Unit 5

(10) (a)

=) Quantum computer is a computer based on a computational model which uses quantum mechanics.

=) It has a new type of hardware with the computational mechanism based on quantum mechanics.

=) In other words; both the hardware and software should be based on the principles of quantum mechanics.

=) Basic unit of information in quantum computer is called a qubit.

=) It is based on superposition of states in which both the states 0 and 1 are overlapped. Current computers represent information by means of a bit which can have only one value of 0 an 1. but quantum computers use qubits which can represent more information than in bits.

=) As a result, Quantum computers can perform the computation of some important problems. whose solutions were regarded impossible in traditional computers.

=) Some people showed that quantum computers can ~~change~~ be used to compute the answers to certain difficult problems much faster than any classical computers

=) Traditional computers can work on and compute only one transaction at a time whereas quantum computers can work and perform multiple transactions at the same time.

(b)

S.NO	BITS	QUBITS
01.	The device computes by manipulating those bits with the help of logical gates (AND, OR, NOT)	or The device computes by manipulating those bits with the quantum logic gates.
02	Each bit in a classical computer holds either 0 or 1.	A qubit can hold a 1, 0 or crucially a superimposition of these
03	Bits are used in classical computer.	Qubits are used in quantum computer.
04	Information is stored in bits, which take the discrete values 0 and 1.	Information is stored in qubits. A qubit can be in states (called $|0\rangle$ and $|1\rangle$, but it can also be in superposition of these states, $a|0\rangle + b|1\rangle$ where a and b are complex numbers.
05	Bits are slow	Qubits are faster.
06	Its ~~crucial~~ circuit behaviour is based on classical physics	Its circuit behaviour based on quantum mechanics.

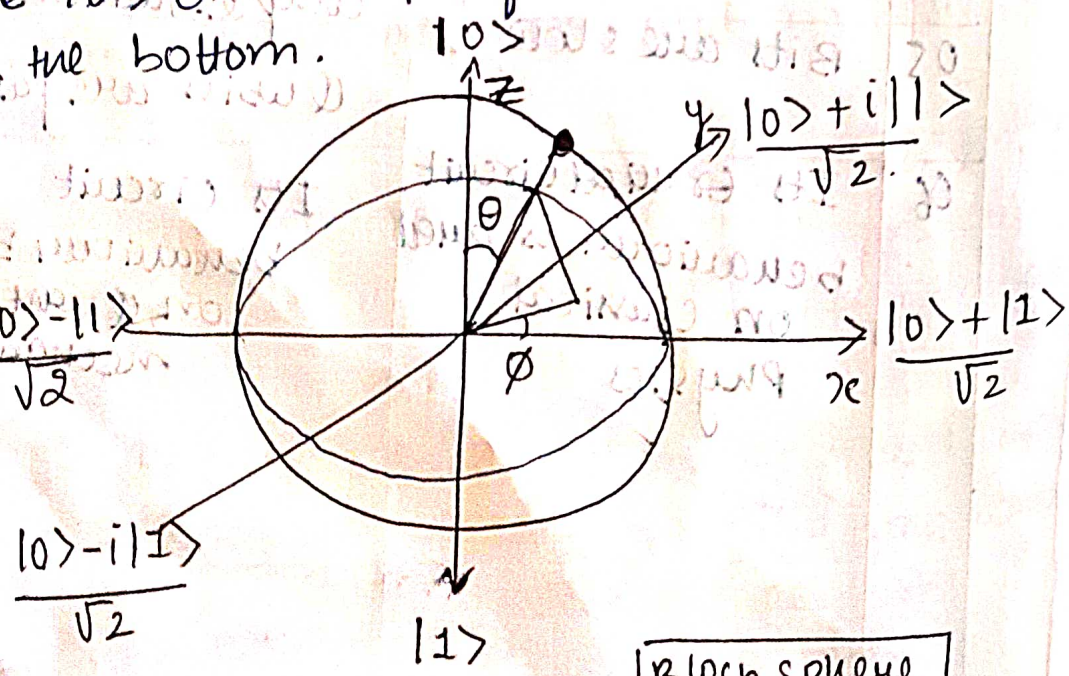
(11) (a) Bloch sphere Representation of Qubits.

