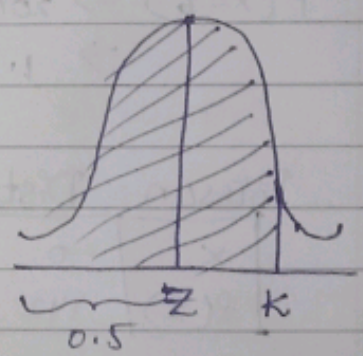
$0.5 - A(K) = 0.2946$

$0.5 - 0.2946 = A(K)$

$0.2054 = A(K)$

$\boxed{K = 0.54}$

b) $P(z < K) = 0.0427$



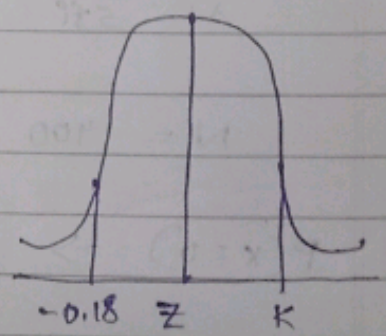
$0.5 + A(K) = 0.0427$

$A(K) = 0.0427 - 0.5$

$A(K) = -0.4573$

$\boxed{K = 1.72}$

c) $P(K < Z < -0.18) = 0.4197$



$P(K) - P(0.18) = 0.4197$

$A(K) - A(0.18) = 0.4197$

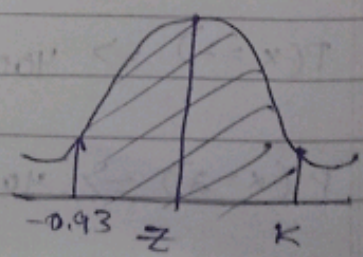
$A(K) - 0.0714 = 0.4197$

$A(K) = 0.4197 + 0.0714$

$A(K) = 0.491$

$\boxed{K = 2.37}$

d) $P(-0.93 < Z < K) = 0.7235$



$P(0.93) + P(K) = 0.7235$

$0.3238 + A(K) = 0.7235$

$A(K) = 0.4$ $\boxed{K = 1.28}$

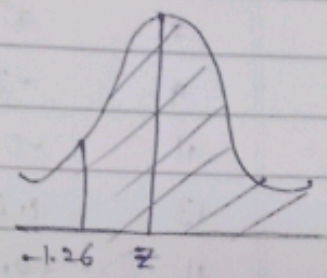
5] $\sigma = 6.3$ $\mu = 40$

a) $(z > 32)$

$$z = \frac{x-\mu}{\sigma}$$

$$z = \frac{32-40}{6.3} = -1.27$$

$P(x \geq 32) = P(z \geq -1.26)$



$= 0.5 + A(1.26)$

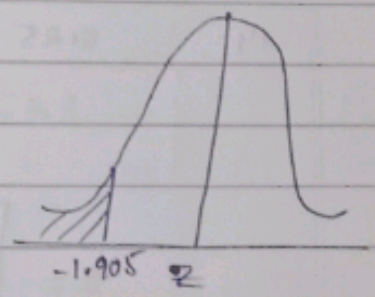
$= 0.5 + 0.3962$

$= 0.896$.

b) $P(x \leq 28)$

$$z = \frac{28-40}{6.3} = -1.905$$

$P(x \leq 28) = P(z \leq -1.905)$



$= 0.5 - A(-1.9)$

$= 0.5 - 0.4713$

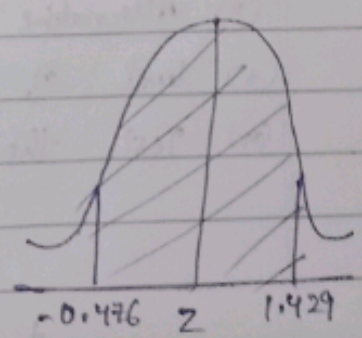
$= 0.029$

c) b/w 37 and 49 months.

$P(37 \leq x \leq 49) \Rightarrow$

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{37-40}{6.3} = -0.476$$

$$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{49-40}{6.3} = 1.429$$



$P(0.476 \leq z \leq 1.429)$

$\Rightarrow A(0.476) + A(1.429)$

$\Rightarrow 0.1808 + 0.4222$

$\Rightarrow 0.603$.

