APPLIED PHYSICS QUESTION BANK WITH ANSWERS

UNIT-1

1. (a) **Explain the processes of absorption, spontaneous and stimulated emission of light and derive the expression for Einstein's coefficients.** (8M)

Answer:

Stimulated Absorption:

An atom in the ground state with energy E1 absorbs a photon of energy hυ and goes to the excited state (Higher state) with energy E2 as shown in fig. This provided the photon energy hυ is equal to the energy difference (E2 - E1). This process is called **Stimulated Absorption** or simply **Absorption**.

Definition: The process in which the atoms transition takes place from the ground state to excited state with supplying of external energy is known as stimulated absorption or absorption.

2.Spontaneous emission:

The atoms in the excited state returns to the ground state by emitting a photon of energy E = (E2 - E1) = hυ, spontaneously without any external triggering as shown in fig. This process is known as **Spontaneous emission**. Such an emission is random and is independent of incident radiation.

Definition: The process in which the atoms transition takes place from the excited state to ground state by emitting photons without supplying of external energy i.e spontaneously is known as spontaneous emission. The emitted photons are not move in the direction of incident photon.

3. Stimulated Emission:

The atom in the excited state can also return to the ground state by external triggering (or) inducement of photon thereby emitting a photon of energy equal to the energy of the incident photon, known as stimulated emission. Thus, results in two photons of same energy, phase difference and of same directionality as shown in Fig.

Definition: The process in which the atoms transition takes place from the excited state to ground state by emitting photons with supplying of external energy is known as spontaneous emission. The emitted photons are having same energy and move with same direction as like incident photon.

EINSTEIN'S COEFFICIENTS (A & B):

In 1916, Albert Einstein proposed that there are three processes occurring in the formation of an atomic spectral line. The three processes are referred to as **spontaneous emission, stimulated emission, and absorption**. With each is associated an Einstein coefficient which is a measure of the probability of that particular process occurring. Einstein considered the case of isotropic radiation of frequency ν, and **energy density $\rho(\nu)$**.

Let us consider N_1 and N_2 be the populations in the energy levels of energies E_1 and E_2 respectively in a system of atoms at a thermal equilibrium of temperature T.

Upward transition:

Absorption is the process by which a photon is absorbed by the atom, causing an electron to jump from a lower energy level E_1 to a higher one E_2. The process is described by the Einstein coefficient B_{12},

The absorption rate is directly proportional to N_1 and $\rho(\nu)$

Therefore, $\boldsymbol{Rate\ of\ absorption = B_{12}N_1\rho(\nu)}$ ---------- **(1)**

Here B_{12} is constant which gives the probability of stimulated absorption transitions per unit time is known as **Einstein's coefficient of absorption**.

Downward transition:

Spontaneous emission is the process by which an electron "spontaneously" (i.e. without any outside influence) decays from E_2 to E_1. The process is described by the Einstein coefficient A_{21}.

Spontaneous emission rate is directly proportional to N_2 only.

Therefore, $\boldsymbol{Rate\ of\ spontaneous\ emission = A_{21}N_2}$ ---------- **(2)**

Here, A_{21} is a constant which gives the probability of spontaneous emission transitions per unit time is known as **Einstein's coefficient of spontaneous emission**.

Stimulated emission is the process by which an atomic electron in the excited E_2 is interacting with a photon of certain frequency may drop to a lower energy level E_1, transferring its energy to that photon. A new photon created in this manner has the same phase, frequency and direction of travel as same as the incident photon. The process is described by the Einstein coefficient B_{21}.

Stimulated emission rate is directly proportional to N_2 and $\rho(\nu)$.

Therefore, $\boldsymbol{Rate\ of\ stimulated\ emission = B_{21}N_2\,\rho(\nu)}$ ---------- **(3)**

Here, B_{21} is a constant which gives the probability of stimulated emissions per unit time is known as **Einstein's coefficient of stimulated emission**.

Einstein was the first to calculate the probability of such transition assuming the atomic

system to be in equilibrium with electromagnetic radiation.

Einstein's theory of absorption and emission of light by an atom is based on Planck's theory of radiation. Also, under thermal equilibrium, the populations of energy levels obey the Maxwell- Boltzmann's distribution law.

(i.e.) under thermal equilibrium,

Rate Absorption = Rate of spontaneous emission + Rate of Stimulated emission

Equation (1) =Equation (2) + Equation (3)

i.e. $B_{12}N_1\rho(\nu) = A_{21}N_2 + B_{12}N_2\rho(\nu)$

$B_{12}N_1\rho(\nu) - B_{12}N_2\rho(\nu) = A_{21}N_2$

$[B_{12}N_1 - B_{21}N_2]\,\rho(\nu) = A_{21}N_2$

$$\rho(\nu) = \frac{A_{21}N_2}{[B_{12}N_1 - B_{21}N_2]} \quad \text{or} \quad \rho(\nu) = \frac{A_{21}N_2}{B_{21}N_2\left\{\left[\frac{N_1}{N_2}\left(\frac{B_{12}}{B_{21}}\right)\right]-1\right\}}$$

$$\rho(\nu) = \frac{A_{21}}{B_{21}}\frac{1}{\left\{\left[\frac{N_1}{N_2}\left(\frac{B_{12}}{B_{21}}\right)\right]-1\right\}} \text{----------}(4)$$

But, in thermal equilibrium, the Boltzmann's distribution law applies, so,

$$N_n = N_0 e^{\frac{-E_n}{K_B T}}$$

If n=1, 2 the above equation modified as,

$$N_1 = N_0 exp\left(\frac{-E_1}{k_B T}\right) \; and \; N_2 = N_0 exp\left(\frac{-E_2}{k_B T}\right)$$

where, N_o is population in ground state and k_B is the Boltzmann's constant.

Therefore, $\frac{N_1}{N_2} = \exp\left[\frac{E_2-E_1}{k_B T}\right] = \exp\left[\frac{h\nu}{k_B T}\right]$ (Since, E_2-E_1= $h\nu$)----------(5)

Substituting eq. (5) in eq. (4), we get

$$\rho(\nu) = \frac{A_{21}}{B_{21}}\frac{1}{\left\{\left[exp\left(\frac{h\nu}{k_B T}\right)\left(\frac{B_{12}}{B_{21}}\right)\right]-1\right\}} \text{---------- (6)}$$

According to Plank's law of black body radiation at temperature T we have the energy density $\rho(\nu)$ at frequency ν is

$$\rho(\nu) = \frac{8\pi h\nu^3}{c^3}\frac{1}{\left\{exp\left(\frac{h\nu}{k_B T}\right)-1\right\}} \text{---------- (7)}$$

Comparing eq. (6) and eq. (7), we get

$$\boxed{\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \quad \text{----- (8)}}$$

\

$$\boxed{\frac{B_{12}}{B_{21}} = 1 \text{ -------- (9)}}$$

Equations (8) and (9) show the relations between Einstein's coefficients B_{12}, B_{21} and A_{21}.

From the above relation eq. (8) shows that the ratio of rate of spontaneous (A_{21}) to rate of stimulated emission (B_{21}) is **proportional to cube of the frequency** of the incident radiation. This is why it is

difficult to achieve the laser action in higher frequency ranges such as X-rays. The eq. (9) shows that the rate of absorption is equal to the rate of stimulated emission, when the system in equilibrium.

(b) **What are Population Inversion and Meta-stable state** **(4M)**

Answer:

Population Inversion creates a situation in which the number of atoms in higher energy state is more than that in the lower energy state.

Usually at thermal equilibrium, the number of atoms N_2 i.e., the population of atoms at higher energy state is much lesser than the population of the atoms at lower energy state N_1.

The Phenomenon of making $N_2 > N_1$ is known as Population Inversion.

Metastable state

Definition: It is an excited state of an atom or other system with a longer life time than other excited states. There could be no population inversion and hence no laser action, if metastable state not exist.

An atom can be excited to a higher level by supplying energy to it. Normally, excited atoms have short life times and release their energy in a matter of nano seconds (10^{-9}) through spontaneous emission. It means atoms do not stay long to be stimulated. As a result, they undergo spontaneous emission and rapidly return to the ground level; thereby population inversion could not be established. In order to do so, the excited atoms are required to ***'wait'*** at the upper energy level till a large number of atoms accumulate at that level. In other words, it is necessary that excited state have a longer lifetime. A ***Meta stable state*** is such a state. Metastable can be readily obtained in a crystal system containing impurity atoms. These levels lie in the forbidden gap of the host crystal. There could be no population inversion and hence no laser action, if metastable states don't exist.

2. (a) Describe the construction of ruby laser and its working with the help of energy level diagram. (8M)

Answer:

PRINCIPLE

A ruby laser is a solid-state laser that utilizes synthetic ruby crystal as its laser medium. In 1960 Maiman designed the first working ruby laser. Ruby laser is among the few solid-state lasers that emit visible light. It produces wavelength 694.3 nm of deep red light.
This is also called solid state laser or three level lasers.

CONSTRUCTION

The Ruby laser consists of a ruby rod whose length is few centimetre and diameter is 0.5 cm. This is made up of with chromium (Cr^{3+}) doped Al_2O_3 material. Both the ends of the ruby rod are silvered such that one end is fully reflecting and the other end is partially reflecting. The ruby rod is surrounded by helical xenon flash lamp tube which provides the optical pumping to raise Cr^{+3} ions to upper energy level. The light from the xenon flash tube was focused by the cylindrical cavity onto the ruby rod, thereby exciting the chromium atoms which were responsible for the laser action. A cooling arrangement is provided to keep the experiment setup at normal temperature.

WORKING (Energy Level Diagram)

The ruby laser is a three-Level system (Adjacent Fig.) since only three energy levels are involved in the process of stimulated emission. The depopulation of the ground state for population inversion is achieved by exciting the atoms of the ruby crystal with intense light from a xenon flash lamp. Thus, the atoms are excited from the ground state (level 1) to an upper state (level 3) by means of absorption. From the energy level 3, the atoms are transferred to energy level 2 without emitting radiation (non-radiative transfer). The energy level 2 is called met stable level since the atoms stay at this level for a longer interval of time. Finally, the atoms return to the ground state from the meta stable level through the process of stimulated emission giving rise to an intense laser light at **6943A°**. The laser beam comes out in the form of a pulse of very short duration (about a millisecond).

CHARACTERISTICS OF RUBY LASER

1. Type: Solid State Laser
2. Active Medium: Al_2O_3: Cr^{3+} ions
3. Pumping method: Optical pumping
4. Pumping source: Xenon flash lamp
5. Optical resonator: Two ends of ruby rod polished with silver used as optical resonator. (one end is fully silvered & other mirror partially silvered)
6. Power Output: 10^4 -10^6 watts (high power output)
7. Nature of output: Pulsed light
8. Output Wavelength: 6943Å or 694.3 nm

Advantages

1. Its output intensity is high
2. It has high acceptance band

Disadvantages

1. The efficiency of ruby laser is very low because only
2. The laser output is not continuous but occurs in the form of pulses of microseconds duration.
3. The defects due to crystalline imperfection are also present in this laserApplications

Applications

1. Many non-destructive testing labs use ruby lasers to create holograms of large objects such as aircraft tires to look for weaknesses in the lining.
2. It is used to find distance between planets.
3. Ruby lasers were used extensively in tattoo and hair removal.

(b) Differentiate the three and four level laser schemes. (4M)

Answer:

In the **three-level lasers** figure, we look for three levels in an atom such that $E_3>E_2>E_1$, with a fast decay between levels 3 and 2 and a slow decay between 2 and 1. Incident radiation of angular frequency $\nu_{31} = (E_3 - E_1)/\hbar$ is used to raise as many atoms as possible from the level 1 to level 3. Note that a population inversion cannot be obtained between levels 3 and 1, because when the number of atoms N_3 in level 3 equals the number N_1 in level 1, absorption will be balanced by stimulated emission and the material will become transparent at the frequency ν_{31}. If level 3 decays rapidly to level 2, a population inversion can be obtained between levels 2 and 1. An example is the ruby laser.

The three-level lasers operate in pulsed mode.

Four level laser schemes

When light is falling on laser medium (active medium), the active centers are readily excited from the ground state to the pumping level E_4. The atoms stay at the E_4 level for only about 10^{-8} s, and quickly drop down to the meta stable state E_3. The spontaneous transitions cannot take place from level E_3 to level E_2, atoms are trapped in the state E_3. The level E_2 is well above the ground state such that $(E_2-E_1) > k_BT$. Therefore, the non-radiative transition (no light) takes place when the atoms are shifted from E_4 to E_3 & E_2 to E_1. From state E_2, the atoms subsequently undergo non-radiative transitions to the ground state E_1 or E_0 and will be once again available for excitation, making it possible for light to be emitted **continuously.**

The four-level laser scheme operate in continuous wave mode

3. (a) Explain the construction and working of Helium-Neon laser with a neat energy level diagram. (8M)

Answer:

HELIUM-NEON (He-Ne) LASER:

PRINCIPLE

The He-Ne laser active medium consists of two gases which do not interact form a molecule. Therefore He-Ne laser is one type of atomic gas laser and also called as Four level laser.

CONSTRUCTION

In 1960 Ali Javan invented the first gas laser that was the Helium-Neon laser. It was the first continuous-light laser and first to operate on the principle of converting electrical energy to a laser light output.

He - Ne gas laser consists of a gas discharge tube of length 30cm and diameter of 1.5cm. The tube is made up of quartz and is filled with a mixture of Neon under a pressure of 0.1mm of Hg. The Helium under the pressure of 1mm of Hg, the ratio of He-Ne mixture of about 10:1, hence the no. of helium atoms is greater than neon atoms. The mixtures are enclosed between a set of parallel mirrors forming a resonating cavity, one of the mirrors is completely reflecting and the other partially reflecting in order to

amplify the output laser beam. The output of the laser depends upon the length of the discharge tube and pressure of the gas mixture.

WORKING

In the He-Ne laser the light produced by atomic transitions within the Neon atom. The Helium does not directly produce laser light but it acts as a buffer gas, this purpose of which is to assist/help the atoms of the other gas to produce lasing action.

The active energy levels of He and Neon atoms are show in adjacent figure. In helium there are three active energy levels named as F_1, F_2 and F_3 where as in Neon, there are six active energy levels named as E_1, E_2, E_3, E_4, E_5, *and* E_6. In Helium, the metastable states are F_2 and F_3, where as in Neon, E_4 and E_6. When a discharge is passed through the gaseous mixture electrons are accelerated in the tube these accelerated electrons collide with the helium atoms and excite them to higher energy levels (F_2 and F_3) since the levels are metastable energy levels, He atoms spend sufficiently long time. Inelastic collision of the excited helium atoms (F_2 and F_3) with the ground state (E_1) Neon atoms results in transfer of energy to the neon atoms, exciting them into E_4 and E_6 states. This is due to the coincidence of energy levels between the helium and neon atoms. Since E_4 and E_6 are metastable states, hence the population inversion takes place between the E_6 and E_4 with respect to E_5 and E_3. The stimulated emission takes place between $E_6 \rightarrow E_3$ gives a laser light of wave length **6328A°** and the stimulated emission between $E_6 \rightarrow E_5$ gives a laser light wavelength of **3.39µm**. Another stimulated emission between $E_4 \rightarrow E_3$ gives a laser light wavelength of **1.15µm**. The neon atoms undergo spontaneous emission from $E_3 \rightarrow E_2$ and $E_5 \rightarrow E_2$. Finally, the neon atoms are returned to the ground state E_1 from E_2 by non-radiative diffusion and collision process.

After arriving the ground state, once again the neon atoms are raised to $E6$ & $E4$ by excited helium atoms thus we can get **continuous wave** output from He-Ne laser. But some optical elements placed insides the laser system are used to absorb the infrared laser wavelengths 3.39µm and1.15µm. Hence the output of He-Ne laser contains only a single wavelength of 6328A° and the output power is about few milliwatts.

Laser transitions

Consider E_1, E_2, E_3, E_4, E_5, E_6 as Ne_1, Ne_2, Ne_3, Ne_4, Ne_5, Ne_6 respectively.

$Ne_6 \longrightarrow Ne_3$ Transition: This transition generates a laser beam of red colour of wavelength 632.8 nm or 6328 Å

$Ne_6 \longrightarrow Ne_5$ transition: During this transition electromagnetic radiation of wave length 3.39 µm or 33900 Å

$Ne_6 \longrightarrow Ne_3$ transition: During this transition electromagnetic radiation of wave length 1.15 µm or 11500 Å is emitted.

CHARACTERISTICS OF Helium-Neon LASER

1. Type: Gas Laser
2. Active Medium: Helium-Neon gases mixture
3. Pumping method: Electrical discharge
4. Pumping source: R.F. Oscillator (DC Power supply)
5. Optical resonator: External setup mirror
6. Power Output: 5-50 milliwatts (high power output)

7. Nature of output: Continuous wave mode
8. Output Wavelength: visible and infrared wavelength (638.2 nm, 3.39 μm & 1.15 μm)

Advantages

1.Helium-neon laser emits laser light in the visible portion of the spectrum.
2.High stability
3.Low cost
4.Operates without damage at higher temperatures

Disadvantages

1.Low efficiency
2.Low gain
3.Helium-neon lasers are limited to low power tasks

Applications

1. Helium-neon lasers are used in industries.
2. Helium-neon lasers are used in scientific instruments.
3. Helium-neon lasers are used in the college laboratories.

The He-Ne Laser is used in Holography in producing the 3D images of objects

(b) **Explain the Electrical and Optical pumping schemes**

Answer: (4M)

PUMPING ACTION

The process to achieve the population inversion in the medium is called **Pumping action.** It is essential requirement for producing a laser beam.

DIFFERENT PUMPING METHODS

The methods commonly used for pumping action are:

1. Optical pumping (Excitation by Photons)
2. Electrical discharge method (Excitation by electrons)
3. Direct conversion
4. In elastic atom – atom collision between atoms

OPTICAL PUMPING

When the atoms are exposed to light radiations energy hυ, atoms in the lower energy state absorb these radiations and they go to the excited state. This method is called Optical pumping. It is used in solid state lasers like ruby laser and Nd-YAG laser. In ruby laser, xenon flash lamp is used as pumping source.

ELECTRIC DISCHARGE

In this method, the electrons are produced in an electrical discharge tube. These electrons are accelerated to high velocities by a strong electrical field. These accelerated electrons collide with the gas atoms.

By the process, energy from the electrons is transferred to gas atoms. Some atoms gain energy and they go to the excited state. This results in population inversion. This method is called Electrical discharge method.

It is represented by the equation

$$A + e^* = A^* + e$$

Where A – gas atom in the ground state
A* = same gas atom in the excited state
e* = Electrons with higher Kinetic energy
e – Same electron with lesser energy.
This method of pumping is used in gas lasers like argon and CO_2 Laser.

4. (a**) Explain the construction and working of Semiconductor diode laser with a neat energy level diagram**.

Answer:

PRINCIPLE

When a p-n junction diode is forward biased, the electrons from n – region and the holes from the p-region cross the junction and recombine with each other.
During the recombination process, the light radiation (photons) is released from a certain specified direct band gap semiconductor like Ga-As. This light radiation is known as recombination radiation or radiative recombination.
The photon emitted during recombination stimulates other electrons and holes to recombine. As a result, stimulated emission takes place which produces laser.