⇒ There are several ways to represent the state of a qubit.

1) we can write out the state in Dirac Notation. For example; if we have a qubit that is prepared in state $|0\rangle$ and then apply the X operator, we will then find the qubit in state $|1\rangle$.

$$X|0\rangle \rightarrow |1\rangle$$

2) we can use the Bloch sphere to represent the state of a single qubit. Any state in a quantum computation can be represented as a vector that begins at the origin and terminates on the surface of the unit Bloch sphere. By applying unitary operators to the state vectors, we can move the state around that sphere. we take as convention that the two ~~antipo~~ antipodes of the sphere are $|0\rangle$ on the top of the sphere and $|1\rangle$ on the bottom.



Bloch sphere

③ As we can see in fig one of the advantages of visualization with the bloch sphere is that we can represent superposition states such as we see at the x axis.

⇒ we can also differentiate btw states that contain different phases as its shown in the states along the x and y axes.

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(11) (b) Dirac Notation

⇒ The Dirac Bra-ket notation is a concise and convenient way to describe quantum states.

⇒ we introduce and define the symbol $|\alpha\rangle$ to represent a quantum state.

⇒ This is called a ket, or a ket vector.

⇒ It is an abstract entity, and serves to describe the "state" of a quantum system.

⇒ we say that a physical system is in quantum state α, where α represents some physical quantity, such as momentum, spin etc, when represented by the ket $|\alpha\rangle$

⇒ The rule is to turn inner products into bra-ket pairs as follows:

$$\langle u, v\rangle \rightarrow \langle u|v\rangle$$
$$\langle u|v\rangle \rightarrow \langle u|\,|v\rangle$$

Here; $|v\rangle$ is called a ket and $\langle u|$ is called a bra.

⇒ Dirac allowed the bra's and ket's to line up back to back, i.e $\langle\alpha|\beta\rangle \equiv (|\alpha\rangle, |\beta\rangle)$

$/ \underline{\text{Ln}\, k_B T}) \, , (E_V - E_F) /$

⇒ The symbol $\langle \alpha | \beta \rangle$ represents a complex number that is equal to the value of the inner product of the ket $|\alpha\rangle$ and $|\beta\rangle$.

Pauli gates:

~~The Pauli gates are based on the better known~~

(12) (Pauli matrices) which are incredibly

⇒ used ~~for~~ to calculate the spin of a single electron.

⇒ One pauli gate / matrix for each axis in space (x, y and z).

$$\text{Pauli} - X \equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_{2\times 2}$$

$$\text{Pauli} - Y \equiv \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}_{2\times 2}$$

$$\text{Pauli} - Z \equiv \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}_{2\times 2}$$

The Pauli X-gate

⇒ The pauli x-gate is a dream come true for those that fear matrix math. No imaginary number. No minus signs. And a simple operation: negation.

⇒ this is only natural, because the pauli x-gate corresponds to a classical NOT gate. For this reason, the X gate is often called the quantum NOT gate as well.

- A —— NOT ——o —— $\bar{A}$

A	$\bar{A}$
0	1
1	0

- $|A\rangle$ —— $\boxed{X}$ —— $|\bar{A}\rangle$

$\|A\rangle$	$\|\bar{A}\rangle$
$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$.

- **gate** : Input —— $\boxed{X}$ —— output

- **Matrix**: $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

- **Example**:- Let $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle.$$

- **Truth table**

Input	Output
$\alpha\|0\rangle + \beta\|1\rangle$	$\beta\|0\rangle + \alpha\|1\rangle$.
$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$

Pauli Y - gate

Gate: Input —— $\boxed{Y}$ —— Output

matrix $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

Example Let $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix} = i\begin{bmatrix} 0 \\ 1 \end{bmatrix} = i|1\rangle$$

Truth Table:

Input	Output
$\alpha\|0\rangle + \beta\|1\rangle$	$-i\beta\|0\rangle + i\alpha\|1\rangle$.
$\|0\rangle$	$i\|1\rangle$
$\|1\rangle$	$-i\|0\rangle$.

