MA 226 : Monte Carlo Simulation
Project

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Q1.

Given $F(x) = (1 + \lambda)F_1(x) - \lambda F_1^2(x)$.

Generate random numbers following the disribution $F(x)$.Here, $F_1(x)$ is the CDF of normal distribution.

a.)Use Inverse-Transformation Method

b.)Use Acceptance-Rejectance Method

Q1(a)

Suppose we want to generate a random variable X with the property that $P(X <= x) = F(x)\ \forall x$. The inverse transform method sets : $X = F^{-1}(U)$, $U \sim U(0, 1)$, where $U(0, 1)$ is a uniform distribution on $[0, 1]$.

So, substituting $F(x) = U$ and assuming $F_1(x) = u_1$, we get a quadratic equation in u_1.

$\lambda u_1^2 - (1 + \lambda)u_1 + U = 0$

On solving the quadratic, we get, $u_1 = ((1 + \lambda) - ((1 + \lambda)^2 - 4\lambda U)^{1/2})/2\lambda$

Now, $u_1 = F_1(X)$. Therefore, $X = F^{-1}(u_1)$

$F^{-1}(u_1)$ is calculated using **qnorm(u_1,mean,sd)**.

qnorm returns the inverse cumulative density function (quantiles).
The idea behind qnorm is that we give it a probability, and it returns the number whose cumulative distribution matches the probability.

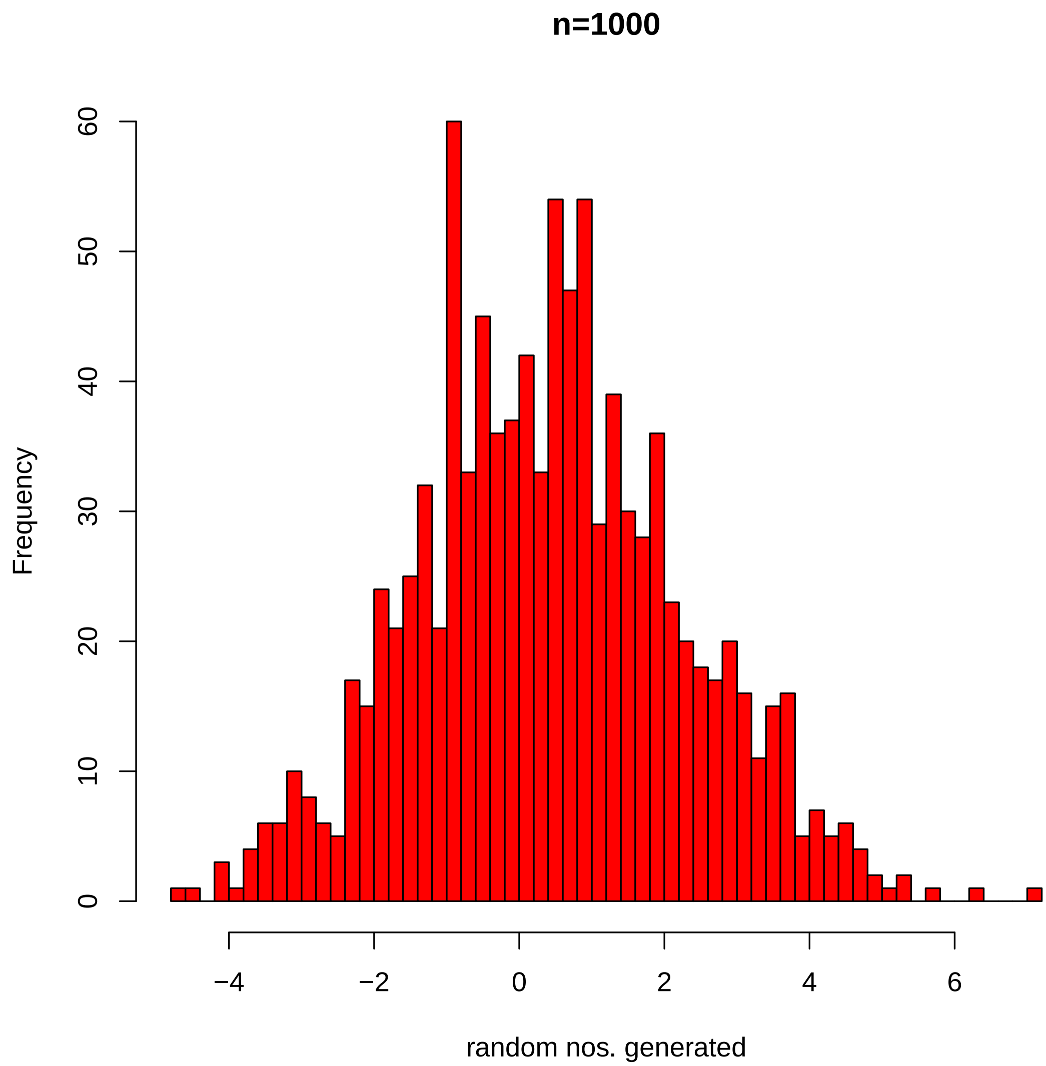
Q1(a) - R Code

```
{
 u=runif(100000);
 l=0.5;

 u1=array(100000);
 x=array(100000);

 for(i in 1:100000)
    {
     u1[i]=((1+l)-sqrt(((1+l)^2)-(4*l*u[i])))/(2*l);
     x[i]=qnorm(u1[i],mean=1,sd=2);
     cat(x[i],"\n")
    }

 print(mean(x));
 print(var(x));
 hist(x,breaks=50,col="red")
}
```