CONSTRUCTION OF HOMOJUNCTION SEMICONDCUTOR DIODE LASER

In this laser system, the active medium is pn junction diode formed between n-GaAs and p-GaAs. The impurities germanium and tellurium are doped into GaAs semiconductor to obtain p-type and n-type GaAs respectively. The thickness of the pn-junction layer is very narrow so that the emitted radiation has large divergence and poor coherence. At the junction two sides which are parallel to each other and are well polished through which laser is emitted and other sides are roughened to avoid laser emission. To provide forward bias two metal contacts are provided in the top and bottom of the diode. This diode is extremely small in size i.e 500 μm long and 100 μm wide and thick.

CONSTRUCTION OF HETEROJUNCTION SEMICONDCUTOR DIODE LASER

The heterojunction diode laser further classified as single and double heterojunction laser diode lasers. It is a multilayer structure designed such that the carriers are confined in a narrow region and population built up at lower current levels.

We study here the structure and working of a double heterojunction diode laser. In a double heterojunction laser, a GaAs layer is sandwiched between two layers of GaAlAs.

The material GaAlAs has a wider energy gap and a lower refractive index than GaAs. In figure, the numbers 1,2,3,4 and 5 are indicating the various layers. The laser emission takes place between the layers 2 and 4 where 1,2,4 are GaAlAs layers and 3,5 are GaAs layers.

WORKING OF HOMOJUNCTION AND HETEROJUNCTION SEMICONDUCTOR DIODE LASER

- The working principle is same for the homo and heterojunction diode laser systems. The population inversion can be obtained by injecting electrons and holes in to the junction from the n-region and p-region by means of forward bias voltage.
- When the forward bias is not connected, no electrons and holes present in the depletion region. When small forward bias voltage is given to the p-n junction then small number of electrons and holes will injected into the depletion region from respective regions.
- When relatively a large current of the order of 10^4 A/cm^2 is passed through the junction then large number of electrons and holes will be injected into the depletion region as shown in above figure. Then the direct recombination processes take place between holes and electrons in the depletion region and release the photons. Further the emitted photons increase the rate of recombination.

Thus, a greater number of photons produced having same phase and frequency of the induced photons.

The wavelength of the emitted radiation depends on the energy band gap of the semiconductor material. The energy gap of the GaAs Semiconductor is 1.44 eV then it emits laser light of wavelength ~8600 A°

The wavelength of laser light is given by,

$$E_g = h\nu = h\frac{c}{\lambda}$$

$$\lambda = \frac{hc}{E_g}$$

Where, h=6.626 x 10 ^-34 J.S

C= 3 x 10^8 m/s

Eg=1.45 eV for GaAs, then wavelength of laser light can be calculated.

CHARACTERISTICS OF HOMOJUNCTION DIODE LASER

1. Type: Solid state semiconductor Laser
2. Active Medium:
A PN junction diode made from single crystal of gallium arsenide is used as an active medium.
3. Pumping method: Direct Conversion
4. Pumping source: High forward bias voltage supplied battery
5. Optical resonator: p-n junctions are polished well act as an optical resonator
6. Power Output: 1 milliwatts
7. Nature of output: Continuous wave mode or pulsed
8. Output Wavelength: 8300 to 8500 Å

CHARACTERISTICS OF HETEROJUNCTION DIODE LASER

1.Type: Solid state semiconductor Laser
2. Active Medium:
A PN junction diode made from different layers is used as an active medium.
3. Pumping method: Direct Conversion
4. Pumping source: High forward bias voltage supplied battery
5. Optical resonator: p-n junctions are polished well act as a optical resonator
6. Power Output: 1 milliwatts
7. Nature of output: Continuous wave mode or pulsed
8. Output Wavelength: Nearly 8000 Å

(b) **Write the applications of Laser in Industry.**

Answer:

Lasers are employed over a wide range of applications from scientific research, biomedicine, and environmental sciences to industrial materials processing, microelectronics, avionics, and entertainment. Lasers are widely used in manufacturing, e.g. for cutting, drilling, welding, cladding, soldering (brazing), hardening, ablating, surface treatment, marking, engraving, micromachining, pulsed laser deposition, lithography, alignment, etc. In most cases, relatively high optical intensities are applied to a small spot, leading to intense heating, possibly evaporation and plasma generation. Essential aspects are the high spatial coherence of laser light, allowing for strong focusing, and often also the potential for generating intense pulses.

Laser processing methods have many advantages, compared with mechanical approaches. They allow the fabrication of very fine structures with high quality, avoiding mechanical stress such as caused by

mechanical drills and blades. A laser beam with high beam quality can be used to drill very fine and deep holes, e.g. for injection nozzles. A high processing speed is often achieved, e.g. in the fabrication of filter sieves. Further, the lifetime limitation of mechanical tools is removed. It can also be advantageous to process materials without touching them.

The main industrial applications of laser are

1. **Welding**: It can be done even at difficult to reach place. There is no chance of impurities reaching while joining of metal parts. CO_2 lasers are widely used in welding of thin sheets and foils.
2. **Drilling:** It takes place through intense evaporation of material heated by a sequence of powerful light pulses of short duration of 10^{-4} to 10^{-3}s. Piercing of holes in a baby bottle nipple and drilling holes in diamond are two interesting examples of drilling.
3. **Heat Treatment for Hardening:** It is used in the tooling and automotive industry to strengthen cylinder blocks, gears, camshafts etc in the automobile industry. In general, CO_2 lasers of about 1 kW output power operating in continuous mode are used for heat treatment.
4. **Cutting:** The materials include paper, wood, cloth, glass, quartz, ceramics, steel etc. The advantage of laser cutting is very fine and precise.
5. **Holography:** It is a technique for recording and reproducing an image of an object without the use of lenses. The three-dimensional image of object can be recorded by using laser.
6. **Electronics Industry:** It uses lasers in the manufacture of electronic components and integrated circuits. Trimming of thick and thin film resistors using lasers is a very common application.
7. **Measurement of atmospheric pollutants:** Laser is a very useful tool for the measurement of the concentration's variations of various atmospheric pollutants such as N_2, CO, SO_2 etc. gases and particulate matter such as dust, smoke and fly ash without sample collection.
8. **Medical Industry:** Lasers are playing important role in diagnostics and surgery. The medical industry utilizes mostly the diode & CO_2 lasers.
9. **Communication Industry:** Laser work as a light source in all fiber optics in use.

5. (a) Explain construction and working principle of optical fiber. (6M)

Answer:

OPTICAL FIBER CONSTRUCTION

A very thin, long, flexible, transparent, cylindrical dielectric medium which guide the light signal propagated through it.

Structure or construction of optical fiber

It consists of three parts:

1. The core
2. The cladding,
3. The outer jacket.

To avoid the damage of optical fiber cable and longer distance communication the outer jacket comprised into several coatings such as silicon coating, strengthen member, buffer coating altogether is called as sheath.

CORE: It consists of an inner cylinder made of glass or plastic called core. The core has high refractive index n1. It is **the inner light-carrying layer. The core diameter range: 8.5 µm to 63.5 µm**

CLADDING: This core is surrounded by cylindrical shell of glass or plastic called cladding. The cladding has low refractive index n2. It is the middle layer. Light is always launched into the core and striking the core-cladding interface. **Diameter range: 125 µm**

SILICON COATING: It is placed between buffer jacket and cladding to improve the quality of transmission of light.

BUFFER JACKET: it is made up of plastic and protects the fiber from moisture and abrasion

STRENGTH MEMBERS: In order to provide the necessary toughness and tensile strength it is surrounded the buffer jacket.

OUTER JACKET: It is the outermost layer. It is covered the fiber cable by black polyurethane

Because of this arrangement it is not damaged during hard pulling, bending, stretching or rolling. **Diameter range including from coating to outer jacket: 250 µm to 900 µm.**

PRINCIPLE OF OPTICAL FIBER

The transmission of light in optical fiber is based on the principle of **Total Internal reflection**.

Let n_1 and n_2 be the refractive indices of core and cladding respectively such that $n_1 > n_2$. Let a light ray travelling from the medium of refractive index n_1 to the refractive index n_2 be incident with an angle of incidence θ and the angle of refraction r. By Snell's law,

$$n_1 \sin \theta = n_2 \sin r \text{ ----------(1)}$$

The refracted ray bends towards the normal as the ray travels from rarer medium to denser medium. On the other hand, the refracted ray bends away from the normal as it travel from denser medium to rarer medium. In Optical fiber, the light ray travelled from core (denser) to cladding (rarer) medium, there is possibility of total internal reflection, if the angle of incidence is greater than the critical angle (θ_c).

Critical Angle: When a light ray moves from high refractive index (core) medium to low refractive index (cladding) medium and for a particular angle of incidence the refraction angle (r) is 90° then the angle of incidence is known as critical angle (θ_c).

n_1 = refractive index of the denser medium n_2 = refractive index of the rarer medium

θ_1 = angle of incidence θ_2 = angle of refraction θ_C = critical angle

Snell's law of refraction
$n_1 \sin\theta_1 = n_2 \sin\theta_2$

For critical angle, $\theta_2 = 90^0$
$n_1 \sin\theta_1 = n_2 \sin 90^0 = n_2 \times 1$
$\theta_1 = \theta_C = \arcsin(n_2/n_1)$

For total internal reflection to occur, $\theta_1 > \theta_C$

i. When $\vartheta < \vartheta_c$, then the ray refracted into the second medium as shown in figure.
ii. When $\vartheta = \vartheta_c$, then the ray travels along the interface of two media as shown in the above figure.
iii. When $\vartheta > \vartheta_c$, then the ray totally reflects into the same medium (core) as shown in the above figure.

The critical angle can be calculated from Snell's law, suppose if $\theta = \theta_c$ then $r = 90^o$ then,

$$n_1 \sin \theta_c = n_2 \sin 90^0$$

$$\sin \theta_c = n_2 / n_1$$

Critical angle $\boxed{\theta_c = \sin^{-1}(n_2/ n_1)}$

The light signal once entered the fiber and takes total internal reflection within the core as shown in figure, it will continue to propagate till the other end of the fiber.

(b) **Distinguish between step index and graded index optical fibers.** **(6M)**

<u>ANSWER</u>

Difference between Step Index fiber and GRIN fiber

Basis of comparison	**Step Index fiber**	**GRIN fiber**
Definition	Step index fiber is a fiber in which the core is of a uniform refractive index and there is a sharp decrease in the index of refraction at the cladding	Graded index fiber is a type of fiber where the refractive index of the core is maximum at the center core and then it decreases towards core-cladding interface.
Types	Step index fiber is found in two types, that is mono or single mode fiber and multi-mode fiber.	Graded index fiber is of only one type, that is, multi-mode fiber.
Index Profiles	Index profiles are in the shape of step.	Index profiles is in the shape of a parabolic curve.

Light Rays Propagation	The path of light propagation is *zig- zag* in manner. The light ray propagation is in the form of meridional rays and it passes through the fiber axis.	The path of light is *helical* in manner. The light propagation is in the form of skew rays and it will not cross fiber axis.
Signal Distortion	Signal distortion is more in case of high-angle rays in multimode step index fiber. In single mode step index fiber, there is no distortion	Signal distortion is very low even though the rays travel with different speeds inside the fiber.
Bandwidth Size & Cost	The fiber has lower bandwidth. Less expensive.	The fiber has higher bandwidth. Higher expensive.
Diameter of the Core	The diameter of the core is between 50-200µm in the case of multimode fiber and 10µm in the case of single mode fiber.	The diameter of the core is about 50µm in the case of multimode fiber.
Attenuation	*Attenuation is more* for multimode step index fiber but for single mode it is very less.	*Attenuation is less.*
Application	Used for short distance communication.	Used for long distance communication.

6. (a**) Define and deduce the expression for acceptance angle and numerical aperture of an optical fiber**. (8M)

ANSWER:

ACCEPTANCE ANGLE

Definition

The maximum angle of incidence to launch the beam at its one end to enable the entire light to pass through the core is called acceptance angle

(or)the maximum angle at or below which the light can suffer Total Internal Reflection is called acceptance angle.

When we launch the light beam in to the fiber at its one end the entire light may not pass through the core and propagate. Only the rays which make the angle of incidence greater than critical angle undergo total internal reflection and propagate through the core and all other rays are lost.

Let us consider a cross-sectional view of an optical fiber along its length as shown in figure. Let n_0, n_1 **and** n_2 are the refractive indices of air medium, core and cladding respectively. Here n_2 is greater than n_0 and n_1 is greater than n_0 and n_2. Let OA be the incident ray on the interface of air medium and core with angle of incidence 'i' or 'α_i'. Then the light refracts into core medium with angle of refraction 'α_r' or 'r' and the refracted ray OB incidents on the interface of core and cladding with angle of incidence (90°-θ). If the value of (90°-θ) is greater than critical angle (θ_c), the total internal reflection takes place then it continues till other end of the fiber through multiple total internal reflections.

According to Snell's law at the point 'A'

$$n_0 \sin \alpha_i = n_1 \sin \alpha_r \text{----------(1)}$$

Let a normal 'BC' be drawn from the point 'B' to the fiber axis. Then from ΔABC, we get

$$\alpha_r = 90° - \theta \rightarrow (2)$$

Substitute eq - (2) in eq - (1)

$$n_o sin\alpha_i = n_1 sin(90° - \theta)$$

$$n_o sin\alpha_i = n_1 cos\theta \rightarrow (3)$$

To get total internal reflection at point B (core-cladding interface) i.e. θ > θc

Let the maximum angle of incidence at point A be α_i(max)for which$\theta > \theta_c$.

$$n_o sin\alpha_i = n_1 cos\theta \rightarrow (4)$$

$\sin \alpha_i = n_1 / n_0 \cos\theta$----------(4)

If θ is less than the critical angle θc, the ray will be lost by refraction. Therefore, limiting value for containing the beam inside the core by total internal reflection is θc. Let 'i' or 'α_i' be the maximum possible angle of incidence at the fiber face A for which θ = θc

If θ = θc; $\alpha_i = \alpha_{i(max)}$, the equation (4) becomes,

$\sin \alpha_{i(max)} = n_1 / n_0 \cos\theta c$----------(5)

$\sin^2\theta + \cos^2\theta = 1$; $\cos\theta = \sqrt{1 - sin^2\theta}$

substituting this value in equation (5) becomes,

$$\sin \alpha_{i(max)} = n_1 / n_0 \sqrt{1 - sin^2\theta_c}$$

Substituting the Critical angle **$\sin\theta c = n_2/n_1$** in the above equation becomes,

$$\sin \alpha_{i(max)} = n_1 / n_0 \sqrt{1 - \frac{n_2^2}{n_1^2}}$$

After deducing the above expression, it can be written as

$$sin\ \alpha_i(\text{max}) = \frac{n_1}{n_0}\sqrt{\frac{n_1^2 - n_2^2}{n_1}}$$

$$\sin \alpha_{i(max)} = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

$$\alpha_{i(max)} = \sin^{-1}\left(\frac{\sqrt{n_1^2 - n_2^2}}{n_0}\right)$$

For air medium $n_0 = 1$; $\alpha_{i(max)} = \sin^{-1}(\sqrt{n_1^2 - n_2^2})$ if $\alpha_{i(max)} = i_o$

Acceptance angle $\alpha_{i(max)} = i_o = \sin^{-1}(\sqrt{n_1^2 - n_2^2})$

Numerical Aperture

The numerical aperture (NA) is defined as the sine of the acceptance angle. Numerical aperture determines the light gathering ability of the fiber. It is a measure of amount of light that can be accepted by a fiber. NA depends only on the refractive indices of the core and cladding materials. A large NA implies that a fiber will accept large amount of light from the source. Its value varies **from 0.13 to 0.5**

$$NA = \sin i_o$$

$$i_o = \sin^{-1}\left(\frac{\sqrt{n_1^2 - n_2^2}}{n_0}\right)$$

The above equation becomes,

$$NA = \sin\left[\sin^{-1}\left(\frac{\sqrt{n_1^2 - n_2^2}}{n_0}\right)\right] \qquad \sin[\sin^{-1}(x)] = x$$

$$NA = \left(\frac{\sqrt{n_1^2 - n_2^2}}{n_0}\right) \text{ If } n_o = 1$$

$$\boxed{NA = \sqrt{n_1^2 - n_2^2}}$$

(b) Explain Liquid level sensor.

<u>ANSWER</u>

A sensor is defined as a device or a module that helps to detect any changes in physical quantity like pressure, force, temperature, displacement and electrical quantity like current or any other form of energy. After observing the changes, sensor sends the detected input to a microcontroller or microprocessor.

A fiber optic sensor in general, consists of a light source. The light source is coupled to an optical fiber. A light detector which receives the signal – carrying light beam is as it emerges from the fiber.

Fiber optic liquid-level sensors can be classified in the following two categories: sensors for continuous measurements and sensors for point measurements. It is based on the total internal reflection of light, which is disturbed by contact with liqu*id.* The system includes an electronic control unit with a transmitter and a receiver, a sensor, and two fibers. A chamfered Optical fiber, containing of core alone is used in a level sensor. The condition here is that the refractive indices of air, core and liquid should such that $\mathbf{n_{air} < n_{core} < n_{liquid}}$**.**

A light signal from the source is fed to the fiber core as shown in fig. (i). The light signal reaches the detector after getting subjected to total internal reflections in the core. It is observed here that the liquid level did not touch the tip of the optical fiber yet and hence the light signal smoothly reaches the detector.

When the liquid raises to sufficient level as shown in figure (ii), at the point of core-liquid interface, total internal reflection cannot take place. The reason is $n_{core} < n_{liquid}$. Therefore, the light signal gets leaked into liquid without reaching the detector.Thus, the light signal reaching the detector indicates lower liquid level while the detector does not receive the light signal, indicates sufficient level of the liquid.

7. (a) **Explain Attenuation losses in Optical fibers** (4M)

Losses in Optical fiber

As light signal propagates through a fiber, it suffers loss of amplitude and change in shape. The loss of amplitude is referred to as attenuation and the change in shape as dispersion.

Attenuation

Definition: The attenuation of optical signal is the ratio of the optical output power from a fiber of length 'L' to the input optical power.

$P_o = P_i\, e^{-\alpha L}$----------(1)

Where 'α' is called the fiber attenuation coefficient expressed in units of km^{-1}.

Taking logarithms on both the sides of the equation (1) we obtain,

$$\alpha = \frac{1}{L} ln \frac{P_i}{P_o}$$

In units of dB/km, *the above equation can* be modified as,

Figure 4.2

$$\alpha_{dB/km} = \frac{10}{L} ln \frac{P_i}{P_o}$$

$$P_o = P_i e^{-\alpha L} \rightarrow (1)$$

where, P_o – Output Power (optical)

P_i – Input Power (optical)

α – attenuation Co-efficient

L – Length of optical fibre [in km]

To find 'α', Take a logarithm on both sides of equation (1), we get

$$\ln P_o = \ln\left[P_i e^{-\alpha L}\right]$$

$$\ln P_o = \ln\left[\frac{P_i}{e^{\alpha L}}\right]$$

$$\ln P_o = \ln P_i - \ln(e^{\alpha L})$$

$$\ln P_o = \ln P_i - \alpha L$$

$$\alpha L = \ln P_i - \ln P_o$$

$$\alpha L = \ln\left[\frac{P_i}{P_o}\right]$$

$$\alpha = \frac{1}{L} \ln\left[\frac{P_i}{P_o}\right]$$

$\therefore e^{-\alpha L} = \frac{1}{e^{\alpha L}}$

$\ln\left[\frac{A}{B}\right] = \ln A - \ln B$

$\ln(e^{x}) = x$

In terms of dB/km unit, the above eqn modified as,

$$\boxed{\alpha = \frac{10}{L} \log_{10}\left[\frac{P_i}{P_o}\right]}$$

Attenuation can be divided into two categories.