Pauli Z-gate

Gate: Input —[Z]— Output.

Matrix: $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Example:- $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$.

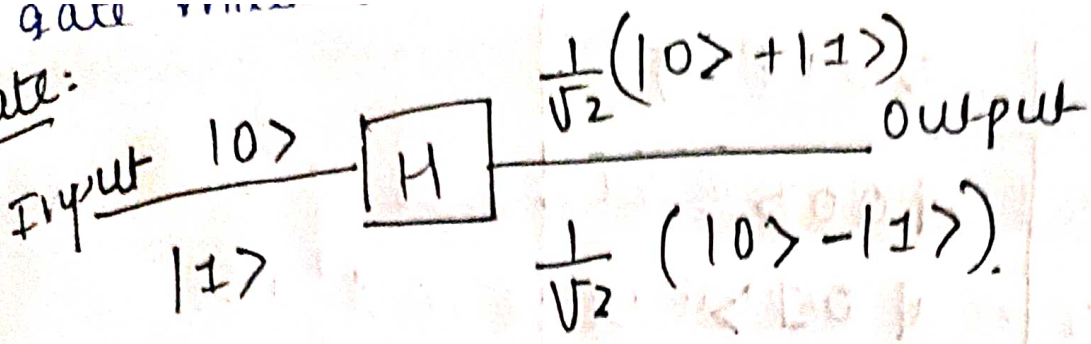
Input	Output
$\alpha\|0\rangle + \beta\|1\rangle$	$\alpha\|0\rangle - \beta\|1\rangle$.
$\|0\rangle$	$\|0\rangle$
$\|1\rangle$	$-\|1\rangle$.

The Hadamard Gate:

⇒ The H-gate is extremely useful for performing the first computation in any ~~program~~ quantum program because it transforms pre-set, or initialized, qubits back into their normal fluid state in order to leverage their quantum powers.

gate

Gate:



Matrix: $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

Example: let $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle).$$

Truth Table

Input	output
$\|0\rangle$	$\frac{1}{\sqrt{2}}(\|0\rangle + \|1\rangle)$
$\|1\rangle$	$\frac{1}{\sqrt{2}}(\|0\rangle - \|1\rangle)$

(25) (a)

⇒ The prototypical multi-qubit quantum gate is the controlled-NOT or CNOT gate.

⇒ This gate has two input qubits, known as the control qubit and the target qubit, respectively.

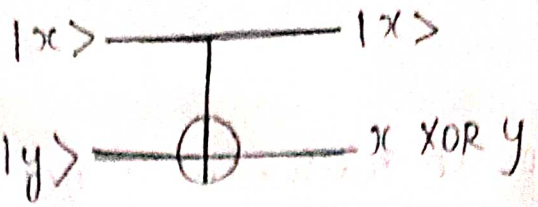
⇒ Another way of describing the CNOT is a generalization of the classical XOR gate, since the action of the gate may be summarized as $|A, B\rangle \rightarrow |A, B \oplus A\rangle$, where $\oplus$ is addition modulo two, which is exactly what the XOR gate does.

⇒ If control bit (first bit) is zero, there is no change in the target bit which is shown in the below table. If the control bit is 1, the flipping of target bit is observed which is also indicated in the following table.

Input state	Output state
$\|00\rangle$	$\|00\rangle$
$\|01\rangle$	$\|01\rangle$
$\|10\rangle$	$\|11\rangle$
$\|11\rangle$	$\|10\rangle$

⇒ In computer science; the controlled NOT gate is a quantum logic gate that is an essential component in the construction of a gate based quantum computer.

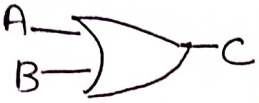
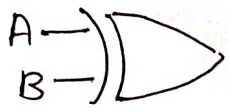
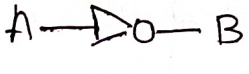
⇒ CNOT is one of universal quantum gates.