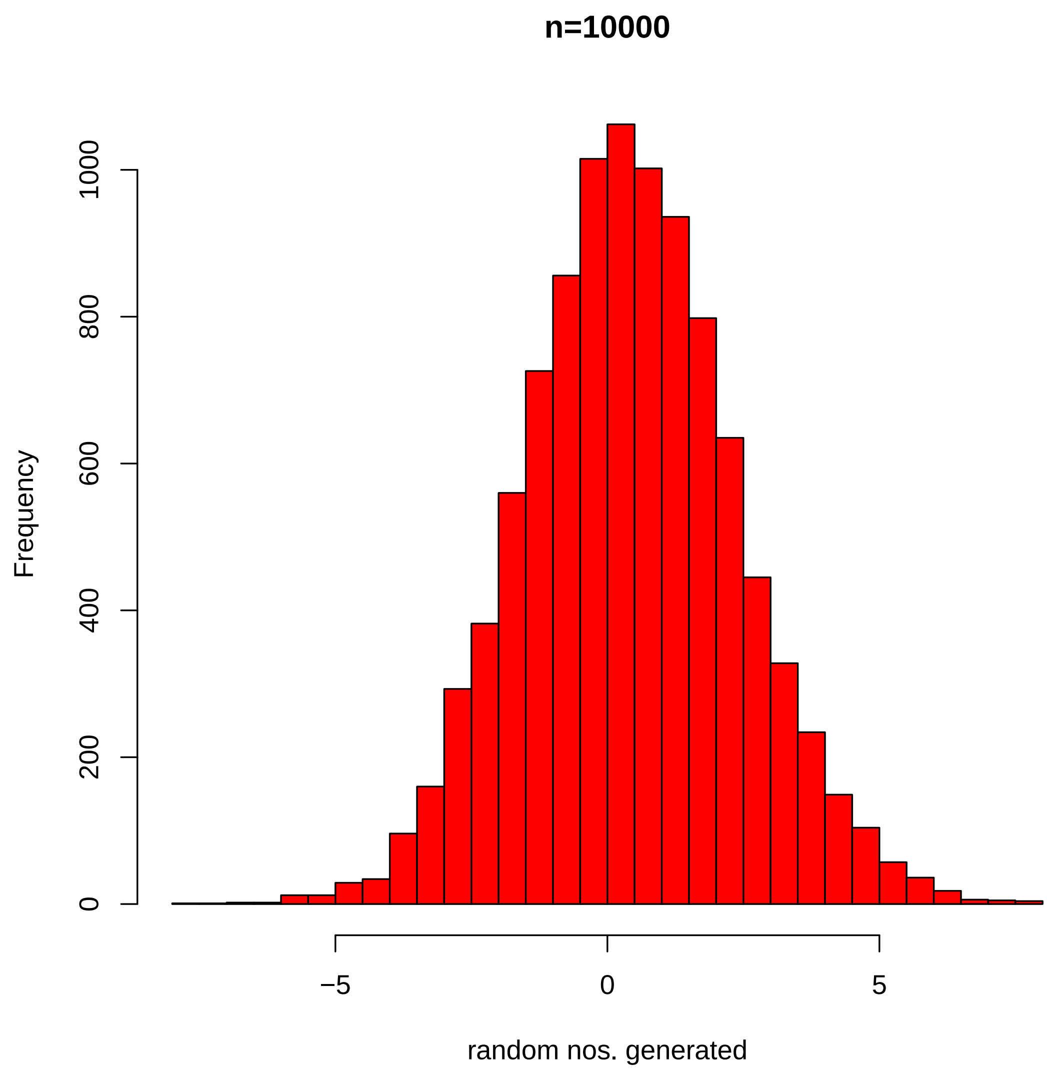
The above code is run for n=1000,taking $\lambda = l = 0.5$. The mean and variance for the normal distribution are taken as 1 and 2 respectively. The following are the plots, mean and variances are for n=1000, n=10000 and n=100000.