1. Intrinsic Attenuation
   - Absorption loss
   - Scattering loss
2. Extrinsic Attenuation
   - Macrobend loss
   - Microbend loss

Absorption Loss:

1. Absorption by material: It is a major cause of signal loss in an optical fiber resulting from the conversion of optical energy into some other energy like heat.

2. Usually, absorption of light occurs due to imperfections of the atomic structure such as missing molecules, (OH-), hydroxyl ions, high density cluster of atoms etc., which absorbs light.

Figure: - 7(Scattering process)

Scattering Loss:

Scattering is also a wavelength dependent loss, which occurs inside the fibers. Since the glass is used in fabrication of fibers, the disordered structure of glass will make some variations in the refractive index inside the fiber. As a result, if light is passed through the atoms in the fiber, a portion of light is scattered (elastic scattering) this type of scattering is called Raleigh scattering.

$$\textbf{Rayleigh scattering } \alpha \frac{1}{\lambda^4}$$

Extrinsic Attenuation

It is classified according to the bending of finite radius of curvature in optical fibers. i.e micro bend and macro bend loss. It's usually caused by improper handling of fibre optic cable.

Macrobending loss: outside the fiber

If the radius of the core is large compared to fiber diameter, it may cause large-curvature at the position where the fiber cable turns at the corner. At these corners the light will not satisfy the condition for total internal reflection and hence it escapes out from the fiber. This is called as macroscopic / macro bending losses. Also note that this loss is negligible for small bends. It is visible to us.

Microbending loss: inside the fiber

Micro-bends losses are caused due to non-uniformities or micro bends inside the fiber as shown. This micro bends in fiber appears due to non-uniform pressures created during the cabling of the fiber or even during the manufacturing itself. This led to loss of light by leakage through the fiber. It is invisible to us.

(b) **Discuss the optical fiber communication system with a neat block diagram. (4M)**

ANSWER:

<u>Optical Fiber Communication System or Fiber Optic Communication System(OFCS)</u>

<u>PARTS OF OFCS</u>

Fig.: Block Diagram of Optical Fiber Communication System

a) **Encoder:** The audio signal (i.e., the words spoken by us) is converted into electrical signal which is an analog signal. Encoder is an electronic circuit that converts this analog signal into binary or digital signals.

b) **Transmitter:** The digital signal from the encoder is fed to the transmitter which consists of two parts- Drive circuit and Light source. Drive circuit receives the digital signal from encoder and feeds it to the light source. Light source is usually LED or a Diode LASER. If digital '0' is received then light source will be turned OFF. If digital '1' is received then the light source will be turned ON. Thus, light source converts electrical signals into optical signals.

c) **Waveguide:** Now the Optical signals generated by the transmitter are fed to an optical fiber which acts as waveguide. The signal travels over longer distances through these waveguides.

d) **Receiver:** On the other side of the waveguide, the optical signal is received by the receiver which consists of Photo detector, amplifier and a signal restorer. The Photo detector receives the optical signal and generates the equivalent electrical signals. These electrical signals are amplified by the amplifier. The signal restorer keeps all the electrical signals in a sequential form and supplies to decoder.

e) **Decoder:** It is an electronic system that converts the digital signal to analog signal.

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7. Optical power of 1mW is launched into an optical fiber of length 100m. If the power emerging from the other end is 0.3mW, Calculate the fibre attenuation.

Solution.

Given: $P_i = 1mW$

$P_o = 0.3\ mW$

$L = 0.1\ km = 100m$

$\alpha = ?$

$$\alpha = \frac{10}{L} \log_{10}\left(\frac{P_i}{P_o}\right) = \frac{10}{0.1km} \log_{10}\left(\frac{1mW}{0.3mW}\right)$$

$$\alpha = \frac{10}{0.1} \log_{10}\left(\frac{1}{0.3}\right) dB/km$$

$$\boxed{\alpha = 52.3\ dB/km}$$

Unit-II

8. (a) What are de-Broglie matter waves and arrive at the expression of de-Broglie wavelength in different forms. (6M)

ANSWER

- The universe is made of radiation (light) and matter (particles). The light exhibits the dual nature (i.e.) it can behave both a wave (interference, diffraction, phenomenon) and as a particle (Compton Effect, photo electric effect etc.)
- In 1923 Louis de-Broglie suggested that an electron or any other material particle must exhibit wave like properties in addition to particle nature.
- The waves associated with a moving material particle are called as matter waves or de-Broglie waves.
- According to his hypothesis, when a material is moving, sometimes it act as like a particle and sometimes it acts as like a wave.

de-Broglie wavelength derivation

From theory of light, considering a photon as a particle the energy of the photon is given by,

$$E = mc^2 \longrightarrow (1)$$

where, m – mass of the particle in kg
c – velocity of light – in m/s

Considering photon as a wave, the energy of the photon is given by,

$$E = h\nu \longrightarrow (2)$$

where, h – planck's constant (6.626×10^{-34} J-s)
ν – frequency of radiation in Hz

From equations (1) & (2).

$$E = mc^2 = h\nu \longrightarrow (3)$$

we know, Momentum $(p) = m \times v$

Here $\boxed{v = c}$

$$p = mc$$

$$mc \times c = h\nu$$

$$p \times c = h\nu$$

we know, $c = \nu\lambda$; $\lambda = \frac{c}{\nu}$; $\nu = \frac{c}{\lambda}$

$$p \times \cancel{c} = \frac{h\cancel{c}}{\lambda}$$

$$p = \frac{h}{\lambda}$$

$$\boxed{\lambda = \frac{h}{p} \longrightarrow (4)}$$

De-Broglie proposed the concept of matter waves, according to which a material particle of mass 'm' moving with velocity 'v' should be associated with de-Broglie wavelength 'λ' given by

$$\lambda = h / m v = h / p \text{ ------(5)}$$

The above eqn. represents de-Broglie wave eqn.

<u>Properties of Matter waves</u>

1. Matter waves are new kind of waves. These waves are produced whether particles are charged or uncharged.
2. It can travel through vacuum as well as medium also.
3. Lighter is a particle, greater is the wavelength associated with it. [$\lambda \alpha 1/m$]
4. Smaller is the velocity of the particle, greater is the wavelength associated with it. [$\lambda \alpha 1/v$]
5. The velocity of the matter waves depends upon the velocity of the material particle i.e. it is not a constant. (em wave velocity is constant)
6. The velocity of matter waves is greater than the velocity of light.

<u>De-Broglie wavelength in different forms</u>

De-Broglie wavelength $\lambda = h/mv$

For different forms of de-Broglie wavelength, need to find the kinetic energy of particle (i.e. Subjected to different external energy like force, electric field, temperature) and find mv then it has to be substituted in the above equation.

<u>1. De-Broglie wavelength in terms of Energy</u>

we know that the kinetic energy of particle i.e. $E = \frac{1}{2}mv^2$ --------- (1)

Multiply Eq-(5) by 'm' on both sides, we get

$$mE = \frac{1}{2}m^2v^2$$
$$2mE = m^2v^2$$
$$mv = \sqrt{2mE}$$

Substituting the above mv in the equation (5) becomes

$$\lambda = \frac{h}{\sqrt{2mE}} \qquad \text{---------- (2)}$$

2. **de-broglie wavelength in terms of voltage(V):**

If a charged particle is accelerated through a potential difference(V), then the kinetic energy of the particle is given as

E = eV ---------- (1)

But we have kinetic energy E) of a particle i.e $E = \frac{1}{2}mv^2$

$$eV = \frac{1}{2}mv^2$$
$$2eV = mv^2$$

Multiply by "m" on both sides we get

$$2meV = m^2v^2 \Rightarrow mv = \sqrt{2meV}$$

$$\boxed{\lambda = \frac{12.27}{\sqrt{V}}\ \text{Å}}$$

h = 6.6× 10^{-34} J-S, m_e = 9.1x10^{-31} kg e = 1.6x 10^{-19} c

We have de-Broglie wavelength $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2meV}}$ or

----------(2)

3**. <u>De-Broglie wavelength in terms of Temperature</u>**

If particles are in thermal equilibrium at temperature 'T'. then the K.E is given by,

$$K.E = 3/2\, K_B T$$
$$1/2\, mv^2 = 3/2\, K_B T$$
$$mv^2 = 3K_BT$$
$$m^2v^2 = 3mK_BT$$
$$\boxed{mv = \sqrt{3mK_BT}}$$

The de-Broglie wavelength $\boxed{\lambda = \frac{h}{\sqrt{3mK_BT}}}$ m

(b) **A proton is moving with a speed of 2.5x 10^{10} m/sec. Find the wavelength of matter wave associated with it. (Mass of proton m = 1. 6 7x10^{-27}kg) (3M)**

(c**) If an electron beam is accelerated by 54 V, Calculate the de-Broglie wavelength of an electron. (3M)**

ANSWER: $\lambda = h / m v$

WILL FORWARD SEPARATELY

9.(a**) Describe Davisson and Germer experiment for confirmation of de-Broglie hypothesis** (8M)

<u>EXPERIMENTAL VERIFICATION OF MATTER WAVES : DAVISSON AND GERMER EXPERIMENT</u>

Principle: Davisson and Germer were studying the reflection (OR) scattering of electrons from nickel metal target (nickel crystal) and measured the intensity of scattered electrons in different directions.

Experimental Arrangement:

It consists of three main parts.

1. ELECTRON GUN
2. NICKEL TARGET SINGLE CRYSTAL
3. MOVABLE DETECTOR OR COLLECTOR

An electron gun, which comprises of a tungsten filament is heated by a low tension battery B1, produces electrons. These electrons are accelerated to desired velocity by applying suitable potential from a high tension source B2. The accelerated electrons are collimated into a fine beam by allowing them to pass through a system of pin holes provided in the cylinder. The whole instrument is kept in an evacuated chamber.

The past moving beam of electrons is made to strike the Nickel target capable of rotating about an axis perpendicular to the plane. The electrons are now scattered in all directions by the atomic planes of crystals. The intensity of the electron beam scattered in a direction can be measured by the electron collector which can be rotated about the same axis as the target. The collector is connected to a galvanometer whose deflection is proportional to the intensity of the electron beam entering the collector.

<u>Observations:</u>

The electron beam is accelerated by 54 V is made to strike the Nickel crystal and a sharp maximum is occurred at angle of 50° with the incident beam. The incident beam and the diffracted beam in this experiment make an angle of 65° with the family of Bragg's planes.

d = 0.091nm (for Ni crystals)

According **to Bragg's law** for maxima in diffracted pattern,

$$2d \sin \theta = n \lambda$$

For n =1, $\lambda = 2d \sin \theta$

$= 2 \times 0.91 \times 10^{-10} \times \sin 65^o = 0.165$ n**m=1.65 Å**

For a 54 V electron, the **de-Brogllie wavelength associated with the electron** is given by

$= 12.25 / \sqrt{V} = (12.25 / \sqrt{54})\ ^oA$

= 0.166 nm=1.66 Å

This value is in agreement with the experimental value. This experiment provides a direct verification of de-Broglie hypothesis of wave nature of moving particles.

(b) **Calculate the de-Broglie wavelength of a electron having kinetic energy of 12.8 MeV. Mass of electron m = 9.1×10^{-37}kg. (4M)**

ANSWER:

$$\lambda = \frac{h}{\sqrt{2mE}}$$

WILL SEND SEAPRATELY

10.(a) **State Heisenberg's uncertainty principle. (4M)**

What is Heisenberg's uncertainty principle? Discuss an experimental illustration of this principle?

According to this principle, macroscopically it is possible to exactly measure the position of a moving particle at any instant and the momentum of the principle at that position but microscopically it is not possible to measure exactly the position of particle and its momentum simultaneously.

"This law can also be stated as the product of change in position (Δx) and momentum (ΔP) of a particle is at least equal to $\hbar/2$". Either the position or the momentum can be measured accurately at a given time but not both.

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

This uncertainty relation also hold true for various other conjugate pairs of physical variable such as energy (E) and time (t) angular moment (J) and angle (θ)

$$\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$$

$$\Delta J \cdot \Delta \theta \geq \frac{\hbar}{2}$$

10 (b)**Apply the Heisenberg's uncertainty principle to prove the non-existence of electron in the nucleus. (8M)**

ANSWER:

<u>Illustration: Non-existence of electron in the nucleus</u>

In this article, we will prove that electrons cannot exist inside the nucleus.

But to prove it, let us assume that electrons exist in the nucleus. As the radius of the nucleus in approximately 10^{-14} m. If electron is to exist inside the nucleus, then uncertainty in the position of the electron is given by

$$\Delta x = 10^{-14} \text{ m}$$

According to uncertainty principle,

$$\Delta x \Delta p_x = h/2\prod$$

Thus $\Delta p_x = h/2\prod \Delta x$

Or $\Delta p_x = 6.62 \times 10^{-34}/2 \times 3.14 \times 10^{-14}$

$$\Delta p_x = 1.05 \times 10^{-20} \text{ kg m/ sec}$$

If this is p the uncertainty in the momentum of electron ,then the momentum of electron should be at least of this order, that is $p=1.05*10^{-20}$ kg m/sec.

An electron having this much high momentum must have a velocity comparable to the velocity of light. Thus, its energy should be calculated by the following relativistic formula

$$E = \sqrt{m^2_0 c^4 + p^2c^2} = 19.6 \text{ MeV or } 20 \text{ MeV}$$

Therefore, if the electron exists in the nucleus, it should have an energy of the order of 19.6 MeV. However, it is observed that beta-particles (electrons) ejected from the nucleus during beta –decay have energies of approximately 3 Me V, which is quite different from the calculated value of 19.6 MeV. Second reason that electron cannot exist inside the nucleus is that experimental results show that no electron or particle in the atom possess energy greater than 4 MeV.

Therefore, it is confirmed that electrons do not exist inside the nucleus.

11. (a) **Derive the Schrödinger time-independent wave equation of matter waves.What is physical significance of wave function? (8M)**

ANSWER:

Schrödinger, in 1926, developed wave equation for the moving particles. One of its forms can be derived by simply incorporating the deBroglie wavelength expression into the classical wave eqn.

If a particle of mass 'm' moving with velocity 'v' is associated with a group of waves.

Let ψ be the wave function of the particle. Also let us consider a simple form of progressing wave like the one represented by the following equation,

$$\Psi = \Psi 0 \sin(\omega t - k x) \text{ --------- } (1)$$

Where $\Psi = \Psi(x, t)$ and $\Psi 0$ is the amplitude.

Differentiating Ψ partially w.r.to x,

$$\partial \Psi / \partial x = \Psi_0 \cos(\omega t - k x)(-k)$$

$$= -k \Psi_0 \cos(\omega t - k x)$$

Once again differentiate w.r.to x

$$\partial^2 \psi / \partial x^2 = (-k) \Psi_0 (-\sin(\omega t - k x))(-k)$$

$$= - k^2 \Psi 0 \sin(\omega t - k x)$$

$$\partial^2 \psi / \partial x^2 = - k^2 \psi \text{ (from eqn (1))}$$

$$\partial^2 \psi / \partial x^2 + k^2 \psi = 0 \text{ ---------- (2)}$$

$$\partial^2 \psi / \partial x^2 + (4 \Pi^2 / \lambda^2) \psi = 0 \text{ --------- (3)} \quad (\text{since } k = 2 \Pi / \lambda)$$

From eqn. (2) or eqn. (3) is the differential form of the classical wave eqn. now we incorporate deBroglie wavelength expression $\lambda = h / m v$.

Thus we obtain

$$\partial^2 \psi / \partial x^2 + (4 \Pi^2 / (h/mv)^2) \psi = 0$$

$$\partial^2 \psi / \partial x^2 + 4 \Pi^2 m^2 v^2 \psi / h^2 = 0 \text{ ---------- (4)}$$

The total energy E of the particle is the sum of its kinetic energy K and potential energy V

$$\text{i.e} \quad E = K + V \text{ ---------- (5)}$$

$$\text{And} \quad K = mv^2 / 2 \text{ ---------- (6)}$$

$$\text{Therefore} \quad m^2 v^2 = 2 m (E - V) \text{ ---------- (7)}$$

From (4) and

$$\boxed{\partial^2 \psi / \partial x^2 + [8\Pi^2 m (E-V) / h^2] \psi = 0 \text{ ---------- (8)}}$$

In quantum mechanics, the value $h / 2 \Pi$ occurs more frequently. Hence we denote,

$$\hbar = h / 2 \Pi$$

Using this notation, we have

$$\boxed{\partial^2 \psi / \partial x^2 + [2 m (E - V) / \hbar^2] \psi = 0 \text{ ---------- (9)}}$$

For simplicity, we considered only one – dimensional wave. Extending eqn. (9) for a three – dimensional, we have

$$\partial^2 \psi / \partial x^2 + \partial^2 \psi / \partial y^2 + \partial^2 \psi / \partial z^2 + [2 m (E - V) / \hbar^2] \psi = 0 \text{ ---------- (10)}$$

$$\text{Where} \quad \Psi = \Psi (x, y, z).$$

Here, we have considered only stationary states of ψ after separating the time dependence of Ψ.

Using the Laplacian operator,

$$\nabla^2 = \partial^2 / \partial x^2 + \partial^2 / \partial y^2 + \partial^2 / \partial z^2 \text{ ---------- (11)}$$

Eqn. (10) can be written as

$$\boxed{\nabla^2 \Psi + [2 m (E - V) / \hbar^2] \psi = 0 \text{ ---------- (12)}}$$

This is the Schrödinger Time Independent Wave Equation in three dimension.

For free particle potential energy is constant or zero V=0

$$\boxed{\nabla^2 \Psi + [2 m E / \hbar^2] \psi = 0 \text{ ---------- (13)}}$$

(or)

$$\boxed{\nabla^2 \Psi + [8\pi^2 m E / h^2] \psi = 0 \text{ ---------- (14)}}$$

11 (b) **Calculate the energy required to excite the electron from its ground state to the fifth excited state if an electron trapped inside a one-dimensional infinite potential well of width 1.5 A^0. Given that h = 6.62 × 10^{-34} J-sec, m = 9.1×10^{-31} Kg.(4M)**

ANSWER

(1) Find the least energy of an electron moving in the dimension in an infinitely high potential box of width $1A^0$, given mass of the electron 9.11×10^{-31} kg

$$E_n = \frac{n^2h^2}{8mL^2} \qquad L = 1\times10^{-10}\,m \qquad m = 9.11\times10^{-31}\,kg$$

The least energy is $n = 1$

$$E_1 = \frac{1^2\times(6.626\times10^{-34})^2}{8\times9.11\times10^{-31}\times(10^{-10})^2}$$

$$E_1 = 9.1\times10^{-19}\,J$$

$$E_1 = \frac{9.1\times10^{-19}}{1.6\times10^{-19}}\,eV$$

$$\boxed{E_1 = 5.68\,eV}$$

WILL SEND SEPARTELY (GIVEN WITH DIFFERENT VALUE)

12. (a) Obtain an expression for energy levels and wave functions of a particle enclosed in one-dimensional potential box of infinite well. (8M)

ANSWER:

PARTICLE IN A ONE-DIMENSIONAL RIGID BOX USING SCHRODINGER TIME INDEPENDENT EQUATION

Consider a particle of mass 'm' placed inside a one-dimensional box of infinite height and width L. Assume that the particle is freely moving inside the box. The motion of the particle is restricted by the walls of the box. The particle is bouncing back and forth between the walls of the box at x= 0 and x = a. For a freely moving particle at the bottom of the potential well, the potential energy is very low. Since the potential energy is very low, moving particle energy is assumed to be zero between x =0 and x = a.

The potential energy of the particle outside the walls is infinite due to the infinite P.E outside the potential well.