UNIT - ~~6~~ 3

1) Given,

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$N = 6$$

(i) without replacement

a) population mean $(\mu) = \frac{1+2+3+4+5+6}{N} = \frac{21}{6}$

$$\boxed{\mu = 3.5}$$

b) population variance $(\sigma^2) = \sum_{i=1}^{6} \frac{(x_i - \mu)^2}{N}$

$$\sigma^2 = \frac{(1-3.5)^2 + (2-3.5)^2 + (3-3.5)^2 + (4-3.5)^2 + (5-3.5)^2 + (6-3.5)^2}{6}$$

$$= \frac{6.25 + 2.25 + 0.25 + 0.25 + 2.25 + 6.25}{6}$$

$$= \frac{17.5}{6} = 2.91$$

population standard Deviation $\sigma = \sqrt{2.91} = 1.706.$

c) Total population $(\mu) = 6$

sample size $= 2$

Sample distribution for without replacement

$$^{N}C_n = {}^{6}C_2 = \frac{6!}{4!\,2!} = 15$$

15 elements

(1,2), (1,3), (1,4) (1,5) (1,6)
(2,3) (2,4) (2,5) (2,6)
(3,4) (3,5) (3,6)
(4,5) (4,6)
(5,6)

$\left(\frac{a+b}{2}\right)$

Sample distribution for mean

$$= \frac{1.5 + 2 + 2.5 + 3 + 3.5 + 2.5 + 3 + 3.5 + 4 + 3.5 + 4 + 4.5 + 4.5 + 5 + 5.5}{15}$$

$$= 3.5$$

(iv) S.D $\sigma = \sqrt{\sum_{i=1}^{6} \frac{(x_i - \mu)^2}{N}}$

$$= \sqrt{\frac{(1.5-3.5)^2 + (2-3.5)^2 + 2(2.5-3.5)^2 + 2(3-3.5)^2 + 3(3.5-3.5)^2 + 2(4-3.5)^2 + 2(4.5-3.5)^2 + (5-3.5)^2 + (5.5-5.5)^2}{15}}$$

$$= \sqrt{\frac{4 + 2.25 + 2 + 0.5 + 0.5 + 2 + 2.25 + 4}{15}}$$

$$= \sqrt{\frac{17.5}{15}} = \sqrt{1.167} = 1.08.$$

(ii) With Replacement

Total population $(\mu) = 6(N)$
Sample size $(n) = 2$
$N^n = 6^2 = 36$
36 elements

{ (1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6) }

$$\text{Mean} = \frac{1 + 1.5 + 2 + 2.5 + 3 + 3.5 + 1.5 + 2 + 2.5 + 3 + 3.5 + 4 + 2 + 2.5 + 3 + 3.5 + 4 + 4.5 + 2.5 + 3 + 3.5 + 4 + 4.5 + 5 + 3 + 3.5 + 4 + 4.5 + 5 + 5.5 + 3.5 + 4 + 4.5 + 5 + 5.5 + 6}{36}$$

$$= \frac{126}{36}$$

$$= 3.5$$

(iv) S.D $\sigma = \sqrt{\sum_{i=1}^{6} \frac{(x_i - \mu)^2}{N}}$

$$= \sqrt{\frac{(1-3.5)^2 + 2(1.5-3.5)^2 + 3(2-3.5)^2 + 4(2.5-3.5)^2 + 5(3-3.5)^2 + 6(3.5-3.5)^2 + 5(4-3.5)^2 + 4(4.5-3.5)^2 + 3(5-3.5)^2 + 2(5.5-3.5)^2 + (6-3.5)^2}{36}}$$

$$= \sqrt{\frac{6.25 + 8 + 6.75 + 4 + 1.25 + 0 + 1.25 + 4 + 6.75 + 8 + 6.25}{36}}$$

$$= \sqrt{\frac{52.5}{36}} = 1.2076$$

2a) Given,

mean $(\mu) = 155$

S.D $\sigma = 15$

$n = 36$.