Q1(a) - Output

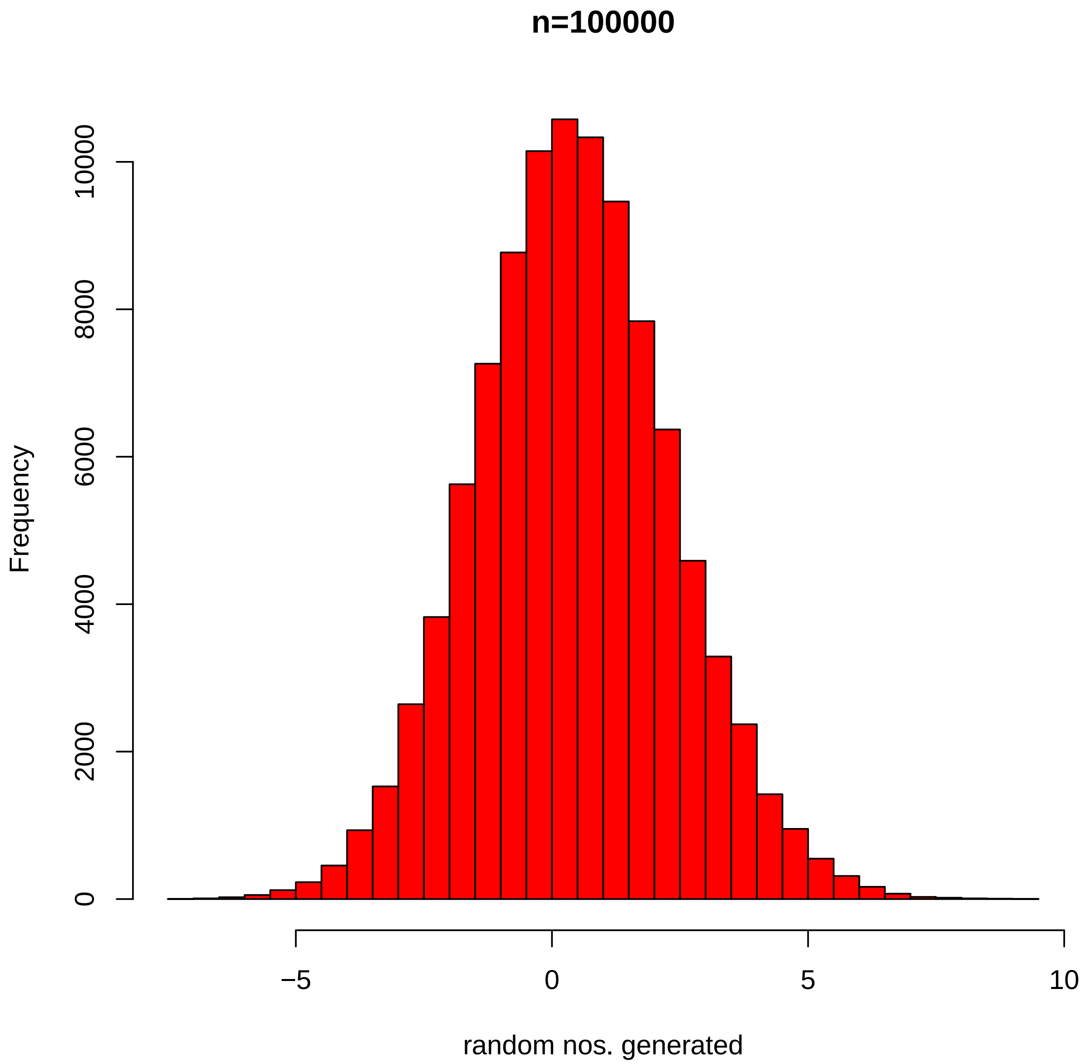
Histograms :



mean $= 0.4240377$
variance $= 3.805862$



mean$=0.4240377$
variance$=3.805862$



mean=0.4322516
variance=3.685975

Q1(b)

$F(x) = (1+\lambda)F_1(x) - \lambda F_1^2(x)$

Since, PDF of a distribution is obtained by differentiating its CDF.