The particle cannot escape from the box

i.e. $V(x) = 0$ for $0 < x < L$

$V(x) = \infty$ for $0 \geq x \geq L$

Since the particle cannot be present outside the box, its wave function is zero

i e $|\psi|^2 = 0$ for $0 > x > L$

$|\psi|^2 = 0$ for $x = L$ & $x = 0$

The Schrödinger one – dimensional time independent eqn. is

$$\nabla^2 \Psi + [2m(E - V)/\hbar^2]\psi = 0 \text{ ----------(1)}$$

For freely moving particle $V = 0$

$$\nabla^2 \Psi + [2mE/\hbar^2]\psi = 0 \text{ ----------(2)}$$

Taking $2mE/\hbar^2 = K^2$ ---------- (3)

Eqn.(1) becomes $\partial^2 \psi / \partial x^2 + k^2 \Psi = 0$ ---------- (4)

Eqn. (1) is similar to eq. of harmonic motion and the solution of above eqn. is written as

$$\Psi = A \sin kx + B \cos kx \text{ ----------(5)}$$

Where A, B and k are unknown quantities and to calculate them it is necessary to construct boundary conditions.

Hence boundary conditions are

When $x = 0$, $\Psi = 0$ => from (5) $0 = 0 + B$ => $B = 0$ -------- (6)

When $x = L$, $\Psi = 0$ => from (5) $0 = A \sin kL + B \cos kL$ ---------- (7)

But from (6) $B = 0$ therefore eqn. (7) may turn as

$$A \sin kL = 0$$

Since the electron is present in the box $L \neq 0$

$$\sin kL = 0$$

$$KL = n\Pi$$

$$k = n\Pi / L \text{ ---------- (8)}$$

Substituting the value of k in eqn. (3)

$$2mE/\hbar^2 = (n\Pi/L)^2$$

$$E = (n\Pi/L)^2 (\hbar^2/2m) = (n\Pi/L)^2 (h^2/8m\Pi^2)$$

$$E = n^2 h^2 / 8mL^2$$

In general $E_n = n^2 h^2 / 8mL^2$ ----------(9)

The wave eqn. can be written as

$$\Psi = A \sin(n\Pi x / L) \text{ ---------- (10)}$$

Let us find the value of A, if an electron is definitely present inside the box, then

$$\Rightarrow \int_{-\infty}^{\infty} |\psi|^2 \, dx = 1$$

$$\Rightarrow \int^{L} A^2 \sin^2 (n \Pi x / L) \, dx = 1$$

$$\Rightarrow \int_0^{L} \sin^2 (n \Pi x / L) \, dx = 1 / A^2$$

$$\Rightarrow \int_0^{L} [1 - \cos (2 \Pi n (x / L)) / 2] \, dx = 1 / A^2$$

$$A = \sqrt{\frac{2}{L}} \qquad \text{---------- (11)}$$

From eqn's. (10) & (11)

$$\Psi_n = \sqrt{2 / a} \sin (n \Pi x / L) \text{ ----------(12)}$$

Eqn. (9) represents an energy level for each value of n. the wave function this energy level is given in eqn. (12). Therefore the particle in the box can have discrete values of energies. These values are quantized. Note that the particle cannot have zero energy.The normalized wave functions $\Psi 1$, Ψ_2, Ψ_3 given by eqn (12) is plotted. the values corresponding to each E_n value is known as Eigen value and the corresponding wave function is known as Eigen function.

The wave function $\Psi 1$, has two nodes at x = 0 & x = L

The wave function $\Psi 2$, has three nodes at x = 0, x = L / 2 & x = L

The wave function $\Psi 3$, has three nodes at x = 0, x = L / 3, x = 2 L / 3 & at x = L

12 (b) **An electron is confined in a one-dimensional potential box of infinite well of width 2.5 x 10^{-10} m. Calculate the first three quantum energy levels of an electron. (4M)**

WILL SEND SEPARTELY

Unit-III

13. (a) Illustrate the salient features of classical free electron theory and summarize the merits and demerits. (6M)

ANSWER:

FREE ELECTRON THEORY

In solids, electrons in outer most orbits of atoms determine its electrical properties. Electron theory is applicable to all solids, both metals and non-metals. In addition, it explains the electrical, thermal and magnetic properties of solids. The structure and properties of solids are explained employing their electronic structure by the electron theory of solids.

It has been developed in three main stages:

1. Classical free electron theory

2. Quantum Free Electron Theory.
3. Zone Theory.

1.Classical free electron theory: The first theory was developed by Drude & Lorentz in 1900. According to this theory, metal contains free electrons which are responsible for the electrical conductivity and metals obey the laws of classical mechanics.

2.Quantum Free Electron Theory: In 1928 Sommerfeld developed the quantum free electron theory. According to Sommerfeld, the free electrons move with a constant potential. This theory obeys quantum laws.

3. Zone Theory: Bloch introduced the band theory in 1928. According to this theory, free electrons move in a periodic potential provided by the lattice. This theory is also called "Band Theory of Solids". It gives complete informational study of electrons.

Classical free electron theory [CFET]

Even though the classical free electron theory is the first theory developed to explain the electrical conduction of metals, it has many practical applications. The advantages and disadvantages of the classical free electron theory are as follows:

POSTULATES OR ASSUMPTIONS OF CFET

(a). In an atom electron revolve around the nucleus and a metal is composed of such atoms.

(b) The valence electrons of atoms are free to move about the whole volume of the metals like the molecules of a perfect gas in a container. The collection of valence electrons from all the atoms in a given piece of metal forms electrons gas. It is free to move throughout the volume of the metal.

(c) These free electrons move in random directions and collide with either positive ions fixed to the lattice or other free electrons. All the collisions are elastic i.e., there is no loss of energy.

(d). The movements of free electrons obey the laws of the classical kinetic theory of gases.

(e). The electron velocities in a metal obey the classical Maxwell –Boltzmann distribution of velocities.

(f) The electrons move in a completely uniform potential field due to ions fixed in the lattice.

(g). When an electric field is applied to the metal, the free electrons are accelerated in the direction opposite to the direction of applied electric field.

Merits (OR) Success (OR) Advantages of CFET

1.It verifies Ohm's law.

2. It explains the electrical and thermal conductivities of metals.

3.It derives Wiedemann –Franz law. (i.e., the relation between electrical conductivity and thermal conductivity $K/\sigma=LT$)

4.It explains optical properties of metals.

Demerits (OR) Disadvantages (OR) Failure of CFET

1. It could not explain the photoelectric effect, Compton Effect and black body radiation.
2. Electrical conductivity of semiconductors and insulators could not be explained.
3. Widemann – Franz law ($K/\sigma T$ = constant) is not applicable at lower temperatures.
4. Ferromagnetism could not be explained by this theory.
5. It fails to give a correct mathematical expression for thermal conductivity.
6. According to classical free electron the electronic specific heat is equal to $3/2\ R$ while the actual

value is 0.01*R.*

13 (b) Illustrate the salient features of quantum free electron theory and summarize the merits and demerits. (6M)

ANSWER

Quantum Free Electron Theory [QFET]

Quantum free electron theory was proposed by Sommerfeld in 1928. It overcomes many of the drawbacks of classical theory. Sommerfeld explained them by choosing Fermi- Dirac statistics instead of Maxwell-Boltzmann statistics. He developed this theory by applying the principles of quantum mechanics.

POSTULATES or ASSUMPTIONS OF QFET

1.In a metal the available free electrons are fully responsible for electrical conduction.

2. Valence electrons move freely in a constant potential within the boundaries of metal and is prevented from escaping the metal at the boundaries (high potential). Hence the electron is trapped in a potential well. The electrons move in a constant potential inside the metal.

3. Electrons have wave nature, the velocity and energy distribution of the electron is given by Fermi-Dirac distribution function.

4. The loss of energy due to interaction of the free electron with the other free electron.

5.Electron's distributed in various energy levels according to Pauli's Exclusion Principle

6. The energy values of free electrons are quantized.

7. To find the possible energy values of electron Schrodinger time independent wave equation is applied. The problem is similar to that of particle present in a potential box.

SUCCESS or Advantages or merits of QFET

1.It successfully explains the electrical and thermal conductivity of metals.

2. It can explain the Thermionic phenomenon.

3. It explains temperature dependence of conductivity of metals.

4. It can explain the specific heat of metals.

5. It explains magnetic susceptibility of metals.

6. It can explain photo electric effect, Compton Effect and block body radiation etc.

7. It gives the correct mathematical expression for the thermal conductivity of metals.

Drawbacks or demerits or disadvantages of QFET

1. It is unable to explain the metallic properties exhibited by only certain crystals.

2. It is unable to explain why the atomic arrays in metallic crystals should prefer certain structures only.

3. This theory fails to distinguish between metal, semiconductor and Insulator.

4. It also fails to explain the positive value of Hall Co-efficient.

5. According to this theory, only two electrons are present in the Fermi level and they are responsible for conduction which is not true.

BAND THEORY OR ZONE THEORY OF SOLIDS

- Bloch developed this theory in which the electrons move in a periodic potential due to positive ions in metals. An electron moves through these ions, it experience varying potentials.
- It explains the mechanisms of conductivity, semi -conductivity on the basis of energy bands and hence band theory.

14. Derive the expression for density of states for conduction electron for unit volume of metal.

ANSWER

DENSITY OF STATES [DOS]

A parameter of interest in the study of conductivity of metals and semiconductors is the density of states. The Fermi function F(E) gives only the probability of filling up of electrons in a given energy state. It does not give the information about the number of electrons that can be filled in a given energy state, to know that we should know the number of available energy states called density of states.

Definition: Density of states Z(E) dE is defined the as the number of energy states per unit volume in an energy interval of a metal. It is used to calculate the number of charge carriers per unit volume of any solid.

N(E) dE= Number of energy states between E and E + dE/Volume of the metal---(1)

N(E) dE = Z(E) dE/V---------(1)

N(E) dE- No.of filled energy states

Fig.1.6 Positive octant of n space

In order to fill the electrons in an energy state we have to first find the number of available energy states within a given energy interval. The number of available energy states can be obtained for various combinations of quantum numbers n_x, n_y, n_z i.e.

Let us construct the sphere of radius 'n' in phase space with different quantum numbers n_x, n_y, n_z.

$$n^2 = n_x^2 + n_y^2 + n_z^2$$

The sphere is further divided into many shells represents a particular combination of quantum numbers and represents particular energy value.

Therefore, the number of energy states within a sphere of radius

$$n = {}^4/_3\, \pi n^3$$

Let us consider two energy values E and E + dE can be found by finding the number of energy states between the shells of radius n and n+ dn from the origin. Since the quantum numbers are positive integers, n values can be defined only in the positive octant of the n – space. The number of available energy states within the sphere of radius "n" due to one octant.

$$n = \frac{1}{8}\left[\frac{4}{3}\pi n^3\right]$$

Similarly, the number of available energy states within the sphere of radius n+dn corresponding energy

$$n + dn = \frac{1}{8}\left[\frac{4}{3}\pi(n + dn)^3\right]$$

The number of available energy states between the shells of radius n and n + dn (or) between the energy levels E and E + dE

Z(E) dE= $\frac{1}{8}\left[\frac{4}{3}\pi(n + dn)^3 - \frac{4}{3}\pi n^3\right]$

$$(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$$

$$Z(E)\,dE = \frac{1}{\cancel{8}\,2}\left\{\frac{\cancel{4}}{3}\pi\left[\cancel{n^3} + dn^3 + 3n^2dn + 3ndn^2 + \cancel{n^3}\right]\right.$$

$$Z(E)\,dE = \frac{\pi}{6}\left[3n^2dn + 3ndn^2 + dn^3\right]$$

Since the higher powers of dn is very small, dn^2, dn^3 terms can be neglected.

$$Z(E)\,dE = \frac{\pi}{6} \cdot 3n^2dn$$

$$Z(E)\,dE = \frac{\pi}{2} n^2\,dn$$

$$Z(E)\,dE = \frac{\pi}{2} n \cdot n\,dn \longrightarrow (2)$$

we know the energy of the electron in a cubical metal box is,

$$E = \frac{n^2h^2}{8mL^2}$$

$$n^2 = \frac{8mL^2}{h^2}E \longrightarrow (3)$$

$$n = \left(\frac{8mL^2}{h^2}E\right)^{1/2} \longrightarrow (4)$$

For equation (2), need to differentiate equation (3) w.r.t E

$$2n\,dn = \frac{8mL^2}{h^2}\,dE$$

$$n\,dn = \frac{8mL^2}{2h^2}\,dE \rightarrow (5)$$

Substituting equations (4) and (5) in (2)

$$Z(E)\,dE = 8\,\frac{\pi}{2}\cdot\left(\frac{8mL^2}{h^2}\right)^{3/2} E^{1/2} \cdot \left(\frac{8mL^2}{h^2}\right)^{1} \frac{1}{2}\,dE$$

$$Z(E)\,dE = \frac{\pi}{4}\cdot\left(\frac{8mL^2}{h^2}\right)^{3/2} E^{1/2}\,dE \rightarrow (6)$$

Volume of the cubical metal piece $V = L^3$

The number of filled energy states

$$N(E)\,dE = \frac{Z(E)\,dE}{V} = \frac{\frac{\pi}{4}\left(\frac{8m}{h^2}\right)^{3/2} \cancel{L^3} E^{1/2}\,dE}{\cancel{L^3}}$$

$$N(E)\,dE = \frac{\pi}{4}\left(\frac{8m}{h^2}\right)^{3/2} E^{1/2}\,dE$$

Since each energy levels provides 2 electron states one with spin up and another spin down according pauli's exclusion principle

$$N(E)\,dE = 2 \cdot \frac{\pi}{4}\left(\frac{8m}{h^2}\right)^{3/2} E^{1/2}\,dE$$

$$\boxed{N(E)\,dE = \frac{\pi}{2} \cdot \left(\frac{8m}{h^2}\right)^{3/2} E^{1/2}\,dE} \longrightarrow (7)$$

Carrier Concentration in metals

To find the carrier concentration in the solid materials

n_c = No. of filled energy states × Fermi-dirac distribution function

$$n_c = N(E)\,dE \times F(E)$$

$$\boxed{n_c = \frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{3/2} E^{1/2}\,dE \cdot F(E)}$$

15. (a) **Explain the differences between three distribution functions (6M)**

ANSWER

Parameter	M-B Statistics	B-E Statistics	Fermi-Dirac Statistics

Particles	Particles are distinguishable	Particles are indistinguishable	Particles are indistinguishable
Restriction on particles	No restriction	No restriction	Restriction due to Pauli's exclusive principle.
Two particles distribution in two cells	The particles are distinguishable and can be arranged in three ways.	The particles are indistinguishable and can be arranged in three ways.	No two particles can occupy the same cell. It can be arranged in one way by restriction
Examples	Molecules of gas	Photons in a cavity Phonons in a solid	Free electrons in conductors
Properties of Particles	Any Spin and wavefunctions do not overlap	Integer spins: 0,1,2,3 Overlap of wave functions	Half-integer spins: 0, ½, 3/2, 5/2…. Overlap of wave functions.
Distribution function	$F_{M-B}(E_i)=\frac{1}{e^{\frac{(E_i-E_F)}{K_BT}}}$	$F_{B-E}(E_i)=\frac{1}{e^{\frac{(E_i-E_F)}{K_BT}}-1}$	$F_{F-D}(E_i)=\frac{1}{e^{\frac{(E_i-E_F)}{K_BT}}+1}$

(b) **What is Fermi energy? Discuss variation of Fermi factor with energy and temperature. (6M)**

ANSWER

Fermi Energy & Fermi level

Definition: The energy of the highest occupied level at 0K is called the fermi energy and the energy level is called as fermi level. The fermi energy is denoted by E_F.

(OR)

The fermi level is that energy level for which the probability of occupation is ½=0.5 or 50% at any temperature.

Effect of Temperature on Fermi-Distribution function

According to Fermi - Dirac distribution, the probability of electron occupying an energy level E_i is

$$F_{F-D}(E_i)=\frac{1}{e^{\frac{(E_i-E_F)}{K_BT}}+1}$$

Ei is the energy of ith level
E_F is the energy of Fermi level,
K_B is a Boltzmann constant
T is the absolute temperature and

$F(E_i)$ is the Fermi function

When the material is at a temperature 'T' 0K, below the Fermi energy levels are totally fulfilled and above the energy levels are empty. When it receives thermal energy from surroundings, the electrons are thermally excited into higher energy levels. The occupation of electrons in energy levels obeys a Fermi Dirac statistical distribution law.

Figure 2 Fermi-Dirac Distribution Function at Different Temperatures

From this curve we can observe the probability of finding an electron at different Temperatures.

Case I. At $T > 0k$ $if\ \varepsilon_i = \varepsilon_F$, : $f(\varepsilon_i) = \frac{1}{2}$

$$f(\varepsilon_i) = \frac{1}{e^{\frac{(\varepsilon_i - \varepsilon_F)}{kT}} + 1} = \frac{1}{e^{\left(\frac{0}{kT}\right)} + 1} = \frac{1}{e^0 + 1} = \frac{1}{1+1} = \frac{1}{2}$$

$$f(\varepsilon_i) = \frac{1}{2}$$

Case II. At $T = 0k$ $if\ \varepsilon_i > \varepsilon_F$, : $f(\varepsilon_i) = 0$

$$f(\varepsilon_i) = \frac{1}{e^{\frac{(\varepsilon_i - \varepsilon_F)}{0}} + 1} = \frac{1}{e^{\frac{(\varepsilon_i - \varepsilon_F)}{0}} + 1} \qquad Science(\varepsilon_i - \varepsilon_F) = +ve\ valye$$

$$= \frac{1}{e^{\infty} + 1} = \frac{1}{\infty + 1} = \frac{1}{\infty} = 0 \qquad since\ \frac{+ve\ value}{0} = \infty$$

$$f(\varepsilon_i) = 0$$

Case III. At $T = 0k$ $if\ \varepsilon_i < \varepsilon_F$, : $f(\varepsilon_i) = 1$

$$f(\varepsilon_i) = \frac{1}{e^{\frac{(\varepsilon_i - \varepsilon_F)}{0}} + 1} = \frac{1}{e^{\frac{-ve\ value}{0}} + 1} \qquad \because\ \varepsilon_i < \varepsilon_F \quad since\ (\varepsilon_i - \varepsilon_F)$$

$$= -ve\ value$$

$$= \frac{1}{e^{-\infty} + 1} = \frac{1}{0+1} = \frac{1}{1} = 1 \qquad \because e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$$

$$f(\varepsilon_i) = 1$$

The probability function F(E) lies between 0 and 1

Hence there are three probabilities namely

1. F(E) =1 100% probability to occupy the energy level by electrons.
2. F(E) =0 No probability to occupy the energy levels by electrons and hence, it is empty.
3. F(E) =0.5 50% probability of finding the electron in the energy level.

<u>Importance of fermi energy level & Fermi energy</u>

1. It acts as a reference level which separates the vacant and filled states at T=0 K
2. It gives the information about the filled electrons states and the empty states.
3. At 0K, below fermi energy level electrons are filled at above fermi energy level it will be empty
4. When the temperature is increased, few electrons gain the thermal energy and it goes to higher energy levels.

16. **(a) Discuss the formation of bands in solids using Kronig–Penny model and E-K diagram**. (6M)

<u>KRONIG-PENNEY MODEL</u>

It is not easy to solve Schrödinger's equation with these potentials. So, Kronig and Penney approximated these potentials inside the crystal to the shape of rectangular steps as shown in Fig. (c). This model is called Kronig-Penney model of potentials.