$P(x < 157)$

when $x = 157$

$$\Rightarrow z = \frac{x-\mu}{\sigma/\sqrt{n}} = \frac{157-155}{15/\sqrt{36}} = \frac{2}{15/6} = \frac{2}{2.5}$$

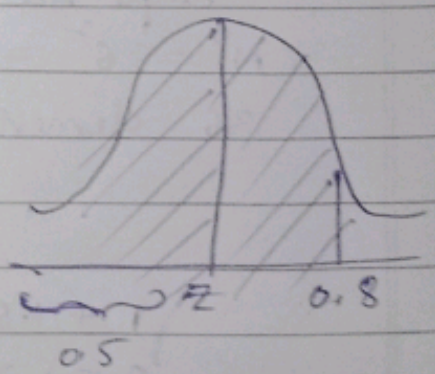
$$z = 0.8$$

$P(z < 0.8)$

$= 0.5 + A(0.8)$

$= 0.5 + 0.2811$

$= 0.7811$



2b) Given, $E = 0.06$

$(1-\alpha)100 = 95$ $\qquad P = \frac{1}{2}$

$(1-\alpha) = \frac{95}{100}$ $\qquad Q = \frac{1}{2}$

$1-\alpha = 0.95$

$\alpha = 0.05$ $\qquad \alpha/2 = 0.025$ $\qquad Z_{\alpha/2} = 1.96$

$$\boxed{n = PQ\left[\frac{Z_{\alpha/2}}{E}\right]^2} = \frac{1}{2}\cdot\frac{1}{2}\left[\frac{1.96}{0.06}\right]^2$$

$$n = \frac{1}{4}\left[32.667\right]^2 = \frac{1067.133}{4} = 266.783.$$

$\therefore n = 266.783$

3] Given,

$\mu = 800$

$\sigma = 40$ $\qquad \alpha = 0.05$

$n = 30$

$\bar{x} = 788.$

1] Null Hypothesis (H_0) : $\mu = 800$

2] Alternative Hypothesis (H_1) : $\mu \neq 800$

3] level of significance $\alpha = 0.05$

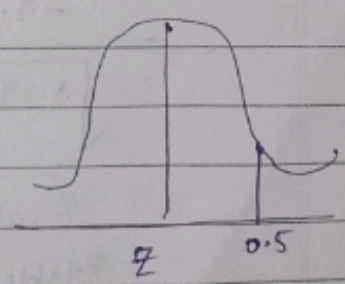
4] Test the statistic $(Z) = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$$Z = \frac{788 - 800}{40/\sqrt{30}} = \frac{-12}{40/5.477} = \frac{-12}{7.303}$$

$t = -1.643$

$|Z_{cal}| = 1.643$

$\therefore Z_{table} = 0.5 - Z_{\alpha/2}$



$= 0.5 - 0.025$

$= 0.475$

$Z_{table} = 1.96$

Con. $\therefore$ $Z_{cal} < Z_{table}$

$\therefore$ H_0 is accepted.

Confidence interval:

$$\left(\bar{x} - Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} , \bar{x} - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}\right)$$

$$= \left(788 - 1.96 \cdot \frac{40}{\sqrt{30}} , 788 + 1.96 \cdot \frac{40}{\sqrt{36}}\right)$$

$$= (788 - 14.314 , 788 + 14.314)$$

$$= (773.686 , 802.314)$$

$n_1 = 71$ $\quad$ $n_2 = 75$

$\bar{x}_1 = 83.2$ $\quad$ $\bar{x}_2 = 90.8$

$\sigma_1 = 19.3$ $\quad$ $\sigma_2 = 21.4$

(1) Null Hypothesis (H_0): $\mu_1 = \mu_2$

(2) Alternative Hypothesis (H_1): $\mu_1 \neq \mu_2$

(3) Level of significiance: $(\alpha) = 0.05$

Test the statistic (Z): $\dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$

$$= \frac{83.2 - 90.8}{\sqrt{\dfrac{(19.3)^2}{71} + \dfrac{(21.4)^2}{75}}} = \frac{-7.6}{\sqrt{\dfrac{372.49}{71} + \dfrac{457.96}{75}}}$$

$$= \frac{-7.6}{\sqrt{5.246 + 6.106}} = \frac{-7.6}{3.369} = 2.256$$

$Z_{cal} = 2.256$

$Z_{table} = 0.5 - Z_{\alpha/2}$

$= 0.475$

$Z_{table} = 1.96$

$\therefore Z_{cal} > Z_{table}$

$\therefore H_0$ is rejected

5] a) Given, $n = 1000$.