Differentiating the above equation, we get :

$$f(x) = (1+\lambda)f_1(x) - 2\lambda F_1(x)f_1(x)$$

Choosing $g(x) = f_1(x)$

$$f(x)/g(x) = (1+\lambda) - 2\lambda F_1(x)$$

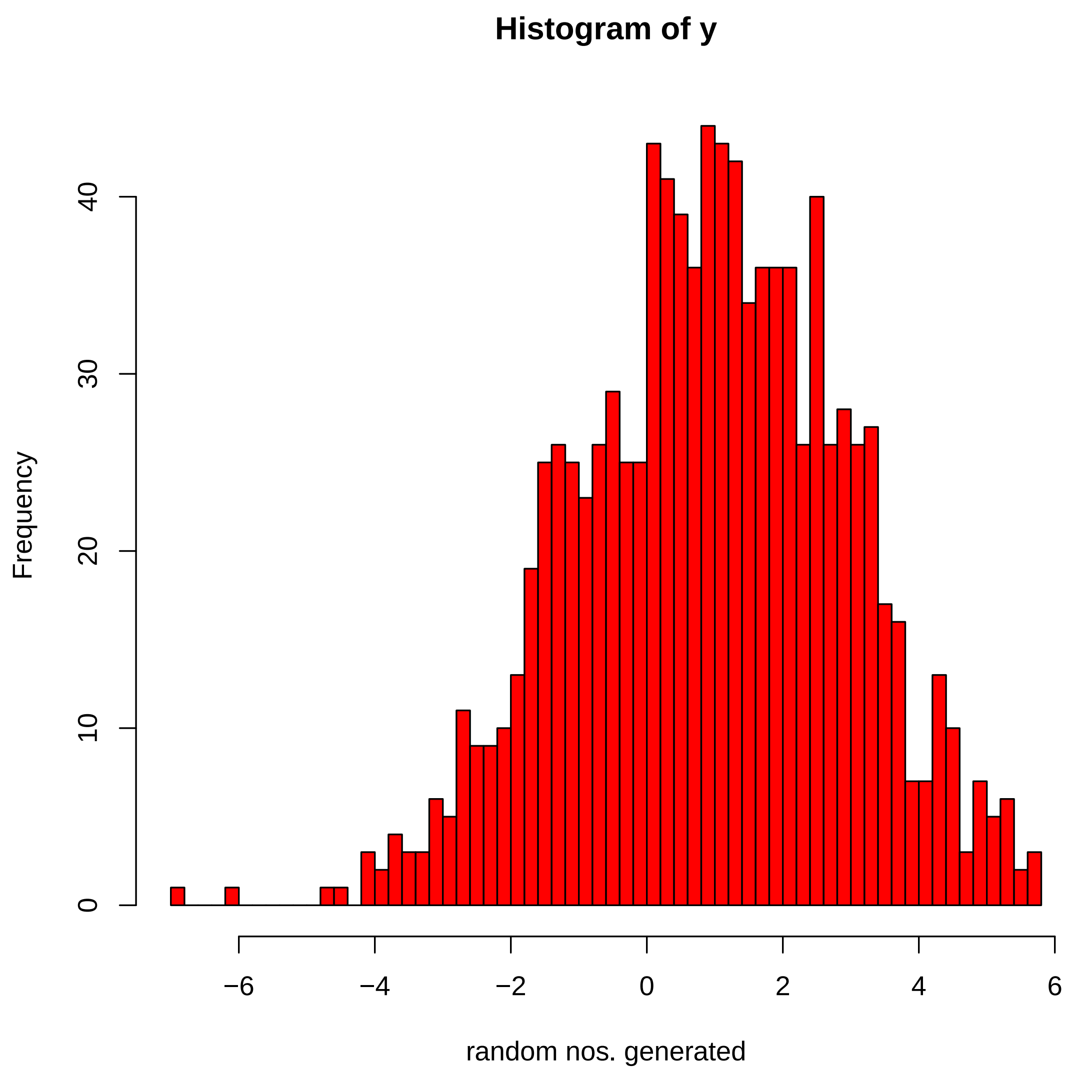
Q1(b) - R Code

```
{
 c=2;
 l=0.1;
 y=NULL;
 n=2000;
 count=0;

 X=rnorm(n,mean=1,sd=2);
 U=runif(n);
 P=ecdf(X);
 k=NULL;

 for(i in 1:n)
     {
       k[i]=((1+l)-2*l*P(X[i]))/c;

       if(U[i]<=k[i])
          {
    y[count]=X[i];
    count=count+1;
          }
     }

 print(y);
 print(count);
 hist(y, col="red",breaks=50, xlab="random nos. generated")
 print(mean(y));
 print(var(y));
}
```