According to Kronig and Penney the electrons move in a periodic square well potential. This potential is produced by the positive ions (ionized atoms) in the lattice. To overcome the above problems, the periodic potentials due to the positive ions in a metal have been considered. shown in Fig. (a), if an electron moves through these ions, it experiences varying potentials.

The potential is zero near to the nucleus of positive ions and maximum between the adjacent nuclei. The variation of potential is shown in figure.

$V(x) = 0 \quad for \quad I\ region \quad 0 < x < a$

$V(x) = V_0 \quad for \quad II\ region \quad -b < x < 0$

Fig. Periodic potential in Kronig-Penny model

The corresponding Schrödinger wave equations for *I* region is

$$\frac{\partial^2\Psi}{\partial x^2} + \frac{8\pi^2 m}{h^2}(E\Psi) = 0 \qquad \because V(x) = 0 \quad for \quad I\ region$$

$$\frac{\partial^2\Psi}{\partial x^2} + \alpha^2\Psi = 0 \qquad (1)$$

$$where \quad \alpha^2 = \frac{8\pi^2 m}{h^2}E \qquad (2)$$

The corresponding Schrödinger wave equations for *II* region is

$$\frac{\partial^2\Psi}{\partial x^2} + \frac{8\pi^2 m}{h^2}(E - V_0)\Psi = 0 \qquad \because V(x) = V_0 \quad for \quad II\ region$$

$$\frac{\partial^2\Psi}{\partial x^2} - \frac{8\pi^2 m}{h^2}(V_0 - E)\Psi = 0 \qquad \because V_0 > E$$

$$\frac{\partial^2\Psi}{\partial x^2} - \beta^2\Psi = 0 \qquad (3)$$

$$where \quad \beta^2 = \frac{8\pi^2 m}{h^2}(V_0 - E) \qquad (4)$$

The general solutions of the equations (1) and (3) are of the form

$$\Psi_1(x) = Ae^{i\alpha x} + Be^{-i\alpha x} \qquad (5)$$

$$\Psi_2(x) = Ae^{\beta x} + Be^{-\beta x} \qquad (6)$$

By applying the boundary conditions and Bloch function, the expressions are reduced into

$$\frac{8\pi^2 m V_0}{2h^2\alpha} b.Sin(\alpha a) + Cos(\alpha a) = Cos(ka)$$

$$\frac{P}{\alpha a} Sin(\alpha a) + Cos(\alpha a) = Cos(ka)$$

Where $P = \frac{8\pi^2 m V_0\, a\, b}{2\, h^2}$ = potential barrier strength ----(7)

The plot of $\frac{P}{\alpha a} Sin(\alpha a) + Cos(\alpha a)$ verses αa is shown in figure.

The left hand side of the equation (7) is plotted as a function of α for the value of P = 3 Π / 2 which is shown in fig, the right hand side one takes values between -1 to +1 as indicated by the horizontal lines in fig. Therefore the equation (7) is satisfied only for those values of ka for which left hand side between ± 1.

From fig , the following conclusions are drawn.

1) The energy spectrum of the electron consists of a number of allowed and forbidden energy bands.

2) The width of the allowed energy band increases with increase of energy values ie increasing the values of αa. This is because the first term of equation(7) decreases with increase of αa.

$$(P / \alpha a) \sin \alpha a + \cos \alpha a == 3 \Pi / 2$$

3) With increasing P, ie with increasing potential barrier, the width of an allowed band decreases. As P→∞, the allowed energy becomes infinitely narrow and the energy spectrum is a line spectrum as shown in fig.

If P→∞, then the equation (7) has solution ie $\sin \alpha a = 0$

$$\alpha a = \pm n \Pi$$

$$\alpha = \pm n \Pi / a$$

$$\alpha^2 = n^2 \Pi^2 / a^2$$

But $\alpha^2 = 2 m E / \hbar^2$

Therefore $2 m E / \hbar^2 = n^2 \Pi^2 / a^2$

$$E = [\hbar^2 \Pi^2 / 2 m a^2] n^2$$

$$E = n h^2 / 8 m a^2 \qquad (\text{since } \hbar = h / 2 \Pi)$$

This expression shows that the energy spectrum of the electron contains discrete energy levels separated by forbidden regions.

4) When P→0 then $\cos \alpha a = \cos ka$

$$\alpha = k , \quad \alpha^2 = k^2$$

but $\alpha^2 = 2 m E / \hbar^2$

$k=2\pi/\lambda$; so, $k^2 = 4\pi^2/\lambda^2$; $4\pi^2/\lambda^2 = 2 m E / \hbar^2$ since $\hbar = h / 2 \Pi$)

$E = P^2 / 2 m \qquad \lambda=h/p$

$$E = 1/2mv^2$$

The equation (7) shows all the electrons are completely free to move in the crystal without any constraints. Hence , no energy level exists ie all the energies are allowed to the electrons and shown in fig(5). This case supports the classical free electrons theory.

4. For $p = 0$ the energy spectrum is continuous as shown in figure (3)

<u>E-k diagram or Brillouin Zones</u>

Using the equation coska= P Sin αa/ αa + Cos αa

it is possible to plot a curve showing the energy E as a function of 'k'as shown in fig. From fig. it is clear that energy of electron is continuously increasing from k=0 to $\frac{\pi}{a}$. The left-hand side of above equation becomes +1 or -1 for values of k=± $\frac{n\pi}{a}$. and hence discontinuity appears in E-k graph, at k=± $\frac{\pi}{a}$. From fig it is seen that energy spectrum of electron consists of allowed regions and forbidden regions. The allowed region extends from $-\frac{n\pi}{a}\ to\ +\frac{n\pi}{a}$. This is known as first Brillouin Zone. After a discontinuity in energy called forbidden gap another allowed region extends from $-\frac{2\pi}{a}\ to -\frac{\pi}{a}\ and +\frac{\pi}{a}\ to +\frac{2\pi}{a}$. This is known as second Brillouin Zone. Similarly, other higher order Brillouin zones can be defined.

16 (b) **Classify the crystalline solids based on band theory of solids. (6M)**

ORIGIN OF ENEGRY BANDS IN SOLIDS

An isolated atom possesses discrete energies of different electrons. Suppose two isolated atoms are brought to very close proximity, then the electrons in the orbits of two atoms interact with each other. So, that in the combined system, the energies of electrons will not be in the same level but changes and the energies will be slightly lower and larger than the original value. So, at the place of each energy level, a closely spaced two energy levels exists. If '*N*' number of atoms are brought together to form a solid and if these atoms' electrons interact and give 'N' number of closely spaced energy levels in the place of discrete energy levels, it is known as bands of allowed energies.

On the basis of band theory how the crystalline solids are classified into metals, semiconductors and Insulators. Band

A set of closely packed energy levels is called as band

Valence band

A band which is occupied by the valence electrons is called as valence band. The valence band may be partially or completely filled up depending on the nature of the material.

Conduction band

The lowest unfilled energy band is called as conduction band. This band may be empty or partially filled. In conduction band the electrons can move freely named as free electrons.

Forbidden energy gap or Forbidden gap

The energy gap between valence band and conduction band is called forbidden energy gap or forbidden gap or band gap.

CLASSIFICATION OF CRYSTALLINE SOLIDS

Based on forbidden band width, the solids are classified into conductors, semiconductors and insulators.

Conductors: The valence band and conduction bands are overlapped with each other and the energy gap Eg is zero. At room temperatures, free electrons already exist in huge number at conduction band. Hence these solids are good electrical conductors as well as good thermal conductors. The electrical resistivity increases at high temperature by collisions among the free electrons.

Ex: Al, Cu, Ag, Au etc.,

Semiconductors: The valence band and conduction bands are separated with a small energy band gap Eg ≈ 1 eV. At low temperatures (0 K), free electrons are not available in conduction band. Hence, they behave like insulators at low temperatures. The electrical conductivity increases at high temperatures by the transition of free electrons from valence band to conduction band. Thus, these solids behave like electrical conductors at high temperatures.

Ex: Silicon Eg = 1.1 ev, Germanium Eg = 0.7ev

Insulators: The valance band and conduction bands are separated by a very large energy gap Eg ³ 3 eV. At room temperatures conduction band is empty and valance band is full of electrons. Hence they these solids are electrical insulators. Even at high temperatures valence electrons are unable to jump in to conduction band.

Ex: Glass, Mica, Ebonite, Rubber etc.,

17. (a**) Derive an expression for the effective mass of an electron moving in energy bands of a solid. Show how it varies with the wave vector.** (6M)

ANSWER

Effective mass of an electron

An electron in crystal may behave as if it had a mass different from the free electron mass m_0. There are crystals in which the effective mass of the carriers is much larger or much smaller than m_0. The effective mass may be anisotropic, and it may even be negative. The important point is that the electron in a periodic potential is accelerated relative to the lattice in an applied electric or magnetic field as if its mass is equal to an effective mass.

Definition: When an electron in a period potential is accelerated by an electric field (or) magnetic field, then the mass of the electron is called effective mass (m*).

The mass of an electron in the periodic potentials of a crystal is different from the free electron mass and is usually referred to as the effective mass. According to de Broglie hypothesis, a moving electron is associated with a wave. The velocity of an electron (v) is equal to the group velocity (v_g) of the associated wave. The group velocity is given by:

The group velocity is given by,

$$V_g = \frac{d\omega}{dk} \longrightarrow (1)$$

where, angular frequency $\omega = 2\pi\nu$

ν - Linear frequency ; $\nu = \omega/2\pi$

In quantum mechanics, the energy of an electron is,

$$\boxed{E = h\nu = \frac{h\omega}{2\pi} = \hbar\omega} \rightarrow (2) \quad \hbar = \frac{h}{2\pi}$$

Substituting $\omega = \frac{E}{\hbar}$ in equation (1) becomes,

$$V_g = \frac{d}{dk}\left(\frac{E}{\hbar}\right) = \frac{1}{\hbar}\frac{dE}{dk} \rightarrow (3)$$

The maximum force experienced by an electron is $F = m^* a \rightarrow (4)$

$$a = \frac{dV_g}{dt} = \frac{d}{dt}\left[\frac{1}{\hbar}\frac{dE}{dk}\right]$$

$$a = \frac{1}{\hbar}\frac{d^2E}{dt.dk} \longrightarrow (5)$$

Multiply and divide 'dk' on RHS of eqn (5)

$$a = \frac{1}{\hbar} \cdot \frac{d^2E}{dk\,dk} \frac{dk}{dt}$$

$$\boxed{a = \frac{1}{\hbar} \frac{d^2E}{dk^2} \frac{dk}{dt}} \rightarrow (6)$$

On comparing equations (6) and (4), we need to convert (6) in the form equation (4)

So, $E = p \quad p = \hbar k$

$$\frac{dp}{dt} = \hbar \frac{dk}{dt}$$

$$\therefore F = \frac{dp}{dt}$$

$$F = \hbar \frac{dk}{dt}$$

$$\frac{dk}{dt} = F/\hbar \rightarrow (7)$$

Substituting equ (7) in eqn. (6) becomes,

$$a = \frac{1}{\hbar^2} \frac{d^2E}{dK^2} \cdot F$$

$$\boxed{F = \frac{\hbar^2}{(d^2E/dK^2)} \cdot a} \rightarrow (8)$$

Comparing equations (4) & (8) $\quad F = m^* a$

$$\boxed{m^* = \frac{\hbar^2}{\left(\frac{d^2E}{dK^2}\right)}}$$

Effective mass of an electron $m^* = \dfrac{\hbar^2}{\left(\frac{d^2E}{dK^2}\right)}$

a. Variation of E with K: Variation of E with K corresponds to first Brillouin Zone. Using this type of variation velocity can be calculated.

(a) *E-K* diagram; (b) Velocity versus *K*; (c) Effective mass of an electron; (d) Degree of freedom of an electron; (e) *E-K* diagram

b. Variation of V with K: For K=0 velocity is zero and as 'k' value increases V also increases reaching its maximum value at Ko known as point of inflexion. Beyond Ko velocity decreases and reaches to 0 at K=π/a.

c. Variation of m* with K: Near K=0, m*=m. As K value increases m* also increases reaching its maximum value at Ko. Above Ko, m* is negative and decreases as K tends to π/a. Beyond Ko velocity decreases and hence retarding force acts on electron and it behaves as positively charged particle referred to as hole.

Therefore: - m* is positive near the bottoms of all bands, - m* is negative near the tops of all bands.

The E-k curve is concave at the bottom of the CB, so m_e* is positive. Whereas, it is convex at the top of the VB, thus me* is negative. This means that a particle in that state will be accelerated by the field in the reverse direction expected for a negatively charged electron. That is, it behaves as if a positive charge and mass. This is the concept of the hole.

For valance band the degenerate band with smaller curvature around k=0 is called the heavy-hole band, and the one with larger curvature is the light-hole band.

d. Variation of f_k: Degree of freedom of an electron is defined as **f_k=m/m*.** fk is measure of the extent to which the electron is free in a state K. If m* is large fk is small, particle behaves as heavy particle. If fk=1, electron behaves as free electron.

(b) **Explain the following: 1) Bloch theorem and 2) E-K diagram. (6M)**

<u>ANSWER</u>

<u>PERIODIC POTENTIAL</u>

To overcome the drawbacks of quantum free electron theory, the periodic potentials due to the positive ions in a metal have been considered. shown in Fig. (a), if an electron moves through these ions, it experiences varying potentials. The potential of an electron at the positive ion site is zero and is maximum in between two ions. The potential experienced by an electron, when it passes along a line through the positive ions is as shown in Fig. (b).

A simplified model of an electron inside a crystal is to consider the electron to move in a periodic potential well structure. The essential features of the problem is brought out by considering the crystal lattice to be a regular arrangement of atoms along a straight line with inter-atomic separation 'a'. The variation of potential energy with distance on a line along the centre of the atoms is as shown in the figure.

<u>BLOCH WAVE FUNCTION</u>

The *Bloch theorem* in essence formulates a condition that *all* solutions Ψ(r), for *any* periodic potential *V*(*r*) whatsoever, have to meet. In one version it ascertains

V(r)= V(r+a)

For potentials that are periodic, the wavefunction satisfies Bloch theorem which states that the form of the wavefunction is

$$\Psi(r)=u(r).\exp(i.k.r)$$

In one dimension,

$$\Psi_k(x)=u_k(x).\exp(i.k.x)$$

$$V(x)=V(x+a)$$

Where, wave vector $k=2\pi/\lambda$

Bloch's theorem has many more forms and does not only apply to electrons in periodic potentials, but for all kinds of waves in periodic structures, e.g. phonons.

18. Derive an expression for the carrier concentration in an intrinsic semiconductor.(12M)

ANSWER:

A pure semiconductor without any impurities is known as an intrinsic semiconductor.

CARRIER CONCENTRATION IN AN INTRINSIC SEMICONDUCTOR

We know that, at 0K intrinsic pure semiconductor behaves as insulator. But as temperature increases some electron move from valance band to conduction band as shown in fig. therefore both electrons in conduction band and holes in valance band will contribute to electrical conductivity. Therefore the carrier concentration (or) density of electrons (n_e) and holes (n_h) has to be calculated.

Fig. Energy band diagram of intrinsic semiconductor

Assume that electron in the conduction band as a free electron of mass m^*_e and the hole in the valance band behave as a free particle of mass m*h. The electrons in the conduction band have energies lying from Ec to ∞ and holes in the valance band have energies from -∞ to Ev as shown in figure. Here Ec represents the lowest energy level of conduction band Ev represents the highest energy level of the valance band.

<u>Concentration of electrons in the conduction band:</u>

In a semiconductor, the conduction band is completely empty and the valence band is completely filled at 0K. If the temperature becomes greater than 0K, some of the electrons in valence band get excited into the conduction band. Let us calculate the electrons available in the conduction band, when T > 0 K.

Concentration of electrons in the conduction band: In a semiconductor, the conduction band is completely empty and the valence band is completely filled at 0K .If the temperature becomes greater than 0K, some of the electrons in valence band get excited into the conduction band. Let us calculate the electrons available in the conduction band, when T > 0 K.

Density of electrons on conduction band $n_e = \int_{Ec}^{\infty} Z(E).F(E)dE \longrightarrow (1)$

From Fermi-Dirac statistics we can write

$$Z(E)dE = 2.\ \pi / 4\ [8m^*e / h^2]^{3/2}\ E^{1/2}\ dE \longrightarrow (2)$$

Considering minimum energy of conduction band as Ec and maximum energy can go upto ∞ we can write equation 2 as

$$Z(E)dE = \pi / 2 \times [8m^*e / h^2]^{3/2} (E - Ec)^{1/2}\ dE \longrightarrow (3)$$

We know Fermi function, probability of finding an electron in a given energy state is

$$F(E) = \frac{1}{1 + e^{(E-E_F)/K_B T}} \longrightarrow (4)$$

Substituting equation 4 & 3 in equation 1 we have Density of electrons in conduction band within the limits Ec to ∞ as

$$n_e = \pi / 2 \times [8m^*e / h^2]^{3/2} \int_{Ec}^{\infty} \frac{(E - Ec)^{1/2}\ .\ dE}{[1 + e^{(E-E_F)/K_B T}]} \longrightarrow (5)$$

Since to move an electron from valavce band to conduction band the energy required is greater then 4 $K_B T$ (i.e) $E - E_F >> K_B T$ (or) $(E - E_F) / K_B T >> 1$

(or) $e^{(E-E_F)/K_B T} >> 1$

$$1 + e^{(E-E_F)/K_B T} = e^{(E-E_F)/K_B T}$$

Equation 5 becomes

$$n_e = \pi / 2 \times [8m^*e / h^2]^{3/2} \int_{Ec}^{\infty} \frac{(E - Ec)^{1/2}}{e^{(E-E_F)/K_BT}} \cdot dE$$

$$n_e = \pi / 2 \times [8m^*e / h^2]^{3/2} \int_{Ec}^{\infty} (E - Ec)^{1/2} e^{-(E-E_F)/K_BT} dE$$

$$n_e = \pi / 2 \times [8m^*e / h^2]^{3/2} \int_{Ec}^{\infty} (E - Ec)^{1/2} e^{(E_F-E)/K_BT} dE \longrightarrow (6)$$

Let us assume that $E - Ec = x K_BT$

Or $E = Ec + x K_BT$

Differentiating we get $dE = K_BT. dx$,

Limits: when $E = Ec$; $x = 0$

When $E = \infty$; $x = \infty$

Limits are 0 to ∞

Equation 6 can be written as

$$n_e = \pi / 2 \times [8m^*e / h^2]^{3/2} \int_0^{\infty} (xK_BT)^{1/2} e^{(E_F - xK_BT - Ec)/K_BT} . K_BT\, dx$$

$$n_e = \pi / 2 \times [8m^*e / h^2]^{3/2} \int_0^{\infty} (x)^{1/2} (K_BT)^{3/2} e^{(E_F - Ec)/K_BT} . e^{-x} dx$$

$$n_e = \pi / 2 \times [8m^*e K_BT / h^2]^{3/2} e^{(E_F - Ec)/K_BT} \int_0^{\infty} (x)^{1/2} . e^{-x} dx$$

$$n_e = \pi / 2 \times [8m^*e K_BT / h^2]^{3/2} e^{(E_F - Ec)/K_BT} \sqrt{\pi}/2 \quad \left\{ \int_0^{\infty} (x)^{1/2} . e^{-x} dx = \sqrt{\pi}/2 \right\}$$

$$n_e = 1/4 \times [8 \pi m^*e K_BT / h^2]^{3/2} e^{(E_F - Ec)/K_BT}$$

Density of electrons in conduction band is

$$n_e = 2 . [2 \pi m^*e K_BT / h^2]^{3/2} . e^{(E_F - Ec)/K_BT} \longrightarrow (7)$$

CONCENTRATION OF HOLES IN THE VALENCE BAND

We know F(E) represents the probability of filled states. As the maximum probability 1, the probability of un filled states will be [1 – F(E)].

