ORACLE NOTES

5/11/2025

Example-1 :- Searching Q_1Q_0 (0 1) from the list

Q_1	Q_0	Y
0	0	0
0	1	1
1	0	0
1	1	0

Boolean equation is $Y = \bar{Q}_1 Q_0$

means $Y=1$ for this particular case.

Q_1	Q_0	Y
0	1	1

start with $Q_1 Q_0$ 0 0 ;

apply hadamard gate,

then creates all states with equal probability

$$\frac{1}{2}\left[|00\rangle + |01\rangle + |10\rangle + |11\rangle\right]$$

then the oracle should do, it should only flip the phase of $|01\rangle$ state as this is what we are looking for:

We need to prepare a oracle logic for this particular logic.

create another Q bit, Qtemp; here what this Qtemp does is, if $Q_1 Q_0$ is 0 1, then it should flip Qtemp

$$\frac{1}{2}\left[|00\rangle + |01\rangle + |10\rangle + |11\rangle\right] \otimes \frac{1}{\sqrt{2}}\left[|0\rangle - |1\rangle\right]$$

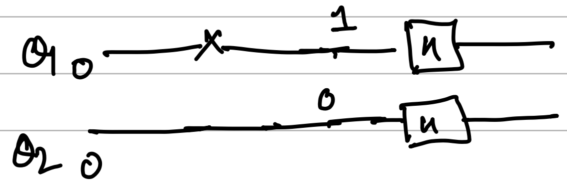
$$= \frac{1}{2\sqrt{2}}\Big[|00\rangle|0\rangle - |00\rangle|1\rangle + |01\rangle|1\rangle - |01\rangle|0\rangle + |10\rangle|0\rangle - |10\rangle|1\rangle + |11\rangle|0\rangle - |11\rangle|1\rangle\Big]$$

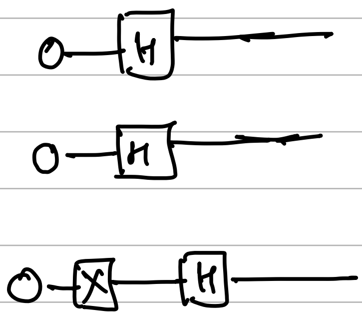
$$= \frac{1}{2\sqrt{2}}\Big[|00\rangle\left[|0\rangle - |1\rangle\right] - |01\rangle\left[|0\rangle - |1\rangle\right] + |10\rangle\left[|0\rangle - |1\rangle\right] + 11\left[|0\rangle - |1\rangle\right]\Big]$$

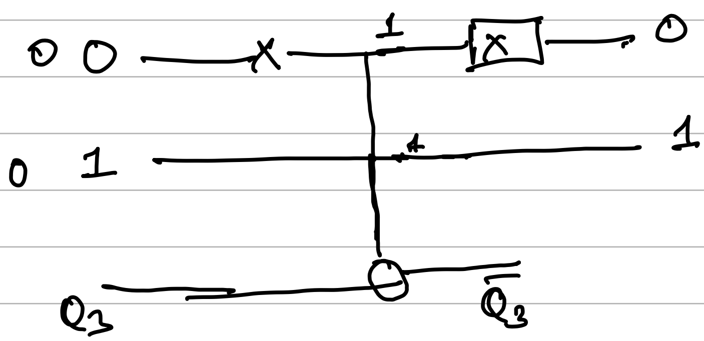
$$= \frac{1}{2\sqrt{2}}\left[|00\rangle - |01\rangle + |10\rangle + |11\rangle\right] \otimes \left[|0\rangle - |1\rangle\right]$$