$p = \frac{540}{1000} = 0.54$ $\quad P = \frac{1}{2} = 0.5$

$Q = 0.5$

(1) Null Hypothesis (H_0) : $P = 0.5$

(2) Alternative Hypothesis (H_1) : $P \neq 0.5$

(3) level of significance (α) = 0.05

(4) Test the statistic $\boxed{Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}} = \frac{0.54 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{1000}}}$

$= \frac{0.04}{\sqrt{\frac{0.25}{1000}}} = \frac{0.04}{\sqrt{0.00025}} = \frac{0.04}{0.0158} = 2.529$

$\therefore Z_{cal} = 2.529$

$\therefore Z_{table} = Z_{\alpha/2}$ at 0.01

$= 0.5 - 0.005$

$= 0.495 \quad Z_{tab} = 2.58$

$Z_{cal} < Z_{table}$

$\therefore H_0$ is accepted.

5b] Given,

$n_1 = 100 \qquad n_2 = 125$

$P_1 = \frac{63}{100} \qquad P_2 = \frac{59}{125}$

$= 0.63 \qquad = 0.472$

$P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = \frac{100(0.63) + 125(0.472)}{100 + 125}$

$= \frac{63 + 59}{225} = \frac{\cancel{98}122}{225} = \cancel{0.422}\ 0.542$

$p = 0.542$ $\qquad q = 1 - p$

$= 1 - 0.542 = 0.458$

(1) Null Hypothesis (H_0): $P_1 = P_2$

(2) Alternative Hypothesis (H_1): $P_1 \neq P_2$

(3) Level of significance (α) = 0.05

(4) Test the statistic $z = \dfrac{P_1 - P_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

$$= \frac{0.63 - 0.472}{\sqrt{(0.542)(0.458)\left(\frac{1}{100} + \frac{1}{125}\right)}} = \frac{0.158}{\sqrt{0.248\,(0.018)}}$$

$$= \frac{0.158}{\sqrt{0.004}} = \frac{0.158}{0.063} = 2.508$$

$Z_{cal} = 2.508$

$Z_{table} = 0.5 - 0.0025$

$= 0.475$

$Z_{table} = 1.96$

$Z_{cal} > Z_{table}$

$\therefore$ H_0 is rejected.

Unit - 4

2]

Sample - I	9	11	13	11	15	9	12	14
Sample - II	10	12	10	14	9	8	10	—

$\bar{x} = \Sigma x_i = 11.75$ $\quad$ $\bar{y} = \Sigma y_i = 10.429$

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	y_i	$(y_i - \bar{y})$	$(y_i - \bar{y})^2$
9	-2.75	7.563	10	-0.429	0.184
11	-0.75	0.563	12	1.571	2.468
13	1.25	1.563	10	-0.429	0.184
11	-0.75	0.563	14	3.571	12.752
15	3.25	10.563	9	-1.429	2.042
9	-2.75	7.563	8	-2.429	5.9
12	0.25	0.063	10	-0.429	0.184
14	2.25	5.063			$\Sigma(y_i - \bar{y})^2$
		$\Sigma(x_i - \bar{x})^2$			$= 23.714$
		$= 33.504$			

$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2 + \Sigma(y_i - \bar{y})^2}{n_1 + n_2 - 2}}$$

$$s = \sqrt{\frac{33.504 + 23.714}{8 + 7 - 2}} = \frac{7.564}{3.606} = 2.098$$

(1) Null Hypothesis (H_0) : $\mu_1 = \mu_2$

(2) Alternative Hypothesis (H_1) $\mu_1 \neq \mu_2$

(3) level of significance (α) : 0.05

(4) Test the statistic t : $\dfrac{\bar{x} - \bar{y}}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

$$t = \frac{11.75 - 10.429}{2.098\sqrt{\frac{1}{8} + \frac{1}{7}}} = \frac{1.321}{2.098\sqrt{0.125 + 0.143}}$$

$$= \frac{1.321}{2.098\sqrt{0.268}} = \frac{1.321}{1.086} = 1.216$$

t_{table} at 13 $= t_{\alpha/2}$ at 13

$= 2.160$

$\therefore t_{cal} < t_{table}$

$\therefore H_0$ is accepted.