Example, if F(E) = 0.8 then 1 – F(E) = 0.2

(i.e) 80% chance of finding an electron in valance band and 20% chance of finding a hole in valance band.

Let the maximum energy in valance band be Ev and the minimum energy be -∞. Therefo density of hole in valance band n_h is given by

$$n_h = \int_{-\infty}^{Ev} Z(E).[\,1 - F(E)]dE \longrightarrow (8)$$

we know

$$Z(E)dE = \pi / 2 \times [8m^*h / h^2]^{3/2} (E\,v - E)^{1/2}\, dE \longrightarrow (9)$$

$$1 - F(E) = 1 - \frac{1}{(E-E_F)/K_BT}$$

$$1 - F(E) = \frac{e^{(E-E_F)/kT}}{1 + e^{(E-E_F)/K_BT}}$$

Here $E - E_F << K_BT$, $(E - E_F)/K_BT << 1$

$$e^{(E-E_F)/K_BT} << 1$$

$$1 + e^{(E-E_F)/K_BT} = 1$$

$$1 - F(E) = e^{(E-E_F)/K_BT} \longrightarrow (10)$$

Substituting equation 10 & 9 in 8, we get

$$n_h = \pi / 2 \times [8m^*h / h^2]^{3/2} \int_{-\infty}^{Ev} (Ev - E)^{1/2}\, e^{(E-E_F)/K_BT}\, dE \longrightarrow (11)$$

Let us assume that

$$Ev - E = x\, K_BT,$$

Differentiating we get $dE = -K_BT.\, dx$,

Limits: when $E = \infty$;

When $Ev - (-\infty) = x$; $x = \infty$

when $E = Ev$; $x = 0$

Limits are ∞ to 0

Equation 11 becomes

$$n_h = \pi / 2 \times [8m^*_h / h^2]^{3/2} \int_{\infty}^{0} (xK_BT)^{1/2} e^{(E_v - xK_BT - E_F)/K_BT} .(-K_BT)\, dx$$

to exclude the negative sign, the limits can be interchanged.

$$n_h = \pi / 2 \times [8m^*_h / h^2]^{3/2} \int_0^{\infty} (x)^{1/2} (K_BT)^{3/2} e^{(E_v - E_F)/K_BT} . e^{-x} dx$$

$$n_h = \pi / 2 \times [8m^*_h K_BT / h^2]^{3/2} e^{(E_v - E_F)/k_BT} \int_0^{\infty} (x)^{1/2} . e^{-x} dx$$

$$n_h = \pi / 2 \times [8m^*_h K_BT / h^2]^{3/2} e^{(E_v - E_F)/K_BT} \sqrt{\pi}/2 \qquad \left(\int_0^{\infty} (x)^{1/2} . e^{-x} dx = \sqrt{\pi}/2\right)$$

$$n_h = 1/4 \times [8 \pi m^*_h K_BT / h^2]^{3/2} e^{(E_v - E_F)/K_BT}$$

Density of holes in valance band is

$$n_h = 2 . [2 \pi m^*_h K_BT / h^2]^{3/2} . e^{(E_v - E_F)/K_BT} \longrightarrow (12)$$

Intrinsic carrier concentration

In an intrinsic semiconductor, the concentration of electrons in the conduction band is equal to the concentration of holes in the valance band.

We can write $n_i = n_e = n_h$ & $n_i^2 = n_e . n_h$

Substituting equation 15 & 16, we have

$$n_i^2 = 2 . [2 \pi m^*e K_BT / h^2]^{3/2} . e^{(E_F - Ec)/k_BT} \times 2 . [2 \pi m^*_h K_BT / h^2]^{3/2} . e^{(E_v - E_F)/k_BT}$$

$$n_i^2 = 4 [2 \pi K_BT / h^2]^3 . e^{(E_F - Ec + Ev - E_F)/k_BT} [m^*e\, m^*_h]^{3/2}$$

$$n_i^2 = 4 [2 \pi K_BT / h^2]^3 . e^{(Ev - Ec)/k_BT} [m^*e\, m^*_h]^{3/2}$$

$$n_i^2 = 4 [2 \pi K_BT / h^2]^3 . e^{-Eg/k_BT} [m^*e\, m^*_h]^{3/2}$$

Where $Ec - Ev = Eg$ is the forbidden energy gap.

$$n_i = 2 [2 \pi K_BT / h^2]^{3/2} . e^{-Eg/2k_BT} [m^*e\, m^*_h]^{3/4}$$

19. (a) **Arrive at an expression for the Fermi level in an intrinsic semiconductor.(6M)**

Fermi level in an Intrinsic Semiconductor

For an intrinsic semiconductor number of electrons (i.e) electron density will be the same as that of the number of holes (i.e) hole density.

$$(\text{i.e}) \quad n_e = n_h$$

Equating equations 7& 12

$$[m^*e]^{3/2} \cdot e^{(E_F - Ec)/k_BT} = [m^*_h]^{3/2} \cdot e^{(E_v - E_F)/k_BT}$$

$$\left(\frac{m^*h}{m^*_e}\right)^{3/2} = \frac{e^{(E_F - E_c)/k_BT}}{e^{(E_v - E_F)/k_BT}}$$

$$= e^{(E_F - E_c - E_v + E_F)/k_BT}$$

$$= e^{(2E_F - (E_v + E_c))/k_BT}$$

Taking log on both on sides we have

$$3/2 \log\left(\frac{m^*h}{m^*_e}\right) = \frac{[2E_F - (E_v + E_c)]}{k_BT}$$

$$2E_F = E_v + E_c + \tfrac{3}{4} K_BT \log\left(\frac{m^*h}{m^*_e}\right)$$

$$E_F = \frac{E_v + E_c}{2} + \tfrac{3}{4} K_BT \log\left(\frac{m^*h}{m^*_e}\right) \longrightarrow (13)$$

If $m^*h = m^*e$, then $\log m^*h / m^*e = 0$ since $\log 1 = 0$

$$E_F = \left(\frac{E_v + E_c}{2}\right) \longrightarrow (14)$$

(i.e) the Fermi energy level lies in the midway between Ec and Ev as shown fig (since at 0K, T = 0).

(b) **Discuss the variation of the Fermi level with temperature and effective masses of electron and holes (6M)**

VARIATION OF FERMI LEVEL WITH TEMPERATURE

The Fermi level in an intrinsic semiconductor may be considered as independent of temperature as staying in the middle of the band gap. If $m_e^* > m_h^*$ displaced downward to the top edge of the valence band. If $m_e^* < m_h^*$ with increase in temperature, the Fermi level gets displaced upward to the bottom edge of the conduction band.

Variation of Fermi level with temperature in an intrinsic semiconductor

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20. What is an extrinsic semiconductor? Derive an expression for density of holes in p-type semiconductor.

Doping: The method of adding impurities to a pure semiconductor is known as doping and impurity added is called doping agent or dopant.

P – type semiconductor is obtained by doping an intrinsic semiconductor with trivalent (3 electrons in valance band) impurity atoms like boron, gallium, indium etc.,

CARRIER CONCENTRATION IN P-TYPE SEMICONDUCTOR

Energy band diagram for a *p*-type semiconductor

For p – type at absolute zero E_F will be exactly between Ea and Ev. At low temperature some electron from valence band fills the holes in the acceptor energy level as shown in figure. We know the density of holes in the valence band,

$$n_h = 2 . [2 \pi m^*_h K_B T / h^2]^{3/2} . \; e^{(E_v - E_F)/k_B T} \longrightarrow (1)$$

Let Na be the number of acceptor energy levels per cm 3 which has energy Ea above valence band. If some electrons are accepted by acceptor energy levels from the valence band, say for example if two electrons are accepted to fill the hole sites in the acceptor levels, then two holes will be created in the valence band as shown in fig.

Therefore in general the electron density in the acceptor energy level can be written as,

$$N(E_a)\, dE = Z(E_a)\, dE \;\; F(E_a)$$

$$\text{(i.e) } n_e = N_a . F(E_a) \longrightarrow (2)$$

$$F(Ea) = \frac{1}{1 + e^{(Ea - E_F)/kT}}$$

$$Ea - E_F >> K_B T \text{ (or) } (Ea - E_F) / K_B T >> 1$$

$$e^{(Ea - E_F)/K_B T} >> 1$$

$$1 + e^{(Ea - E_F)/K_B T} = e^{(Ea - E_F)/K_B T}$$

$$F(Ea) = \frac{1}{e^{(Ea - E_F)/kT}} \longrightarrow (3)$$

Substituting equation (3) in (2), we get Na.

$$n e = \frac{N_a .}{e^{(Ea - E_F)/kT}} \longrightarrow (4)$$

At equilibrium condition

Number of electrons per unit volume in conduction band (electron density) = Number of holes per unit volume in conduction band (hole density)

Equating equation (1) and equation (4) we get

$$2 . [2 \pi m^*_h K_B T / h^2]^{3/2} . e^{(E_v - E_F)/k_B T} = \frac{Na}{e^{(Ea-E_F)/kT}}$$

$$e^{(E_v - E_F + Ea - E_F)/k_B T} = \frac{Na}{2. [2 \pi m^*_h K_B T / h^2]^{3/2}}$$

$$e^{(E_v + Ea - 2E_F)/k_B T} = \frac{Na}{2. [2 \pi m^*_h K_B T / h^2]^{3/2}}$$

Taking log on both sides

$$\log [e^{(E_v + Ea - 2E_F)/k_B T}] = \log \left(\frac{Na}{2. [2 \pi m^*_h K_B T / h^2]^{3/2}} \right)$$

$$\frac{(E_v + E_a) - 2E_F}{K_B T} = \log \left(\frac{Na}{2. [2 \pi m^*_h K_B T / h^2]^{3/2}} \right)$$

$$2E_F = E_v + E_a - K_B T \log \left(\frac{N_a}{2 [2 \pi m^* h K_B T / h^2]^{3/2}} \right)$$

By substituting the Fermi energy level E_F , we have

$$Density\ of\ holes = (2N_a)^{\frac{1}{2}} \left[\frac{2\pi m^*{}_h K_B T}{h^2}\right]^{\frac{3}{4}} exp \left[\frac{E_V - E_a}{2K_B T}\right]$$

The above eq. represents the concentration of carriers in a P-type semiconductor.

21. What is an extrinsic semiconductor? Derive an expression for carrier concentration in n-type semiconductor.

When a small amount of pentavalent impurity (group V element) is added to a pure semiconductor, it becomes a n – semiconductor. Such impurities are known as donor impurities because they donate the free electrons to the semiconductor crystal

CARRIER CONCENTRATION OF N-TYPE SEMICONDUCTOR

The energy band structure of n – type semiconductor is shown in fig. At 0 K, E_F will lie exactly between Ec and Ed, but even at low temperature some electrons may go from Ed to Ec. Let us assume that $Ec - E_F > K_BT$. Then the density of electrons in conduction band can be written as

Energy band diagram of n-type semiconductor

$$n_e = 2 . [2 \pi m^* e K_B T / h^2]^{3/2} . e^{(E_F - Ec)/k_B T} \longrightarrow (1)$$

Let N_d be the number of donor energy levels per cm^3 (i.e) density of state Z(Ed) dE. If some electrons donated from donor energy level to conduction band say for example if two electrons goes to conduction band then two vacant sites (holes) will be created in E_d levels. Thus in general we can write the density of holes in donor energy level as

$$N(E_d)\,dE = Z(E_d)\,dE\,.(1 - F(E_d))$$

$$(\text{i.e})\ n_h = N_d\,.(1 - F(E_d)) \longrightarrow (2)$$

$$1 - F(E_d) = 1 - \frac{1}{1 + e^{(E_d - E_F)/kT}}$$

$$1 - F(E) = \frac{e^{(E_d - E_F)/kT}}{1 + e^{(E_d - E_F)/kT}} \longrightarrow (3)$$

Here $E_d - E_F << K_B T$, $(E_d - E_F)/K_B T << 1$

$$e^{(E_d - E_F)/K_B T} << 1$$

$$1 + e^{(E_d - E_F)/K_B T} = 1$$

Equation 3 becomes

$$1 - F(E_d) = e^{(E_d - E_F)/k_B T} \longrightarrow (4)$$

$$1 - F(E_d) = e^{(E_d - E_F)/k_B T}$$

Substituting equation (4) in (2)

$$n_h = N_d\,.\,e^{(E_d - E_F)/k_B T} \longrightarrow (5)$$

At equilibrium condition

Number of electrons per unit volume = Number of holes per unit volume in conduction band (electron density) in conduction band (hole density)

Equating equation (1) and equation (5) we get

$$2\,.\,[2\pi m^* e K_B T / h^2]^{3/2}\,.\ e^{(E_F - E_c)/k_B T} = N_d\,.\,e^{(E_d - E_F)/k_B T} \longrightarrow (6)$$

$$\frac{e^{(E_F - E_c)/k_B T}}{e^{(E_d - E_F)/k_B T}} = \frac{Nd}{2\,.\,[2\pi m^* e K_B T / h^2]^{3/2}}$$

$$e^{(E_F - E_c - E_d + E_F)/k_B T} = \frac{N_d}{2 \,.\, [2\pi m^* e K_B T / h^2]^{3/2}}$$

$$e^{(2E_F - (E_c + E_d))/k_B T} = \frac{N_d}{2 \,.\, [2\pi m^* e K_B T / h^2]^{3/2}}$$

Taking log on both sides

$$\text{Log}\ [e^{(2E_F - (E_c + E_d))/k_B T}] = \log \left(\frac{N_d}{2 \,.\, [2\pi m^* e K_B T / h^2]^{3/2}} \right)$$

$$\frac{2E_F - E_c + E_d}{K_B T} = \log \left(\frac{N_d}{2\, [2\pi m^* e K_B T / h^2]^{3/2}} \right)$$

$$2E_F = E_c + E_d + K_B T \log \left(\frac{N_d}{2\, [2\pi m^* e K_B T / h^2]^{3/2}} \right)$$

$$E_F = \frac{E_c + E_d}{2} + \frac{K_B T \log}{2} \left(\frac{N_d}{2\, [2\pi m^* e K_B T / h^2]^{3/2}} \right) \longrightarrow (7)$$

When T = 0, we can write the above equation

$$E_F = \left(\frac{E_c + E_d}{2.} \right) \longrightarrow (8)$$

Equation (7) shows that, at 0K, E_F will lie expertly in the midway between Ec and E_d.

$$e^{(E_F - E_c)/k_B T} = \exp. \left(\frac{E_c + E_d}{2 k_B T} - \frac{E_c}{k_B T} + \frac{1}{2} \log \frac{N_d}{2\, [2\pi m^* e K_B T / h^2]^{3/2}} \right)$$

$$e^{(E_F - E_c)/k_B T} = \exp.\left(\frac{E_d - E_c}{2 k_B T} + \frac{1}{2} \log \frac{N_d}{2\,[2\pi m^* e K_B T / h^2]^{3/2}} \right)$$

$$= \exp.\left(\frac{E_d - E_c}{2 k_B T} \right) \left(\frac{N_d}{2\,[2\pi m^* e K_B T / h^2]^{3/2}} \right) \longrightarrow (9)$$

By substituting equation (9) in (1), we have,

$$n = 2\,[2\pi m^* e K_B T / h^2]^{3/2} \exp\left(\frac{E_d - E_c}{2 k_B T} \right) \left(\frac{N_d}{2\,[2\pi m^* e K_B T / h^2]^{3/2}} \right)^{1/2}$$

$$n = (2 N_d)^{1/2} [2\pi m^* e K_B T / h^2]^{3/4} \exp\left(\frac{E_d - E_c}{2 k_B T} \right) \longrightarrow (10)$$

Equation (10) gives the carrier concentration in n-type semiconductor.

22. a)**Explain the variation Fermi level with temperature and doping concentration of p type and n type semiconductor.(6M)**

<u>VARIATION OF FERMI LEVEL WITH TEMPERATURE AND CONCENTRATION</u>

N-TYPE SEMICONDUCTOR

At 0 K the Fermi level lies in between the conduction band and the donor level.

As the temperature increases from 0 K , the Fermi level falls.

At higher temperature, it falls below the donor level and hence, it approaches the centre of the forbidden gap. It shows that the material behaving as an intrinsic semiconductor.

As the donor concentration is increased, the Fermi level will move up.

VARIATION OF FERMI LEVEL WITH TEMPERATURE AND CONCENTRATION IN P-TYPE SEMICONDUCTOR

At T=0K, the Fermi energy level lies at the middle of the VB and the acceptor level. At very high temperatures, it lies at middle of the energy gap. It shows that at very high temperature, the p-type material behaves as an intrinsic material.

As the acceptor concentration is increased the increase in Fermi energy is minimized and hence the Fermi level is moves down.

23. b) Write a short on direct and indirect band gap semiconductors

DIRECT AND INDIRECT BAND GAP SEMICONDCUTORS:

According to the band theory of solids, the energy spectrum of electrons consists of a large number of energy bands and is separated by forbidden regions. The band gap is the difference in the energy between the lowest point of conduction band and highest point of valence band. Based on the structure of energy bands and type of energy emission, semiconductors are classified into two types.

• Direct band gap semi conductor

• Indirect band gap semiconductor

Direct band gap semiconductor

The maximum of VB and the minimum of CB exists at the same value of wave number (k) Such semiconductors are called direct band gap semiconductor.

In this type during the recombination of holes and electron, a photon of light is released. This process is known as radiative recombination and also called as spontaneous emission.

It is more effective, because the direction of motion of electron remains unchanged

In this type of semiconductors life time (recombination time) of charge carrier is very less.

Electron – hole pair can be easily generated because it requires less momentum.

Recombination probability is much high. These are used to fabricate LEDs and laser diodes.

These are mostly from the compound semiconductors.

Ex: InP, GaAs …….

Figure Direct and Indirect band gap semiconductor

The maximum value of VB and the minimum value of CB exist at the different values of wave number (k). Such semiconductors are called Indirect Band Gap Semiconductor.

In this type, heat energy is produced during the recombination of holes and electrons. This process is known as non radiative recombination.

It is less efficient, because the direction of motion of electron changed. In this type of semiconductors life time (recombination time) charge carrier is high.

The incident process is slower, because it requires an electron, hole and phonon for interaction. Recombination occurs through some defect states.

Due to the longer life time of charge carriers, these are used to amplify the signals as in the case of diodes and transistors.

These are mostly from the elemental semiconductors. Exx: Si (1.15eV), Ge (0.7 eV).

(or) with diagram

DISTINGUISH BETWEEN DIRECT AND INDIRECT BAND GAP SEMICONDUCTORS

S.No.	Elemental or Indirect band gap semiconductors	Compound of Direct band gap semiconductors

1.	These are made from single element	These are made from compound (mixed) element.
2.	These are made from IV group and VI group elements	These are made from III and V [or] II and VI elements.
3.	These are called as indirect band gap semiconductor (electron-hole recombination takes place through traps)	These are called as direct band gap semiconductor (electron-hole recombination takes place directly)
4.	Heat is produced in the recombination	Photons are emitted during recombination
5.	Life time of charge carriers is more due to indirect recombination	Life time of charge carriers is less due to direct recombination.
6.	These are used for making diodes, transistor, etc.	These are used for making LED, laser diodes, etc.
7.	Example : Ge, Si	Example : GaAs, GaP, CdS, MgO

24. (a**) What is the Hall Effect? Derive the expression for Hall coefficient (8M).**

Hall Effect

Measurement of conductivity will not determine whether the conduction is due to electron or holes and therefore it will be very difficult to distinguish between p – type and n- type semiconductors. Therefore Hall Effect is used to distinguish between the two types of charge carriers and their carrier densities and is used to determine the mobility of charge carriers.