Q1(b) - Output

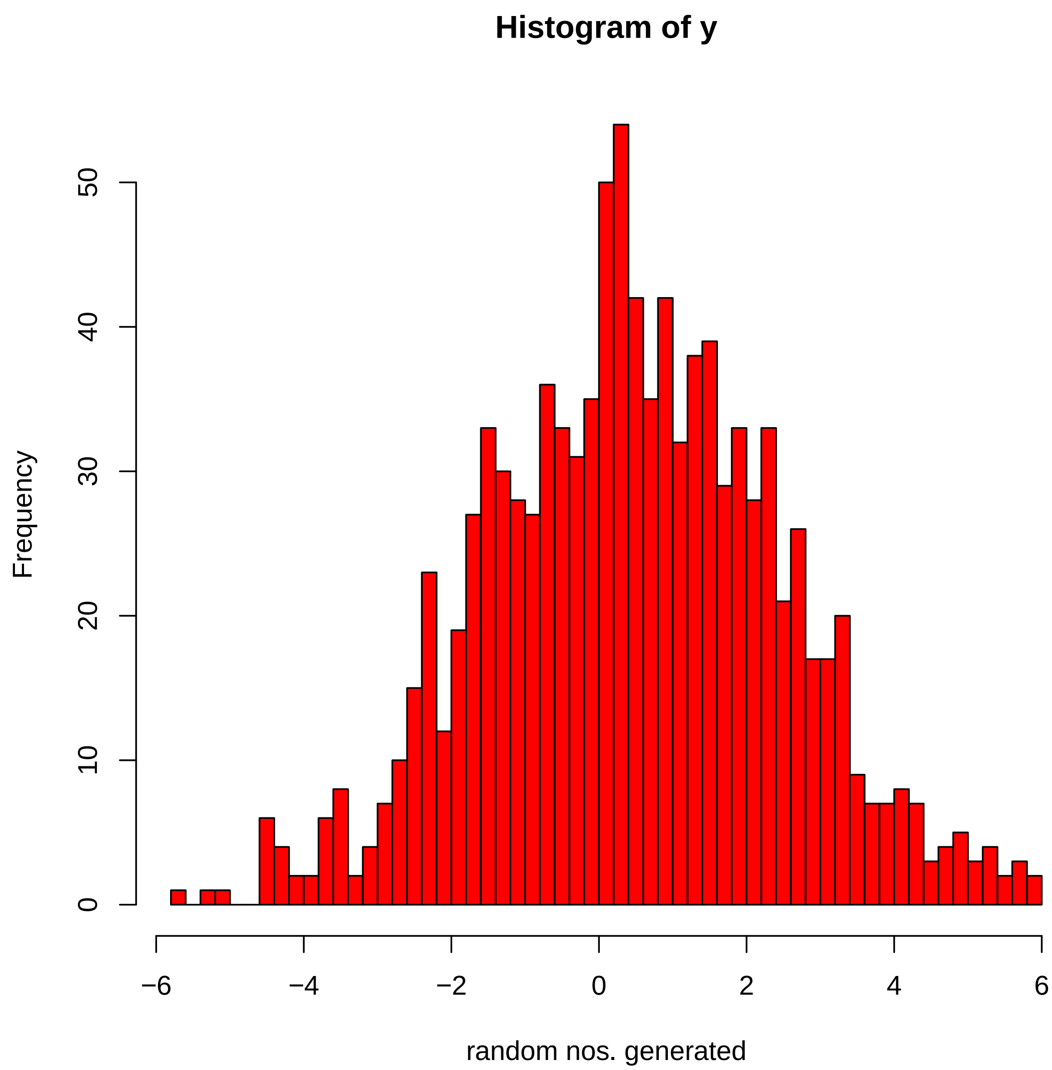
Histograms :