1] Given

63, 63, 66, 67, 68, 69, 70, 70, 71, 71.

$\mu = 66$

$n = 10$

$\nu = n-1 = 9$

Sample mean $\bar{x} = \frac{\Sigma x_i}{n} = 67.8$

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
63	-4.8	23.04
63	-4.8	23.04
66	-1.8	3.24
67	-0.8	0.64
68	0.8	0.04
69	1.2	1.44
70	2.2	4.84
70	2.2	4.84
71	3.2	10.24
71	3.2	10.24

$\Sigma(x_i - \bar{x})^2 = 81.6$

$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{81.6}{9}} = 3.011$$

(1) Null Hypothesis : $\mu = 66$

(2) Alternative Hypothesis : $\mu \neq 66$

(3) level of significance : $\alpha = 0.05$

(4) Test the statistic $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

$$t = \frac{67.8 - 66}{3.011/\sqrt{10}} = \frac{1.8}{\cancel{1.004}\ 0.952} = 1.891$$

$\therefore t_{cal} = 1.891$

$t_{table} = t_{\alpha/2}$ at 9

$t_{table} = 2.262$

$\therefore t_{cal} < t_{table}$

$\therefore H_0$ is accepted.

3]

unit - A	14.1	10.1	14.7	13.7	14.0
unit - B	14.0	14.5	13.7	12.7	14.1

$n_1 = 5$ $\qquad n_2 = 5$

$\bar{x} = \dfrac{66.6}{5}$ $\qquad \bar{y} = 13.8$

$= 13.32$

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$	y_i	$(y_i - \bar{y})$	$(y_i - \bar{y})^2$
14.1	0.78	0.608	14.0	0.2	0.04
10.1	-3.22	10.368	14.5	0.7	0.49
14.7	1.38	1.904	13.7	-0.1	0.01
13.7	0.38	0.144	12.7	-1.1	1.21
14.0	0.68	0.462	14.1	0.3	0.09
		$\Sigma(x_i - \bar{x})^2 = 13.486$			$\Sigma(y_i - \bar{y})^2 = 1.84$

$$s_1^2 = \Sigma \frac{(x_i - \bar{x})^2}{n_1 - 1} \qquad s_2^2 = \Sigma \frac{(y_i - \bar{y})^2}{n_2 - 1}$$

$s_1^2 = \frac{13.486}{4}$ $\qquad$ $s_2^2 = \frac{1.84}{4}$

$= 3.372$ $\qquad$ $= 0.460$

(1) Null Hypothesis (H_0): $\sigma_1^2 = \sigma_2^2$ (2) Alternative Hypo (H_1): $\sigma_1^2 \neq \sigma_2^2$ (3) level of significance $= 0.05$

(4) Test Statistic (F) $= \frac{s_1^2}{s_2^2} = \frac{3.372}{0.46} = 7.33$

$\therefore$ F_{table} at 0.05 at $(4,4)$ $= 15.98$

$\therefore$ $F_{cal} < F_{table}$

$\therefore$ H_0 is accepted.

(4)

Digits	0	1	2	3	4	5	6	7	8	9
frequency	18	19	23	21	16	25	22	20	21	15

$E_i = \frac{\Sigma f_i}{n} = 20$

O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$	$(O_i - E_i)^2 / E_i$
18	20	-2	4	0.2
19	20	-1	1	0.05
23	20	3	9	0.45
21	20	1	1	0.05
16	20	-4	16	0.8
25	20	5	25	1.25
22	20	2	4	0.2
20	20	0	0	0
21	20	1	1	0.05
15	20	-5	25	1.25

$\Sigma \frac{(O_i - E_i)^2}{E_i}$

$= 4.3$

(1) Null Hypothesis = The digits occured frequently in directory
(2) Alternative Hypothesis = The digits didn't occur frequently "
(3) Test the statistic $(x^2) = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$

$x^2_{cal} = 4.3$

(4) level of significance $(\alpha) = 0.05$

t-table at 9 = 16.919

$\therefore$ $t_{cal} < t_{table}$

$\therefore$ H_0 is accepted.