STATEMENT

When a conductor (metal or semiconductor) carrying current (I) is placed perpendicular to a magnetic field (B), a potential difference (electric field) is developed inside the conductor in a direction perpendicular to both current and magnetic field. This phenomenon is known as Hall Effect and the voltage thus generated is called Hall voltage

Hall Effect in n-type Semiconductor

THEORY

Hall Effect in n- type semiconductor

Let us consider a p-type semiconductor material in the form of rectangular slab. In such a material current flows in X – direction and magnetic field B applied in Z- direction. As a result, Hall voltage is developed along Y – direction as shown in figure.

Since the direction of current is from left to right the electrons moves from right to left in X- direction as shown in fig .

Now due to magnetic field applied the electron moves towards downward direction with velocity v and cause negative charge to accumulate at face (1) of the material as shown in fig. therefore the potential difference is established between the face (2) and face (1) of the specimen which gives rise to EH in the negative Y direction.

This separation of charge carriers creates an electric field EH in the upward direction (negative Y direction).

Theses charge carrier are moving in a magnetic field in the semiconductor, they experience Lorentz force

----------(1)

Where, Vd-drift velocity of the electrons

(θ = 900, because force acting on charge perpendicularly)

q = e, the equation 1 becomes

FL= -eVdB----------(2)

The force experienced by the charge due to electric field is given by,

FH=-eEH----------(3)

Where EH-Hall electric field

After some time both the forces become in equal in magnitude and act in opposite direction,

the situation is said to be under equilibrium.

At equilibrium Eqn. (2) = Eqn. (3)

FL= FH

-eVdB= -eEH

EH=VdB---------- (4)

Vd= EH/B----------(5)

The current density is given by, J=-neVd ----------(6)

Substituting eqn.(5) in eqn.(6) becomes,

J=-neEH/B

EH= BJ/ne----------(7)

The current density is given by, J=I/A----------(8)

Substituting equation (8) in eqn. (7) becomes,

EH=BI/neA-----(9)

If VH be the hall voltage at equilibrium, the hall electric field EH=VH/d

The equation (9) can be written as, VH/d=BI/neA

If RH=-1/ne, the above equation becomes,

VH/d= RHBI/A ----------(10)

By substituting area of cross section A=Width (w) x Thickness (d) in the above equation

VH/d= RHBI/w x d

VH= RHBI/w

RH = VH w/B I----------(11)

Eq. (11) gives the value of RH the by measuring I, B, w, and VH

From RH = 1/ne one can find the value of the concentration of the carriers.

By knowing the concentration of the carriers the mobility of the charge carriers is determined

using the relation. σ=neμ

μ=RHσ

Where $\boldsymbol{\sigma}$ is the electrical conductivity of the materials.

The above RH equations are derived by assuming that the velocity of the electron is constant.

But due to thermal agitation the velocity of the electron is not a constant. It is randomly distributed. So, a correction factor 3$\boldsymbol{\pi}$/8 is introduced in the Hall coefficient. Therefore RH

can be written as,

RH= -3π/8 (1/ne) for n-type

RH= 3π/8 (1/ne) for p-type

RH= -1.18/ne or 1.18/pe

For n-type material RH is negative and for p-type material RH is positive.

(b) **What are the applications of Hall Effect?** (4M)

APPLICATIONS OF HALL EFFECT:

- It is used to determine whether the material is p-type or n-type semiconductor. (ie) if R_H is negative then the material n-type. If the R_H is positive then the material p-type.

- It is used to find the carrier concentration
- It is used to find the mobility of charge carrier's μe, μh.
- It is used to find the sign of the current carrying charges.
- It is used to design magnetic flux meters and multipliers on the basis of Hall voltage.
- It is used to find the power flow in an electromagnetic wave.
- It is used as a magnetic field sensor.
-

25. (a) What is p-n junction? Explain the formation of a p-n junction.(8 M)

Definition: A p-n junction is an interface or a boundary between two semiconductor material types, namely the p-type and the n-type, inside a semiconductor.

The p-side or the positive side of the semiconductor has an excess of holes and the n-side or the negative side has an excess of electrons. In a semiconductor, the p-n junction is created by the method of doping.

Formation of PN junction

- **Definition: A p-n junction is an interface or a boundary between two semiconductor material types, namely the p-type and the n-type, inside a semiconductor.**
- **The p-side or the positive side of the semiconductor has an excess of holes and the n-side or the negative side has an excess of electrons. In a semiconductor, the p-n junction is created by the method of doping**
- **A PN-junction diode is formed when a p-type semiconductor is fused to an n-type semiconductor creating a potential barrier voltage across the diode junction**

As a result a narrow charged region on either side of the junction is formed where there are no mobile charge carriers.

Since this region is depleted of mobile charge carriers it is called ***depletion region or charge free region or space charge region or transition region.***

The thickness of this region is usually in the order of 0.5 µm to 1 µm

<u>P-n junction formula</u>

$$V_{built\text{-}in} = \frac{kT}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right)$$

$V_{built\text{-}in}$ → Built-in potential in volts

N_D → n type - donor atoms concentration

N_A → p type - acceptor atoms concentration

n → concentration of electrons

kT/q → thermal voltage

T → temperature in Kelvin

q → charges in coulombs

Biasing conditions for the p-n Junction Diode

There are two operating regions in the p-n junction diode:

- P-type
- N-type

There are three biasing conditions for p-n junction diode and this is based on the voltage applied:

- Zero bias: There is no external voltage applied to the p-n junction diode.
- Forward bias: The positive terminal of the voltage potential is connected to the p-type while the negative terminal is connected to the n-type.
- Reverse bias: The negative terminal of the voltage potential is connected to the p-type and the positive is connected to the n-type.

PN Junction Diode Forward Biasing

When the p-n junction is forward biased, the depletion region thickness is decreased and hence large current flows in the circuit because of movement of majority charge carriers from p to n or n to p side. At higher voltage the thickness of depletion region is negligible

Under Forward bias condition, the conduction through the diode is more. A large forward current in the order of Ampere flows in the circuit.

<u>Reverse Bias of the pn junction</u>

When the p-type is connected to the negative terminal of the battery and the n-type is connected to the positive side then the p-n junction is said to be reverse biased.

Reverse bias

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Under reverse biased condition, the thickness of depletion region is increased hence very fewer current flows in the circuit because of movement of minority charge carriers.

Under Reverse bias condition, the conduction through the diode is almost negligible. A small reverse current (µA) flows in the circuit.

(b) Explain the I-V characteristics and applications of a p-n junction. (4M)

VI characteristics of PN junction diode is a curve between the voltage and current through the circuit. Voltage is taken along the x-axis while the current is taken along the y-axis. The above graph is the VI characteristics curve of the PN junction diode. With the help of the curve we can understand that there are three regions in which the diode works, and they are:

- Zero bias
- Forward bias- **The voltage beyond which the current increases rapidly is called Knee voltage or Threshold voltage**
- Reverse bias- **Even for large increase in bias voltage (say 20 V), there is negligible increase in reverse current this current is called Reverse saturation current**

26. (a) **What is LED? Explain the construction and working of LED. (8M)**

Principle:

It is a semiconductor solid state device or optoelectronic device is working under forward biased p-n junction by emitting the light energy in the visible or infrared region. This phenomenon is known as injection electroluminescence.

The amount of light energy depends upon the energy band gap of semiconductor material.

$$Eg=hc/\lambda;\ \lambda=hc/\ Eg$$

Various impurities are added during the doping process to vary the colour output

To get Visible color LED need to choose the direct bandgap semiconductor material which is having Energy bandgap between 1.4 to 3 eV.

LED Symbol

Below is the demonstration of the LED symbol. The symbol is similar to that of the p-n junction diode. The difference between these two symbols is that the two arrows indicate that the diode is emitting the light.

LED Symbol

Construction

1. The methods used to construct LED are to deposit three semiconductor layers on the substrate.
2. The three semiconductor layers deposited on the substrate are n-type semiconductor, p-type semiconductor and active region.
3. Active region is present in between the n-type and p-type semiconductor layers.
4. When LED is forward biased, free electrons from n-type semiconductor and holes from p-type semiconductor are pushed towards the active region.
5. When free electrons from n-side and holes from p-side recombine with the opposite charge carriers (free electrons with holes or holes with free electrons) in active region, an invisible or visible light is emitted.
6. In LED, most of the charge carriers recombine at active region. Therefore, most of the light is emitted by the active region. The active region is also called as depletion region.

The above figure shows the two different ways of structuring LED p-n junction. The p-type layer is made thin and is grown on the n-type substrate. Metal electrodes attached on either side of the p-n junction serve as nodes for external electrical connection. The Light emitting diode p-n junction is encased in a dome-shaped transparent case so that light is emitted uniformly in all directions and minimum internal reflection to take place.

The PN junction of an LED is surrounded by a transparent, rigid plastic epoxy resin shell.

The larger leg of LED represents the positive electrode or anode.

Working

1. A P-N junction can convert absorbed light energy into a proportional electric current. The same process is reversed here (i.e. the P-N junction emits light when electrical energy is applied to it).

2. This phenomenon is generally called electroluminescence, which can be defined as the emission of light from a semi-conductor under the influence of an electric field.

3. The charge carriers recombine in a forward-biased P-N junction as the electrons cross from the N-region and recombine with the holes existing in the P-region. Free

electrons are in the conduction band of energy levels, while holes are in the valence energy band.

4. Thus the energy level of the holes will be lesser than the energy levels of the electrons. Some portion of the energy must be dissipated in order to recombine the electrons and the holes. This energy is emitted in the form of heat and light.

5. The electrons dissipate energy in the form of heat for silicon and germanium diodes but in gallium arsenide phosphide (GaAsP) and gallium phosphide (GaP) semiconductors, the electrons dissipate energy by emitting photons.

6. If the semiconductor is translucent, the junction becomes the source of light as it is emitted, thus becoming a light-emitting diode, but when the junction is reverse biased no light will be produced by the LED and, on the contrary, the device may also be damaged.

The wavelength of emitted photon can be calculated by,

$$E=h\upsilon \text{ -----(1)} \qquad [c=\upsilon\lambda;\ \upsilon=c/\lambda]$$

By substituting the values of $E=E_g$; $\upsilon=c/\lambda$ in the above equation becomes,

$$E_g=hc/\lambda;\ \lambda=hc/E_g \text{ ----------(2)}$$

By substituting the values of Planck's constant $h=6.623 \times 10^{-34}$ J/S, velocity of the light $c=3 \times 10^{8}$ m/s the equation (2) can be modified as,

$$\lambda=12400/E_g \text{ Å}$$

Therefore, color of the emitted light depends on the type of material (Eg-energy band gap of given material) used.

(b**) Explain the I-V characteristics of LED (4M)**

The characteristics curve of the LED shows that the forward bias of **1 V** is sufficient to increase the current exponentially.

The output characteristics curve shows that radiant power of LED is directly proportional the forward current in LED.

27. (a) **What is photo diode? Explain construction and working of a photodiode. (8M)**

(b) Explain I-V characteristics of photo diode. (4M)

A special type of p-n junction device that generates current when exposed to light is known as Photodiode. It is also known as photo detector or photo sensor.

It operates in reverse biased mode and converts light energy into electrical energy. It works on the principle of **Photoelectric effect**. The operating principle of the photodiode is such that when the junction of this two-terminal semiconductor device is illuminated then the electric current starts flowing through it. Only minority current flows through the device when the certain reverse potential is applied to it.

Construction

The photodiodes are available in a metallic package. The diode is a p n junction, mounted in an insulated plastic substrate. The PN junction of the device placed inside a glass material. This is done to order to allow the light energy to pass through it. As only the junction is exposed to radiation, thus, the other portion of the glass material is painted black or is metallised. The overall unit is of very small dimension nearly about **2.5 mm**. **The development of a P+ diffusion layer can be done over the heavily doped N-type epitaxial layer. The contacts are designed with metals to make two terminals like anode and cathode. The front region of the diode can be separated into two types like active & non-**

active surfaces. The designing of the non-active surface can be done with silicon dioxide (SiO2). On an active surface, the light rays can strike over it whereas, on a non-active surface, the light rays cannot strike. & the active surface can be covered through the material of anti-reflection so that the energy of light cannot lose and the highest of it can be changed into the current

It is noteworthy that the current flowing through the device is in **micro-ampere** and is measured through an ammeter. Two leads, anode and cathode of the diode come out from the bottom of the metal case. A tab extending from the side of the bottom portion of the metal case identifies the cathode lead.

Working of Photodiode

In the photodiode, a very small reverse current flows through the device that is termed as **dark current**. It is called so because this current is totally the result of the flow of minority carriers and is thus flows when the device is not exposed to radiation.

The working principle of a photodiode is, when a photon of ample energy strikes the diode, it makes a couple of an electron-hole. This mechanism is also called as **the inner photoelectric effec**t. If the absorption arises in the depletion region junction, then the carriers are removed from the junction by the inbuilt electric field of the depletion region. Therefore, holes in the region move toward the anode, and electrons move toward the cathode, and a photocurrent will be generated. The entire current through the diode is the sum of the absence of light and the photocurrent. So the absent current must be reduced to maximize the sensitivity of the device.

The performance of photodiode depends upon the parameters such as responsivity, quantum efficiency and transit time or response time.

V-I Characteristics of Photodiode

Here, the vertical line represents the reverse current flowing through the device and the horizontal line represents the reverse-biased potential.

The first curve represents the dark current that generates due to minority carriers in the absence of light.

As we can see in the above figure that the entire curve shows almost equal spacing in between them. This is so because current proportionally increases with the luminous flux.

Characteristics curve of Photodiode

Applications of Photodiode

1. Photodiodes majorly find its use in counters and switching circuits.
2. Photodiodes are extensively used in an optical communication system.
3. Logic circuits and encoders also make use of photodiode.
4. It is widely used in burglar alarm systems. In such alarm systems, until exposure to radiation is not interrupted, the current flows. As the light energy fails to fall on the device, it sounds the alarm.

Unit-V

1. What is the need of quantum computing? (6M)

ANSWER

Quantum computer is a computer whose computation is performed quantum mechanically. It has a new type of hardware with the computational mechanism based on quantum mechanics. In other words, both the hardware and software should be constructed based on the principles of quantum mechanics. The basic unit of information in quantum computers is called a qubit. It is based on superposition of states in which both the state's 0 and 1 are overlapped. Current computers represent information by means of a bit which can have only one value of 0 and 1. On the other hand, quantum computers use qubits which can represent more information than in bits. Thus, if a number of qubits are given and parallely processed, then quantum computers enable super speed computation. As a result, quantum computers can perform the computation of some important problems whose solutions were regarded impossible or intractable in traditional computers. It is generally thought that the advent of quantum computers can change the world of computers and our life. Some people showed that quantum computers can be used to compute the answers to certain difficult problems much faster than any classical computers. It is in fact a very interesting fact. Now, work for implementing quantum computers, though at the experimental level, is in progress all over the world.

Quantum computer works differently from the traditional computers. The traditional computers can work on and compute only one transaction at a time. On the other hand the quantum computers can work on and perform multiple transactions at the same time which increase their speed more than traditional computers.

Quantum computing began in 1980 when physicist Paul Benioff proposed a quantum mechanical model of the Turing machine. ... In 1994, Peter Shor developed a quantum algorithm for factoring integers with the potential to decrypt RSA-encrypted communications.

In 1998, Gershenfeld and Chuang at MIT developed a 2-qubits quantum computer based on NMR (Nuclear Magnetic Resonance); see Gershenfeld and Chuang. This is the first actual implementation of a quantum computer. In 2001, IBM succeeded in the development of 7-qubits NMR quantum computer and implemented Shor's Algorithm, thus showing the effectiveness of NMR for the implementation for quantum computers; see Vandersypen et.al. There are also other approaches to the hardware of quantum computers. They include trapped ion, quantum dots, and Josephson junction. Currently, it is possible to develop quantum computers with more than 10-qubits and further investigations are expected. We note that fundamental theories for applications of quantum computing are being established. The following table shows the comparison between classical and quantum computing.

Once Benioff, Manin and Feynman opened the doors, researchers began to investigate the nature of the algorithms that could be run on QCs. David Deutsch, a physicist at Oxford, suggested a more comprehensive framework for quantum computing in his 1985 paper [65]. In this work, he describes in detail what a quantum algorithm would look like and anticipates that "one day it will become technologically possible to build quantum computers." Deutsch

then went on to develop an example of an algorithm that would run faster on a quantum computer. He then further generalized this algorithm in collaboration with Richard Jozsa

Table 1. Classical vs Quantum Computing terminology

Classical vs Quantum Computing Terms

Source: Caltech Entrepreneurs Forum, Feb 23, 2019

	Classical	Quantum
Basic Unit	Binary Bit (1 or 0)	**Qbit** (vector)
Computing	Logical Operation	**Unitary** Operation
Description	Truth Table (True/False)	**Unitary** Matrix
Direction	Most Gates Run Forward	Gates are Reversible
Copying	Easy	Impossible
Noise	Minimal w/Error Correction	**Quantum Error Correction** (Very Difficult)
Storage	n-bit storage holds 1 value. from 0 to 2**n - 1	n-qbits storage holds 2**n values
Computation	n-bit processor = 1 operation	n-qbit processor = 2**n operations

(b**) Write differences between classical bits and quantum qubits. (6M)**

Bits and Qubits

Classical bit:

0 or 1

Coin on table

Digital: self-correcting

Quantum bit:

$$\psi = \cos\theta|0\rangle + \sin\theta e^{i\phi}|1\rangle$$

Coin in space

Analog: sensitive to small errors

The difference between bit and qubit

BIT

0

1

Is the smallest unit of information in current computers. It represents only one of two values, **0** or **1**

QUBIT

0

1

Quantum analogue of the classical bit. It can take on two values simultaneously, 0 and 1. This characteristic expands possibility of producing parallel calculations

Classical vs Quantum Computing Terms

Source: Caltech Entrepreneurs Forum, Feb 23, 2019

	Classical	Quantum
Basic Unit	Binary Bit (1 or 0)	**Qbit** (vector)
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Storage	n-bit storage holds 1 value. from 0 to 2**n - 1	n-qbits storage holds 2**n values
Computation	n-bit processor = 1 operation	n-qbit processor = 2**n operations

Describe the superposition and entanglement of qubits

SUPERPOSITION

Superposition - The ability of quantum particles to be a combination of all possible states.
Quantum measurement - The act of observing a quantum particle in superposition and resulting in one of the possible states.
Entanglement - The ability of quantum particles to correlate their measurement results with each other.
Qubit - The basic unit of information in quantum computing. A qubit represents a quantum particle in superposition of all possible states.
Interference - Intrinsic behavior of a qubit due to superposition to influence the probability of it collapsing one way or another.

A superposition of states is a linear combination of the computational basis vectors. The linear combination of two or more state vectors is another state vector in the same Hilbert space and describes another state of the system. A quantum system can exist in two or more states at once, referred to as a “superposition” of states or a “superposition state”. As an example, let us

consider a property of light that illustrates a superposition of states. Light has an intrinsic property called polarization which we can use to illustrate a superposition of states. In almost all of the light we see in everyday life from the sun, for example there is no preferred direction for the polarization. Polarization states can be selected by means of a polarizing filter, a thin film with an axis that only allows light with polarization parallel to that axis to pass through.