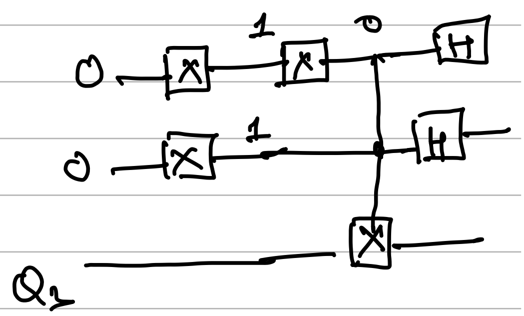
phase change

Quantum gates only naturally control on $|1\rangle$









target

control

$$\frac{1}{2}\left[|00\rangle + |01\rangle + |10\rangle + |11\rangle\right]$$

$$\frac{1}{2}\left[|00\rangle + |11\rangle + |10\rangle + |01\rangle\right]$$

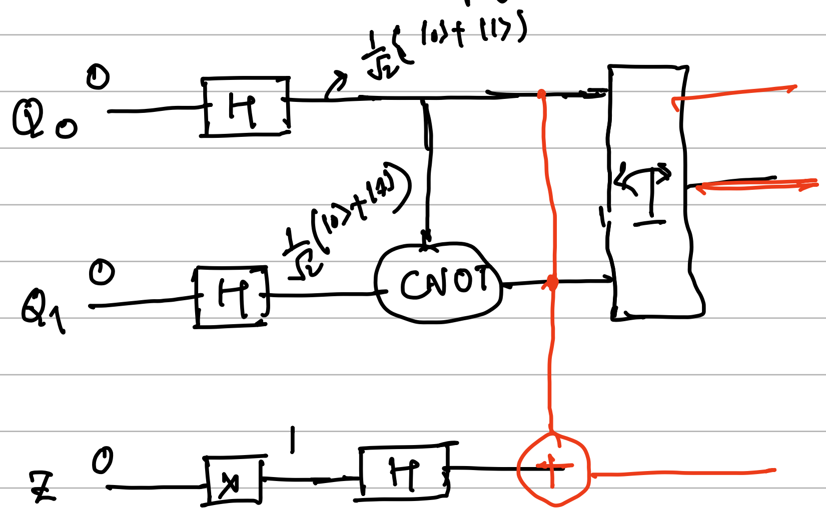
$$\frac{1}{\sqrt{2}}\left[|0\rangle - |1\rangle\right]$$

$Q_0\ Q_1$ at $0\ 0$:-

→ apply hadamard gate

→ Apply CNOT gate as Q_0 as Control gate

→ Then apply Toffil gate



This will provide :-

$$= \frac{1}{2\sqrt{2}} \left[|00\rangle - |01\rangle + |10\rangle + |11\rangle \right] \otimes \left[|0\rangle - |1\rangle \right]$$

phase change

This example is finding a pattern (01) from the list.

Example - 2 :- Searching Even number from the list

Assume $|Q_2 Q_1 Q_0\rangle$ is the 3-bit number and we want to search for even numbers,

Q_2	Q_1	Q_0	
0	0	0	– Even
0	0	1	
0	1	0	– Even
0	1	1	
1	0	0	– Even
1	0	1	
1	1	0	– Even
1	1	1	

if $Q_0 = 0$ it is an even

The oracle logic here is, if it is even we should flip the "phase".

How to do ?

If make Q_0 as control bit and other $Q_1\, Q_2$ as target bits

When $Q_0 = 0$, the phase of $Q_2 Q_1$ should be flipped. We have to use "toffoli gate".

But Quantum gates can naturally control on $|1\rangle$

We need some thing like below: (oracle)

$$\frac{1}{2\sqrt{2}}\Big[\,|001\rangle + |011\rangle + |101\rangle + |111\rangle - |000\rangle - |010\rangle - |110\rangle\,\Big]$$