5]

	No. of units	No. of defectives
Machine 1	375	17
Machine 2	450	22

	A	B	Total
C	375	17	392
D	450	22	472
Total	825	39	864

	A	B	Total
C	374.306	17.694	392
D	450.694	21.306	472
Total	825	39	864

O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$	$(O_i - E_i)^2 / E_i$
375	374.306	0.694	0.482	0.001
17	17.694	-0.694	0.482	0.027
450	~~21.306~~ 450.694	-0.694	0.482	0.001
22	21.306	0.694	0.482	0.023

$\frac{\Sigma (O_i - E_i)^2}{E_i} = 0.051$

$\chi^2_{cal} = 0.0516$

(1) Null Hypothesis :

(2) Alternative Hypothesis :

(3) level of significance (α) $= 0.05$

(4) Test the statistic $\chi^2 = 0.0516$

$\chi^2_{table} = 3.841$

$\therefore \chi^2_{cal} < \chi^2_{table}$

$\therefore$ H_0 is accepted.

UNIT-5

1] Karl Pearson's coefficient of Correlation.

X	1	3	4	5	7	8	10
Y	2	6	8	10	14	16	20

$\bar{x} = \frac{38}{7} = 5.429$ $\qquad \bar{y} = \frac{76}{7} = 10.857$

x	$X = x - \bar{x}$	X^2	y	$Y = y - \bar{y}$	Y^2	XY
1	-4.429	19.616	2	-8.857	78.446	39.228
3	-2.429	5.9	6	-4.857	23.59	11.798
4	-1.429	2.042	8	-2.857	8.162	4.083
5	-0.429	0.184	10	-0.857	0.734	0.368
7	1.571	2.468	14	3.143	9.878	4.938
8	2.571	6.610	16	5.143	26.45	13.223
10	4.571	20.894	20	9.143	83.594	41.793
		$\Sigma X^2 = 57.714$			$\Sigma Y^2 = 230.854$	$\Sigma XY = 115.431$

$$r = \frac{\Sigma XY}{\sqrt{\Sigma X^2 \Sigma Y^2}} = \frac{115.431}{\sqrt{(57.714)(230.854)}} = \frac{115.431}{115.427} = 1.$$

2) rank correlation

x	75	30	60	80	53	35	15	40	38	48
y	85	45	54	91	58	63	35	43	45	44

x	r_1	y	r_2	$d = r_1 - r_2$	d^2
75	9	85	9	0	0
30	2	45	4.5	-2.5	6.25
60	8	54	6	2	4
80	10	91	10	0	0
53	7	58	7	0	0
35	3	63	8	-5	25
15	1	35	1	0	0
40	5	43	2	3	9
38	4	45	4.5	-0.5	0.25
48	6	44	3	3	9

$\Sigma d^2 = 53.5$

$$m_1 = \frac{4+5}{2} = 4.5$$

$$r = 1 - \frac{6\left[\Sigma d^2 + \frac{1}{12}(m_1^3 - m_1)\right]}{n(n^2-1)}$$

$$= 1 - \frac{6\left[53.5 + \frac{1}{12}(8-2)\right]}{10(100-1)}$$

$$= 1 - \frac{6\left[53.5 + 0.5\right]}{10(99)} = 1 - \frac{(324)}{990}$$

$$= 1 - 0.327 = 0.673$$

3] Regression y on x x on y

x	10	12	18	24	23	27
y	13	18	12	25	30	10

y on x

$y = mx + c$

$(y-\bar{y}) = b_{yx}(x-\bar{x})$

$b_{yx} = \frac{\Sigma XY}{\Sigma X^2}$

x on y

$x = my + c$

$(x-\bar{x}) = b_{xy}(y-\bar{y})$

$b_{xy} = \frac{\Sigma XY}{\Sigma Y^2}$

$\bar{x} = 19$ $\bar{y} = 18$

x	$X = x - \bar{x}$	X^2	y	$Y = y - \bar{y}$	Y^2	XY
10	−9	81	13	−5	25	45
12	−7	49	18	0	0	0
18	−1	1	12	−6	36	6
24	5	25	25	7	49	35
23	4	16	30	12	144	48
27	8	64	10	−8	64	64

$\Sigma X^2 = 236$ $\Sigma Y^2 = 318$ $\Sigma XY = 70$

y on x

$b_{yx} = \frac{\Sigma XY}{\Sigma X^2} = \frac{70}{236} = 0.297$

$(y-18) = 0.297(x-19)$

$y - 18 = 0.297x - 5.643$

$y = 0.297x + 12.357$

x on y.