For l = 0.1



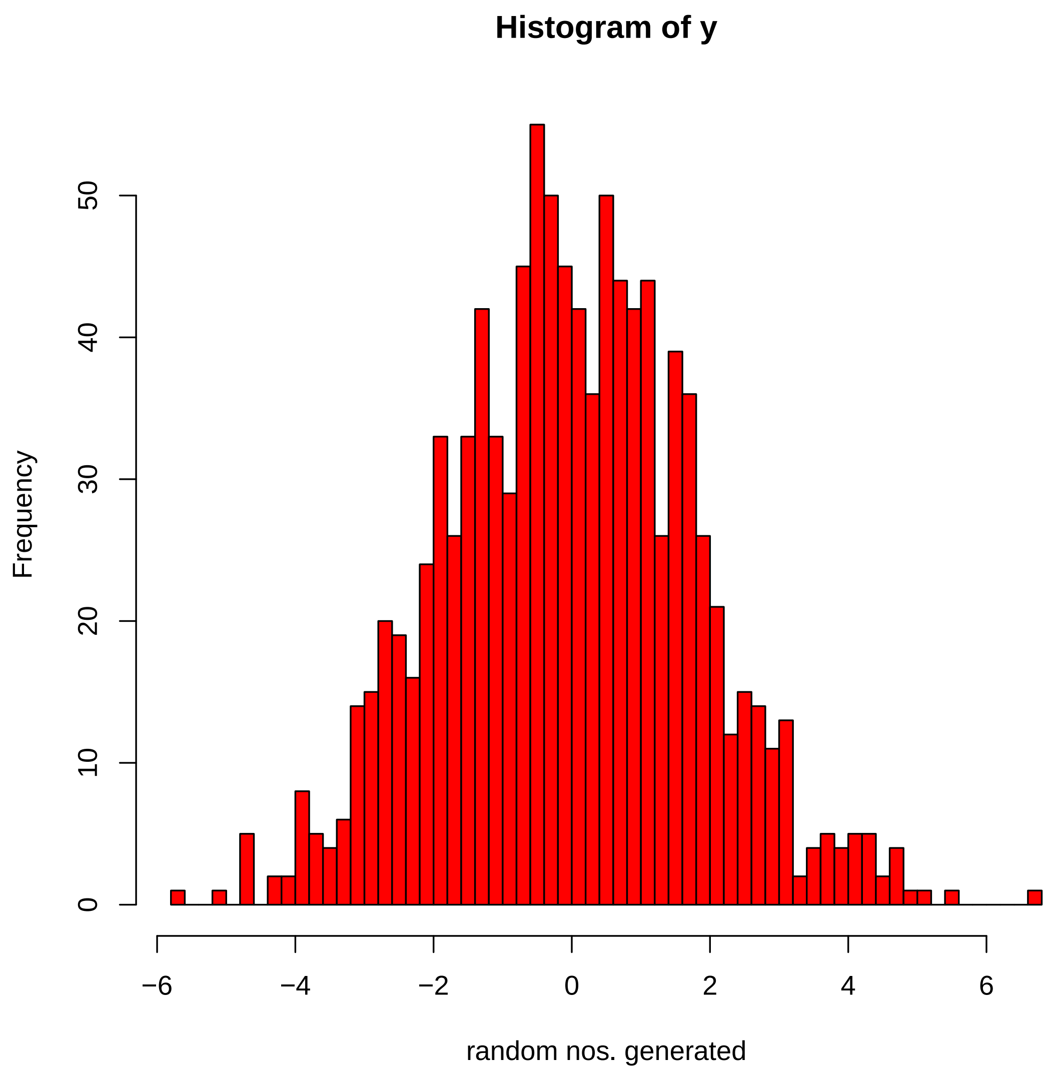
mean = 0.4240377
variance = 3.805862

For l = 0.5



mean=0.4240377
variance=3.805862

For l = 0.9



mean=0.4322516
variance=3.685975