With a single polarizing filter, we can select one polarization of light, for example vertical polarization, which we can denote as $|\uparrow\rangle$. Horizontal polarization, which we can denote as $|\rightarrow\rangle$, is an orthogonal state to vertical polarization. Together, these states form a basis for any polarization of light. That is, any polarization state $|\psi\rangle$ can be written as linear combination of these states. We use the Greek letter ψ to denote the state of the system

$$|\psi\rangle = \alpha\,|\uparrow\rangle + \beta\,|\rightarrow\rangle$$

The coefficients α and β are complex numbers known as amplitudes. The coefficient α is associated with vertical polarization and the coefficient β is associated with horizontal polarization. These have an important interpretation in quantum mechanics which we will see shortly. After selecting vertical polarization with a polarizing filter, we can then introduce a second polarizing filter after the first.

Imagine we oriented the axis of the second filter perpendicular to the axis of the first. Would we see any light get through the second filter? If you answered no to this question, you would be correct. The horizontal state $|\rightarrow\rangle$ is orthogonal to the first, so there is no amount of horizontal polarization after the first vertical filter.

Suppose now we oriented the axis of the second polarizing filter at 45° (i.e., along the diagonal

$\nearrow$ between vertical $\uparrow$ and horizontal $\rightarrow$ to the first instead of horizontally. Now we ask the same question would we see any light get through the second filter? If you answered no to this question, you may be surprised to find the answer is yes. We would, in fact, see some amount of light get through the second filter. How could this be if all light after the first filter has vertical polarization? The reason is that we can express vertical polarization as a superposition of diagonal components. That is, letting $|\nearrow\rangle$ denote 45° polarization and $|\nwarrow\rangle$ denote 45° polarization, we may write

$$|\uparrow\rangle = \frac{1}{\sqrt{2}}|\nearrow\rangle + \frac{1}{\sqrt{2}}|\nwarrow\rangle$$

As you may expect from geometric intuition, the vertical state consists of equal parts of $|\nearrow\rangle$ and $|\nwarrow\rangle$. This value is determined from the amplitudes of the superposition state by a law known as Born's rule.

Max Born demonstrated in his 1926 paper that the modulus squared of the amplitude of a state is the probability of that state resulting after measurement. In this case, since the amplitude is $1/\sqrt{2}$ the probability of obtaining that state is $(1/\sqrt{2})^2=1/2$ so the probability of measuring the light in either the vertical or horizontal polarization state is 50%. Note that we chose an amplitude of $1/\sqrt{2}$ in order to normalize the states so that the sum of the modulus squared of the amplitudes will equal one; this enables us to connect the amplitudes to probabilities of measurement with the Born rule.

ENTANGLEMENT

Two systems are in a special case of quantum mechanical superposition called entanglement if the measurement of one system is correlated with the state of the other system in a way that is stronger than correlations in the classical world. In other words, the states of the two systems are not separable.

Entangled states

- **Two quantum mechanical particles** can interact with each other and create an entangled state
- This means that they share common properties; for example, the total outcome could be one, meaning that if you would measure the particle to be one then the other particles would be zero.

Explain the qubits operators bra-ket notation

QUBITS OPERATORS AND MEASUREMENT

A physical qubit is a two-level quantum mechanical system. As we will see in the chapter on building quantum computers, there are many ways to construct a physical qubit. We can represent a qubit as a two-dimensional complex Hilbert space, C^2. The state of the qubit at any given time can be represented by a vector in this complex Hilbert space.

Bra-Ket (Dirac) Notation

The Dirac Bra-Ket notation is a concise and convenient way to describe quantum states. We introduce and define the symbol $|\alpha>$ to represent a quantum state. This is called a ket, or a ket vector. It is an abstract entity, and serves to describe the "state" of the quantum system. We say that a physical system is in quantum state α, where α represents some physical quantity, such as momentum, spin etc, when represented by the ket $|\alpha>$.

The rule is to turn inner products into bra-ket pairs as follows.

$$<u,v> \rightarrow <u|v>$$

$$<u|v> \rightarrow <u| \ |v>$$

Here $|v>$ is called a **ket** and $<u|$ is called a bra.

Dirac allowed the the bra's and ket's to line up back to back, i.e.

$$\langle\alpha|\beta\rangle \equiv (|\alpha>,|\beta>)$$

The symbol $\langle\alpha|\beta\rangle$ represents a complex number that is equal to the value of the inner product of the ket $|\alpha>$ with $|\beta>$. We note, according to the above definition, that,

$$\langle\alpha|\beta\rangle \equiv \langle\beta>|\alpha\rangle^*$$

HILBERT SPACE

In **quantum mechanics** the state of a physical system is represented by a vector in a **Hilbert space**: a complex vector **space** with an inner product. The term "**Hilbert space**" is often reserved for an infinite-dimensional inner product **space** having the property that it is complete or closed.

In other words, a qubit is represented by a vector space more specifically, a Hilbert space. A Hilbert space is a vector space H over either the field of real or complex numbers equipped with an inner product, that is a complete metric space with respect to the metric induced by the inner product.

Basically this is the generalization of the notion of Euclidean space i.e. it extends the methods of algebra and calculus of 2D Euclidean plane and 3D space to space of any finite or infinite dimensions. Hilbert space is an abstract vector space with inner product defined in it, which allows length and angle to be measured. Hilbert space must be complete.

The Hilbert space is equipped with the inner product which allows us to determine the relative position of two vectors representing qubit states. We denote the inner product of vectors |u>, |v> as <u|v> ; this will equal 0 if |u> and |v> are orthogonal and 1 if |u> = |v>. To represent two or more qubits we can tensor product Hilbert spaces together to represent the combined states of the qubits. As we shall see, we have methods to represent separable states, where the qubits are independent of one another, and entangled states such as a Bell state, where we cannot separate the two qubit states.

We can represent the states |0> and |1> with vectors as shown below. We call these two the computational basis of a two-level system. We can then apply operators in the form of matrices to the vectors in the state space.

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The table below lists the postulates briefly and their manifestation in the spin 1/2 system as presented in the module.

5. **Explain the bloch sphere representation of qubit**

BLOCH SPHERE REPRESENTATION OF QUBITS

In quantum mechanics and computing, the Bloch sphere is a geometrical representation of the pure state space of a two-level quantum mechanical system (qubit), named after the physicist Felix Bloch. Quantum mechanics is mathematically formulated in Hilbert space or projective Hilbert space.

The Bloch sphere is a geometric representation of qubit states as points on the surface of a unit sphere. Many operations on single qubits that are commonly used in quantum information processing can be neatly described within the Bloch sphere picture.

Reversibility is an important concept in quantum computing due to the following reasons.

1. Qubits are limited, short-lived and expensive resources.
2. Initialization of Qubit is an expensive irreversible operation.
3. So, we must reverse the computations and take Qubits back to their original states rather than initializing again.

There are several ways to represent the state of a qubit:

1. We can write out the state in Dirac notation. For example, if we have a qubit that is prepared in state |0> and then apply the X operator, we will then find the qubit in state |1> (assuming no outside noise)

X |0>→|1>

2. We can use the Bloch sphere to represent the state of a single qubit. Any state in a quantum computation can be represented as a vector that begins at the origin and terminates on the surface of the unit Bloch sphere. By applying unitary operators to the state vectors, we can move the state around the sphere. We take as convention that the two antipodes of the sphere are |0> on the top of the sphere and |1> on the bottom.

3. As we can see in Figure one of the advantages of visualization with the Bloch sphere is that we can represent superposition states such as

$$\frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

Figure 5. Bloch sphere

as we see at the X axis. We can also differentiate between states that contain different phases as is shown in the states along the X and Y axes. Let us return to computational universality which we treated above. Now that we have introduced the Bloch sphere, another way to think about a set of gates that satisfies universal computation is one which enables us to reach any point on the Bloch sphere.

6. Difference between classical and quantum logic gates

CHARACTARISTICS	CLASSICAL GATES	QUANTUM GATES
Basic unit of information	Binary bits: {0,1}	Quantum bits: $\{\|0\rangle, \|1\rangle\}$ in the form of superposition $\alpha\|0\rangle + \beta\|1\rangle$
Dynamics	Deterministic	Probabilistic
Measurements	Never influence the system	Influence the system
Feedback loop	Cyclic	Acyclic
Role of Fan-In and Fan-Out when more than one gate combine	Possible	Not possible
Reversibility	Not always	Always

Table-7

<table>
<tr><th>Logic gate</th><th>Symbol</th><th>Boolean function</th><th>Table</th></tr>
<tr><td>AND</td><td>
</td><td>F=AB</td><td>2 Input AND gate
<table>
<tr><th>A</th><th>B</th><th>AB</th></tr>
<tr><td>0</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>1</td><td>0</td></tr>
<tr><td>1</td><td>0</td><td>0</td></tr>
<tr><td>1</td><td>1</td><td>1</td></tr>
</table></td></tr>
<tr><td>OR</td><td>
</td><td>F=A+B</td><td>2 Input OR gate
<table>
<tr><th>A</th><th>B</th><th>A+B</th></tr>
<tr><td>0</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>1</td><td>1</td></tr>
<tr><td>1</td><td>0</td><td>1</td></tr>
<tr><td>1</td><td>1</td><td>1</td></tr>
</table></td></tr>
<tr><td>XOR</td><td>
</td><td>F=A⊕B</td><td>2 Input EXOR gate
<table>
<tr><th>A</th><th>B</th><th>A⊕B</th></tr>
<tr><td>0</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>1</td><td>1</td></tr>
<tr><td>1</td><td>0</td><td>1</td></tr>
<tr><td>1</td><td>1</td><td>0</td></tr>
</table></td></tr>
<tr><td>NOT</td><td>
</td><td>F= A'</td><td>NOT gate
<table>
<tr><th>A</th><th>$\bar{A}$</th></tr>
<tr><td>0</td><td>1</td></tr>
<tr><td>1</td><td>0</td></tr>
</table></td></tr>
</table>

6. Explain the single and multiple quantum logic gates

QUANTUM GATES

A quantum gate or quantum logic gate is a rudimentary quantum circuit operating on a small number of qubits. They are the analogues for quantum computers to classical logic gates for conventional digital computers. Quantum logic gates are reversible, unlike many classical logic gates. Some universal classical logic gates, such as the Toffoli gate, provide reversibility and can be directly mapped onto quantum logic gates. Quantum logic gates are represented by unitary matrices.

SINGLE QUBIT GATES

Classical computer circuits consist of wires and logic gates. The wires are used to carry information around the circuit, while the logic gates perform manipulations of the information, converting it from one form to another. Consider, for example, classical single bit logic gates. The only non-trivial member of this class is the gate, whose operation is defined by its truth table, in which 0 → 1 and 1 → 0, that is, the 0 and 1 state are interchanged. Can an analogous quantum gate for qubits be defined? Imagine that we had some process which took the state |0> to the state |1>, and vice versa. Such a process would obviously be a good candidate for a quantum analogue to the gate.

Pauli gates

The Pauli gates are based on the better-known Pauli matrices (aka *Pauli spin matrices)* which are incredibly useful for calculating changes to the spin of a single electron. Since electron spin is the favored property to use for a qubit in today's quantum gates, Pauli matrices and gates are right up our alley. In any event, there's essentially one Pauli gate/matrix for each axis in space (X, Y and Z).

Pauli's X-gate:

It is the resemblance of classical not gate.

The Pauli's x gate and its matrix is represented below.

Input	Output
$\alpha\|0\rangle + \beta\|1\rangle$	$\beta\|0\rangle + \alpha\|0\rangle$
0	1
1	0

Gate	Input — X — Output
Matrix	$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
State transition	
Reversibility	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} X \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$
Example	$Let\ \|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}\ and\ \|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \|1\rangle$

Pauli's Y-gate

It is a classical NOT gate with I multiple.

Input	Output
$\alpha\|0\rangle + \beta\|1\rangle$	$-i\,\beta\|0\rangle + i\|\alpha\rangle$
$\|0\rangle$	$i\,\|0\rangle$
$\|1\rangle$	$-i\|1\rangle$

Gate	Input — Y — Output
Matrix	$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Reversibility	$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \times \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \begin{bmatrix} -i^2 & 0 \\ 0 & -i^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$
Example	$Let\ \lvert 0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}\ and\ \lvert 1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix}$

Pauli's Z gate

It is flipping the sign of second entangled state.

Input	Output
$\alpha\lvert 0\rangle + \beta\lvert 1\rangle$	$\alpha\lvert 0\rangle - \beta\lvert 1\rangle$
0	0
1	−1

Gate	Input — Z — Output
Matrix	$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Reversibility	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$
Example	$Let\ \lvert 0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}\ and\ \lvert 1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Hadamard gate (H-gate)

Hadamard gate is a very useful gate which is used to create superpositions. By giving a fundamental quantum state (|0> or |1>) to the Hadamard gate we can get superposed states.

Input	Output
0	$\frac{1}{\sqrt{2}}(\lvert 0\rangle + \lvert 1\rangle)$
1	$\frac{1}{\sqrt{2}}(\lvert 0\rangle - \lvert 1\rangle)$

Gate	Input — H — Output
Matrix	$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$
State transition	
Reversibility	$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \times \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} = \frac{1}{2}\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix} = I$
Example	$Let\ \lvert 0\rangle = \begin{bmatrix}1\\0\end{bmatrix}\ and\ \lvert 1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$ $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \times \begin{bmatrix}1\\0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} = \frac{1}{\sqrt{2}}(\lvert 0\rangle + \lvert 1\rangle)$

Pauli Matrices

$$\text{Pauli-X} \equiv \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$

$$\text{Pauli-Y} \equiv \begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}$$

$$\text{Pauli-Z} \equiv \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$$

Hadamard Gate

$$\text{Hadamard} \equiv \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

Pauli-X : Not gate
Pauli-Y: Not gate with i multiple
Pauli-Z: Flips sign of second entangled state

<u>Multiple Q-gates or controlled gates</u>

<u>C-NOT gate</u>

The prototypical multi-qubit quantum logic gate is the controlled-NOT or CNOTgate. This gate has two input qubits, known as the control qubit and the target qubit, respectively. Another way of describing the CNOT is as a generalization of the classical XOR gate, since the action of the gate may be summarized as |A, B→|A, B ⊕ A, where ⊕ is addition modulo two, which is exactly what the XOR gate does.

If control bit (first bit) is zero, there is no change in the target bit which is shown in the below table. If the control bit is 1, the flipping of target bit is observed which is also indicated in the following table.

Gate	
Matrix	$CNot = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
Reversibility	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I$
Example	$Let\ \lvert x\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}\ and\ \lvert y\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\lvert x\rangle \bigoplus \lvert y\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \bigoplus \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \bigoplus \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

4. What are the applications of quantum computing?

Artificial intelligence: Faster calculations could improve perception, comprehension, and circuit fault diagnosis/binary classifiers;

Chemistry: New fertilizers, catalysts, battery chemistry will all drive improvements in resource utilization;

Biochemistry: New drugs, tailored drugs, and maybe even hair restorer;

Finance: Quantum computing could enable faster, more complex Monte Carlo simulations; for example, trading, trajectory optimization, market instability, price optimization and hedging strategies;

Healthcare: DNA gene sequencing, such as radiotherapy treatment optimization/brain tumor detection, could be performed in seconds instead of hours or weeks;

Materials: super strong materials; corrosion proof paints; lubricants and semiconductors;
Computer science: Faster multidimensional search functions; for example, query optimization, mathematics and simulations.

Artificial Intelligence & Machine Learning:

Artificial intelligence and machine learning are some of the prominent areas right now, as the emerging technologies have penetrated almost every aspect of humans' lives. Some of the widespread applications we see every day are in voice, image and handwriting recognition. However, as the number of applications increased, it becomes a challenging task for traditional computers, to match up the accuracy and speed. And, that's where quantum computing can help in processing through complex problems in very less time, which would have taken traditional computers thousands of years.

2. Computational Chemistry:

One of the most promising quantum computing applications will be in the field of computational chemistry. It is believed that the number of quantum states, even in a tiniest of a molecule, is extremely vast, and therefore difficult for conventional computing memory to process.

3. Drug Design & Development: These advancements in computing could enhance efficiency dramatically, by allowing companies to carry out more drug discoveries to uncover new medical treatments for the better pharmaceutical industry.

4. Cyber security & Cryptography:

The online security space currently has been quite vulnerable due to the increasing number of cyber-attacks occurring across the globe cyber security has continued to be an essential concern around the world. With our increasing dependency on digitization, we are becoming even more vulnerable to these threats. Quantum computing with the help of machine learning can help in developing various techniques to combat these cyber security threats. Additionally, quantum computing can help in creating encryption methods, also known as, quantum cryptography.

5.Financial Modeling:

For a finance industry to find the right mix for fruitful investments based on expected returns, the risk associated, and other factors are important to survive in the market. To achieve that, the techniques of 'Monte Carlo' simulations are continually being run on conventional computers, which, in turn, consume an enormous amount of computer time.

6.Weather Forecasting:

Currently, the process of analyzing weather conditions by traditional computers can sometimes take longer than the weather itself does to change. But a quantum computer's ability to crunch vast amounts of data, in a short period, could indeed lead to enhancing weather system modeling allowing scientists to predict the changing weather patterns in no time and with excellent accuracy.

7. Explain the applications of quantum computing in artificial intelligence

Quantum computing in Artificial Intelligence and Machine Learning

Artificial Intelligence is difficult to define. Probably because intelligence, by itself, is difficult to define. AI is often defined as the study of any device that perceives its environment and takes actions that maximize its chance of successfully achieving its goals. In day-to-day situations, the term AI applies when a machine performs tasks that normally require human intelligence, such as learning, reasoning, problem solving, decision-making, planning, visual perception, knowledge representation, natural language processing, competing at the highest level in strategic game systems or driving cars.

The AI field draws upon computer science, mathematics, psychology, linguistics and many others. Approaches to AI include statistical methods, computational intelligence, or traditional symbolic AI. To reach its goals, many tools are used in AI, including search and mathematical optimization, artificial neural networks, and methods based on statistics, probability and economics.

Along its history, AI has seen many ups and downs. As of 2018, AI techniques have experienced a resurgence following concurrent advances in computer power, large amounts of data, and theoretical understanding. Nowadays, AI techniques have become an essential part of the industry.

Machine learning (ML) was first defined by Arthur Samuel in 1959. It is a subset of artificial intelligence that uses statistical techniques to give computers the ability to learn (i.e., progressively improve performance on a specific task) with data, without being explicitly programmed. Machine learning algorithms operate by constructing a model with parameters that can be learned from a large amount of example inputs, called a training set.

Machine learning is typically classified into three broad categories (leading to different typical applications):

Supervised learning: the computer is presented with example inputs and their desired outputs, given by a "teacher", and the goal is to learn a general rule that maps inputs to outputs. As special cases, the input signal can be only partially available, or restricted to special feedback:

Unsupervised learning: no labels are given to the learning algorithm, leaving it on its own to find structure in its input. Unsupervised learning can be a goal in itself (discovering hidden patterns in data) or a means towards an end (feature learning);

Reinforcement learning: inspired by behaviorist psychology, reinforcement learning is concerned with how software agents ought to take actions in an environment so as to maximize some notion of cumulative reward.

Deep learning (DL) is a subset of machine learning. It utilizes a hierarchical level of artificial neural networks to carry out the process of machine learning. While traditional programs build

analysis with data in a linear way, the hierarchical function of deep learning systems enables machines to process data with a nonlinear approach.

DL is a specific approach used for building and training neural networks. An algorithm is considered to be deep if the input data is passed through a series of nonlinear transformations before it becomes output.