$b_{xy} = \frac{\Sigma XY}{\Sigma Y^2} = \frac{70}{318} = 0.22$

$(x-19) = 0.22(y-18)$

$x - 19 = 0.22y - 3.96$

$x = 0.22y + 15.04$

(4) eqn of st. line by least square method.

x	10	12	13	12	16	15
y	40	38	43	45	37	43

4]

x	10	12	13	12	16	15
y	40	38	43	45	37	43

x	y	xy	x^2
10	40	400	100
12	38	456	144
13	43	559	169
12	45	540	144
16	37	592	256
15	43	645	225
$\Sigma x = 78$	$\Sigma y = 246$	$\Sigma xy = 3192$	$\Sigma x^2 = 1038$

$$y = a + bx \longrightarrow (1)$$

$$\Sigma y = na + b\Sigma x \longrightarrow (2)$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 \longrightarrow (3)$$

$$246 = 6a + 78b \longrightarrow (4) \times 13$$

$$3192 = 78a + 1038b \longrightarrow (5)$$

$$3198 = 78a + 1014b \longrightarrow (6)$$

$$3192 = 78a + 1038b \longrightarrow (5)$$

$$6 = -24b$$

$$b = \frac{-6}{24} = -0.25$$

$$\boxed{b = -0.25}$$

$$246 = 6a + 78(-0.25)$$

$$246 = 6a - 19.5$$

$$6a = \cancel{226.5}\ 265.5$$

$$a = \cancel{37.75}$$

$$a = 44.25$$

$$\therefore \boxed{y = 44.25 - 0.25x}$$

5]

x	10	12	13	16	17	20	25
y	10	22	24	27	29	33	37

$$y = a + bx + cx^2$$

$$\Sigma y = na + b\Sigma x + c\Sigma x^2 \longrightarrow (1)$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \longrightarrow (2)$$

$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 \longrightarrow (3)$$

y	x	x^2	x^3	x^4	xy	x^2y
10	10	100	1000	10000	100	1000
22	12	144	1728	20736	264	3168
24	13	169	2197	28561	312	4056
27	16	256	4096	65536	432	6912
29	17	289	4913	83521	493	8381
33	20	400	8000	160000	660	13200
37	25	625	15625	390625	925	23125
$\Sigma y =$	$\Sigma x =$	$\Sigma x^2 =$	$\Sigma x^3 =$	$\Sigma x^4 =$	$\Sigma xy =$	$\Sigma x^2 y =$
182	113	1983	37559	758979	3186	59842

solving (1), (2) & (3) :

$$182 = 7a + 113b + 1983c \longrightarrow (1)$$

$$3186 = 113a + 1983b + 37559c \longrightarrow (2)$$

$$59842 = 1983a + 37559b + 758979c \longrightarrow (3)$$

$$B = \begin{bmatrix} 182 \\ 3186 \\ 59842 \end{bmatrix} \quad A = \begin{bmatrix} 7 & 113 & 1983 \\ 113 & 1983 & 37559 \\ 1983 & 37559 & 758979 \end{bmatrix} \quad X = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$AX = B$ Augumented matrix

$$AX = B \Rightarrow \left[\begin{array}{ccc|c} 7 & 113 & 1983 & 182 \\ 113 & 1983 & 37559 & 3186 \\ 1983 & 37559 & 758979 & 59842 \end{array}\right]$$

On solving,

we get $a = -29.58$

$b = 5.34$

$c = -0.11$

Eqn of second degree

$$y = a + bx + cx^2$$

$$y = -29.58 + 5.34x - 0.11x^2$$