GATE	
MATRIX	$CNot = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
REVERSIBILITY	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I$
EXAMPLE	let $\|x\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $\|y\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\|x\rangle \oplus \|y\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \oplus \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \oplus \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(25) (b).

Characteristics	Classical gates	Quantum gates
Basic unit of information	Binary bits : (0, 1)	quantum bits (|0>, |1>) in the form of superposition $\alpha\|0\rangle + \beta\|1\rangle$
Dynamics	Deterministic	Probabilistic
measurements	Never influence the system	influence the system
Feedback loop	Cyclic	Acyclic
Reversibility	NOT Always	Always

CLASSICAL GATES

Logic gate	Symbol	Boolean Function	Table.
AND		$X = A \cdot B$	A B C 1 1 1 1 0 0 0 1 0 0 0 0
OR		$X = A + B$	A B C 1 1 1 1 0 1 0 1 1 0 0 0
XOR		$X = A \oplus B$	A B C 1 1 0 1 0 0 0 1 1 0 0 0
NOT		$X = \bar{A}$	A B 1 0 0 1

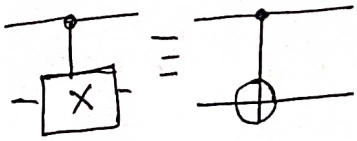
Quantum gates

X-Gate $-\boxed{X}-$ $\equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \beta|0\rangle + \alpha|1\rangle.$

Y-Gate $-\boxed{Y}-$ $\equiv \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = -i\beta|0\rangle + i\alpha|1\rangle.$

Z-Gate $-\boxed{Z}-$ $\equiv \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \alpha|0\rangle - \beta|1\rangle.$

H-Gate $-\boxed{H}-$ $\equiv \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \frac{\alpha + \beta|0\rangle + \alpha - \beta|1\rangle}{\sqrt{2}}.$

C-Not  $\equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = a|00\rangle + b|01\rangle + d|10\rangle + c|11\rangle$

(14) (a) Superposition:

=) A superposition of states is a linear combination of the computational basis vectors

=) the linear combination of two or more state vectors is another state vector in the same Hilbert space and describes another state of the system

=) A quantum system can exist in two or more states at once, ~~(referred to)~~ referred to as a 'superposition' of states or a "superposition state."

(b) Entanglement

=) Two systems are in a special case of quantum mechanical superposition called entanglement if the measurement of one system is correlated with the state of the other system in a way that is stronger

than correlations in the classical world.
In other words, the states of two systems are not separable.

Entangled states:

=) Two quantum mechanical particles can interact with each other and create an entangled state.

=) This means that they share common properties, for example, the total outcome could be 0, meaning that if you would measure the particle to be one, then the other particles would be zero.

(b) Applications

(1) Artificial Intelligence and machine learning:

=) AI and ML are prominent areas, as the emerging technologies have penetrated almost every aspect of human's lives.

=) However, as the no. of applications increased, it becomes a challenging task for traditional computers, to match up accuracy and speed.

=) that's where quantum computing can help in processing through complex problems in very less time, which would have taken traditional computers thousands of years.

② Computational chemistry:-

=) It is belived that the number of quantum states, even in a tiniest of a molecule, is extremely vast, and therefore difficult for conventional computing memory to process.

③ Drug design and development:-

These advancements in computing could enhance efficiency, by allowing companies to carry out more drug discoveries to uncover new medical treatments for the better pharmaceutical industry

④ Cyber security and cryptography

=) Quantum computing with the help of machine learning can help in developing various techniques to combat cyber security threats.

=) Additionally, quantum computing can help in creating encryption methods, also known as, quantum cryptography.

⑤ Weather Forecasting:-

=) A quantum computer's ability to crunch vast amounts of data, in a short period, could indeed lead to enhancing weather system modelling allowing scientists to predict the changing weather patterns in no time and with excellent accuracy.