This is some thing looking like

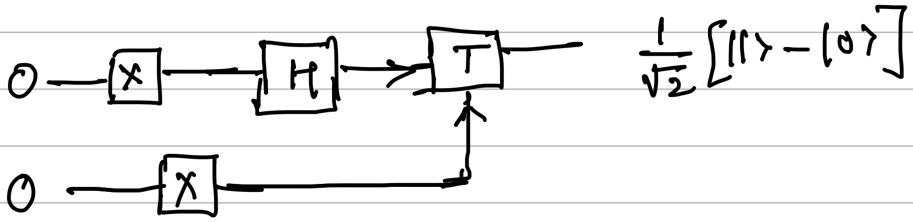
$$= \frac{1}{\sqrt{2}}\big[|0\rangle + |1\rangle\big] \otimes \frac{1}{\sqrt{2}}\big[|0\rangle + |1\rangle\big] \otimes \underbrace{\frac{1}{\sqrt{2}}\big[|1\rangle - |0\rangle\big]}_{\Uparrow\ \text{need to generate}}$$

$$= \frac{1}{2\sqrt{2}}\left[\,|00\rangle|0\rangle + |01\rangle|0\rangle + |10\rangle|0\rangle + |11\rangle|0\rangle\right.$$

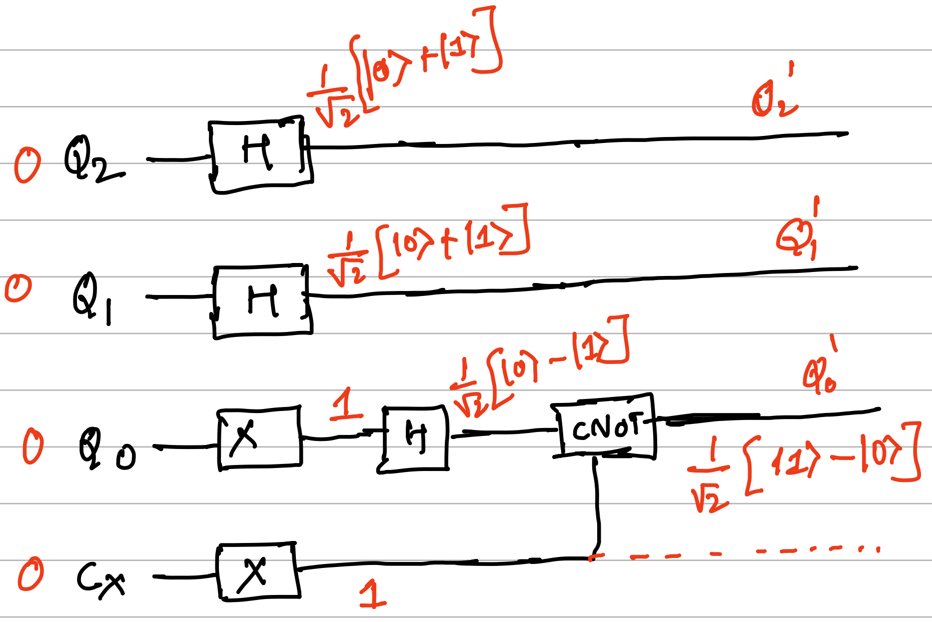
$|1\rangle = \frac{1}{\sqrt{2}}\left[|0\rangle - |1\rangle\right]$ I need $\frac{1}{\sqrt{2}}\left[|1\rangle - |0\rangle\right]$

Generating $\frac{1}{\sqrt{2}}\left[|1\rangle - |0\rangle\right]$:-

$\frac{1}{\sqrt{2}}\left[|0\rangle - |1\rangle\right]$



ORACLE FOR EVEN NUMBER SEARCH :-



$$Q_0'\, Q_1'\, Q_2' = \frac{1}{2\sqrt{2}}\left[|00\rangle + |01\rangle + |10\rangle + |11\rangle\right] \otimes \left[\,|1\rangle - |0\rangle\right]$$

$$= \frac{1}{2\sqrt{2}}\left[|00\rangle|1\rangle + |011\rangle + |101\rangle + |111\rangle - |000\rangle - |010\rangle - |100\rangle - |110\rangle\right]$$